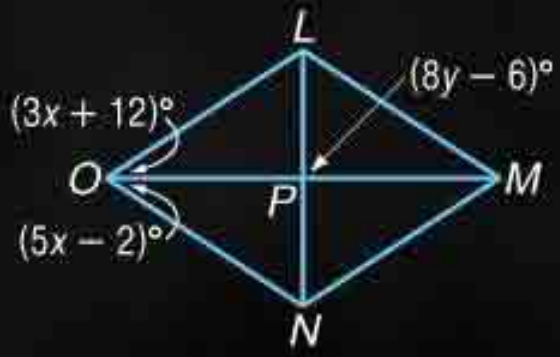
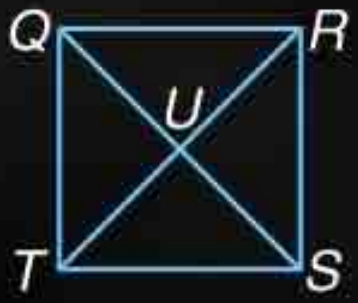
***LMNO* is a rhombus.**

1. Find x.
2. Find y.



***QRST* is a square.**

3. Find n if $m\angle TQR = 8n + 8$.
4. Find w if $QR = 5w + 4$ and $RS = 2(4w - 7)$.
5. Find QU if $QS = 16t - 14$ and $QU = 6t + 11$.

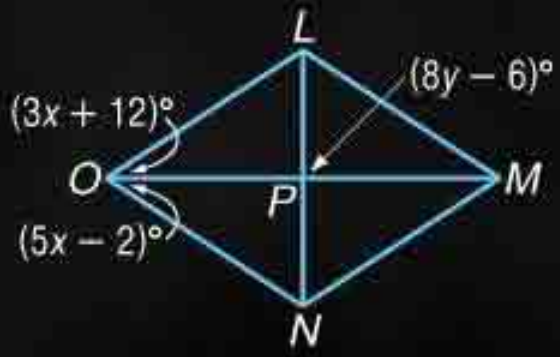


6. **Standardized Test Practice** What property applies to a square, but not a rhombus?

A. Opposite angles are congruent.
B. Opposite sides are congruent.
C. Diagonals bisect each other.
D. All angles are right angles.

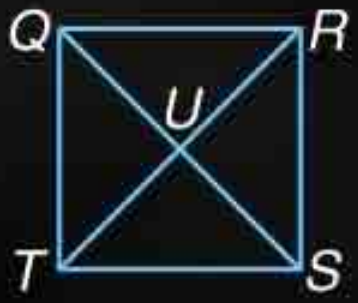
5-Minute Check (over Lesson 8-5) Transparency 8-6

***LMNO* is a rhombus.**



1. Find x. 7
2. Find y. 12

***QRST* is a square.**



3. Find n if $m\angle TQR = 8n + 8$. 10.25
4. Find w if $QR = 5w + 4$ and $RS = 2(4w - 7)$. 6
5. Find QU if $QS = 16t - 14$ and $QU = 6t + 11$. 65

6. **Standardized Test Practice** What property applies to a square, but not a rhombus?

A. Opposite angles are congruent.
B. Opposite sides are congruent.
C. Diagonals bisect each other.
D. All angles are right angles.

8-6 Objectives

You will learn to:

Recognize and apply the properties of trapezoids.

Solve problems involving the medians of trapezoids.

Vocabulary

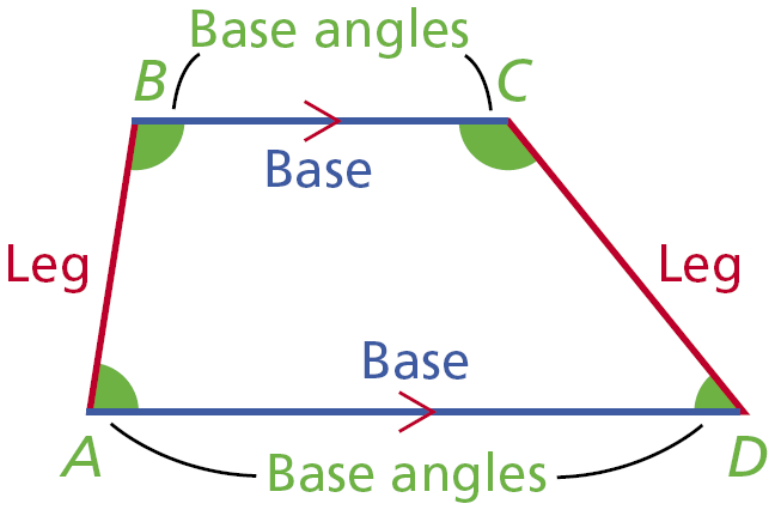
trapezoid

Isosceles Trapezoid

Median

Trapezoid

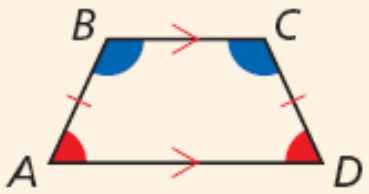
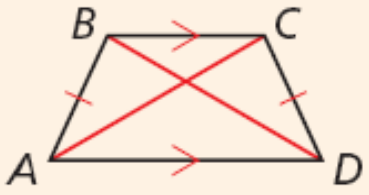
A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. Each of the parallel sides is called a **base**. The nonparallel sides are called **legs**. **Base angles** of a trapezoid are two consecutive angles whose common side is a base.



If the legs of a trapezoid are congruent, the trapezoid is an **isosceles trapezoid**. The following theorems state the properties of an isosceles trapezoid.

Isosceles Trapezoid

Theorems **Isosceles Trapezoids**

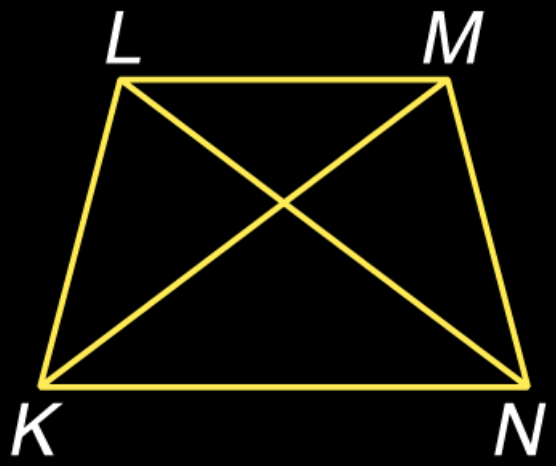
	THEOREM	DIAGRAM	EXAMPLE
8.18	If a quadrilateral is an isosceles trapezoid, then each pair of base angles are congruent. (isosc. trap. → base ∡s ≅)		$\angle A \cong \angle D$ $\angle B \cong \angle C$
8.19	A trapezoid is isosceles if and only if its diagonals are congruent. (isosc. trap. ↔ diags. ≅)		$\overline{AC} \cong \overline{DB} \leftrightarrow$ $ABCD$ is isosceles.

Example

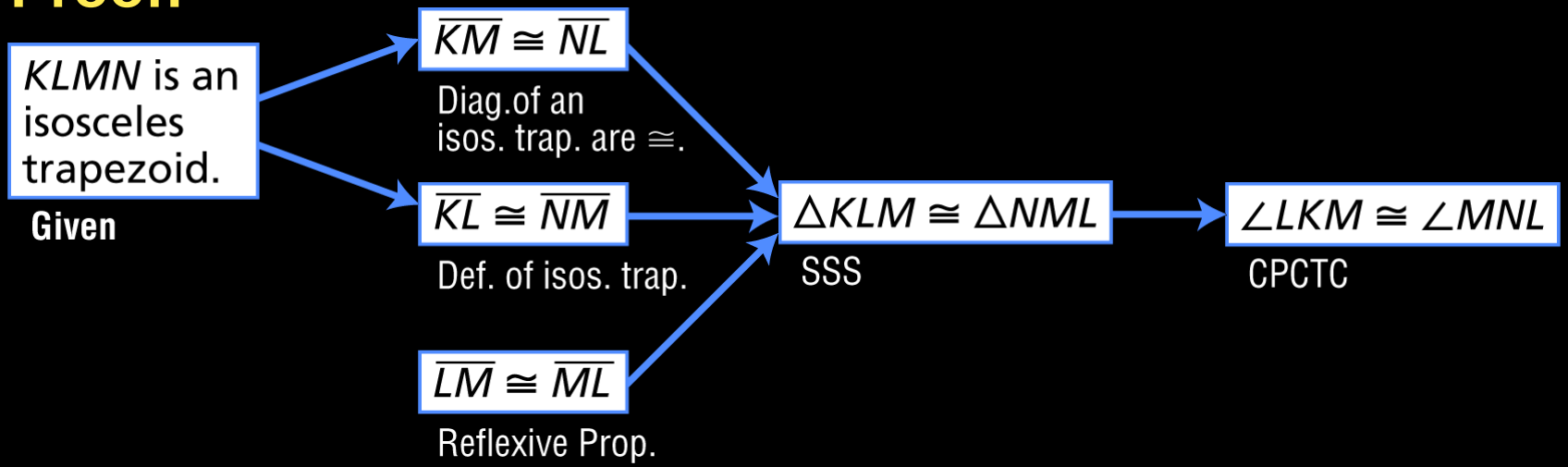
Write a flow proof.

Given: *KLMN* is an isosceles trapezoid.

Prove: $\angle LKM \cong \angle MNL$



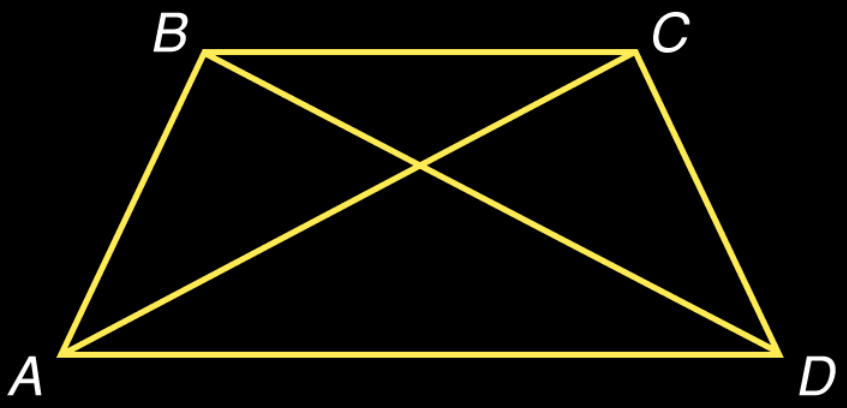
Proof:



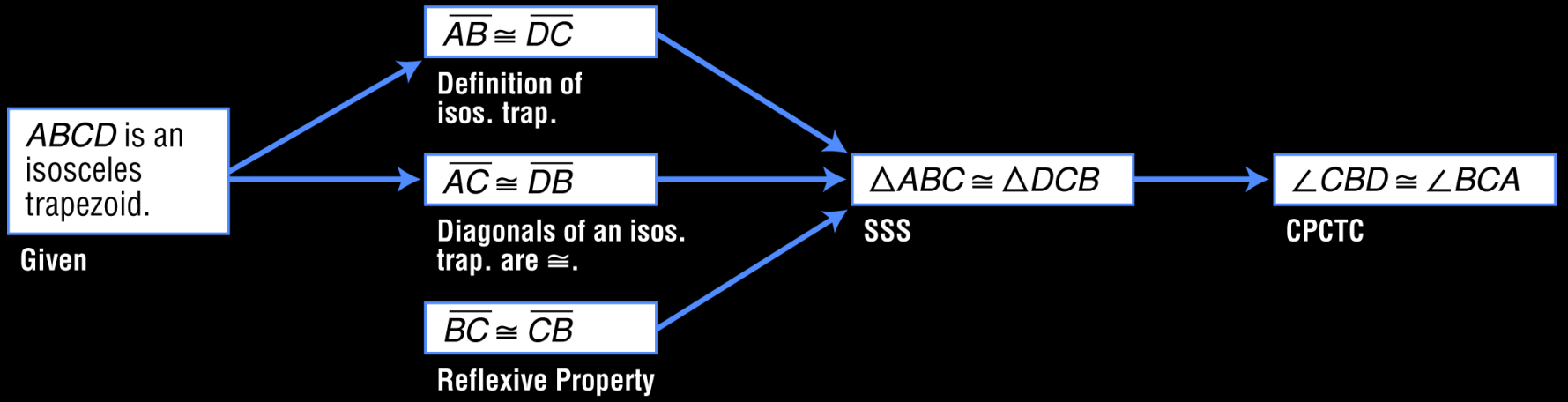
Your Turn **Write a flow proof.**

Given: *ABCD* is an isosceles trapezoid.

Prove: $\angle CBD \cong \angle BCA$

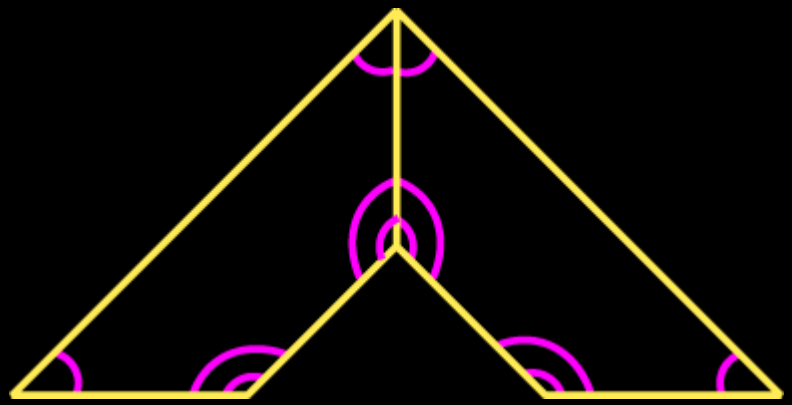


Proof:



Example

The top of this work station appears to be two adjacent trapezoids. Determine if they are isosceles trapezoids.

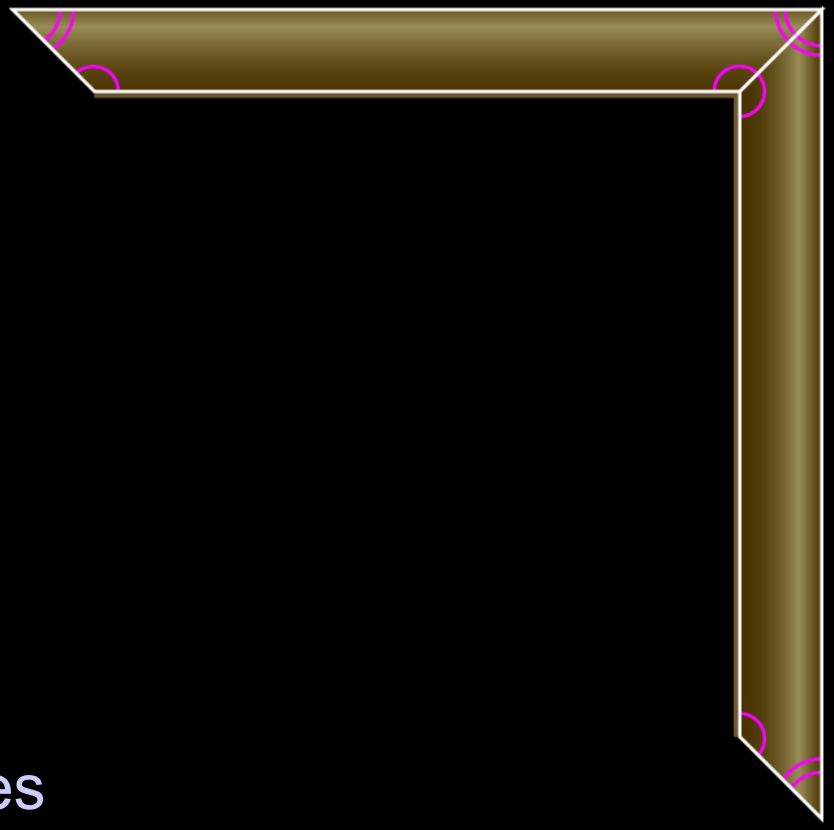


Each pair of base angles is congruent, so the legs are the same length.

Answer: Both trapezoids are isosceles.

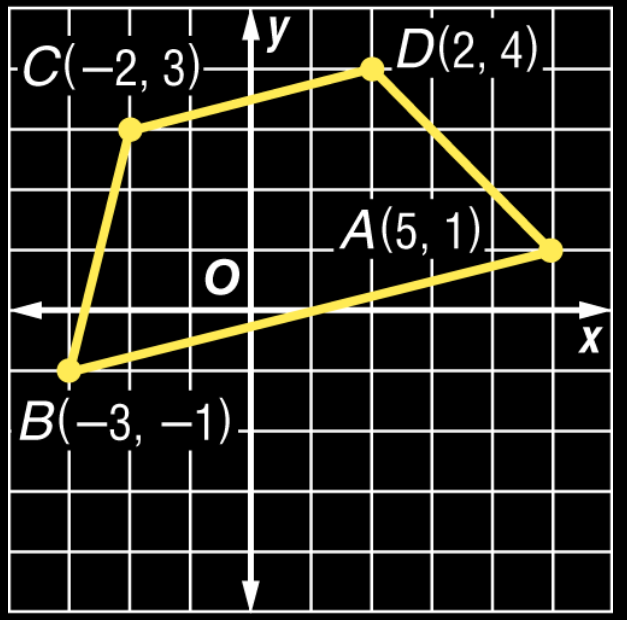
Your Turn

The sides of a picture frame appear to be two adjacent trapezoids. Determine if they are isosceles trapezoids.



Answer: yes

Example ***ABCD*** **is a quadrilateral with vertices** $A(5, 1)$, $B(-3, -1)$, $C(-2, 3)$, **and** $D(2, 4)$. **Verify that** ***ABCD*** **is a trapezoid.**



$$\text{slope of } \overline{AB} = \frac{-1-1}{-3-5} = \frac{-2}{-8} \text{ or } \frac{1}{4}$$

$$\text{slope of } \overline{CD} = \frac{4-3}{2-(-2)} = \frac{1}{4}$$

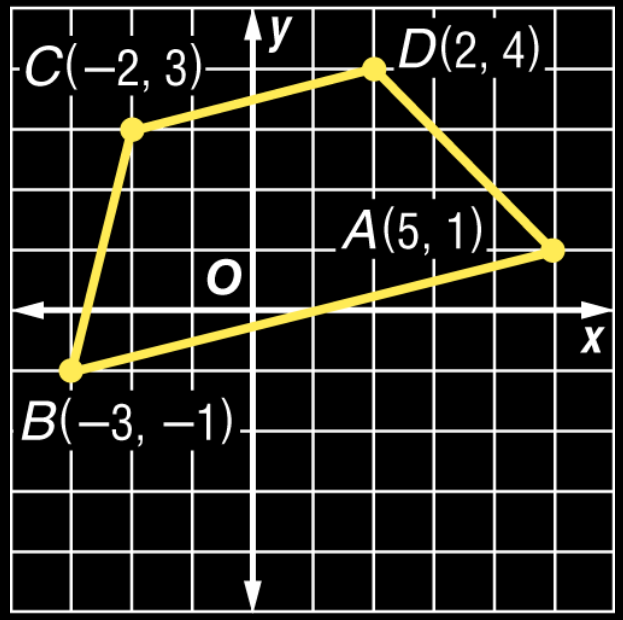
$$\text{slope of } \overline{DA} = \frac{1-4}{5-2} = \frac{-3}{3} \text{ or } -1$$

$$\text{slope of } \overline{BC} = \frac{3-(-1)}{-2-(-3)} = \frac{4}{1} \text{ or } 4$$

A quadrilateral is a trapezoid if exactly one pair of opposite sides are parallel. Use the Slope Formula.

Answer: Exactly one pair of opposite sides are parallel, $\overline{AB}$ and $\overline{CD}$. So, *ABCD* is a trapezoid.

Example ***ABCD*** **is a quadrilateral with vertices** $A(5, 1)$, $B(-3, 1)$, $C(-2, 3)$, **and** $D(2, 4)$. **Determine whether** ***ABCD*** **is an isosceles trapezoid. Explain.**



First use the Distance Formula to show that the legs are congruent.

$$DA = \sqrt{(2-5)^2 + (4-1)^2} = \sqrt{9+9} = \sqrt{18}$$

$$BC = \sqrt{[-2-(-3)]^2 + [3-(-1)]^2} = \sqrt{1+16} = \sqrt{17}$$

Answer: Since the legs are not congruent, *ABCD* is not an isosceles trapezoid.

Your Turn

QRST **is a quadrilateral with vertices** *Q*(–3, –2), *R*(–2, 2), *S*(1, 4), **and** *T*(6, 4).

a. Verify that *QRST* is a trapezoid.

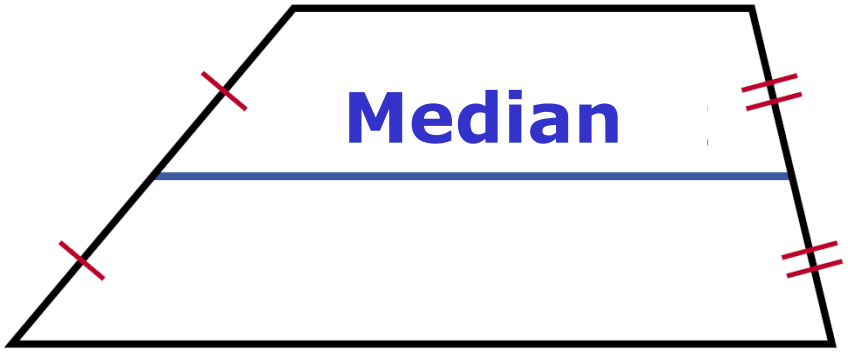
Answer: Exactly one pair of opposite sides is parallel. Therefore, *QRST* is a trapezoid.

b. Determine whether *QRST* is an isosceles trapezoid. Explain.

Answer: Since the legs are not congruent, *QRST* is not an isosceles trapezoid.

Median

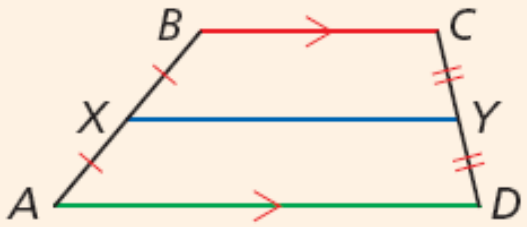
The **midsegment of a trapezoid (also known as the *median*)**is the segment whose endpoints are the midpoints of the legs.



Median Theorem

Theorem 8.20 **Trapezoid Median Theorem**

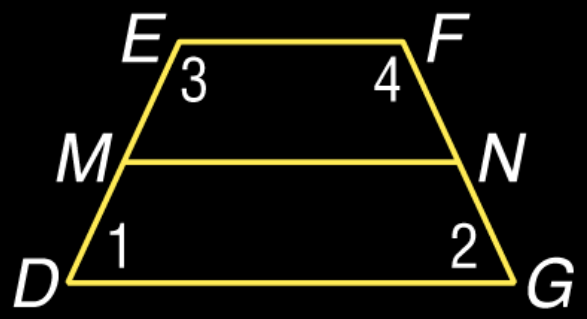
The midsegment of a trapezoid is parallel to each base, and its length is one half the sum of the lengths of the bases.



$\overline{XY} \parallel \overline{BC}, \overline{XY} \parallel \overline{AD}$

$XY = \frac{1}{2}(BC + AD)$

Example $DEFG$ **is an isosceles trapezoid with median** $\overline{MN}$. **Find** DG **if** $EF = 20$ **and** $MN = 30$.



$MN = \frac{1}{2}(EF + DG)$ Theorem 8.20

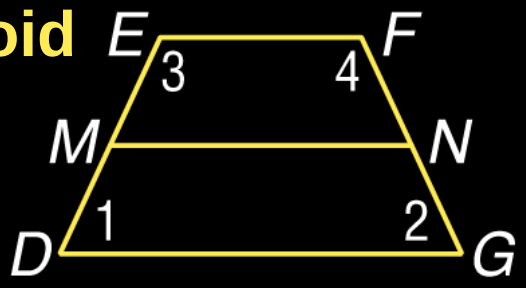
$30 = \frac{1}{2}(20 + DG)$ Substitution

$60 = 20 + DG$ Multiply each side by 2.

$40 = DG$ Subtract 20 from each side.

Answer: $DG = 40$

Example **$DEFG$ is an isosceles trapezoid with median $\overline{MN}$. Find $m\angle 1, m\angle 2, m\angle 3$, and $m\angle 4$ if $m\angle 1 = 3x + 5$ and $m\angle 3 = 6x - 5$.**



Since $\overline{EF} \parallel \overline{DG}$, $\angle 1$ and $\angle 3$ are supplementary. Because this is an isosceles trapezoid, $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$.

$m\angle 1 + m\angle 3 = 180$ Consecutive Interior Angles Theorem

$3x + 5 + 6x - 5 = 180$ Substitution

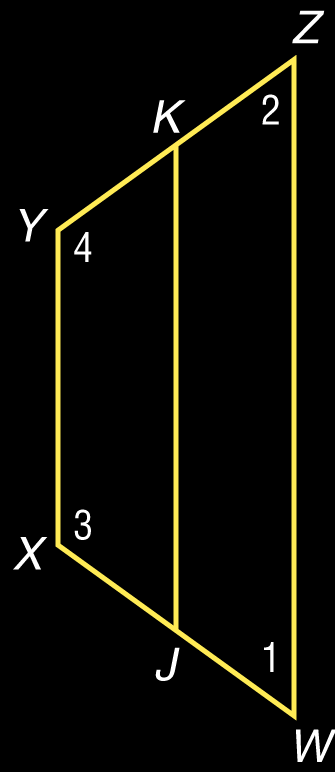
$9x = 180$ Combine like terms.

$x = 20$ Divide each side by 9.

Answer: If $x = 20$, then $m\angle 1 = 65$ and $m\angle 3 = 115$. Because $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$, $m\angle 2 = 65$ and $m\angle 4 = 115$.

Your Turn

$WXYZ$ is an isosceles trapezoid with median $\overline{JK}$.



a. Find XY if $JK = 18$ and $WZ = 25$.

Answer: $XY = 11$

b. Find $m\angle 1$, $m\angle 2$, $m\angle 3$, and $m\angle 4$ if $m\angle 2 = 2x - 25$ and $m\angle 4 = 3x + 35$.

Answer: Since $x = 34$, $m\angle 2 = 43$ and $m\angle 4 = 137$. Because $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$, $m\angle 1 = 43$ and $m\angle 3 = 137$.

What did you learn today?

How to:

Recognize and apply the properties of trapezoids.

Solve problems involving the medians of trapezoids.

Assignment:

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9 – 12, 22-26, 29, 46, 50, 54