Using Klauder's Enhanced Quantization to set a bound to the Cosmological constant, in Pre Planckian space- to obtain a bound on the graviton mass , in the early universe, and generation of (heavy) gravity

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Abstract

We are looking at comparison of two action integrals and we identify the Lagrangian multiplier as setting up a constraint equation (on cosmological expansion).. What we have done is to replace the Hambler Quantum gravity reference based action integral with a result from John Klauder's "Enhanced Quantization" tome. In doing so, with Thanu Padamabhan's treatment of the inflaton, we then commence to initiate an explicit bound upon the cosmological constant. The further approximation is to use the inflaton results, and conflate them with John Klauder's Action principle for a way to , if we have the idea of a potential well, generalized by Klauder, with a wall of space time in the Pre Planckian regime to ask what bounds the Cosmological constant prior to inflation.. And, get an upper bound on the mass of a graviton.

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1. Basic idea, can two First Integrals give equivalent information?

We admit this paper has some similarity to [1], what we will do is instead of using the Hambler result of [2] as to a first integral we are instead using what John Klauder wrote in [3]as to form a first integral in order to make a 1 to 1 equivalence with the first integral associated with general relativity [4], [5] As what was done in [1] we have a 1 to 1 relationship between two first action integrals, i.e. and the idea is to avoid a point cosmic singularity, but to instead have a regime of space-time incorporating the idea of a cosmic bounce, as given in [6] with interior and exterior regimes, i.e. this also over laps with work done by the author in [7] with the caveat that there is a barrier between interior and exterior regimes of space-time and that we are evaluating the space in the interior of a space-time bubble. Having said that. The Integrands in the two integrals are assumed to have a 1-1 and onto relationship to one another. And we will in the next section identify the **two first integrals.**

2. Now for the General relativity First integral. From [1]

We use the Padmabhan 1st integral [8] of the form , with the third entry of Eq. (1) having a Ricci scalar defined via [9] and usually the curvature $\aleph$ set as extremely small, with the general relativity version of , from   [1]

$$S_1 = \frac{1}{2\kappa} \int \sqrt{-g} \cdot d^4x \cdot (\mathfrak{R} - 2\Lambda)$$

$$\& -g = -\det g_{uv} \qquad (1)$$

$$\& \mathfrak{R} = 6 \cdot \left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 + \frac{\aleph}{a^2} \right)$$

Also, the variation of $\delta g_{tt} \approx a_{\min}^2 \phi$ as given by [10, 11] will have an inflaton, ϕ given by [9]

$$a \approx a_{\min} t^{\gamma}$$

3.
$$\Leftrightarrow \phi \approx \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln \left\{ \sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot t \right\} \qquad (2)$$

$$\Leftrightarrow V \approx V_0 \cdot \exp \left\{ -\sqrt{\frac{16\pi G}{\gamma}} \cdot \phi(t) \right\}$$

Leading to [1, 9] to the inflaton which is combined into other procedures for a solution to the cosmological constant problem.

$$\phi \approx \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln \left\{ \sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot t \right\} \qquad (3)$$

Here, we have that $a_{\min}$ is a minimum value of the scale factor presumably given by [12] as a tiny but non zero value. Or at least a quantum bounce as given by [1]

3. Next for the idea from Klauder

We are going to go to page 78 by Klauder [3] as to his idea of what he calls on page 78 a restricted Quantum action principle which he writes as: S_2 where we then write a 1-1 equivalence as in [1] so that

$$S_2 = \int_0^T dt \cdot \left[p(t)\dot{q}(t) - H_N(p(t), q(t)) \right]$$

$$\approx S_1 = \frac{1}{2\kappa} \int \sqrt{-g} \cdot d^4x \cdot (\mathfrak{R} - 2\Lambda) \qquad (4)$$

Our assumption is that Λ is a constant, hence we assume then the following, i.e. a Pre Planckian instant of time, say some power of Planck Time length, hence getting the following approximation

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$$\Lambda \approx \frac{-\left[p(\tilde{t})\dot{q}(\tilde{t}) - H_N\left(p(\tilde{t}), q(\tilde{t})\right)\right]}{\frac{1}{\kappa}\int \sqrt{-g} \cdot d^3x}$$

$$+ \frac{\frac{1}{2\kappa}\int \sqrt{-g} \cdot d^3x \cdot \left(\Re = 6 \cdot \left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\aleph}{a^2}\right)\right)\Big|_{t=\tilde{t}}}{\frac{1}{2\kappa}\int \sqrt{-g} \cdot d^3x} \qquad (5)$$

4. Filling in the details of the above using details from [3] with explanations

To do this, we are making several assumptions.

a. That the two mentioned integrals are evaluated from a Pre Planckian to Planckian space-time domain. i.e. in the same specified integral of space-time. $S_2 \approx S_1$

b. That in doing so, the Universe is assumed to avoid the so called cosmic singularity. In doing so assuming a finite "Pre Planckian to Planckian" regime of space time similar to that given in [1]. With reference also, to the cosmic bounce given in [7]

c. assuming that even in the Pre Planck-Planck regime that curvature $\aleph$ will be a very small part of Ricci scalar $\Re$ and that to first approximation even in the Plank time regime, that to first order [13] has a value altered to be

$$\Re = 6 \cdot \left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\aleph}{a^2}\right) \sim 6 \cdot \left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right) \qquad (6)$$

Furthermore we can make assumptions as to the nature of the cosmic bubble, in assuming that there is a barrier between the Pre-Planckian to Planckian physics regimes so that we have a quantum mechanical style potential well, so to speak in evaluation of the [7] reference which has then if we use Klauder's [3] notation that N represents the strength of the wall, i.e. the Pre Planckian to Planckian bubble boundary

$$\frac{p_0^2}{2} = \frac{p_0^2(N)}{2} + N; \quad for \quad 0 < N \le \infty$$

$$q = q_0 \pm p_0 t$$

$$V_N(x) = 0; \quad for \quad 0 < x < 1 \qquad (7)$$

$$V_N(x) = N; \quad otherwise$$

$$H_N\left(p(t), q(t)\right) = \frac{p_0^2}{2} + \frac{(\hbar \cdot \pi)^2}{2} + N; \quad for \quad 0 < N \le \infty$$

Our innovation is to then equate $q = q_0 \pm p_0 t \sim \phi$ and to assume small time step values. Then

$$\Lambda \approx \frac{-\left[\dfrac{V_0}{3\gamma-1} + 2N + \dfrac{\gamma\cdot(3\gamma-1)}{8\pi G\cdot\tilde{t}^2}\right]}{\dfrac{1}{\kappa}\int\sqrt{-g}\cdot d^3x} + \frac{\int\sqrt{-g}\cdot d^3x\cdot\left(6\cdot\left(\dfrac{\ddot{a}}{a} + \left(\dfrac{\dot{a}}{a}\right)^2\right)\right)\Big|_{t=\tilde{t}}}{\int\sqrt{-g}\cdot d^3x}$$

$$\sim \frac{-\left[\dfrac{V_0}{3\gamma-1} + 2N + \dfrac{\gamma\cdot(3\gamma-1)}{8\pi G\cdot\tilde{t}^2}\right]}{\dfrac{1}{\kappa}\int\sqrt{-g}\cdot d^3x} + \left(6\cdot\left(\dfrac{\ddot{a}}{a} + \left(\dfrac{\dot{a}}{a}\right)^2\right)\right)\Big|_{t=\tilde{t}}$$

(8)

These are terms within the bubble of space-time given in [7] using the same inflaton potential. The scale factor is presumed here to obey the value of the scale factor given in [12]

5. Why this is linked to gravity/massive gravitons

Klauder's program is to isolate a regime of space time for a proper canonical quantization of a classical system. i.e. what we did is to utilize the ideas of [3] so as to make the identification of Eq. (7) which when combined with inflaton physics to have enhanced quantization of the often assumed to be classical inflaton, as given in Eq.(3). I.e. to embed via Eq.(7) as a quantum mechanical well for a Pre Planckian system for inflaton physics as given by Eq. (3).

In short, the scaling of our problem for a bound as to the cosmological constant, in Pre Planckian space-time, as given in Klauders treatment of the action integral as of page 87 of [3] where Klauder talks of the weak correspondence principle, where an enhanced classical Hamiltonian, is given 1-1 correspondence with quantum effects, in a non vanishing fashion. This, in page 87 is in Klauder's description, in his Eq. 5.15

I.e. for the sake of Argument we will make the following assumptions which may be debatable, i.e.

$$\sqrt{-g} \text{ is approximately a constant} \tag{9}$$

For extremely small time intervals (in the boundary between Pre Planckian to Planckian physicboundary regime). As given in [11]. This approximation is why the author assumes Eq. (9).

$$g_{tt} \sim \delta g_{tt} \approx a_{min}^2 \phi \tag{10}$$

If so, if we through this procedure, make a linkage directly to the mass of a graviton, as given by Novello, [13]

$$m_g = \frac{\hbar\cdot\sqrt{\Lambda}}{c} \tag{11}$$

This is a way, then to ascertain a bound, based upon the early universe conditions so set forth, as a way to ascertain a bound to the effective heavy graviton

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6. Conclusion, how this ties in with this contest?

Eq. (11) and Eq. (8) center on gravity, directly.

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