The Analysis of Harold White applied to the Natario Warp Drive Spacetime. From 10 times the mass of the Universe to the mass of the Mount Everest

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Abstract

Warp Drives are solutions of the Einstein Field Equations that allows superluminal travel within the framework of General Relativity. There are at the present moment two known solutions: The Alcubierre warp drive discovered in 1994 and the Natario warp drive discovered in 2001. However as stated by both Alcubierre and Natario themselves the warp drive violates all the known energy conditions because the stress energy momentum tensor is negative implying in a negative energy density. While from a classical point of view the negative energy is forbidden the Quantum Field Theory allows the existence of very small amounts of it being the Casimir effect a good example as stated by Alcubierre himself.The major drawback concerning negative energies for the warp drive is the huge amount of negative energy able to sustain the warp bubble.Ford and Pfenning computed the amount of negative energy needed to maintain an Alcubierre warp drive and they arrived at the result of 10 times the mass of the entire Universe for a stable warp drive configuration rendering the warp drive impossible.However Harold White manipulating the parameter @ in the original shape function that defines the Alcubierre spacetime demonstrated that it is possible to low these energy density requirements.We repeat here the Harold White analysis for the Natario spacetime and we arrive at similar conclusions.From 10 times the mass of the Universe we also manipulated the parameter @ in the original shape function that defines the Natario spacetime and we arrived at a result of 10 billion tons of negative mass to maintain a warp drive moving with a speed 200 times faster than light.Our result is still a huge number about the weight of the Everest Mountain but at least it is better than the original Ford-Pfenning result of 10 times the mass of the Universe.The main reason of this work is to demonstrate that Harold White point of view is entirely correct.

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1 Introduction

The Warp Drive as a solution of the Einstein Field Equations of General Relativity that allows superluminal travel appeared first in 1994 due to the work of Alcubierre.([1]) The warp drive as conceived by Alcubierre worked with an expansion of the spacetime behind an object and contraction of the spacetime in front.The departure point is being moved away from the object and the destination point is being moved closer to the object.The object do not moves at all[1].It remains at the rest inside the so called warp bubble but an external observer would see the object passing by him at superluminal speeds(pg 8 in [1])(pg 1 in [2]).

Later on in 2001 another warp drive appeared due to the work of Natario.([2]).This do not expands or contracts spacetime but deals with the spacetime as a "strain" tensor of Fluid Mechanics(pg 5 in [2]). Imagine the object being a fish inside an aquarium and the aquarium is floating in the surface of a river but carried out by the river stream.The warp bubble in this case is the aquarium whose walls do not expand or contract. An observer in the margin of the river would see the aquarium passing by him at a large speed but inside the aquarium the fish is at the rest with respect to his local neighborhoods.

However the major drawback that affects the warp drive is the quest of large negative energy requirements enough to sustain the warp bubble.While from a classical point of view negative energy densities are forbidden the Quantum Field Theory allows the existence of very small quantities of such energies but unfortunately the warp drive requires immense amounts of it.Ford and Pfenning computed the negative energy density needed to maintain a warp bubble and they arrived at the conclusion that in order to sustain a stable configuration able to perform interstellar travel the amount of negative energy density is of about 10 times the mass of the Universe and they concluded that the warp drive is impossible.(see pg 10 in [3] and pg 78 in [5]).

However recently Harold White discovered that by a manipulation of the parameter @ in the original shape function that defines the Alcubierre spacetime the amounts of negative energy density needed to maintain the warp drive can be lowered to more reasonable levels.(see pg 4 fig 2 in [8]).

In this work we repeat the analysis of Harold White for the Natario warp drive spacetime and by a manipulation of the parameter @ in the original shape function that defines the Natario spacetime we arrive at similar conclusions.

In order to sustain a stable warp bubble moving with a speed 200 times faster then light "only" 10 billion tons of negative mass are needed.We agree with the fact that our result is still a huge number perhaps outside the scope of the capability of the Quantum Field Theory to generate negative energy and although this places the warp drive in the same magnitude of the weight of the Everest Mountain our result at least it is better than the original Ford-Pfenning number of 10 times the mass of the Universe

We adopted the International System of Units where $G = 6,67 \times 10^{-11}\ \frac{Newton \times meters^2}{kilograms^2}$ and $c = 3 \times 10^8$ $\frac{meters}{seconds}$ and not the Geometrized System of units in which $c = G = 1$.

[1] do not violates Relativity

We consider here a Natario warp drive with a radius $R = 100$ meters a thickness parameter $@ = 400$ moving with a speed 200 times faster than light implying in a $vs = 2 \times 10^2 \times 3 \times 10^8 = 6 \times 10^{10}$ and a $vs^2 = 3,6 \times 10^{21}$

This work is a companion work to our works [4],[6] and [7] and we advise the readers of this work to read these companion works in order to get acquaintance with our line of reason specially [6] and [7] in the question of the Quantum Inequalities (QI) and the problem of the infinite Doppler Blueshifts in the Horizon.

Our value of π is $\pi = 3,1415926536$

2 The Problem of the Negative Energy in the Natario Warp Drive Spacetime-The Unphysical Nature of Warp Drive

The negative energy density for the Natario warp drive is given by(see pg 5 in [2])

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2\sin^2\theta\right] \tag{1}$$

Converting from the Geometrized System of Units to the International System we should expect for the following expression(see eqs 21 and 23 pg 6 in [4]):

$$\rho = -\frac{c^2}{G}\frac{vs^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{rs}{2}n''(rs)\right)^2\sin^2\theta\right]. \tag{2}$$

Rewriting the Natario negative energy density in cartezian coordinates we should expect for[2]:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2 + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2(\frac{y}{rs})^2\right] \tag{3}$$

In the equatorial plane:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{4}$$

Note that in the above expressions the warp drive speed vs appears raised to a power of 2. Considering our Natario warp drive moving with $vs = 200$ which means to say 200 times light speed in order to make a round trip from Earth to a nearby star at 20 light-years away in a reasonable amount of time(in months not in years) we would get in the expression of the negative energy the factor $c^2 = (3 \times 10^8)^2 = 9 \times 10^{16}$ being divided by $6,67 \times 10^{-11}$ giving $1,35 \times 10^{27}$ and this is multiplied by $(6 \times 10^{10})^2 = 36 \times 10^{20}$ coming from the term $vs = 200$ giving $1,35 \times 10^{27} \times 36 \times 10^{20} = 1,35 \times 10^{27} \times 3,6 \times 10^{21} = 4,86 \times 10^{48}$!!!

A number with 48 zeros!!!

Our Earth have a mass[3] of about $6 \times 10^{24} kg$ and multiplying this by c^2 in order to get the total positive energy "stored" in the Earth according to the Einstein equation $E = mc^2$ we would find the value of $54 \times 10^{40} = 5,4 \times 10^{41} Joules$.

Earth have a positive energy of $10^{41} Joules$ and dividing this by the volume of the Earth(radius $R_{Earth} =$ 6300 km approximately) we would find the positive energy density of the Earth.Taking the cube of the Earth radius $(6300000m = 6,3 \times 10^6)^3 = 2,5 \times 10^{20}$ and dividing $5,4 \times 10^{41}$ by $(4/3)\pi R_{Earth}^3$ we would find the value of $4,77 \times 10^{20} \frac{Joules}{m^3}$. So Earth have a positive energy density of $4,77 \times 10^{20} \frac{Joules}{m^3}$ and we are talking about negative energy densities with a factor of 10^{48} for the warp drive while the quantum theory allows only microscopical amounts of negative energy density.

So we would need to generate in order to maintain a warp drive with 200 times light speed the negative energy density equivalent to the positive energy density of 10^{28} Earths!!!!

[2] see Appendix A

[3] see Wikipedia:The free Encyclopedia

A number with 28 zeros!!!.Unfortunately we must agree with the major part of the scientific community that says:"Warp Drive is impossible and unphysical!!"

However looking better to the expression of the negative energy density in the equatorial plane of the Natario warp drive:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{5}$$

We can see that a very low derivative and hence its square can perhaps obliterate the huge factor of 10^{48} ameliorating the negative energy requirements to sustain the warp drive.By manipulating the term @ in the original Alcubierre shape function Harold White lowered these requirements for the Alcubierre warp drive.

In the next section we will repeat the White analysis for the Natario warp drive manipulating also the term @ in the original Natario shape function ameliorating the negative energy requirements from 10 times the mass of the Universe that would render the warp drive as impossible and unphysical to "only" 10 billion tons rendering the warp drive possible but with a magnitude of about the weight of the Everest Mountain.

3 The Analysis of Harold White applied to the Natario Warp Drive Spacetime

According to Natario(pg 5 in [2]) any function that gives 0 inside the bubble and $\frac{1}{2}$ outside the bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region is a valid shape function for the Natario warp drive.

The Natario warp drive continuous shape function can be defined by:

$$n(rs) = \frac{1}{2}[1 - f(rs)] \tag{6}$$

$$n(rs) = \frac{1}{2}[1 - \frac{tanh[@(rs + R)] - tanh[@(rs - R)]}{2tanh(@R)}] \tag{7}$$

This shape function gives the result of $n(rs) = 0$ inside the warp bubble and $n(rs) = \frac{1}{2}$ outside the warp bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region according with our Microsoft Excel simulations .(see pg 5 in [2])

Note that its derivative is almost similar to the derivative of the Alcubierre shape function and as a matter of fact the Alcubierre shape function is being used to define its Natario counterpart.

It is easy to see that the derivatives of the Natario shape function are given by:

$$n'(rs) = -\frac{1}{2}f'(rs) \tag{8}$$

$$n''(rs) = -\frac{1}{2}f''(rs) \tag{9}$$

The square of the derivatives are

$$n'(rs)^2 = \frac{1}{4}f'(rs)^2 \tag{10}$$

$$n''(rs)^2 = \frac{1}{4}f''(rs)^2 \tag{11}$$

In the Natario shape function the term $f(rs)$ is the Alcubierre shape function.For more details about the geometrical features of the Alcubierre shape function see pg 4 in [1].

$$rs = \sqrt{(x - xs)^2 + y^2 + z^2} \tag{12}$$

$$f(rs) = \frac{tanh[@(rs + R)] - tanh[@(rs - R)]}{2tanh(@R)} \tag{13}$$

xs is the center of the warp bubble where the ship resides.

R is the radius of the warp bubble and @ is the Alcubierre parameter related to the thickness.According to Alcubierre these can have arbitrary values.We outline here the fact that according to pg 4 in [1] the parameter @ can have arbitrary values.This is very important for the White analysis as we will see later.

The shape function $f(rs)$ have a value of 1 inside the warp bubble and zero outside the warp bubble while being $0 < f(rs) < 1$ in the warp bubble walls.Since $f(rs)$ is always 1 inside the warp bubble and zero outside the warp bubble we need to take only the derivatives where $f(rs)$ varies from 1 to zero which means to say that we take the derivatives of $f(rs)$ in the region where $0 < f(rs) < 1$.This region is known as the Alcubierre warped region.

rs is the path of the so-called Eulerian observer that starts at the center of the bubble xs and ends up outside the warp bubble.In our case we consider the equatorial plane and we have for rs the following expression[4].

$$rs = \sqrt{(x - xs)^2} \tag{14}$$

$$rs = x - xs \tag{15}$$

It is easy to figure out when $f(rs) = 1$(interior of the Alcubierre bubble) then $n(rs) = 0$(interior of the Natario bubble) and when $f(rs) = 0$(exterior of the Alcubierre bubble)then $n(rs) = \frac{1}{2}$(exterior of the Natario bubble).

For the Alcubierre shape function

$$f(rs) = \frac{tanh[@(rs + R)] - tanh[@(rs - R)]}{2tanh(@R)} \tag{16}$$

the derivative is:

$$f'(rs) = \frac{1}{2tanh(@R)}[\frac{@}{cosh^2[@(rs + R)]} - \frac{@}{cosh^2[@(rs - R)]}] \tag{17}$$

For a warp bubble of radius $R = 100$ meters and an Alcubierre parameter $@ = 400$ giving a thickness of $0,07$ meters, the factors of $\varepsilon^{[@(rs+R)]}$ are very large numbers because we are raising $2,718^{400\times100} = 2,718^{40000}$ even when $rs = 0$ the center of the warp bubble.This number is enormous.On the other hand the factors of $\varepsilon^{[-@(rs+R)]} = \frac{1}{\varepsilon^{[@(rs+R)]}} = \frac{1}{2,718^{40000}}$ and this factor reduces to zero.Hence the term in $cosh^2[@(rs + R)]$ reduces to $\varepsilon^{[@(rs+R)]}$ and dividing an Alcubierre parameter $@ = 400$ by $2,718^{40000}$ will reduces this term to zero so the term in $cosh^{-2}[@(rs+R)]$ can be neglected.Then the term of the derivative that really accounts for is the following given below:

$$f'(rs) = \frac{1}{2tanh(@R)}[-\frac{@}{cosh^2[@(rs - R)]}] \tag{18}$$

its square would then be:

$$f'(rs)^2 = \frac{1}{4tanh^2(@R)}[\frac{@^2}{cosh^4[@(rs - R)]}] \tag{19}$$

The term $tanh(@R) = 1$ according with our Excel simulations. Hence the square of the derivative of the Alcubierre shape function is given by:

$$f'(rs)^2 = \frac{1}{4}[\frac{@^2}{cosh^4[@(rs - R)]}] \tag{20}$$

[4]see Appendix A

In the equatorial plane $y = 0$ and we can neglect the second order derivative of the Natario shape function and consequently its square.The square of the first order derivative is then given by:

$$n'(rs)^2 = \frac{1}{4}f'(rs)^2 \tag{21}$$

$$n'(rs)^2 = \frac{1}{4}(\frac{1}{4}[\frac{@^2}{cosh^4[@(rs-R)]}]) \tag{22}$$

$$n'(rs)^2 = \frac{1}{16}[\frac{@^2}{cosh^4[@(rs-R)]}] \tag{23}$$

An interesting feature is the fact that the square of the derivative of the Natario shape function in the equatorial plane is 4 times lower than its Alcubierre counterpart.

Back again to the negative energy density in the Natario warp drive:

$$\rho = T_{\mu\nu}u^\mu u^\nu = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{24}$$

The total energy needed to sustain the Natario warp bubble is obtained by integrating the negative energy density ρ over the volume of the Natario warped region.
Since we are in the equatorial plane then only the term in rs accounts for and the total energy integral can be given by:

$$E = \int(\rho)drs = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\int(3(n'(rs))^2)drs = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\int((n'(rs))^2)drs \tag{25}$$

Above we placed the constant terms c,G and vs^2 outside the integral. But

$$\int((n'(rs))^2)drs = \int(\frac{1}{16}[\frac{@^2}{cosh^4[@(rs-R)]}])drs = \frac{@^2}{16}\int([\frac{1}{cosh^4[@(rs-R)]}])drs \tag{26}$$

Since @ ia also a constant.Then the total energy integral for the Natario warp drive is given by:

$$E = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\int((n'(rs))^2)drs = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\frac{@^2}{16}\int([\frac{1}{cosh^4[@(rs-R)]}])drs \tag{27}$$

Fortunately integrals of these forms are available in tables of integrals of hyperbolic functions and the one needed to compute the integration above is given below($n = 4$)[5]:

$$\int\frac{dx}{cosh^n(ax)} = \frac{sinh(ax)}{a(n-1)cosh^{n-1}(ax)} + \frac{n-2}{n-1}\int\frac{dx}{cosh^{n-2}(ax)} \tag{28}$$

[5]Wikipedia:The free Encyclopedia

In our Excel simulations we consider a Natario warp bubble with a radius $R = 100$ meters and an Alcubierre thickness parameter $@ = 400$.The warped region of both Alcubierre and Natario warp drives [6] starts at $rs = 99,96$ meters and ends up at $rs = 100,03$ meters giving a thickness of $0,07$ meters.

We start to analyze the behavior of $f(rs)$,$n(rs)$ and its square of the derivatives at a distance $rs = 99,56$ meters from the center of the bubble ($rs = 0$) well inside the warp bubble and we end our analysis at a distance $rs = 100,44$ meters from the center of the bubble well outside the warp bubble varying rs by $0,01$. From $rs = 99,56$ meters to $rs = 99,95$ meters the value of $f(rs) = 1$ and $n(rs) = 0$ as expected for the warp bubble interior. From $rs = 100,05$ meters to $rs = 100,44$ meters the value of $f(rs) = 0$ and $n(rs) = \frac{1}{2}$ as expected for the warp bubble exterior. When $rs = 100,04$ meters the value of the Alcubierre shape function is $f(rs) = 1,2656542481 \times 10^{-14}$ while its Natario counterpart is $n(rs) = \frac{1}{2}$.This means to say that the Natario warped region is thinner than its Alcubierre counterpart.

Our warped region is the Natario one defined by the Natario shape function as being $0 < n(rs) < \frac{1}{2}$ and begins at $rs = rs(beg) = 99,96$ meters from the center of the warp bubble and ends up at $rs = rs(end) = 100,03$ meters from the center of the bubble with a thickness of $0,07$ meters.The values of the square of the derivative of the Natario shape function in the beginning and in the end of the Natario warped region are respectively given by:

$$n'(rs)^2 = n'(rs(beg))^2 = 2,5660974290 \times 10^{-23} \tag{29}$$

$$rs = rs(beg) = 99,96 meters \tag{30}$$

$$n'(rs)^2 = n'(rs(end))^2 = 2,2802625283 \times 10^{-16} \tag{31}$$

$$rs = rs(end) = 100,03 meters \tag{32}$$

In order to compute the total energy needed to maintain our Natario warp bubble with 100 meters of radius moving at a 200 times faster than light with a thickness of $0,07$ meters we could integrate analytically the total energy integral already given before but in order to illustrate the power of the idea of Harold White we will employ the numerical method of integration given in Integral Calculus known as the Trapezoidal Rule.

The Trapezoidal Rule formula to integrate $\int_a^b f(x)dx$ is given by[7]:

$$\int_a^b f(x)dx = (b-a)\frac{f(b)+f(a)}{2} \tag{33}$$

In the above expression b is the end of the warped region and a is the beginning. $f(x)$ is our Natario square of the derivative $n'(rs)^2$- If the warped region have a very small thickness due to manipulation of the parameter @ then b is almost so close to a lying in its neighborhoods and the difference $b - a \simeq 0$.

[6]Remember that we are using the Alcubierre shape function to define its Natario counterpart
[7]see Wikipedia:The free Encyclopedia

If $b - a \simeq 0$ then the Trapezoidal Rule gives the following result:

$$\int_a^b f(x)dx = (b-a)\frac{f(b)+f(a)}{2} \simeq 0 \quad (34)$$

Regardless of the values of the square of the derivatives of the shape function.

This was exactly what Harold White did:by manipulating @ to very large values he created Alcubierre warped regions so thinner that the point b is infinitely closed to a giving a very low (and acceptable) value for the negative energy in the Alcubierre warp drive able to obliterate factors of 10^{48}.

Repeating the White analysis for the Natario Warp drive we have:

$$b - a = 0,07 \quad (35)$$

$$f(b) + f(a) = 2,2802627849 \times 10^{-16} \quad (36)$$

$$\int_a^b f(x)dx = (b-a)\frac{f(b)+f(a)}{2} = 7,9809197470 \times 10^{-18} \quad (37)$$

Above is the numerical result of the integral of the square of the derivative of the Natario shape function obtained by the Trapezoidal Rule. Taking the constant factors of the Natario negative energy density ρ and considering only the magnitude of its values and not the minus sign we have:

$$\frac{c^2}{G}\frac{v_s^2}{8\pi} = \frac{9 \times 10^{16}}{6,67 \times 10^{-11}} \times \frac{3,6 \times 10^{21}}{8\pi} \quad (38)$$

$$\frac{c^2}{G}\frac{v_s^2}{8\pi} = 1,3493253373 \times 10^{27} \times 1,4323944878 \times 10^{20} = 1,9327661755 \times 10^{47} \quad (39)$$

Multiplying the value obtained above by the value of the integral of the square of the derivative of the Natario shape function obtained by Trapezoidal Rule we arrive at the following result for the total energy (the minus sign now appears):

$$-1,5425251736 \times 10^{30} Joules \quad (40)$$

Dividing by c^2 to get the total amount of negative mass we get:

$$-1,7139168596 \times 10^{13} Kilograms \quad (41)$$

Dividing by 10^3 we get:

$$-1,7139168596 \times 10^{10} Tons \quad (42)$$

10 billions of tons.A value much better than the original Ford-Pfenning result of 10 times the mass of the Universe. This places the Natario warp drive with the parameter @ $= 400$ in the same magnitude of the mass of the Everest Mount. With a factor @ $= 4000$ the results could be better improved however we are limited by the CPU of our laptop computers and the numerical precision floating point of Microsoft Excel that although not designed to perform scientific calculations of this magnitude it performed extremely well.

4 Conclusion

In this work we demonstrated that the analysis of Harold White can be applied also to the Natario warp drive spacetime. From 10 times the mass of the Universe we lowered the negative energy density requirements to 10 billion tons nearly the magnitude of the weight of the Everest Mountain.We do not know if Quantum Field Theory can encompass negative energies of this order and only the future can tell us about that but at least 10 billion tons is better than 10 times the mass of the Universe.White point of view is entirely correct. Since the Natario warp drive can encompasses a QI sampling time that can last long enough to allow interstellar space travel with low energy densities and can bypass the physical problem of the infinite Doppler Blueshifts (see refs [6] and [7]) we regard it a valid candidate for interstellar space travel. In this work we mentioned an interstellar trip of 20 light years.It was inspired by the first planet discovered in the habitable zone of another star:the Gliese 581 at 20 light years away.Given the huge number of planets in the habitable zones of their parent stars example:Kepler-22 at 600 light years away or Kepler-47 at 4950 light years away these planets can only be accessible to our space exploration if faster than light space travel could ever be developed.

5 Appendix A:The Natario Warp Drive Negative Energy Density in Cartezian Coordinates

The negative energy density according to Natario is given by(see pg 5 in [2])[8]:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2\sin^2\theta\right] \tag{43}$$

In the bottom of pg 4 in [2] Natario defined the x-axis as the polar axis.In the top of page 5 we can see that $x = rs\cos(\theta)$ implying in $\cos(\theta) = \frac{x}{rs}$ and in $\sin(\theta) = \frac{y}{rs}$

Rewriting the Natario negative energy density in cartezian coordinates we should expect for:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2 + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2(\frac{y}{rs})^2\right] \tag{44}$$

Considering motion in the equatorial plane of the Natario warp bubble (x-axis only) then $[y^2 + z^2] = 0$ and $rs^2 = [(x - xs)^2]$ and making $xs = 0$ the center of the bubble as the origin of the coordinate frame for the motion of the Eulerian observer then $rs^2 = x^2$ because in the equatorial plane $y = z = 0$.

Rewriting the Natario negative energy density in cartezian coordinates in the equatorial plane we should expect for:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{45}$$

[8] $n(rs)$ is the Natario shape function.Equation written in the Geometrized System of Units $c = G = 1$

6 Epilogue

- "The only way of discovering the limits of the possible is to venture a little way past them into the impossible."-Arthur C.Clarke[9]
- "The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction. There is no logical path to these laws; only intuition, resting on sympathetic understanding of experience, can reach them"-Albert Einstein[10][11]

7 Remarks

Reference [8] "Warp Field Mechanics 101" by Harold "Sonny" White of NASA Lyndon B.Johnson Space Center Houston Texas is available at NASA Technical Reports Server (NTRS)[12] however we can provide a copy in PDF Acrobat reader for those interested.

We performed all the numerical calculus of our simulations for the Natario Warp Drive using Microsoft Excel[13].We can provide our Excel files to those interested[14] and although Excel is a licensed program there exists another program that can read Excel files available in the Internet as a freeware for those that perhaps may want to examine our files:the OpenOffice[15] at http://www.openoffice.org

[9] special thanks to Maria Matreno from Residencia de Estudantes Universitas Lisboa Portugal for providing the Second Law Of Arthur C.Clarke

[10] "Ideas And Opinions" Einstein compilation, ISBN $0-517-88440-2$, on page 226."Principles of Research" ([Ideas and Opinions],pp.224-227), described as "Address delivered in celebration of Max Planck's sixtieth birthday (1918) before the Physical Society in Berlin"

[11] appears also in the Eric Baird book Relativity in Curved Spacetime ISBN $978-0-9557068-0-6$

[12] browse Google for NASA Technical Report Server-20110015936

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