SAT Subject Math Level 2 Facts & Formulas

Numbers, Sequences, Factors

Integers: ..., -3, -2, -1, 0, 1, 2, 3, ...
Reals: integers plus fractions, decimals, and irrationals ($\sqrt{2}$, $\sqrt{3}$, π, etc.)

Order Of Operations: PEMDAS
(Parentheses / Exponents / Multiply / Divide / Add / Subtract)

Arithmetic Sequences: each term is equal to the previous term plus d

Sequence: $t_1, t_1 + d, t_1 + 2d, \ldots$
The n$^{\text{th}}$ term is $t_n = t_1 + (n-1)d$
Number of integers from i_n to $i_m = i_m - i_n + 1$
Sum of n terms $S_n = (n/2) \cdot (t_1 + t_n)$

Geometric Sequences: each term is equal to the previous term times r

Sequence: $t_1, t_1 \cdot r, t_1 \cdot r^2, \ldots$
The n$^{\text{th}}$ term is $t_n = t_1 \cdot r^{n-1}$
Sum of n terms $S_n = t_1 \cdot (r^n - 1)/(r-1)$
Sum of infinite sequence ($r < 1$) is $S_\infty = t_1/(1-r)$

Prime Factorization: break up a number into prime factors (2, 3, 5, 7, 11, ...)

$$200 = 4 \times 50 = 2 \times 2 \times 2 \times 5 \times 5$$
$$52 = 2 \times 26 = 2 \times 2 \times 13$$

Greatest Common Factor: multiply common prime factors

$$200 = 2 \times 2 \times 2 \times 5 \times 5$$
$$60 = 2 \times 2 \times 3 \times 5$$
$$\text{GCF}(200, 60) = 2 \times 2 \qquad \times 5 = 20$$

Least Common Multiple: check multiples of the largest number

LCM$(200, 60)$: 200 (no), 400 (no), 600 (yes!)

Percentages: use the following formula to find part, whole, or percent

$$\text{part} = \frac{\text{percent}}{100} \times \text{whole}$$

Averages, Counting, Statistics, Probability

$$\text{average} = \frac{\text{sum of terms}}{\text{number of terms}}$$

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$\text{sum} = \text{average} \times (\text{number of terms})$$

$$\text{mode} = \text{value in the list that appears most often}$$

$$\text{median} = \text{middle value in the list (which } \textit{must} \text{ be sorted)}$$

Example: median of $\{3, 10, 9, 27, 50\} = 10$
Example: median of $\{3, 9, 10, 27\} = (9 + 10)/2 = 9.5$

Fundamental Counting Principle:

If an event can happen in N ways, and another, independent event can happen in M ways, then both events together can happen in $N \times M$ ways. (Extend this for three or more: $N_1 \times N_2 \times N_3 \ldots$)

Permutations and Combinations:

The number of permutations of n things is ${}_nP_n = n!$

The number of permutations of n things taken r at a time is ${}_nP_r = n!/(n-r)!$

The number of permutations of n things, a of which are indistinguishable, b of which are indistinguishable, etc., is ${}_nP_n/(a!\,b!\ldots) = n!/(a!\,b!\ldots)$

The number of combinations of n things taken r at a time is ${}_nC_r = n!/\big((n-r)!\,r!\big)$

Probability:

$$\text{probability} = \frac{\text{number of desired outcomes}}{\text{number of total outcomes}}$$

The probability of two different events A and B *both* happening is $P(A \text{ and } B) = P(A) \cdot P(B)$, as long as the events are independent (not mutually exclusive).

If the probability of event A happening is $P(A)$, then the probability of event A *not* happening is $P(\text{not } A) = 1 - P(A)$.

Logic (Optional):

The statement "event A implies event B" is logically the same as "*not* event B implies *not* event A". However, "event A implies event B" is *not* logically the same as "event B implies event A". To see this, try an example, such as $A = \{\text{it rains}\}$ and $B = \{\text{the road is wet}\}$. If it rains, then the road gets wet ($A \Rightarrow B$); alternatively, if the road is not wet, it didn't rain (not $B \Rightarrow$ not A). However, if the road is wet, it didn't necessarily rain ($B \not\Rightarrow A$).

Powers, Exponents, Roots

$$x^a \cdot x^b = x^{a+b} \qquad x^a/x^b = x^{a-b} \qquad 1/x^b = x^{-b}$$

$$(x^a)^b = x^{a \cdot b} \qquad (xy)^a = x^a \cdot y^a \qquad (-1)^n = \begin{cases} +1, & \text{if } n \text{ is even;} \\ -1, & \text{if } n \text{ is odd.} \end{cases}$$

$$x^0 = 1 \qquad \sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$$

$$\text{If } 0 < x < 1, \text{ then } 0 < x^3 < x^2 < x < \sqrt{x} < \sqrt[3]{x} < 1.$$

Factoring, Solving

$$(x+a)(x+b) = x^2 + (b+a)x + ab \qquad \text{“FOIL”}$$

$$a^2 - b^2 = (a+b)(a-b) \qquad \text{“Difference Of Squares”}$$

$$a^2 + 2ab + b^2 = (a+b)(a+b)$$

$$a^2 - 2ab + b^2 = (a-b)(a-b)$$

$$x^2 + (b+a)x + ab = (x+a)(x+b) \qquad \text{“Reverse FOIL”}$$

You can use Reverse FOIL to factor a polynomial by thinking about two numbers a and b which add to the number in front of the x, and which multiply to give the constant. For example, to factor $x^2 + 5x + 6$, the numbers add to 5 and multiply to 6, i.e., $a = 2$ and $b = 3$, so that $x^2 + 5x + 6 = (x+2)(x+3)$.

To solve a quadratic such as $x^2 + bx + c = 0$, first factor the left side to get $(x+a)(x+b) = 0$, then set each part in parentheses equal to zero. E.g., $x^2 + 4x + 3 = (x+3)(x+1) = 0$ so that $x = -3$ or $x = -1$.

The solution to the quadratic equation $ax^2 + bx + c = 0$ can always be found (if it exists) using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Note that if $b^2 - 4ac < 0$, then there is no solution to the equation. If $b^2 - 4ac = 0$, there is exactly one solution, namely, $x = -b/2a$. If $b^2 - 4ac > 0$, there are two solutions to the equation.

To solve two linear equations in x and y: use the first equation to substitute for a variable in the second. E.g., suppose $x + y = 3$ and $4x - y = 2$. The first equation gives $y = 3 - x$, so the second equation becomes $4x - (3 - x) = 2 \Rightarrow 5x - 3 = 2 \Rightarrow x = 1, y = 2$.

Solving two linear equations in x and y is geometrically the same as finding where two lines intersect. In the example above, the lines intersect at the point $(1, 2)$. Two parallel lines will have no solution, and two overlapping lines will have an infinite number of solutions.

Functions

A function is a rule to go from one number (x) to another number (y), usually written

$$y = f(x).$$

The set of possible values of x is called the *domain* of $f()$, and the corresponding set of possible values of y is called the *range* of $f()$. For any given value of x, there can only be one corresponding value y.

Translations:

> The graph of $y = f(x - h) + k$ is the translation of the graph of $y = f(x)$ by (h, k) units in the plane.

Absolute value:

$$|x| = \begin{cases} +x, & \text{if } x \geq 0; \\ -x, & \text{if } x < 0. \end{cases}$$

$$\begin{aligned} |x| < n \quad &\Rightarrow \quad -n < x < n \\ |x| > n \quad &\Rightarrow \quad x < -n \quad \text{or} \quad x > n \end{aligned}$$

Parabolas:

A parabola parallel to the y-axis is given by

$$y = ax^2 + bx + c.$$

If $a > 0$, the parabola opens up. If $a < 0$, the parabola opens down. The y-intercept is c, and the x-coordinate of the vertex is $x = -b/2a$.

Note that when $x = -b/2a$, the y-value of the parabola is either a minimum ($a > 0$) or a maximum ($a < 0$).

Ellipses:

An ellipse is essentially a squashed circle. The equation of an ellipse centered on the origin which intersects the x-axis at $(\pm a, 0)$ and the y-axis at $(0, \pm b)$ is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Hyperbolas (Optional):

A hyperbola looks like two elongated parabolas pointed away from one another. The equation of a hyperbola centered on the origin, pointing down the positive and negative x-axes, and which intersects the x-axis at $(\pm a, 0)$ is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

Compound Functions:

A function can be applied directly to the y-value of another function. This is usually written with one function inside the parentheses of another function. For example:

$f(g(x))$	means:	apply g to x first, then apply f to the result
$g(f(x))$	means:	apply f to x first, then apply g to the result
$f(x)g(x)$	means:	apply f to x first, then apply g to x, then multiply the results

For example, if $f(x) = 3x - 2$ and $g(x) = x^2$, then $f(g(3)) = f(3^2) = f(9) = 3 \cdot 9 - 2 = 25$.

Inverse Functions:

Since a function $f()$ is a rule to go from one number (x) to another number (y), an inverse function $f^{-1}()$ can be defined as a rule to go from the number y back to the number x. In other words, if $y = f(x)$, then $x = f^{-1}(y)$.

To get the inverse function, substitute y for $f(x)$, solve for x in terms of y, and substitute $f^{-1}(y)$ for x. For example, if $f(x) = 2x + 6$, then $x = (y - 6)/2$ so that $f^{-1}(y) = y/2 - 3$. Note that the function $f()$, given $x = 1$, returns $y = 8$, and that $f^{-1}(y)$, given $y = 8$, returns $x = 1$.

Usually, even inverse functions are written in terms of x, so the final step is to substitute x for y. In the above example, this gives $f^{-1}(x) = x/2 - 3$. A quick recipe to find the inverse of $f(x)$ is: substitute y for $f(x)$, interchange y and x, solve for y, and replace y with $f^{-1}(x)$.

Two facts about inverse functions: 1) their graphs are symmetric about the line $y = x$; and 2) if one of the functions is a line with slope m, the other is a line with slope $1/m$.

Logarithms (Optional):

Logarithms are basically the inverse functions of exponentials. The function $\log_b x$ answers the question: b to what power gives x? Here, b is called the logarithmic "base". So, if $y = \log_b x$, then the logarithm function gives the number y such that $b^y = x$. For example, $\log_3 \sqrt{27} = \log_3 \sqrt{3^3} = \log_3 3^{3/2} = 3/2 = 1.5$. Similarly, $\log_b b^n = n$.

The natural logarithm $\ln x$ is just the usual logarithm function, but with base equal to the special number e (approximately 2.718).

A useful rule to know is: $\log_b xy = \log_b x + \log_b y$.

Complex Numbers

A complex number is of the form $a + bi$ where $i^2 = -1$. When multiplying complex numbers, treat i just like any other variable (letter), except remember to replace powers of i with -1 or 1 as follows (the pattern repeats after the first four):

$$\begin{array}{llll} i^0 = 1 & i^1 = i & i^2 = -1 & i^3 = -i \\ i^4 = 1 & i^5 = i & i^6 = -1 & i^7 = -i \end{array}$$

For example, using "FOIL" and $i^2 = -1$: $(1 + 3i)(5 - 2i) = 5 - 2i + 15i - 6i^2 = 11 + 13i$.

Lines (Linear Functions)

Consider the line that goes through points $A(x_1, y_1)$ and $B(x_2, y_2)$.

Distance from A to B: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Mid-point of the segment $\overline{AB}$: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

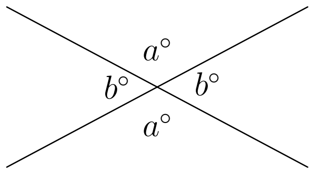
Slope of the line: $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\text{rise}}{\text{run}}$

Point-slope form: given the slope m and a point (x_1, y_1) on the line, the equation of the line is $(y - y_1) = m(x - x_1)$.

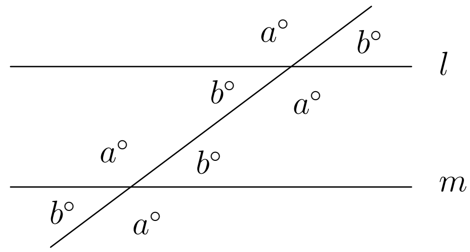
Slope-intercept form: given the slope m and the y-intercept b, then the equation of the line is $y = mx + b$.

To find the equation of the line given two points $A(x_1, y_1)$ and $B(x_2, y_2)$, calculate the slope $m = (y_2 - y_1)/(x_2 - x_1)$ and use the point-slope form.

Parallel lines have equal slopes. Perpendicular lines (i.e., those that make a 90° angle where they intersect) have negative reciprocal slopes: $m_1 \cdot m_2 = -1$.



Intersecting Lines



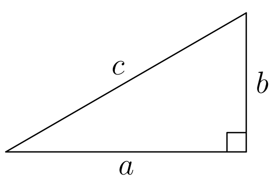
Parallel Lines ($l \parallel m$)

Intersecting lines: opposite angles are equal. Also, each pair of angles along the same line add to 180°. In the figure above, $a + b = 180°$.

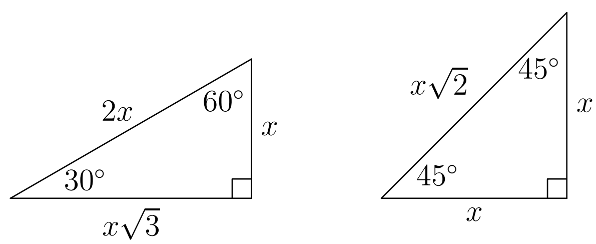
Parallel lines: eight angles are formed when a line crosses two parallel lines. The four big angles (a) are equal, and the four small angles (b) are equal.

Triangles

Right triangles:



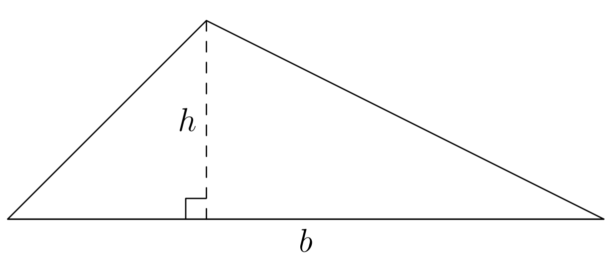
$a^2 + b^2 = c^2$



Special Right Triangles

A good example of a right triangle is one with $a = 3$, $b = 4$, and $c = 5$, also called a 3–4–5 right triangle. Note that multiples of these numbers are also right triangles. For example, if you multiply these numbers by 2, you get $a = 6$, $b = 8$, and $c = 10$ (6–8–10), which is also a right triangle.

All triangles:



$$\text{Area} = \frac{1}{2} \cdot b \cdot h$$

Angles on the inside of any triangle add up to 180°.

The length of one side of any triangle is always *less* than the sum and *more* than the difference of the lengths of the other two sides.

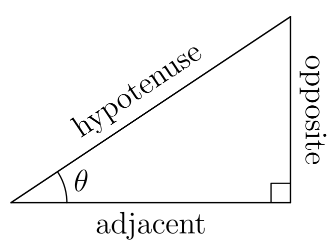
An exterior angle of any triangle is equal to the sum of the two remote interior angles.

Other important triangles:

Equilateral:	These triangles have three equal sides, and all three angles are 60°. The area of an equilateral triangle is $A = (\text{side})^2 \cdot \sqrt{3}/4$.
Isosceles:	An isosceles triangle has two equal sides. The "base" angles (the ones opposite the two sides) are equal (see the 45° triangle above).
Similar:	Two or more triangles are similar if they have the same shape. The corresponding angles are equal, and the corresponding sides are in proportion. The ratio of their areas equals the ratio of the corresponding sides squared. For example, the 3–4–5 triangle and the 6–8–10 triangle from before are similar since their sides are in a ratio of 2 to 1.

Trigonometry

Referring to the figure below, there are three important functions which are defined for angles in a right triangle:



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

"SOH" "CAH" "TOA"

(the last line above shows a mnemonic to remember these functions: "SOH-CAH-TOA")

An important relationship to remember which works for any angle θ is:

$$\sin^2\theta + \cos^2\theta = 1.$$

For example, if $\theta = 30°$, then (refer to the Special Right Triangles figure) we have $\sin 30° = 1/2$, $\cos 30° = \sqrt{3}/2$, so that $\sin^2 30° + \cos^2 30° = 1/4 + 3/4 = 1$.

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The functions $y = a\sin bx$ and $y = a\cos bx$ both have periods $T = 360°/b$, i.e., they repeat every $360/b$ degrees. The function $y = a\tan bx$ has a period of $T = 180°/b$.

There are three other trig functions (cosecant, secant, and cotangent) that are related to the usual three (sine, cosine, and tangent) as follows:

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

$$= 1/\sin\theta \qquad\qquad = 1/\cos\theta \qquad\qquad = 1/\tan\theta$$

It is very useful to know the following trigonometric identities:

$$\begin{aligned}
\sin(A+B) &= \sin A\cos B + \cos A\sin B \\
\cos(A+B) &= \cos A\cos B - \sin A\sin B \\
\tan(A+B) &= \frac{\tan A + \tan B}{1 - \tan A\tan B}
\end{aligned}$$

The following double-angle and half-angle trigonometric identities are optional to memorize, but they can be easy shortcuts to solve problems if you are good at memorization.

Double-Angle (Optional):

$$\begin{aligned}
\sin 2\theta &= 2\sin\theta\cos\theta \\
\cos 2\theta &= \cos^2\theta - \sin^2\theta \\
\tan 2\theta &= \frac{2\tan\theta}{1-\tan^2\theta}
\end{aligned}$$

Half-Angle (Optional):

$$\begin{aligned}
\sin\frac{\theta}{2} &= \pm\sqrt{\frac{1-\cos\theta}{2}} \\
\cos\frac{\theta}{2} &= \pm\sqrt{\frac{1+\cos\theta}{2}} \\
\tan\frac{\theta}{2} &= \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}
\end{aligned}$$

For any triangle with angles A, B, and C, and sides a, b, and c (opposite to the angles), two important laws to remember are the Law of Cosines and the Law of Sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad \text{“Law of Sines”}$$

$$c^2 = a^2 + b^2 - 2ab\cos C \qquad \text{“Law of Cosines”}$$

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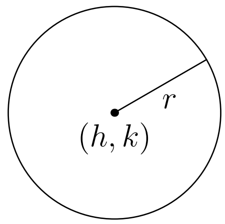
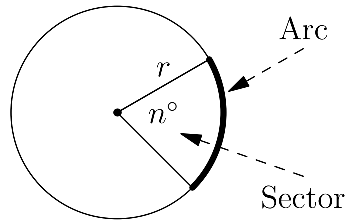
Circles





Area $= \pi r^2$
Circumference $= 2\pi r$
Full circle $= 360°$

Length Of Arc $= (n°/360°) \cdot 2\pi r$
Area Of Sector $= (n°/360°) \cdot \pi r^2$

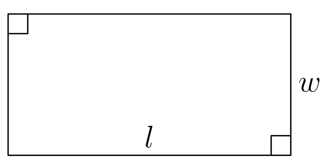
Equation of the circle (above left figure): $(x - h)^2 + (y - k)^2 = r^2$.

Another way to measure angles is with radians. These are defined such that π radians is equal to $180°$, so that the number of radians in a circle is 2π (or $360°$).

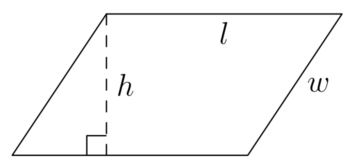
To convert from degrees to radians, just multiply by $\pi/180°$. For example, the number of radians in $45°$ is 0.785, since $45° \cdot \pi/180° = \pi/4$ rad ≈ 0.785 rad.

Rectangles And Friends

Rectangles and Parallelograms:

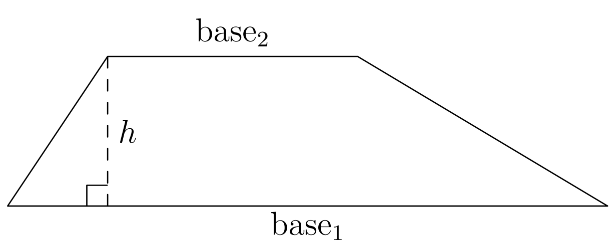


Rectangle
(Square if $l = w$)
Area $= lw$



Parallelogram
(Rhombus if $l = w$)
Area $= lh$

Trapezoids:



$$\text{Area of trapezoid} = \left(\frac{\text{base}_1 + \text{base}_2}{2}\right) \cdot h$$

Polygons:

Regular polygons are n-sided figures with all sides equal and all angles equal.

The sum of the inside angles of an n-sided regular polygon is $(n-2) \cdot 180°$.

The sum of the outside angles of an n-sided regular polygon is always $360°$.

Solids

The following five formulas for cones, spheres, and pyramids are given in the beginning of the test booklet, so you don't have to memorize them, but you should know how to use them.

Volume of right circular cone with radius r and height h: $V = \frac{1}{3}\pi r^2 h$

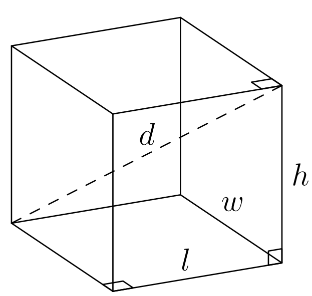
Lateral area of cone with base circumference c and slant height l: $S = \frac{1}{2}cl$

Volume of sphere with radius r: $V = \frac{4}{3}\pi r^3$

Surface Area of sphere with radius r: $S = 4\pi r^2$

Volume of pyramid with base area B and height h: $V = \frac{1}{3}Bh$

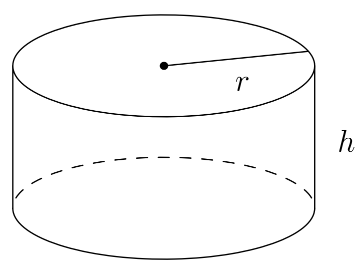
You should know the volume formulas for the solids below. The area of the rectangular solid is just the sum of the areas of its faces. The area of the cylinder is the area of the circles on top and bottom ($2\pi r^2$) plus the area of the sides ($2\pi rh$).



Rectangular Solid

Volume $= lwh$

Area $= 2(lw + wh + lh)$



Right Cylinder

Volume $= \pi r^2 h$

Area $= 2\pi r(r + h)$

The distance between opposite corners of a rectangular solid is: $d = \sqrt{l^2 + w^2 + h^2}$.

The volume of a uniform solid is: $V = (\text{base area}) \cdot \text{height}$.