# FRACTAL ARTS:
# A 2D-MFDFA APPROACH

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## Abstract

"*Art attracts us only by what it reveals of our most secret self.*"- *Alfred North Whitehead*

The basic building blocks upon which the natural world is built are Fractals. Recognizing these patterns in Nature is essential-because of these patterns Nature is so aesthetically pleasing. We try to find these patterns everywhere instinctively. In our work we look forward to find the fractality in Abstract art-the paintings of the renowned artist Jackson Pollock using a novel approach of 2D-Multifractal Detrended Fluctuation Analysis in his paintings.

## Introduction

"*In the mind's eye, a fractal is a way of seeing infinity.*"- *James Gleick*

What are Fractals? A fractal can be defined as a mathematical set exhibiting a repeating structure or a pattern displayed at every scale, also known as expanding symmetry. An object is called a self-similar one if the repetition is same at each scale. A famous example of such a pattern is the Mandelbrot set itself which gained popularity because of its aesthetic charisma. Magnifying or zooming an image of a Mandelbrot set reveals its self-repeating properties.

The word 'fractal' was coined by Benoit Mandelbrot [1] and this word became popular within a short span of time. The idea of the word 'fractal' was taken from the Latin word 'fractus' which means 'to create irregular objects'. These concepts of fractals, irregularities in objects, self-similarities, the patterns attracted artists all over the world. This resulted in a genesis of what we know as the *Fractal Art* [2]. Researchers from various domains related to Signal Processing and Composition started using the idea of fractal dimension in their works. New algorithms developed including the famous Higuchi Fractal Dimension and Katz Dimension to exploit the fractal dimension in Audio Signals. This idea then extended to analyze the business trends, social trends to extract information regarding whether there will be a rise or fall in the sales.

The foundation of Chaos Theory inspired mathematicians to study the nature-to unveil the Nature. Nature has ample examples that needs to be studied and Chaos Theory accompanied with this fresh concept of Fractals be major tools for the scientists to conduct those researches [1]. We are aware of the butterfly effect-most of us know about this phenomenon because of the movie of the same name. Nevertheless, it is an exciting topic-the underlying concept is Chaos. The weather system, weather forecast all of these can be studied, the underlying principle being Chaos Theory. The aim is to find pattern in these Chaos. [3] This is the idea of Fractals-to find regularities in the existing irregularities.

Mathematics inspired artists from time to time. The Euclidean geometry, the concept of Fibonacci Sequence and the Golden Ratio attracted painters and sculptors. This fact is evident from their creations. In the recent past the artists are imbued with this newly emerging concept of Fractals and these are reflected in their arts. Digital media has progressed a lot in this respect.

The concept of dream within a dream and within a dream in the movie 'Inception' was nothing but a repeating structure-the concept of Fractals. What we find in nature is in a way has some form of fractal dimension in it. Good examples of fractals found in nature are broccoli, cauliflowers and so on. Careful observation into the structure of broccoli reveals its self-repeating properties. Same applies for cauliflowers as well. These are nothing but fractals. A leaf has self-repeating patters, our skin, fish scales-all of these are fractals. An interesting experiment can be done like this. Let us consider a simple structure. If we go on repeating it eventually it will lead to a different structure-a bigger one, a better one with a more aesthetic appeal. This is how music is made. At a very small scale if we observe a music signal we can get repeating patterns of the same signal. A music which seems pleasing to our ears, that is has a more aesthetic value has more fractality than that of one which has less aesthetic value. A simple comparison in this regard can be made by considering to music signals like a simple noise and a piano cover of Bohemian Rhapsody. Which sounds more pleasing? Of course the piano cover sounds more pleasing. Studies show that this simple fact can be shown by comparing the Hurst exponent values-a parameter for judging self-similarity in a signal; the piano cover had a Hurst exponent value much greater than that of the noise suggesting the same as judged by our ears.

Scientific ventures into studying the emotions associated with Audio signals, both Music and Speech signals have been made recently and the results came out to be satisfactory. What a layman thinks as a music associated with sad emotion or happy emotion can be identified properly with these approaches involving the estimation of the fractal dimensions [4, 5].

Ever noticed the temples? All of them incorporate self-repeating structures. This idea was innate in the artists but then there was little scope for mathematical interpretation. The Eiffel Tower looks fascinating only because of the existence of fractality in it. Bridges have fractality in the structures. This domain is called Fractal Architecture. That which is not fractal has no aesthetic value, been regarded as noise and we discard them. Nature approves of it-so most of the things existing in nature, including the man-made things are fractals.

A drop of ink when placed on a piece of moist paper when gets soaked takes a shape which is self-repeating. The branches of a tree are self-repeating, nothing but fractals. The famous paintings of artists like Jackson Pollock also have fractal elements in it. [6] One of the examples can be *The Autumn Rhythm*. Recent advancements in the field of bio-medical research works have been made with the concept of fractals. [7] Recent studies have been conducted with famous literatures of eminent authors. The idea is to check the frequency of occurrences of words in a span of say around hundred pages or an entire composition. By this approach we get a histogram which will have a pattern, we need to identify the fractal dimension and the associated emotion of the word that is reflected in the composition. These approaches can be applied to lyrics of a song as well.

In our work we focus on finding the Hurst exponents of 2-D signals, the paintings of Jackson Pollock applying the existing techniques of 2D-Multifractal Detrended Fluctuation Analysis [8] and exploited the patterns using the Hurst exponents to find out the existing fractality in them.

## A study on Fractal Paintings of Jackson Pollock

Jackson Pollock was an American artist and a major figure in the abstract expressionist movement. He gained popularity for one of the most unique styles he used to employ in his paintings popularly known as '*drip painting*'. His notable works include: *Number 30* later known as *Autumn Rhythm*, *Blue Poles, Shimmering Substance, The She Wolf, Eyes in The Heat, One Number 31, 1950* and so on. Pollock used to name the paintings using with the help of numbers. He believed that titles affected with perception of his paintings. From the picture it is well recognized that

the same patterns and colors are being used repeatedly over and over again which enhanced the aesthetic beauty.

Fig. 1

The picture shows the use of black, white and brown paints splattered and interlaced throughout the canvas. Attempts to study the fractal patterns of paintings of Jackson Pollock have been made by the scientists. They got several interesting results regarding the fractal nature possessed by the paintings-why these abstract arts caught the attention of the people, what made them so much special. The general notion of people is to get accustomed to what they see around them. They try to find similarity between things so that they can be related in some way or the other. Pollock somehow succeeded in investigating the inherent property of Nature-the self-similarity and started applying them in his paintings.

We look forward to a constructive study of some of his paintings using a very recent approach of 2-D MFDFA (Two Dimensional Multifractal Detrended Fluctuation Analysis) the algorithm of which has been discussed extensively in many recent literatures based on fractal studies. MFDFA has been applied in bio-medical studies, to study trends in stock markets and its 2D versions been applied to images recently shows magnificent results. We have tried to plot the results of MFDFA in a log-log curve as shown in Fig. 2, the x-axis being the $\log_{10}(w)$ where *w* stands for the

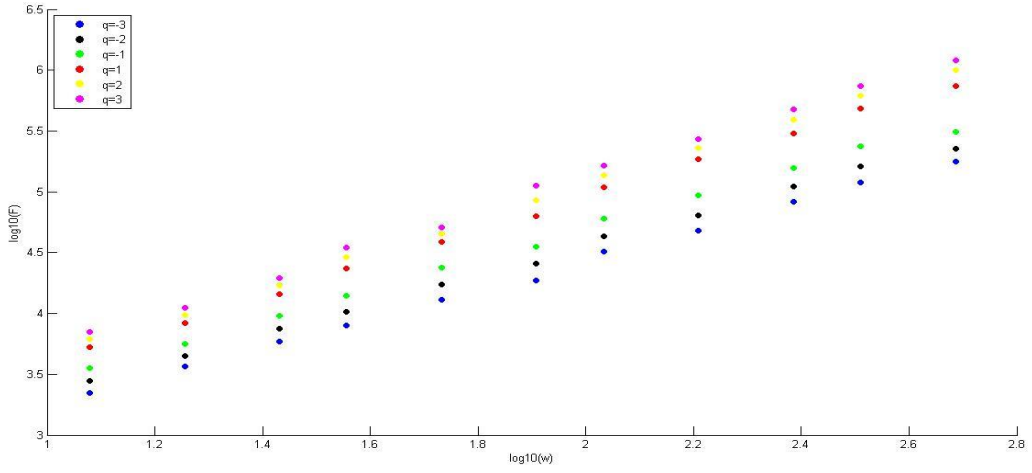


Fig. 2

windows in which the image pixels are portioned and y-axis represents the $\log_{10}(F)$. We have the equation as $F=w^h$.

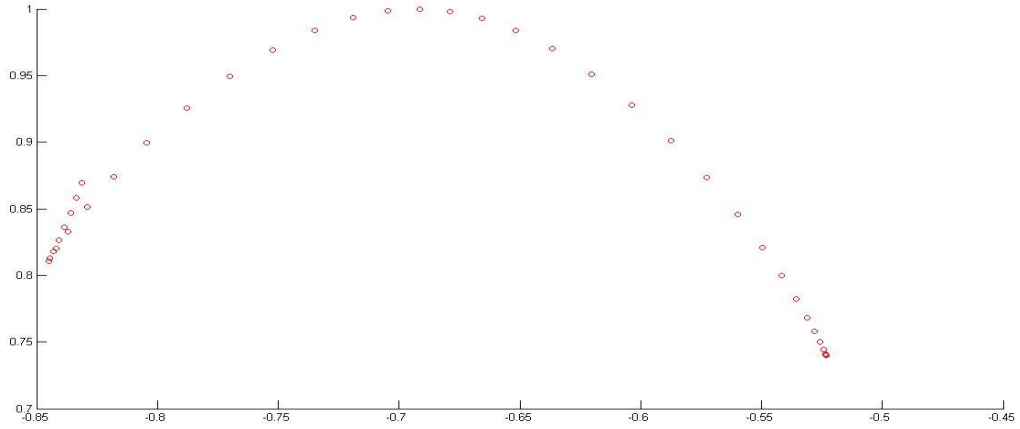


Fig. 3

The multifractal q-order Hurst exponent Hq is used to parameterize the multifractal structure of time series. Hq is first converted to q-order mass exponent tq and tq is thereafter converted to q-order singularity exponent hq and q-order singularity dimension Dq. We show the plot of hq versus Dq in Fig. 3 which is referred to as the multifractal spectrum. The difference between the maximum and the minimum values of hq gives the value of the multifractal spectrum width.

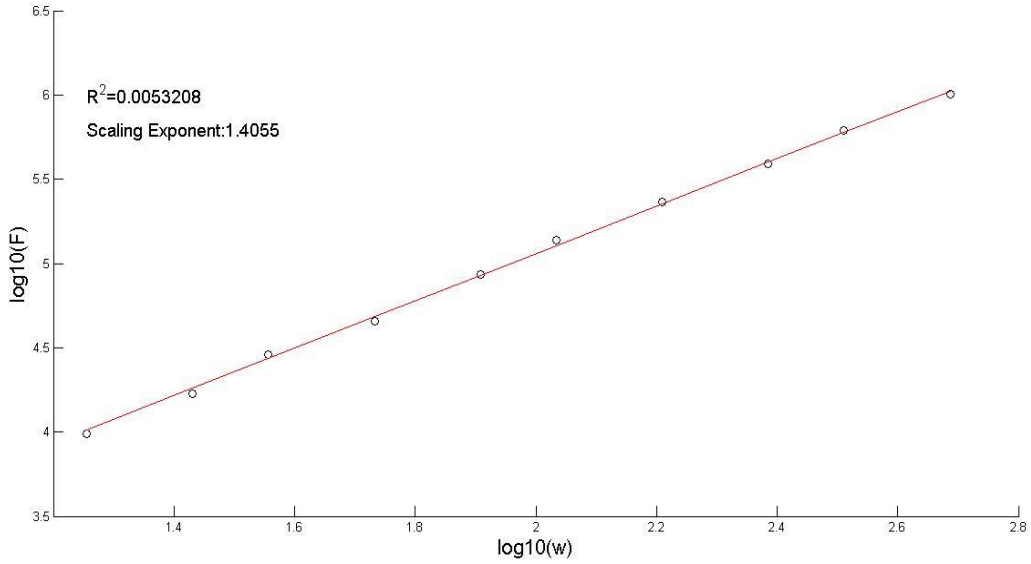


Fig. 4

In the next figure, Fig. 4 we showed the correctness of the fit, the $R^2$ value is nothing but the error of the fit. The value is close to zero indicates that the linear fit is accurate. This is the plot where we considered the q value equals to 2, thus the scaling exponent or the Hurst exponent here is nothing but the DFA scaling exponent for H(q=2).

## Results

Next we tabulated the results we got after applying 2D-MFDFA to some of Pollock's paintings.

| Serial Number | Name of Painting | Hurst Exponent (H(q=2)) | Fit Error | Multifractal-Width |
|---|---|---|---|---|
| 1 | Autumn Rhythm, (Number 30) | 1.4055 | 0.0053 | 0.4892 |
| 2 | Blue Poles | 1.1352 | 0.0258 | 0.5710 |
| 3 | Number 5 | 1.0176 | 0.0081 | 0.4048 |

## Conclusion

"*Every act of perception, is to some degree an act of creation, and every act of memory is to some degree an act of imagination"- Oliver Sacks*

Pollock immensely contributed to Modern Art through the new form of drip painting [8]. He could paint fractals on canvas, so people might well think about how he came to know about fractals which seems that Pollock was an extremely talented artist who exploited the Nature and found out what makes it all the more impressive, all the more attractive. What Mandelbrot did was just formulating these patterns using mathematics. Nature has fractals as the building blocks-the flowers, waves, clouds, leaves, trees and every wonder of the Nature possess this fractality. The singing of birds to the roaring of the waves all seems pleasing, when analyzed they possess inherent properties of fractals. Our studies involved analysis of these patterns in abstract art forms-to find the intricate patterns obeying fractal nature with excellent precision. Human beings have an affinity for fractals which is reflected in their perception and cognition and why not-the brain (EEG) response is itself a signal showing fractal nature [7]. This is a novel field of research involving Psychology, Arts, Music, Mathematics, Physics and Philosophical aspects. Fractal approaches have been used in the recent times to distinguish between the forged and the actual paintings of Pollock. More interesting studies are yet to be conducted in this emerging field of fractals arts.

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