Modern Control Systems

Code: PGT312/3

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Course Outcomes

CO1: Ability to **EXPLAIN** control systems and control systems design.

CO2: Ability to **DESCRIBE** the mathematical model models for such mechanical, electrical and electromechanical systems. (state variable systems, stability in feedback control systems, and frequency domain).

CO3: Ability to **ANALYZE AND DESIGN** linear feedback systems using root locus method, digital control systems.

Evaluation

Mid-Term:	20%
Final Examination:	40%
Quizzes:	5%
Assignment:	10%
Laboratory:	
Report and Assessment:	15%
Mini Project:	10%

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References

Text Book:

Richard C. Dorf, Robert H. Bishop, Modern Control Systems, 12th edition, Prentice Hall, 2011.

Norman S Nise, Control System Engineering, 6th Ed, 2011.

Other references:

K. Ogata, Modern Control Engineering, 5th edition, Prentice Hall, 2009.

G. Franklin, J.D. Powell and A. Emani-Naeini, Feedback Control and Dynamic Systems, 6th edition, Prentice Hall, 2009.

Constantine H. Houpis, Stuart N. Sheldon, Linear Control System Analysis and Design with MATLAB, 6th edition, CRC Press, 2014.

Teaching Plan

Week	Course Contents
1	Introduction to Control Systems
2-3	Mathematical Models of Systems
4-5	State Variable Models
6-7	Feedback Control Systems And Characteristics, Performance and Stability of Linear Feedback Systems.
9	Root Locus Method
10-11	Frequency Response Method
12-13	Stability in the frequency domain.
14-15	Digital Control Systems

Chapter 1: Learning Outcomes

Define a Control System and describe some application

Describe control system analysis and design objectives

Describe a control system's design process

Describe the benefit from studying control system

Definition on standard terms used:

Control: The process of causing or regulate the system to to desired value.

System: A collection of components which are coordinated together to perform a function.

Control System:

Interconnection of components forming a system configuration that will provide a desired system response.

Figure 1.1: Simplified description of a control system

Example 1: Elevator

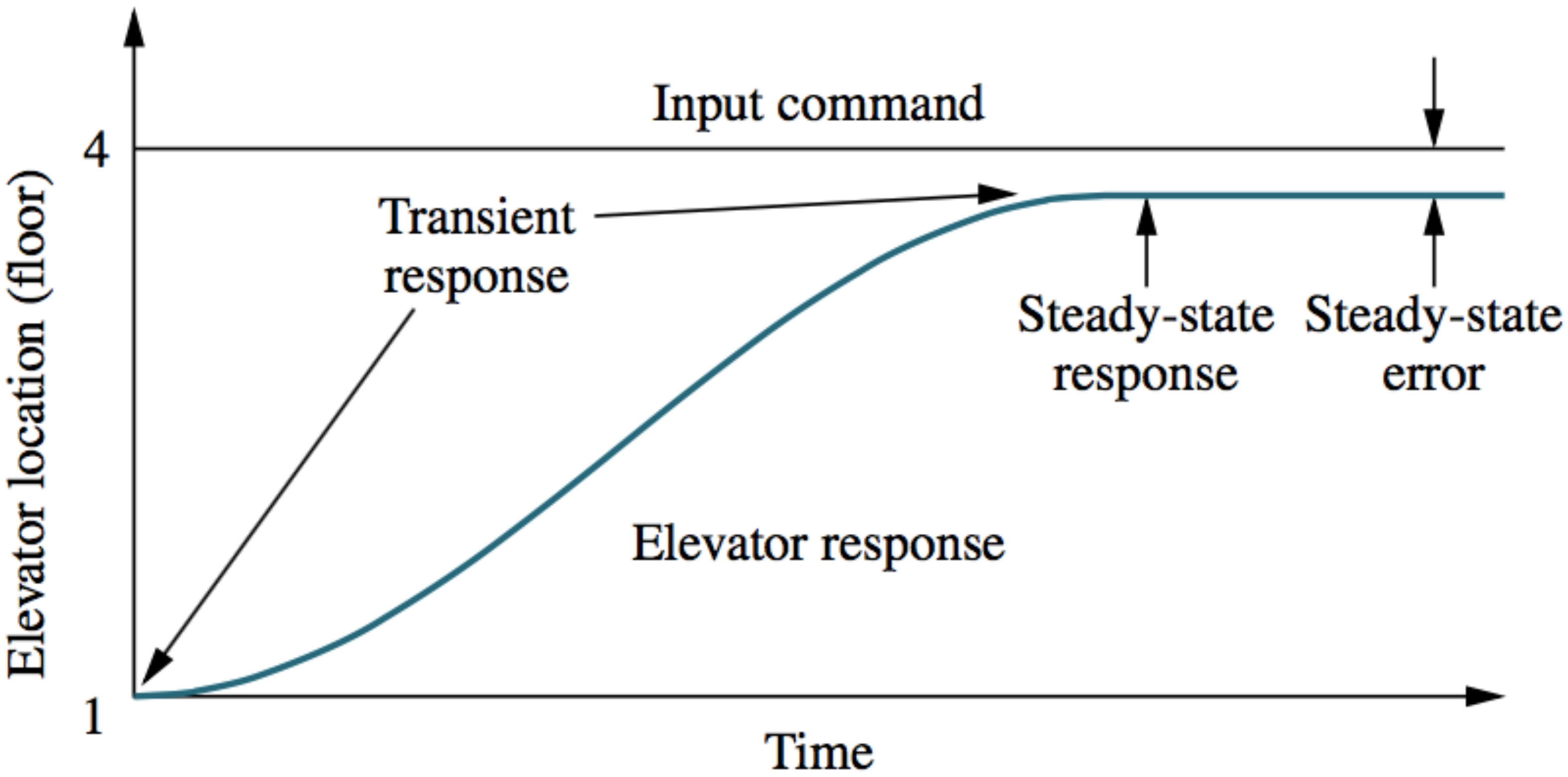

Figure 1.2 Elevator response

German train crash investigators focus on signal controller – source 10 Feb 2016
Image adapted from Google news.

...s kakinya tersepit di eskalator KL Sentral, Isnin lalu.

Kuala Lumpur

DOSH perlu tambah baik ciri keselamatan eskalator

Kementerian Pembangunan Wanita, Keluarga dan Masyarakat (KPWKM) meminta Jabatan Keselamatan dan Kesihatan Pekerja (DOSH) menambah baik ciri keselamatan eskalator pada masa hadapan.

Menterinya, Datuk Seri Rohani Abdul Karim berkata, pengurusan perlu sentiasa mengadakan pemantauan terhadap eskalator dalam jadual rutin.

"Perkara ini perlu dipandang serius kerana kita tidak mahu insiden menimpa orang lain pada masa hadapan.

"Ia perlu dijadikan pengajaran dan iktibar kepada semua pihak dalam aspek penjagaan keselamatan anak kecil termasuk ibu bapa supaya berwaspada," katanya selepas melawat Dzil Mikhail Nasaruddin yang putus sebahagian tapak kaki kiri akibat tersepit di eskalator di Hospital Kuala Lumpur (HKL), di sini, semalam.

Pada masa sama, Rohani menyampaikan bantuan segera daripada KPWKM.

Rohani berkata, setakat ini, punca insiden masih belum diketahui, tetapi pihaknya difahamkan DOSH datang berjumpa, keluarga mangsa dan siasatan berjalan.

Menurutnya, keluarga mangsa memaklumkan kementerian laporan penyiasatan itu dijangka siap minggu depan.

"Kita akan minta laporan daripada DOSH apabila siap nanti," katanya.

Pengarah HKL, Datuk Dr Zaininah Mohd Zain berkata, pihaknya melakukan rawatan lakaran kulit mangsa bagi menstabilkan pergerakan kaki kirinya.

"Dia akan dipantau dan diberi rehabilitasi sehingga memasuki usia belasan tahun," katanya.

Figure: Escalator incident. Adapted from Harian Metro, 13 Feb 2016.

Advantages of Control Systems

- Can move large equipment with precision that otherwise be impossible.
- Can point huge antennas toward the farthest reaches of the universe to pick up faint radio signals
- Elevators (timing) carry us quickly to our destination.

Control systems built for four (4) primary reason:

- Power amplification
- Remote control
- Convenience to input form
- Compensation to disturbances
- Others?...

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Control System: System Configuration

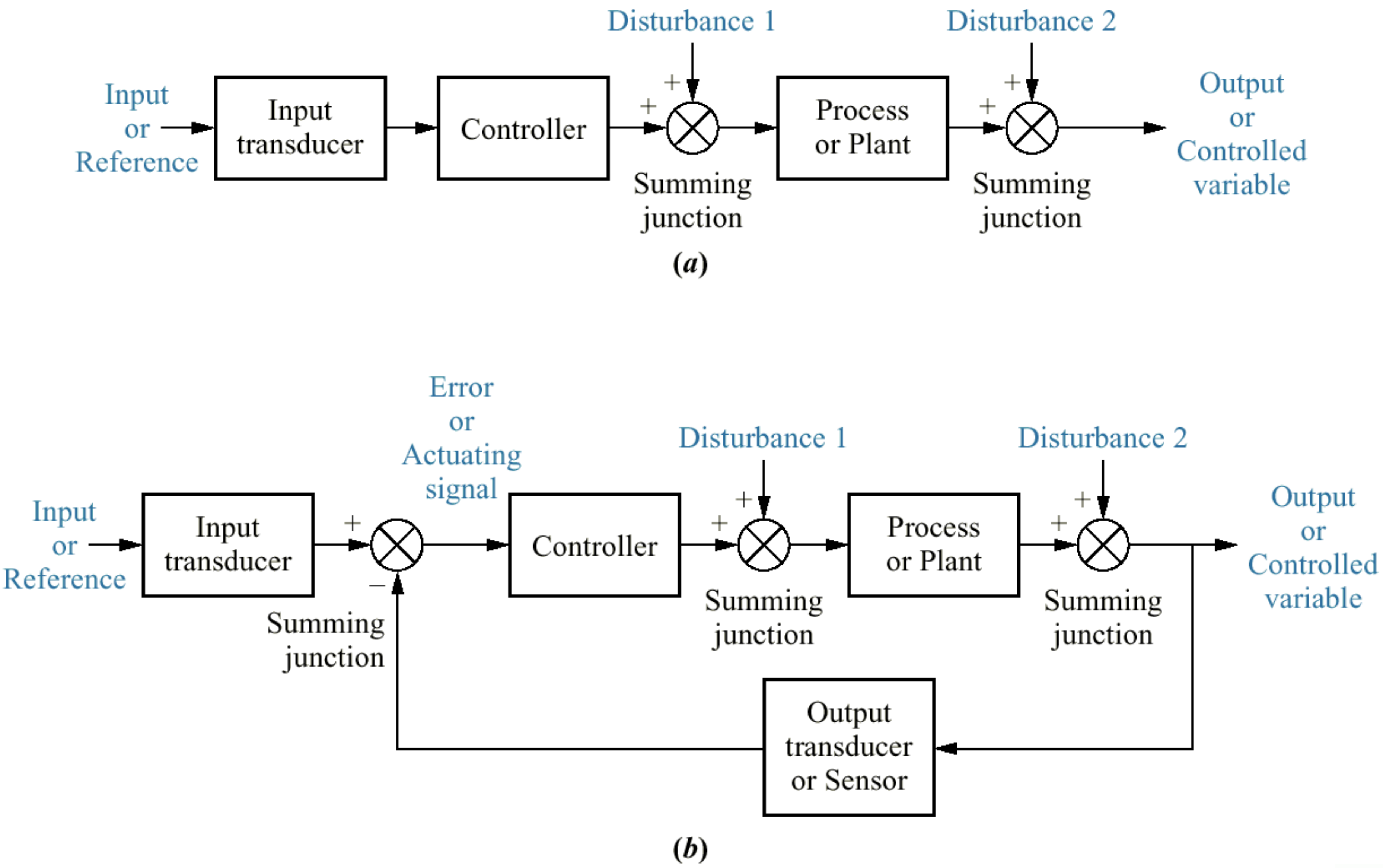
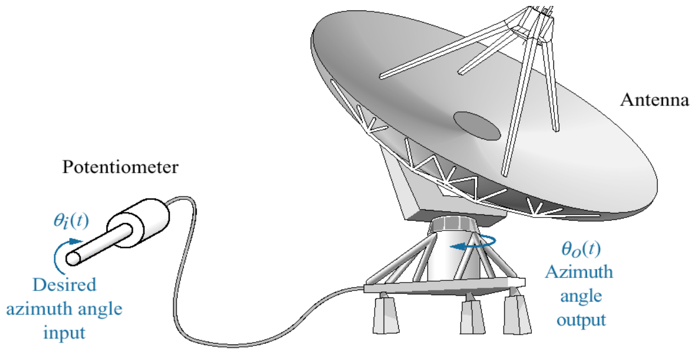
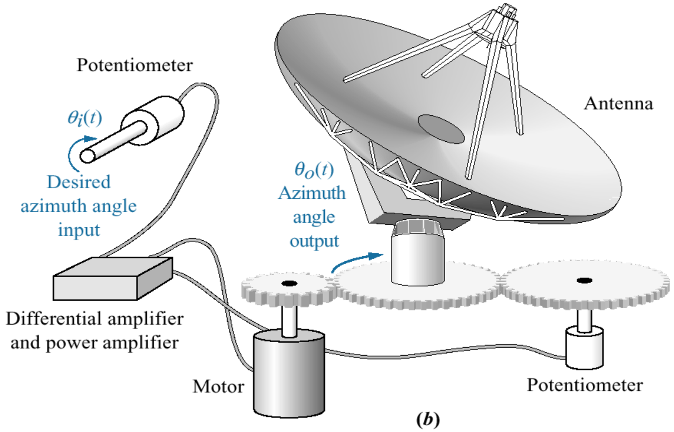


Figure 1.3: Block diagram of control system, (a) Open loop system; (b) Closed-loop system

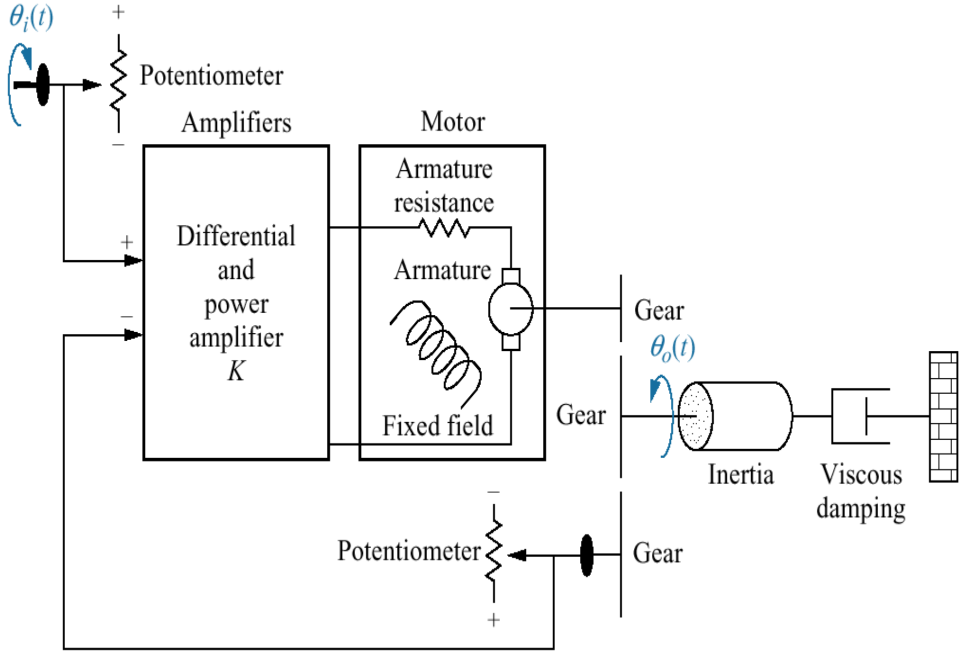
Example 2: Antenna azimuth position control



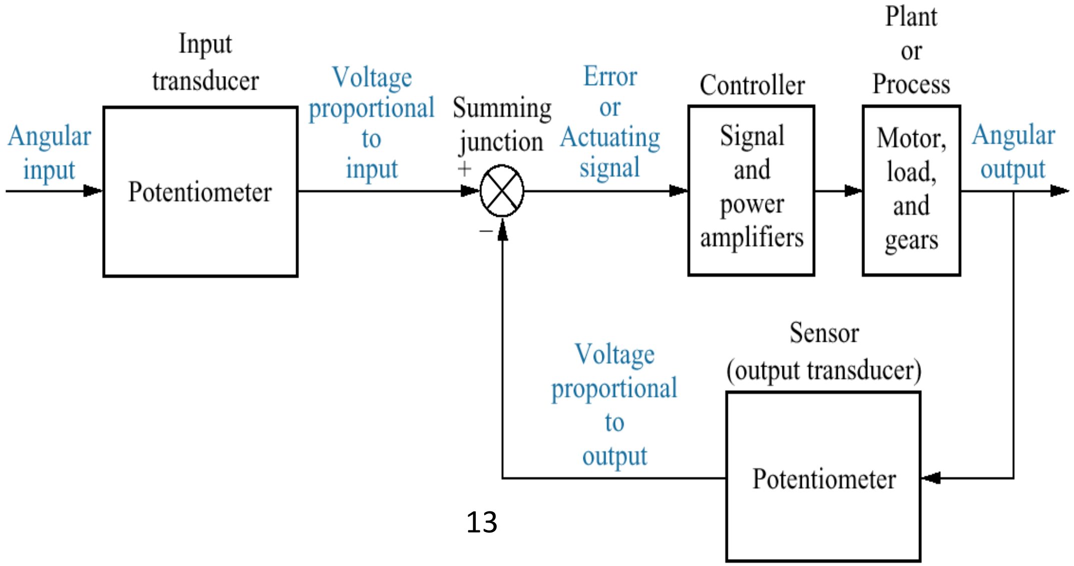
(a)



(b)



(c)



(d)

Figure 1.4 (a): System concept
Figure 1.4 (b): Detailed layout
Figure 1.4 (c): Schematic
Figure 1.4 (d): Functional block diagram

Analysis and Design Objectives

- Analysis is the process by which a system's performance is determine.
- Design is the process by which a system's performance is created or changed.
- Familiar with Transient, Steady-State, and Stability *(please refer Fig 1.2)*
- Other considerations: Hardware selection, finances, and robustness

**please describe based on your understanding*

The Design Process

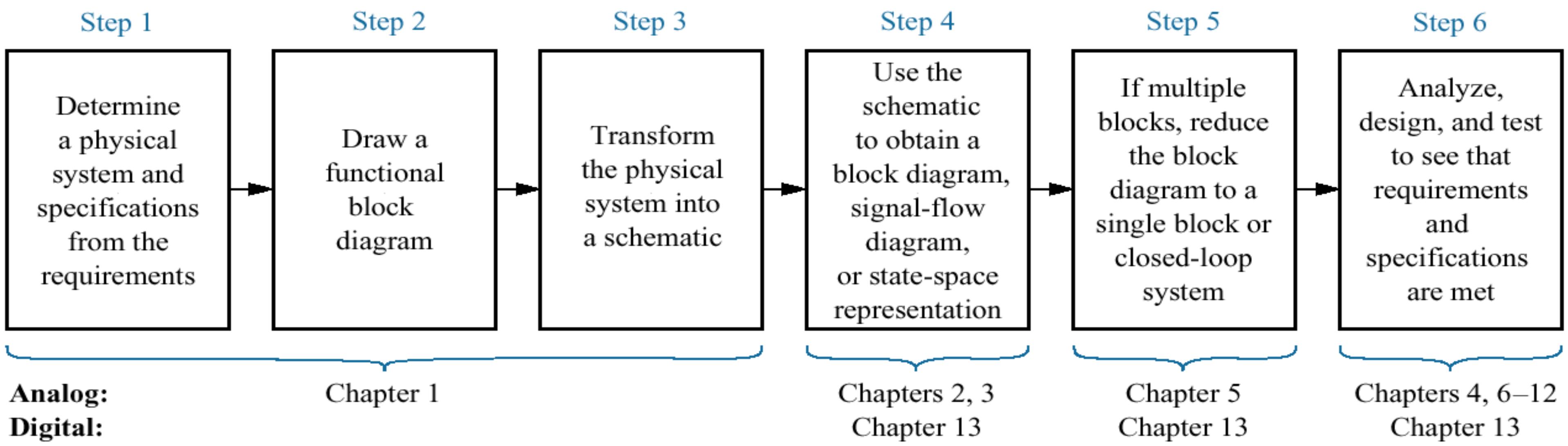


Figure 1.5: The control system design process

Summary

What have you learned from this topic?

Review Questions

1. Name three applications for feedback control systems.

Guided missiles, automatic gain control in radio receivers, satellite tracking antenna

2. Name three reasons for using feedback control systems and at least one reason for not using them.

Yes - power gain, remote control, parameter conversion; No - Expense, complexity

3. Give three examples of open-loop systems.

Motor, low pass filter, inertia supported between two bearings

4. Functionally, how do closed-loop systems differ from open-loop systems?

Closed-loop systems compensate for disturbances by measuring the response, comparing it to the input response (the desired output), and then correcting the output response.

5. Name the three major design criteria for control systems.

Stability, transient response, and steady-state error

6. Name the two parts of a system's response.

Steady-state, transient

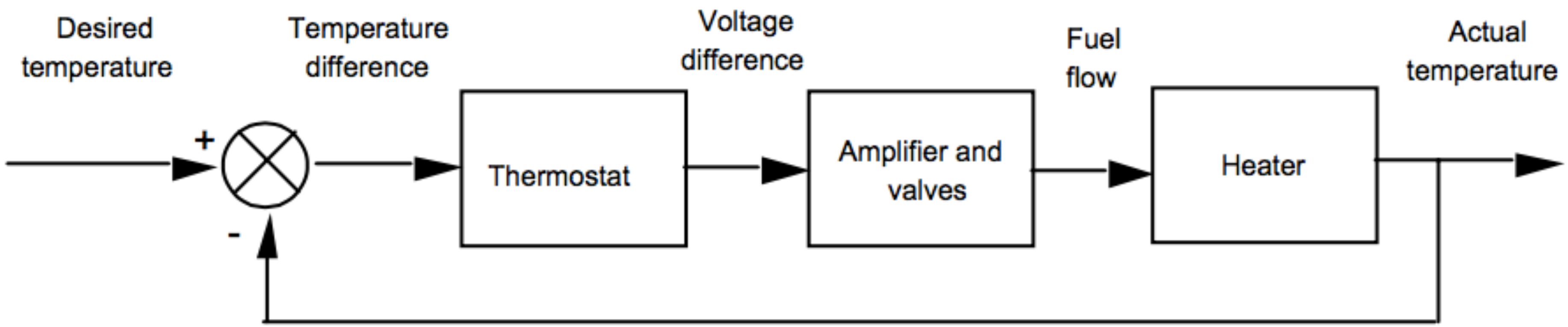
7. Physically, what happens to a system that is unstable?

It follows a growing transient response until the steady-state response is no longer visible. The system will either destroy itself, reach an equilibrium state because of saturation in driving amplifiers, or hit limit stops.

Exercise 1

A temperature control system operates by sensing the difference between the thermostat setting and the actual temperature and then opening a fuel valve an amount proportional to this difference. Draw a functional closed-loop block diagram similar to Figure 1.9(d) identifying the input and output transducers, the controller, and the plant. Further, identify the input and output signals of all subsystems previously described.

Answer to Exercise 1



CHAPTER 2:
MATHEMATICAL MODELS OF SYSTEM

Learning Outcomes:

After completing this chapter, you will be able to:

- Find the Laplace transform of time functions and the inverse Laplace transform.
- Find the transfer function from a differential equation and solve the differential equation using the transfer function
- Find the transfer function for linear, time-invariant electrical networks.

2.1 Introduction:

Last lecture:

We discussed the analysis and design sequence that included Obtaining the system's schematic and demonstrated with several applications.

Why?

Because, to obtain schematic, the control system engineer must often make many simplifying assumptions in order to keep the ensuing model manageable and still approximate physical reality.

Two methods: Trans. functions in frequency domain (CH2)

State equation in time domain (CH3)

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2.2 Laplace Transform

$$\mathscr{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}\,dt$$

Where: $s = \sigma + j\omega$

$$\mathscr{L}^{-1}[F(s)] = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} F(s)e^{st}\,ds = f(t)u(t)$$

Laplace Transform

Table 2.1: Laplace transform table:

Item no.	$f(t)$	$F(s)$
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^n + 1}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

Table 2.2: Laplace Transform theorems

1.	$\mathscr{L}[f(t)] = F(s)$	$= \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)]$	$= kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t) + f_2(t)]$	$= F_1(s) + F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)]$	$= F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)]$	$= e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)]$	$= \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right]$	$= sF(s) - f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right]$	$= s^2F(s) - sf(0-) - f'(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right]$	$= s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)d\tau\right]$	$= \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty)$	$= \lim_{s \to 0} sF(s)$	Final value theorem[1]
12.	$f(0+)$	$= \lim_{s \to \infty} sF(s)$	Initial value theorem[2]

[1]For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts, and no more than one can be at the origin.

[2]For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (that is, no impulses or their derivatives at $t = 0$).

Example 2.1

Find the Laplace Transform of:

$$Ae^{-at}u(t)$$

$$e^{-at}\sin\omega t u(t)$$

$$e^{-at}\cos\omega t u(t)$$

$$t^3 u(t)$$

Table 2.3: Voltage current, voltage-charge, and impedance relationships for capacitors, resistors, and inductors

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
Capacitor	$v(t) = \frac{1}{C}\int_0^1 i(\tau)d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^1 v(\tau)d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Note: The following set of symbols and units is used throughout this book: $v(t)$ – V (volts), $i(t)$ – A (amps), $q(t)$ – Q (coulombs), C – F (farads), R – Ω (ohms), G – Ω (mhos), L – H (henries).

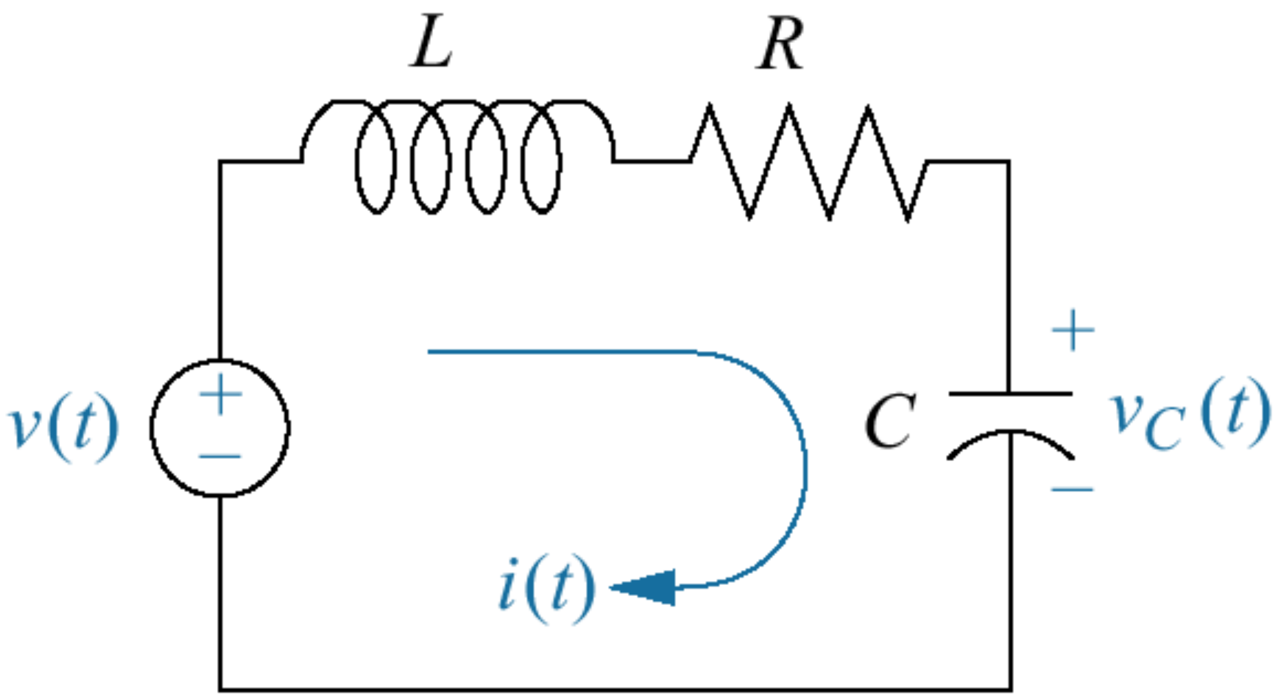
Given the electric network shown in figure below. Assume:

$$R = 1\Omega \qquad L = 1H \qquad \frac{1}{LC} = 25$$

a) Write the differential equation for the network if v(t)= u(t).

b) Solve the differential equation for the current, i(t) if there is no initial energy in the network

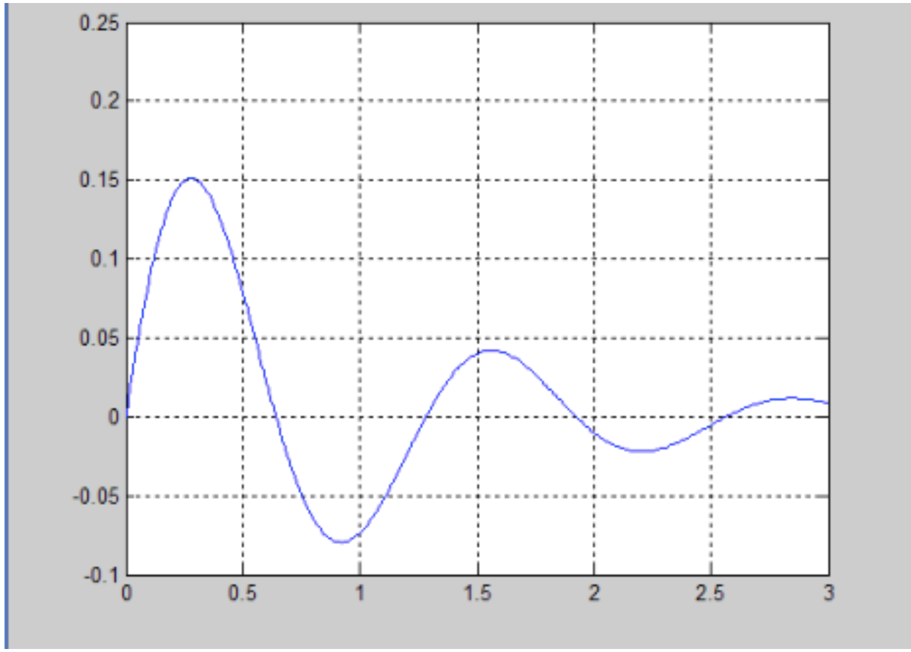
c) Make plot of your solution if R/L=1.



Answer:

a. Writing the loop equation, $Ri + L\frac{di}{dt} + \frac{1}{C}\int idt + v_C(0) = v(t)$

b. Differentiating and substituting values, $\frac{d^2i}{dt^2} + 2\frac{di}{dt} + 25i = 0$



Writing the characteristic equation and factoring,

$$M^2 + 2M + 25 = (M + 1 + \sqrt{24}i)(M + 1 - \sqrt{24}i).$$

The general form of the solution and its derivative is

$$i = Ae^{-t}\cos(\sqrt{24}t) + Be^{-t}\sin(\sqrt{24}t)$$

$$\frac{di}{dt} = (-A + \sqrt{24}B)e^{-t}\cos(\sqrt{24}t) - (\sqrt{24}A + B)e^{-t}\sin(\sqrt{24}t)$$

Using $i(0) = 0; \frac{di}{dt}(0) = \frac{v_L(0)}{L} = \frac{1}{L} = 1$

$$i(0) = A = 0$$

$$\frac{di}{dt}(0) = -A + \sqrt{24}B = 1$$

Thus, $A = 0$ and $B = \frac{1}{\sqrt{24}}$.

The solution is

$$i = \frac{1}{\sqrt{24}}e^{-t}\sin(\sqrt{24}t)$$

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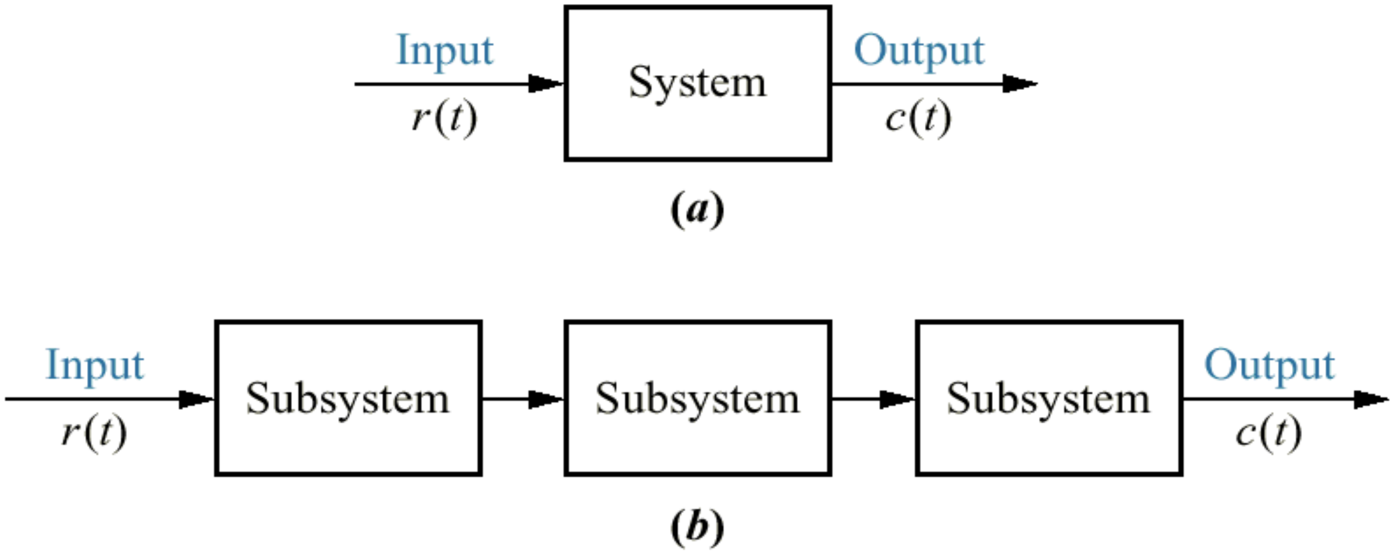
Transfer Function:

To formulate the system representation that algebraically relates a system’s output to it input.



Note: The input, $r(t)$, stands for *reference input*.
The output, $c(t)$, stands for *controlled variable*.

Example 2.6:

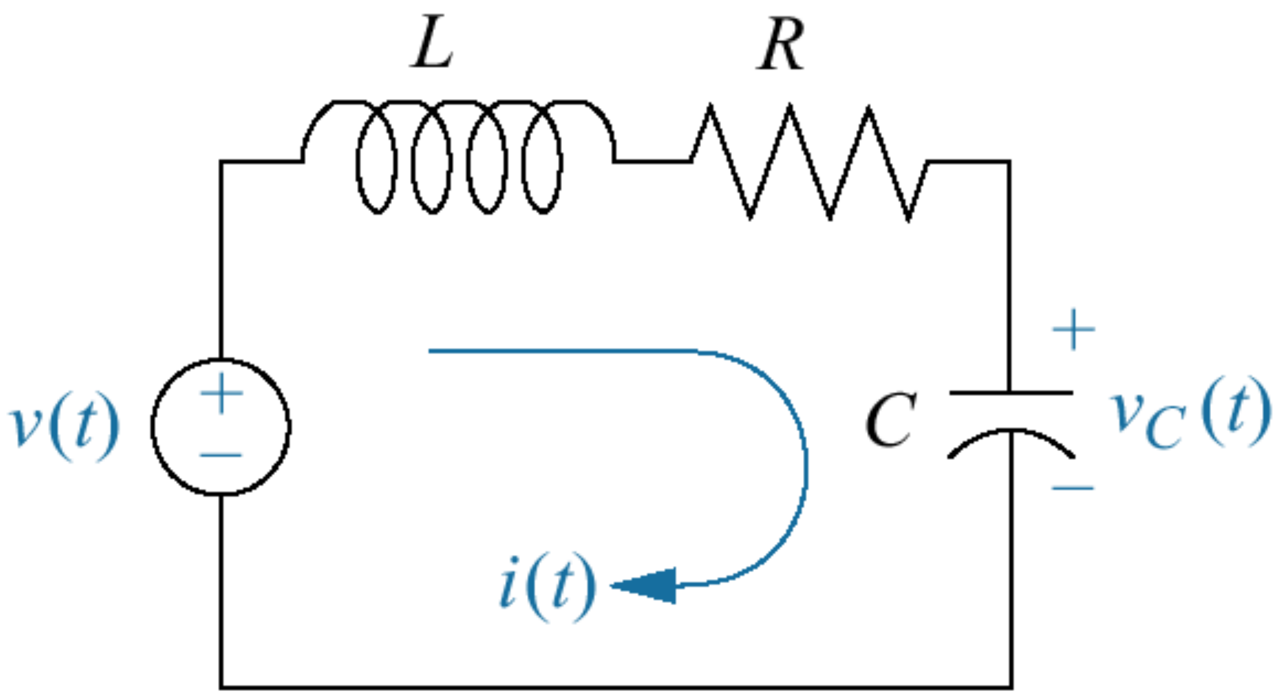


Figure 2.3: Series RLC network

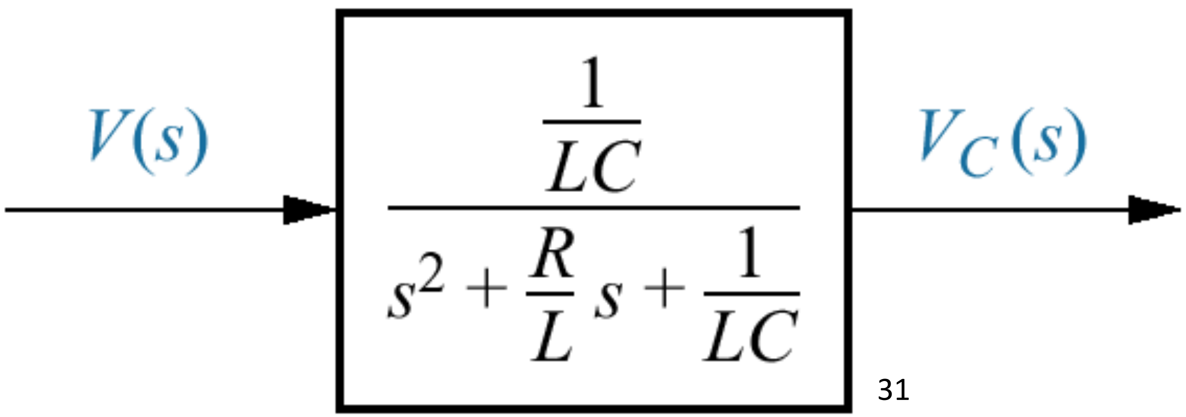
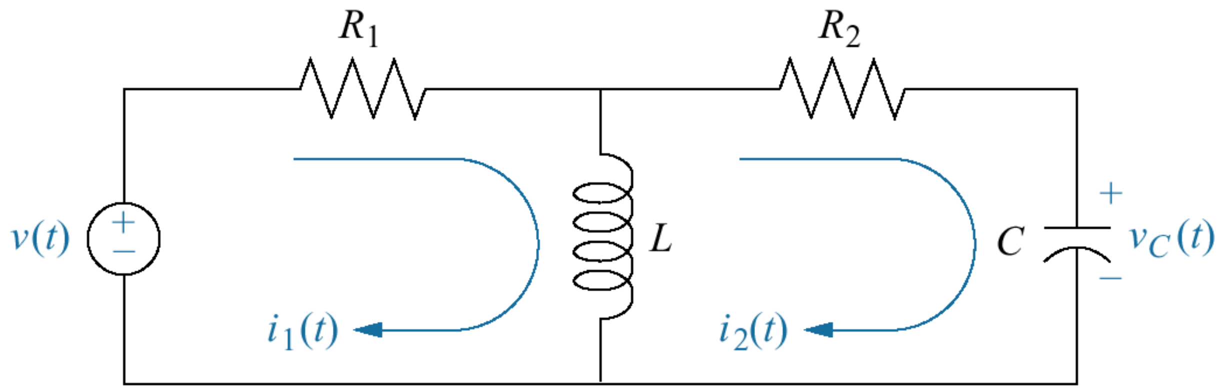
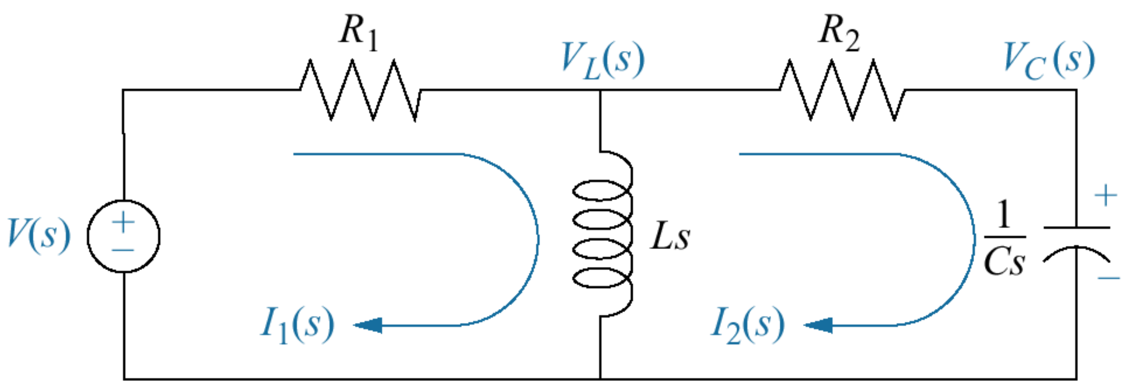


Figure 2.4: Block diagram of series RLC electrical network

Example 2.10:



(a)



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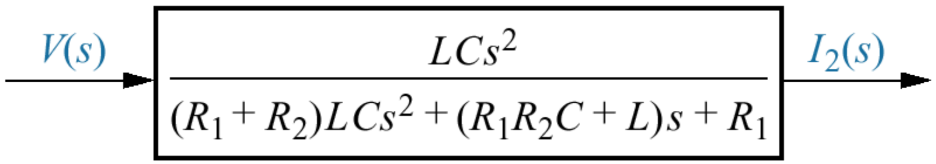
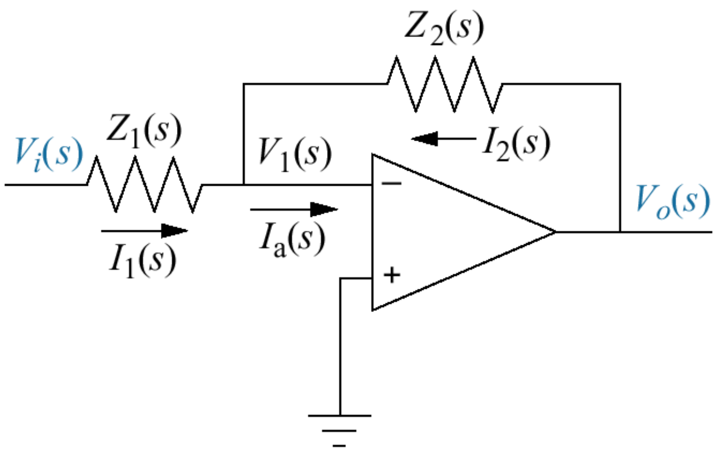


(c)

Note: Please prove the transfer function obtained

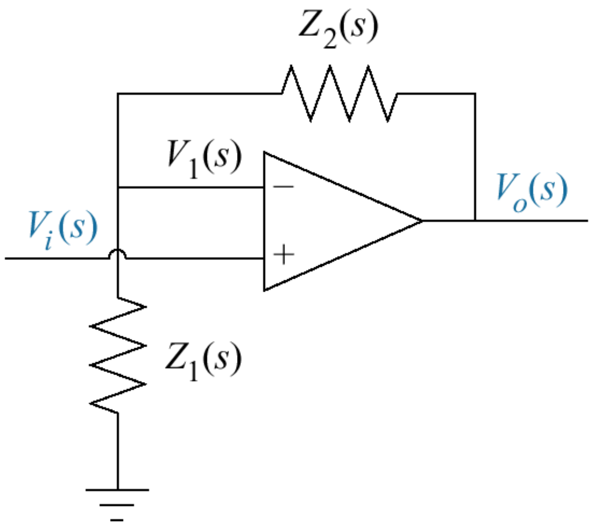
Operational Amplifier

Inverting Amplifier



$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

Non-inverting Operational Amplifier

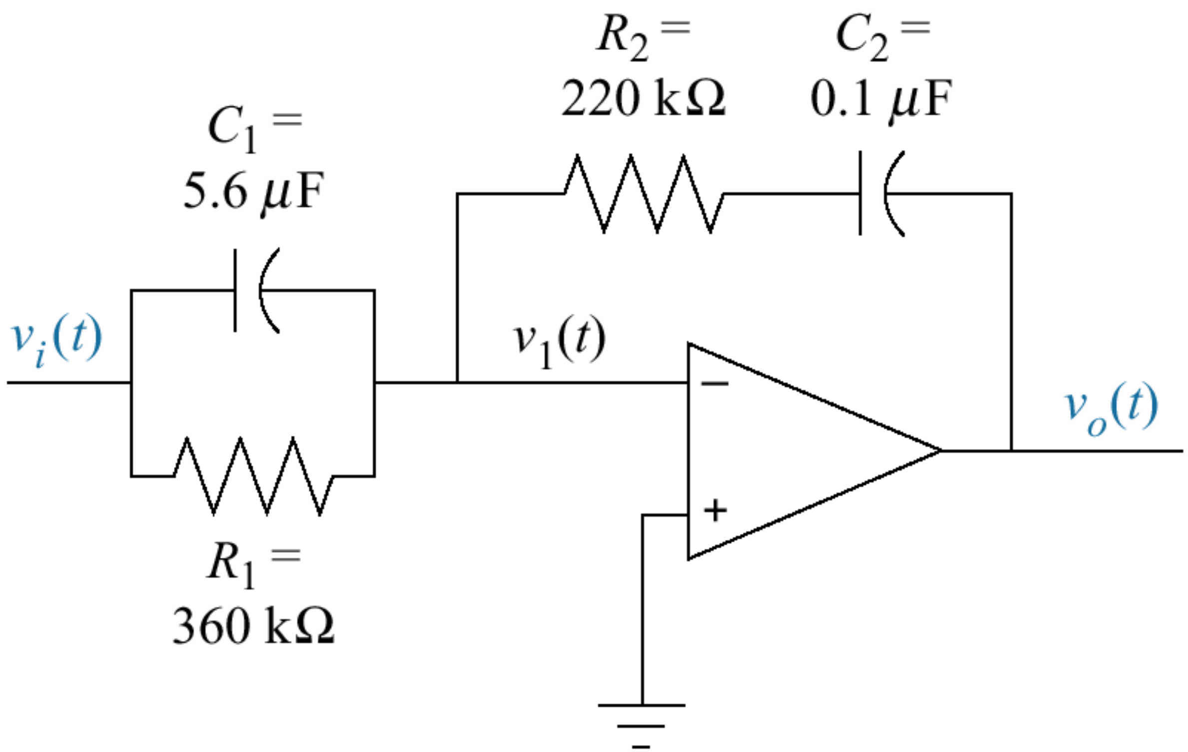


$$\frac{V_o(s)}{V_i(s)} = \frac{Z_1(s) + Z_2(s)}{Z_1(s)}$$

Note: Please prove the transfer function

Example 2.14:

Figure 2.11:
Inverting operational amplifier circuit for Example 2.14



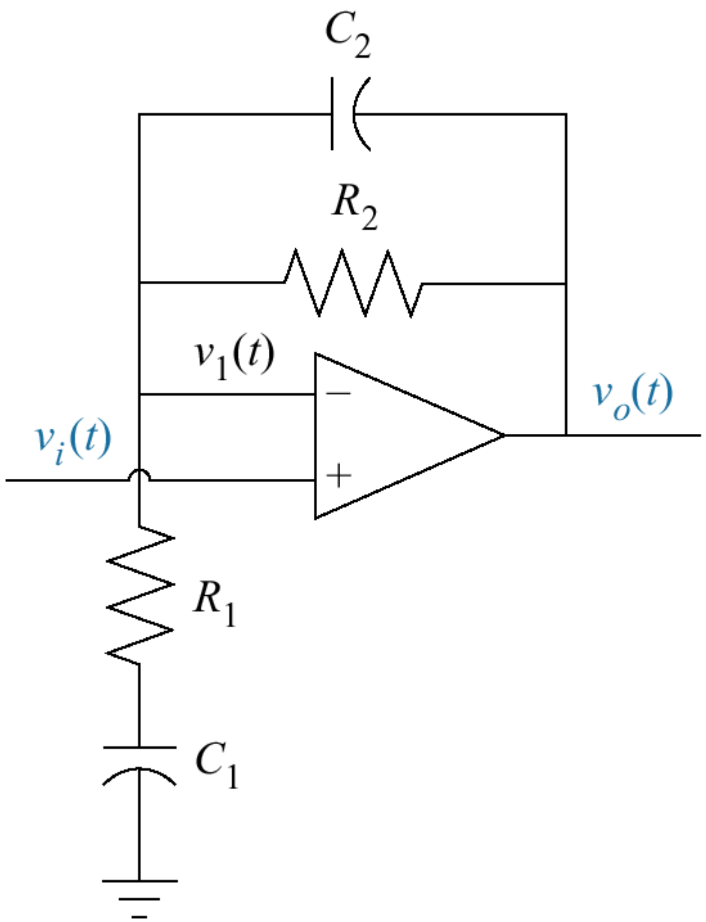
Find the transfer function, $V_o(s)/V_i(s)$

Answer:

$$\frac{V_o(s)}{V_i(s)} = -1.232\frac{s^2 + 45.95s + 22.55}{s}$$

Example 2.15:

Figure 2.13
Noninverting operational amplifier circuit for Example 2.15



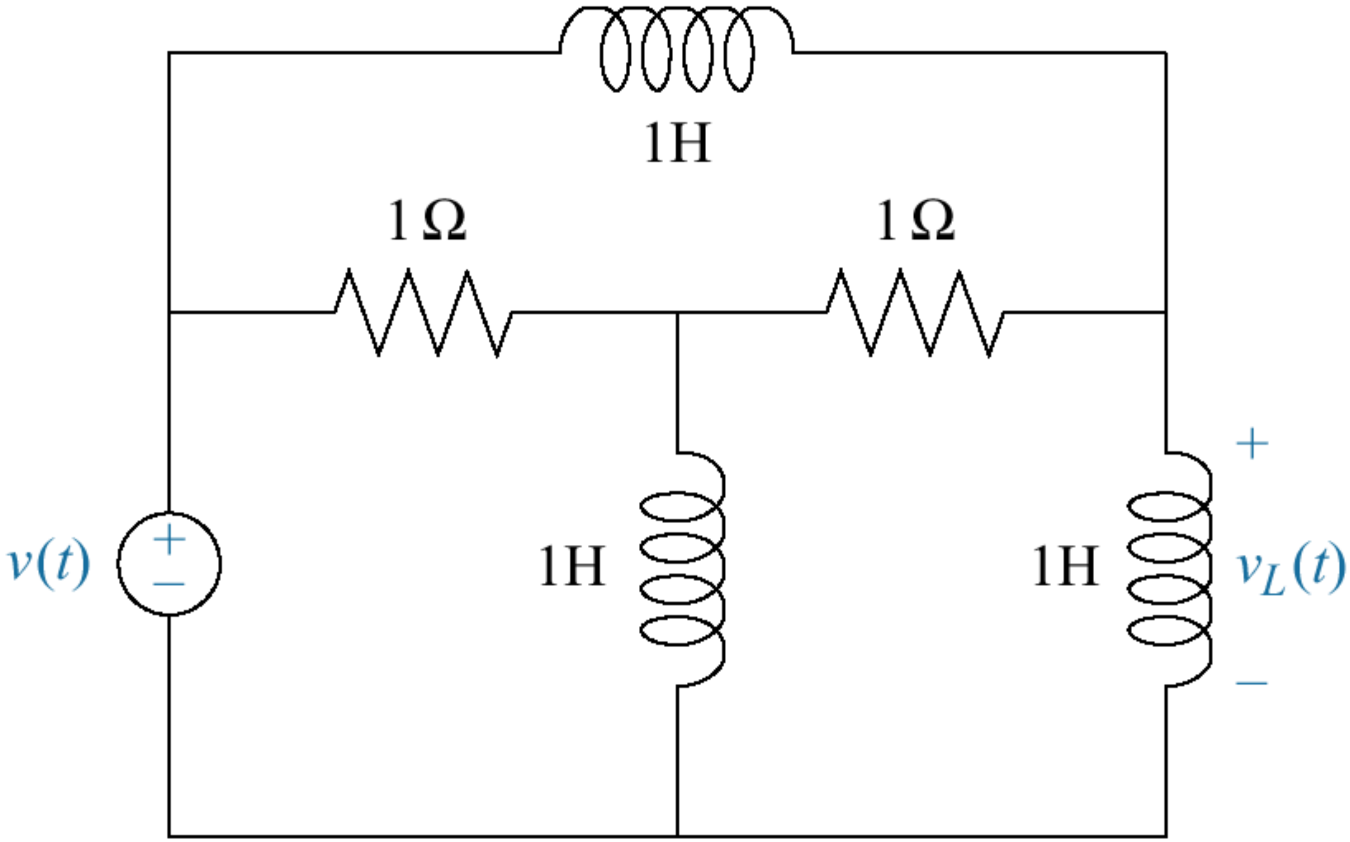
Find the transfer function, $V_o(s)/V_i(s)$

Answer:

$$\frac{V_o(s)}{V_i(s)} = \frac{C_2C_1R_2R_1s^2 + (C_2R_2 + C_1R_2 + C_1R_1)s + 1}{C_2C_1R_2R_1s^2 + (C_2R_2 + C_1R_1)s + 1}$$

Exercise @ Assignment:

Figure 2.14
Electric circuit for Skill-Assessment Exercise 2.6



Find the transfer function, $V_L(s)/V(s)$

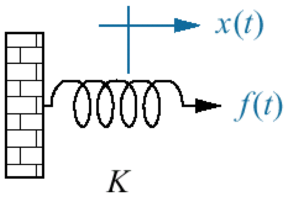
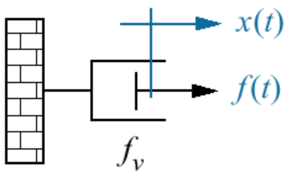
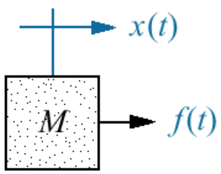
Translational Mechanical System Transfer Function:

Learning Objectives:

- Find the transfer function for linear, time-invariant translational mechanical systems
- Find the transfer function for linear, time-invariant rotational mechanical systems
- Find the transfer functions for gear systems with no loss and for gear systems with loss
- Find the transfer function for linear, time-invariant electromechanical systems
- Produce analogous electrical and mechanical circuits
- Linearise a nonlinear system in order to find the transfer function

Translational Mechanical System Transfer Function:

In this section we concentrate on translational mechanical systems

Component	Force-velocity	Force-displacement	Impedance $Z_M(s) = F(s)/X(s)$
Spring 	$f(t) = K\int_0^t v(\tau)\,d\tau$	$f(t) = Kx(t)$	K
Viscous damper 	$f(t) = f_v v(t)$	$f(t) = f_v \frac{dx(t)}{dt}$	$f_v s$
Mass 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

Note: The following set of symbols and units is used throughout this book: $f(t)$ = N (newtons), $x(t)$ = m (meters), $v(t)$ = m/s (meters/second), K = N/m (newtons/meter), f_v = N-s/m (newton-seconds/meter), M = kg (kilograms = newton-seconds^2/meter).

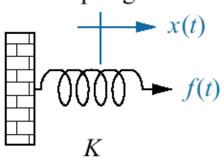
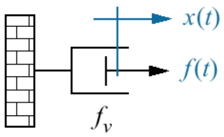
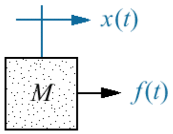
Figure 2.4: Force-velocity, force-displacement, and impedance translational relationships for springs, viscous dampers, and mass.

What can we relate between electrical and mechanical system analogies?

Mechanical: Also have three passive, linear components.

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
Capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Note: The following set of symbols and units is used throughout this book: $v(t)$ – V (volts), $i(t)$ – A (amps), $q(t)$ – Q (coulombs), C – F (farads), R – Ω (ohms), G – Ω (mhos), L – H (henries).

Component	Force-velocity	Force-displacement	Impedance $Z_M(s) = F(s)/X(s)$
Spring 	$f(t) = K\int_0^t v(\tau)\,d\tau$	$f(t) = Kx(t)$	K
Viscous damper 	$f(t) = f_v v(t)$	$f(t) = f_v\frac{dx(t)}{dt}$	$f_v s$
Mass 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

Note: The following set of symbols and units is used throughout this book: $f(t)$ = N (newtons), $x(t)$ = m (meters), $v(t)$ = m/s (meters/second), K = N/m (newtons/meter), f_v = N-s/m (newton-seconds/meter), M = kg (kilograms = newton-seconds²/meter).

K: spring constant

f_v : coefficient of viscous friction

M: mass

Force-velocity= voltage-current

Force displacement= voltage charge

Spring= capacitor

Viscous damper= resistor

Mass= Inductor

Example 2.16: Find the transfer function X(s)/F(s) for the system of Figure 2.15.

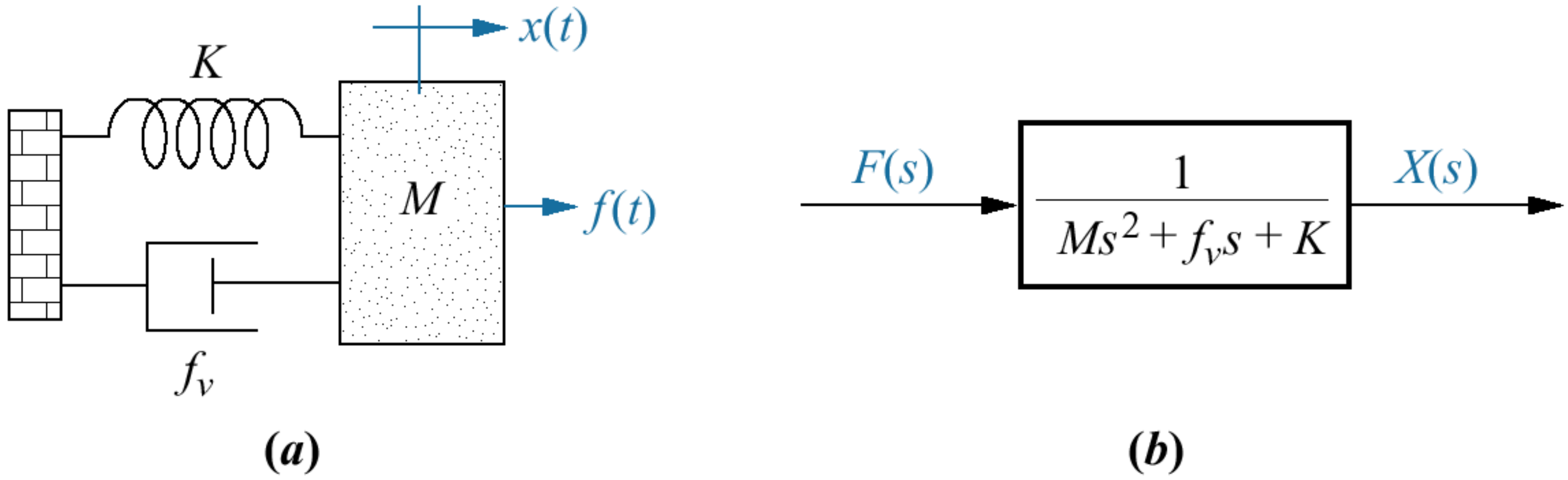
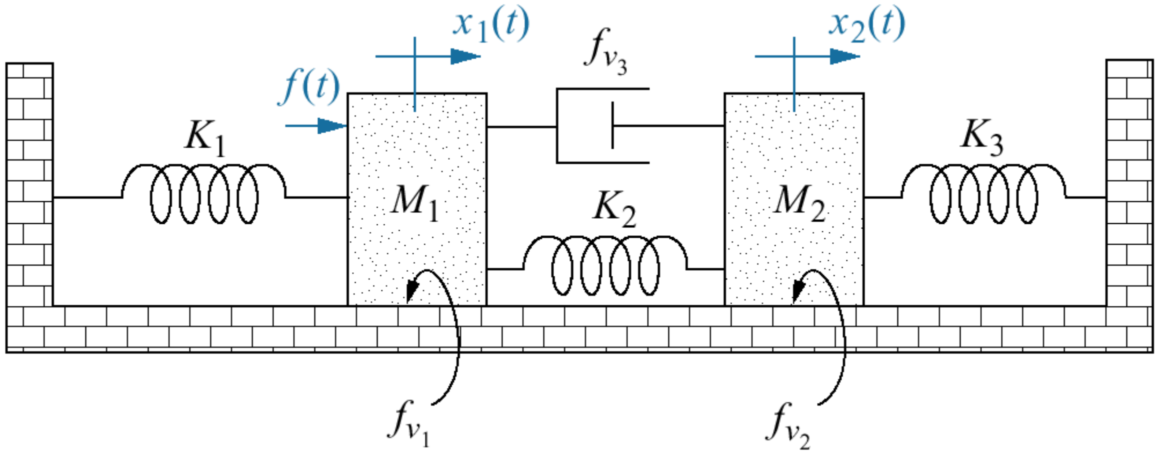
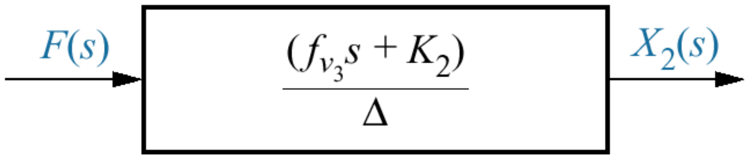


Figure 2.15: Mass spring and damper system

Example 2.16: Find the transfer function $X_2(s)/F(s)$ for the system of Figure 2.15.



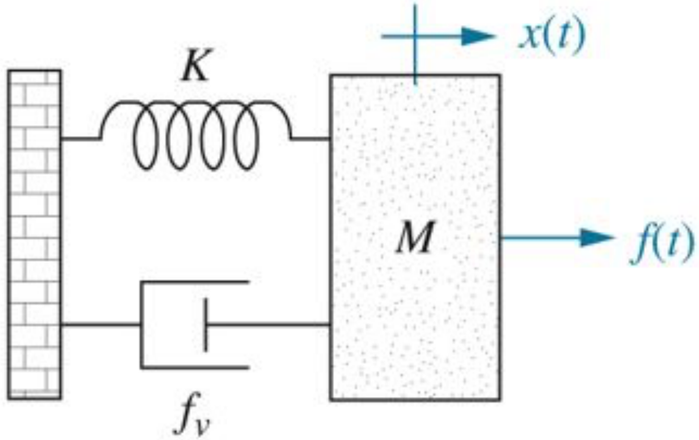
(a)



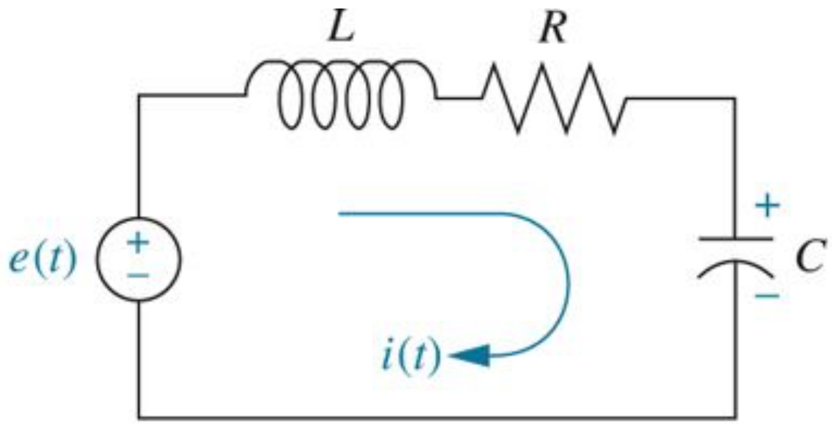
(b)

$$\left[M_1 s^2 (f_{v_1} + f_{v_3})s + (K_1 + K_2)\right]X_1(s) - (f_{v_3}s + K_2)X_2(s) = F(s)$$

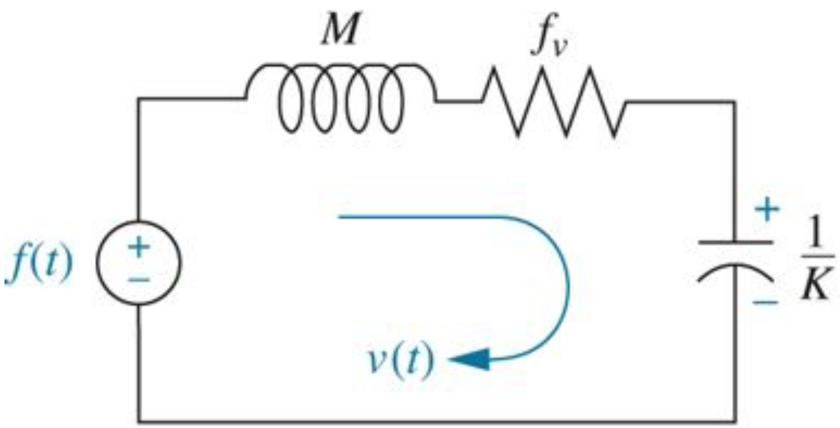
$$-(f_{v_3}s + K_2)X_1(s) + \left[M_2 s^2 + (f_{v_2} + f_{v_3})s + (K_2 + K_3)\right]X_2(s) = 0$$



(a)



(b)



(c)

mass = M	⟶	inductor	=	M henries
viscous damper = f_v	⟶	resistor	=	f_v ohms
spring = K	⟶	capacitor	=	$\frac{1}{K}$ farads
applied force = $f(t)$	⟶	voltage source	=	$f(t)$
velocity = $v(t)$	⟶	mesh current	=	$v(t)$

(d)

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CHAPTER 3: MODELING IN TIME DOMAIN (STATE-SPACE FORM)

Objectives:

- Find a mathematical model, called a ***state-space representation, for a linear, time-invariant system***
- **Model** electrical and mechanical systems in state space
- **Convert** a transfer function to state space
- **Convert a state-space** representation to a transfer function
- **Linearise** a state-space representation

What have we learnt in Chapter 2?
classical frequency-domain

What is the advantages and disadvantages of this system?

- simplifies the representation, also simplifies modeling interconnected system.
- Limited applicability: only to LTI systems

- So what about **non-linear system**: backlash, saturation, dead zone,… non-zero initial condition?

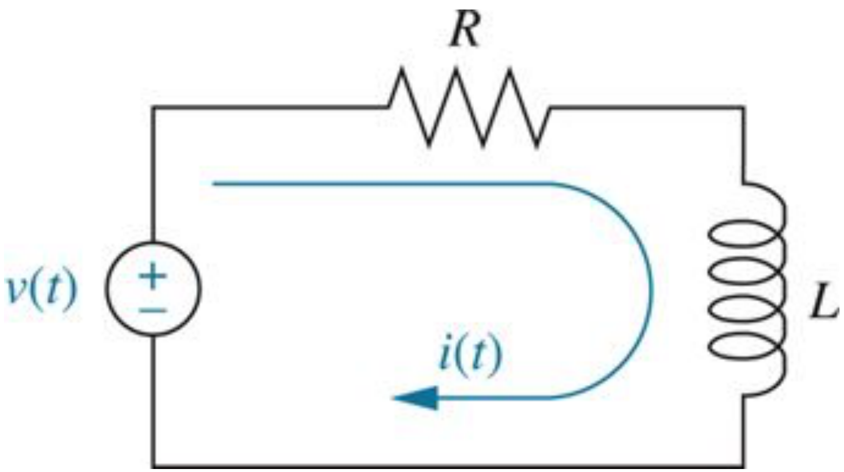
Solution: State-space (Time domain) approach
The state-space approach is a **unified method** for modeling, analyzing, and designing a wide range of systems.

Time varying systems: Missiles with varying fuel levels or lift in an aircraft flying through a wide range of altitudes

Multiple input and output: vehicle with input direction and input velocity yielding an output direction and an output velocity

The state-space approach is also attractive because of the availability of numerous **state-space software packages** for the personal computer.

Example 3.1:



$$L\frac{di}{dt} + Ri = v(t) \qquad (3.1)$$

$$L[sI(s) - i(0)] + RI(s) = V(s) \qquad (3.2)$$

$$I(s) = \frac{1}{R}\left(\frac{1}{s} - \frac{1}{s+\frac{R}{L}}\right) + \frac{i(0)}{s+\frac{R}{L}} \qquad (3.3)$$

$$i(t) = \frac{1}{R}\left(1 - e^{-(R/L)t}\right) + i(0)e^{-(R/L)t} \qquad (3.4)$$

The function *i(t)* is a subset of all possible network variables that we are able to find from Eq. (3.4) if we know its initial condition, i(0), and the input, v(t). Thus, ***i(t)*** is **a state variable**, and the **differential equation (3.1) is a state equation**.

Example 3.2:

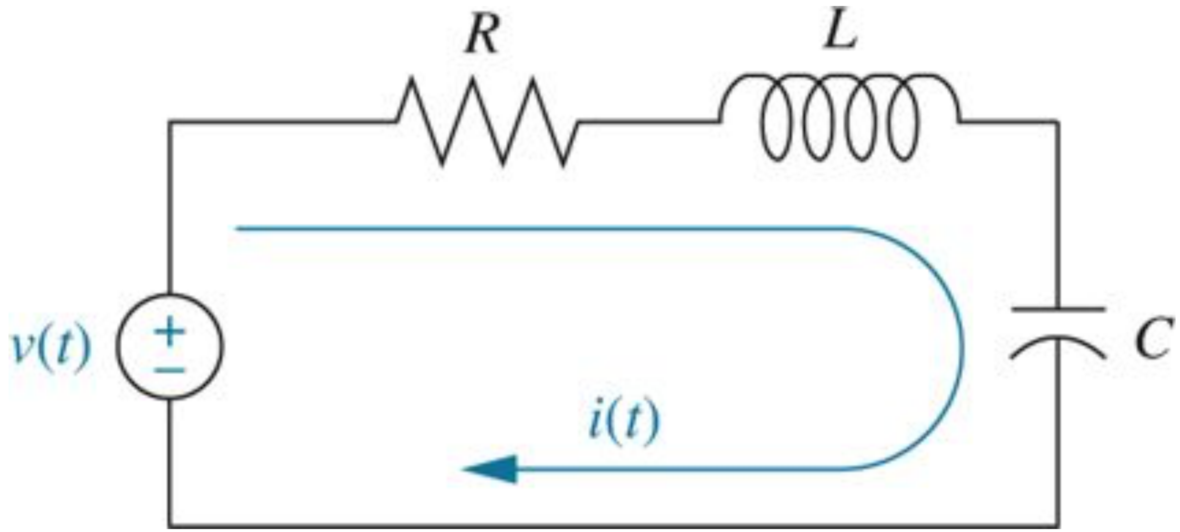


Figure 3.2: RLC circuit

Please create the state space equation for the above figure with:

i. i(t) and q(t) as a state variable

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = v(t) \tag{3.9}$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = v(t) \tag{3.10}$$

$$\frac{dq}{dt} = i \tag{3.12a}$$

$$\frac{di}{dt} = -\frac{1}{LC}q - \frac{R}{L}i + \frac{1}{L}v(t) \tag{3.12b}$$

$$v_L(t) = -\frac{1}{C}q(t) - Ri(t) + v(t) \tag{3.13}$$

$$\dot{\mathbf{x}} = \begin{bmatrix} dq/dt \\ di/dt \end{bmatrix}; \quad \mathbf{A} = \begin{bmatrix} 0 & 1 \\ -1/LC & -R/L \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} q \\ i \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 0 \\ 1/L \end{bmatrix}; \; u = v(t)$$

$$y = v_L(t); \quad \mathbf{C} = [-1/C \quad -R]; \quad \mathbf{x} = \begin{bmatrix} q \\ i \end{bmatrix}; \quad D = 1; \quad u = v(t)$$

Chapter 4: Time Response

Course Outcome:

- Use poles and zeros of transfer functions to determine the time response of a control system.
- Describe quantitatively the transient response of first order systems.
- Write the general response of second-order systems given the pole location.
- Find the damping ratio and natural frequency of a second-order system.
- Find the settling time, peak time, percent overshoot, and rise time for under-damped second order system.
- Approximate higher-order systems and systems with zeros as first- or second-order systems.
- Describe the effects of nonlinearities on the system time response.
- Find the time response from the state-space representation.

4.1 Introduction:

We have learn mathematical modeling in frequency domain and state-space form.

From that mathematical representation of a subsystem, the subsystem is analyzed for its transient and steady state responses:- to see if these characteristics yield the desired behavior.

This chapter is devoted to the analysis of system transient response.

4.2: Poles, Zero and System Response

The output response of a system is the sum of two responses: the forced response and the natural response.

We analyze the output- model in mathematics – Derivative or Inverse Laplace transform- time consuming.

Concept poles and zeros- simplifies the evaluation of a system's response.

Poles: refer to denominator (s-variable)
Zeros: refer to nominator (s-variable)
(the exact and detail meaning can be referred to text book)

Example 4.0 (Explanation…)
Given the transfer function G(s) in Figure 4.1(a). To show the properties of the poles and zeros, let us find the unit step response of the system.

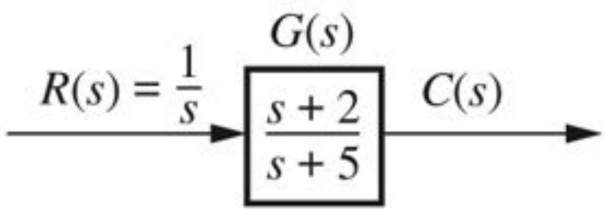


Figure 4.1(a)

$$C(s) = \frac{(s+2)}{s(s+5)} = \frac{A}{s} + \frac{B}{s+5} = \frac{2/5}{s} + \frac{3/5}{s+5} \quad (4.1)$$

$$c(t) = \frac{2}{5} + \frac{3}{5}e^{-5t} \quad (4.2)$$

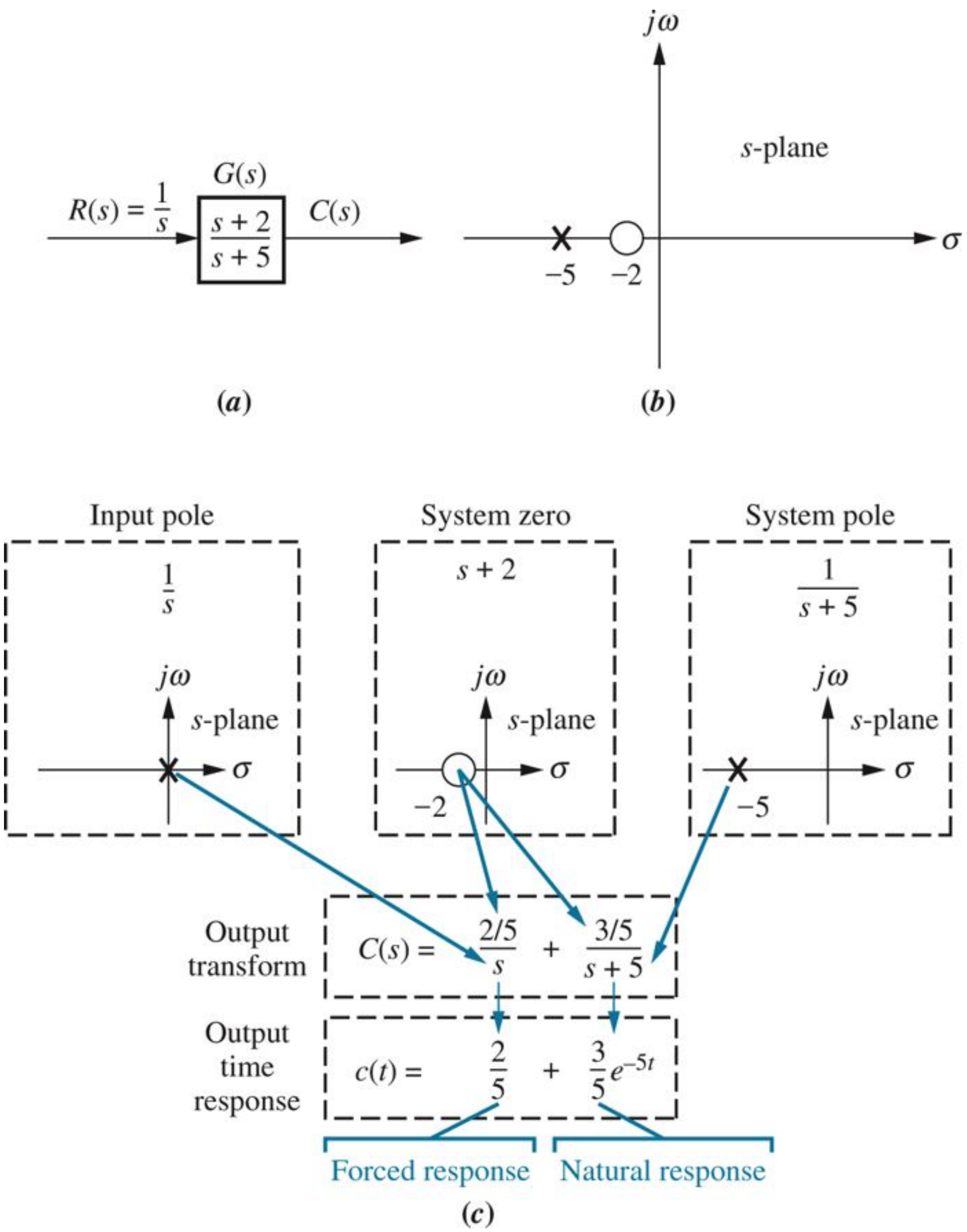


Figure 4.1:
a. System showing input and output
b. pole-zero plot of the system
c. evolution of a system response.

Follow blue arrows to see the evolution of the response component generated by the pole or zero.

Example 4.1: Evaluating Response Using Poles

Given the system of Figure 4.3, write the output, c(t), in general terms. Specify the forced and natural parts of the solution.

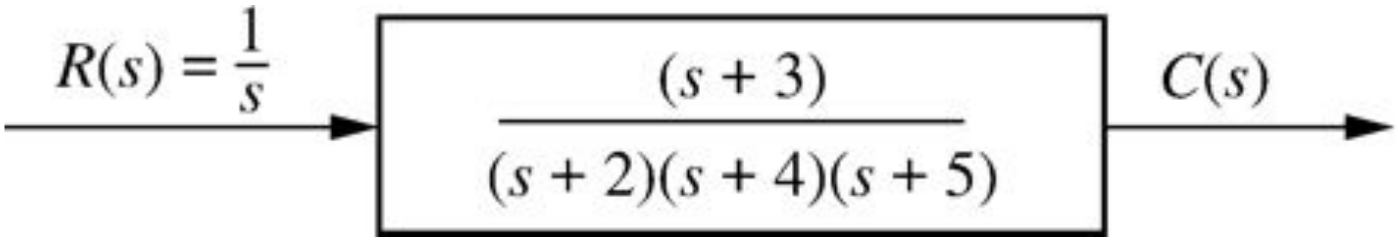


Figure 4.3: System for example 4.1

SOLUTION: By inspection, each system pole generates an exponential as part of the natural response. The input's pole generates the forced response. Thus,

$$C(s) \equiv \underbrace{\frac{K_1}{s}}_{\text{Forced response}} + \underbrace{\frac{K_2}{s+2} + \frac{K_3}{s+4} + \frac{K_4}{s+5}}_{\text{Natural response}} \qquad (4.3)$$

Find the value of all K's.

Exercise: A system has a transfer function

$$G(s) = \frac{10(s+4)(s+6)}{(s+1)(s+7)(s+8)(s+10)}.$$

Write, by inspection, the output, c(t), in general terms if the input is a unit step.

$c(t) \equiv A + Be^{-t} + Ce^{-7t} + De^{-8t} + Ee^{-10t}$

4.3: First-Order Systems

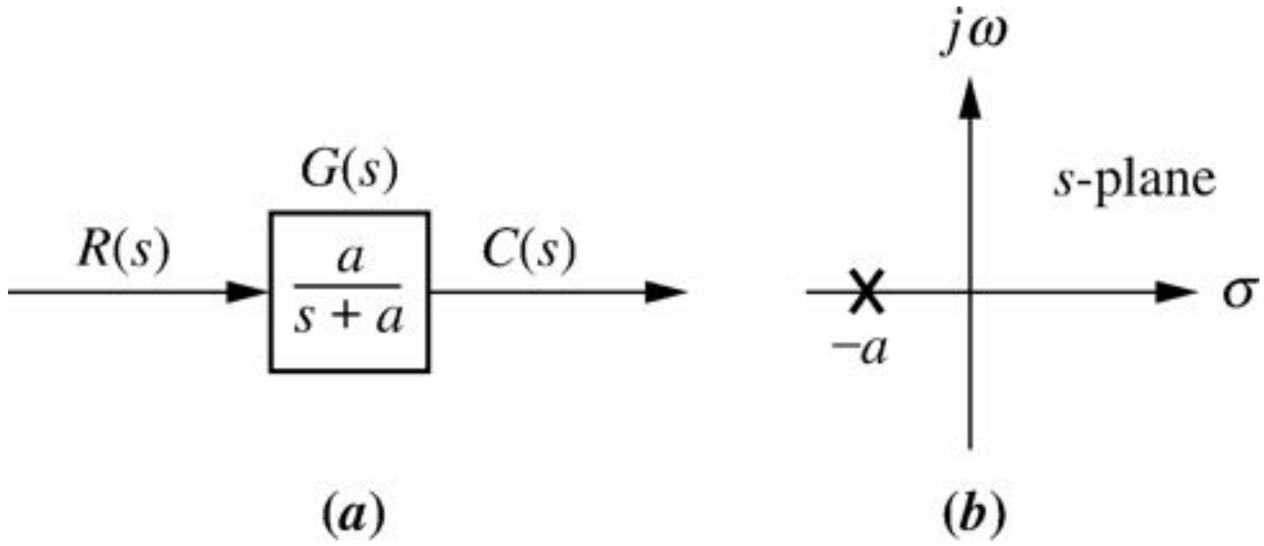


Figure 4.3: a) First order system
b) Pole plot

If the input is a unit step, where R(s) = 1/s, the Laplace transform of the step response is C(s), where

$$C(s) = R(s)G(s) = \frac{a}{s(s+a)}$$

$$c(t) = c_f(t) + c_n(t) = 1 - e^{-at}$$

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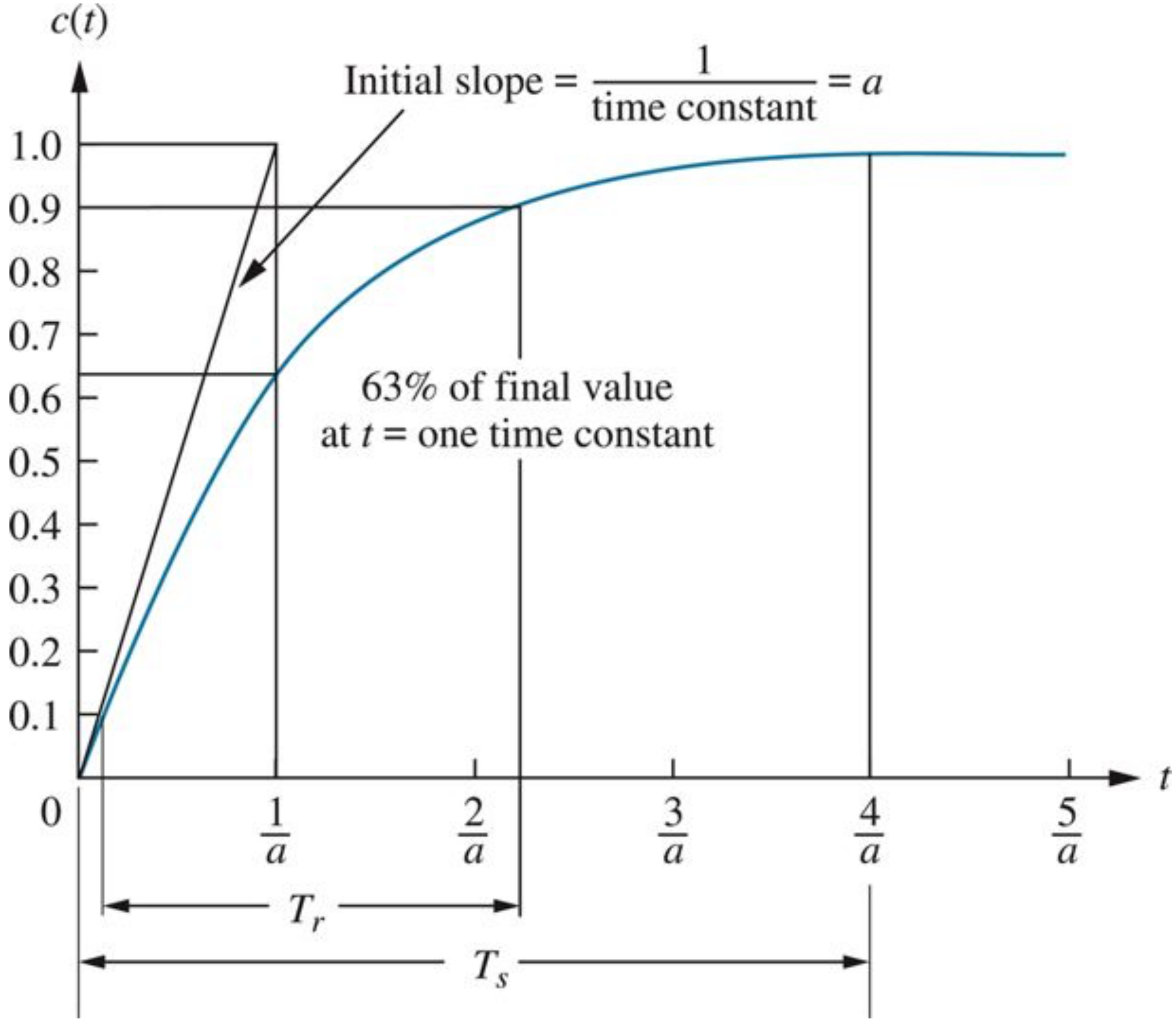
Figure 4.5: First-order system response to a unit step



Time Constant

We call 1/a the time constant of the response. From Eq. (4.7), the time constant can be described as the time for e^{-at} to decay to 37% of its initial value. Alternately, from Eq. (4.8) the time constant is the time it takes for the step response to rise to 63% of its final value.

$$e^{-at}|_{t=1/a} = e^{-1} = 0.37 \qquad (4.7)$$

$$c(t)|_{t=1/a} = 1 - e^{-at}|_{t=1/a} = 1 - 0.37 = 0.63 \qquad (4.8)$$

Rise Time, T_r

Rise time is defined as the time for the waveform to go from 0.1 to 0.9 of its final value.

$$T_r = \frac{2.31}{a} - \frac{0.11}{a} = \frac{2.2}{a}$$

Settling time is defined as the time for the response to reach, and stay within, 2% of its final value.

$$T_s = \frac{4}{a}$$

Exercise:

A system has a transfer function, Find the time constant, $T_{c,}$ settling time T_s, and rise time $T_{r.}$

$$G(s) = \frac{50}{s + 50}.$$

$T_c = 0.02$ s, $Ts = 0.08$ s, and $Tr = 0.044$ s.

Forced response: For linear systems, that part of the total response function due to the input. It is typically of the same form as the input and its derivatives.

Natural response: That part of the total response function due to the system and the way the system acquires or dissipates energy.

Review

A **control system** is defined as an **interconnection of components** forming a system that will **provide a desired system response**.

An **open-loop system** operates without feedback and **directly generates the output in response to an input signal**.

A **closed-loop system** uses a **measurement of the output signal and a comparison with the desired output** to generate an **error signal** that is used by the controller to **adjust the actuator**.

Second-Order Systems

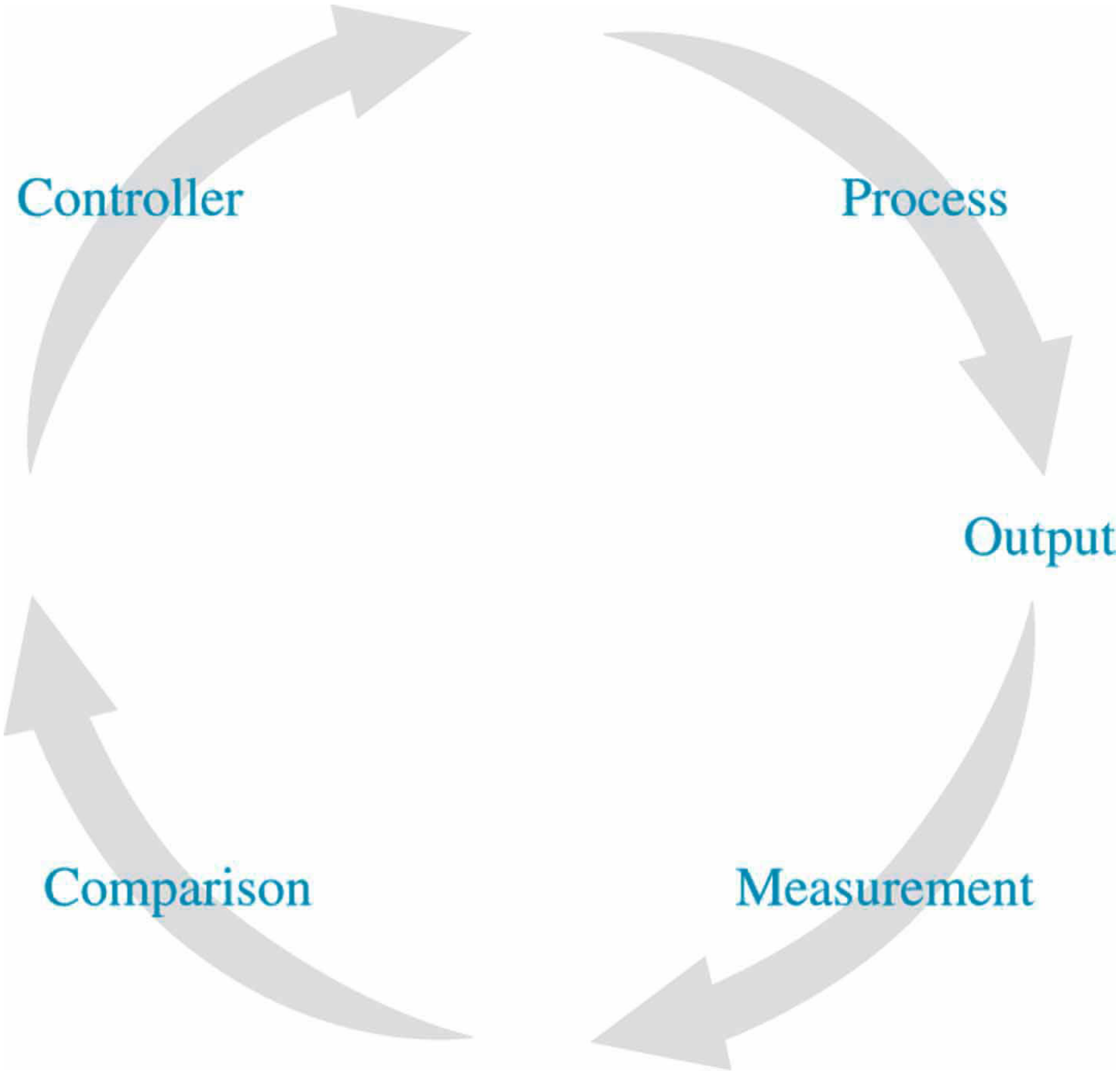


Figure 4.1 A closed-loop system.

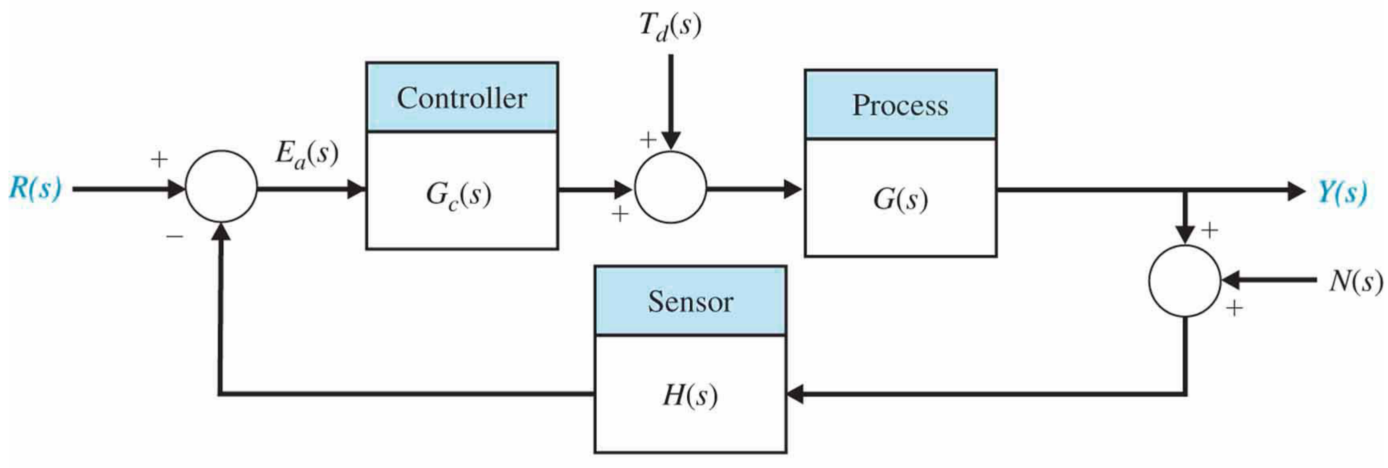


Figure 4.3: Standard Closed-loop system block diagram

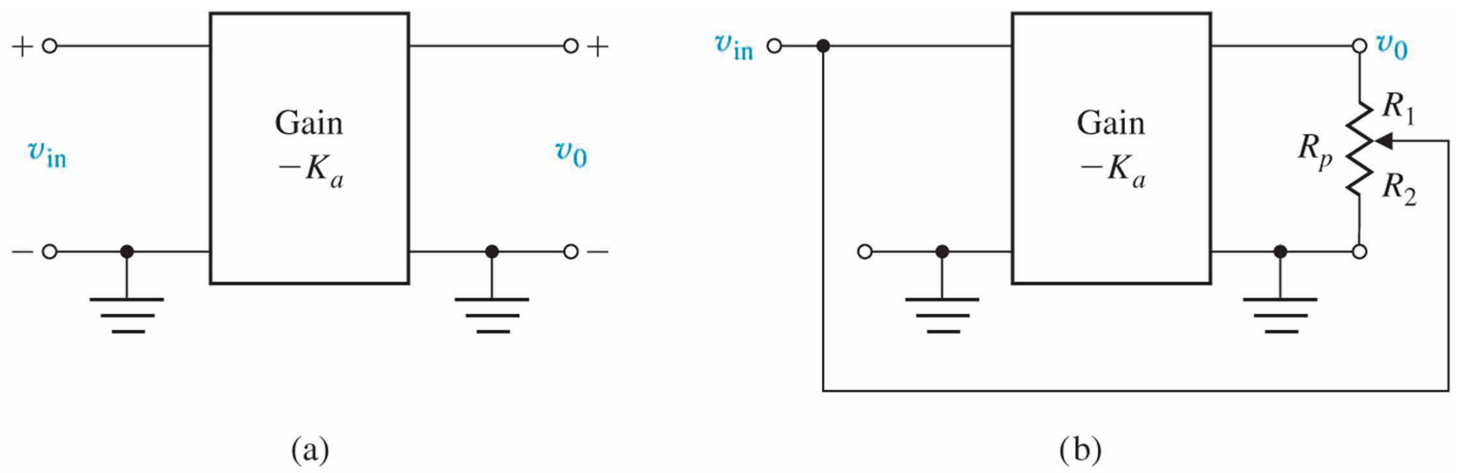
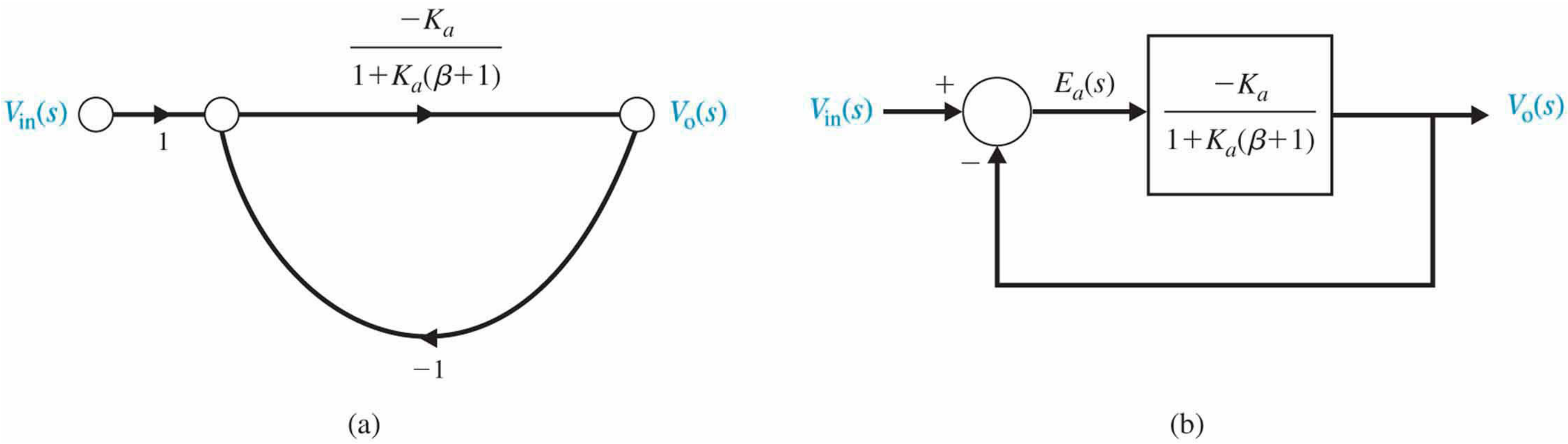


Figure 4.4 (a) Open-loop amplifier. (b) Amplifier with feedback.

vo=-Ka*vin vo=-Ka/(1+KaB)*vi *B=R2/R1

Figure 4.5 Block diagram model of feedback amplifier assuming $R_p >> R_0$ of the amplifier.



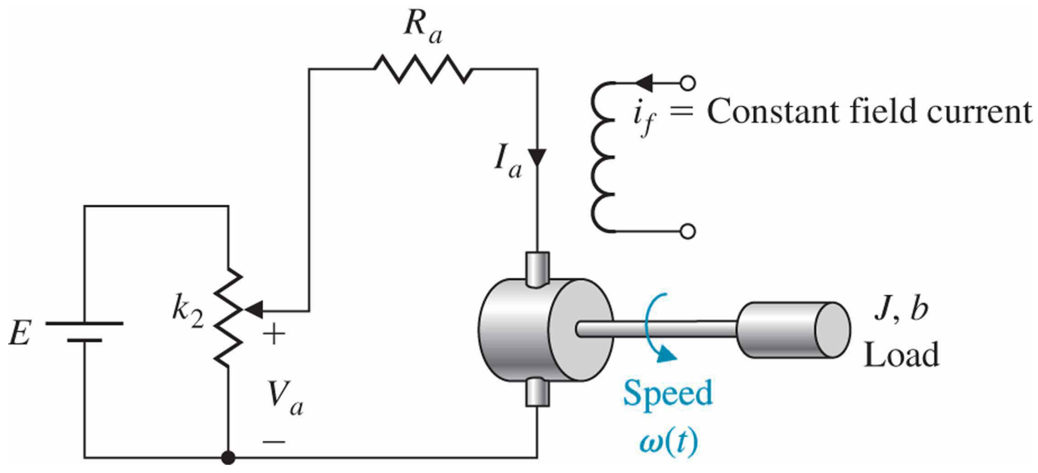


Figure 4.13 Open-loop speed control system (without feedback).

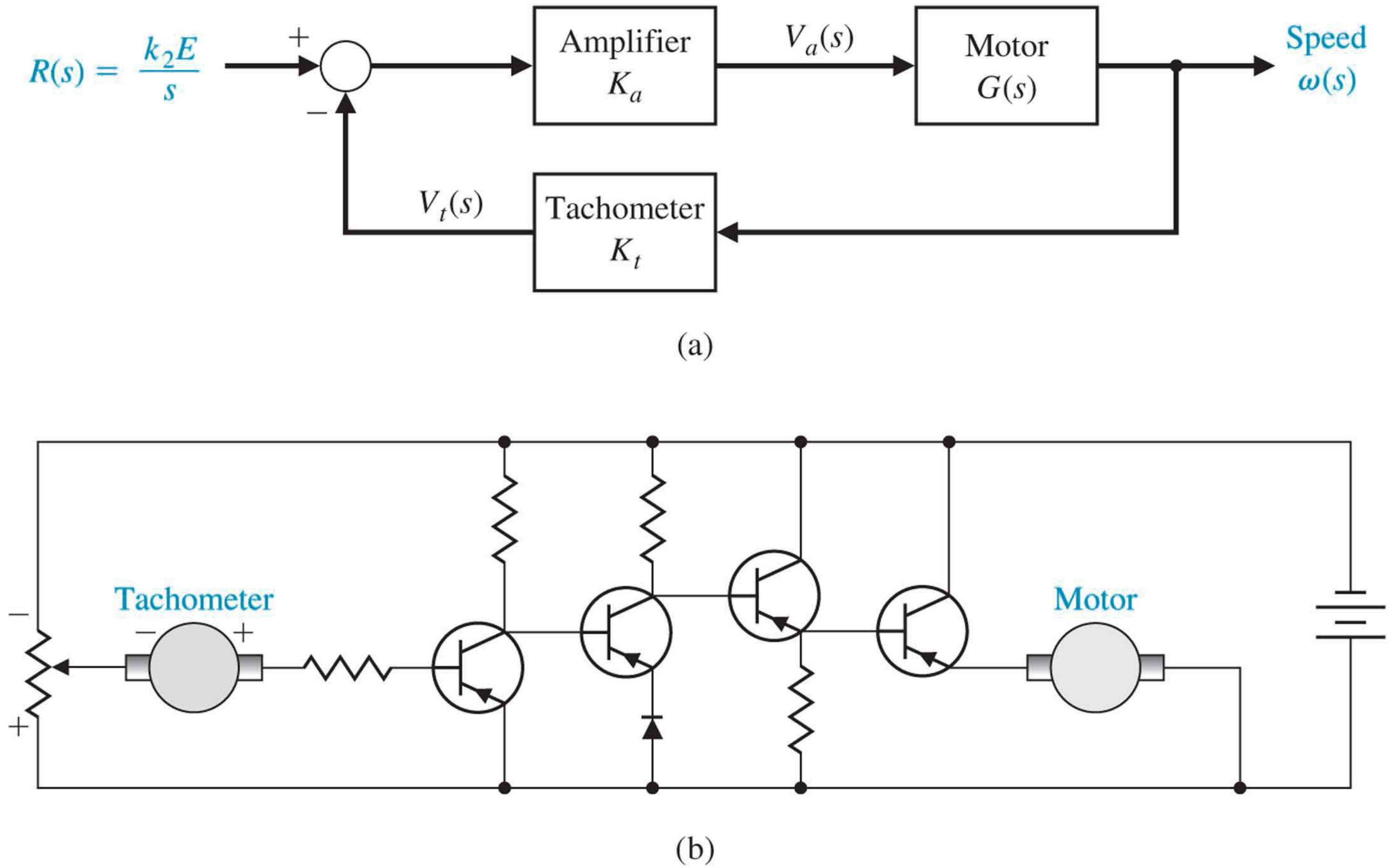


Figure 4.14 (a) Closed-loop speed control system. (b) Transistorised closed-loop speed control system.

Transient Response of Second-Order Systems- Performance

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$Y(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} R(s).$$

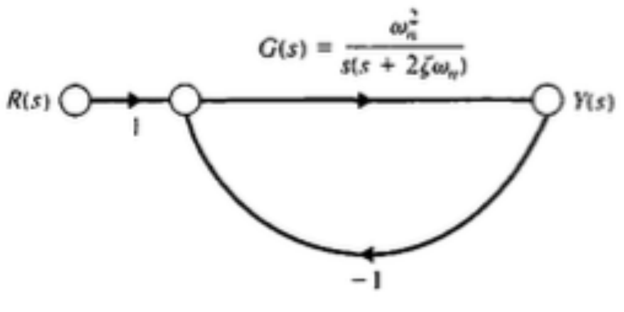
With a unit step input, we obtain

$$Y(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}, \tag{5.8}$$

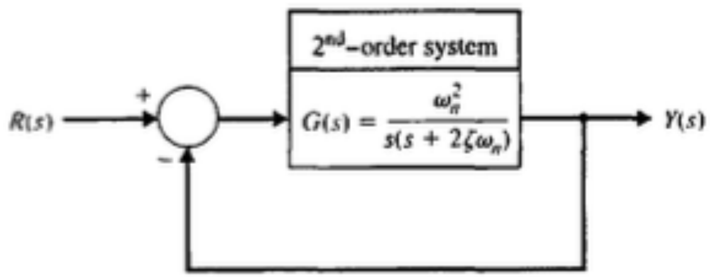
for which the transient output, as obtained from the Laplace transform table in Table 2.3, is

$$y(t) = 1 - \frac{1}{\beta} e^{-\zeta\omega_n t} \sin(\omega_n \beta t + \theta), \tag{5.9}$$

where $\beta = \sqrt{1 - \zeta^2}$, $\theta = \cos^{-1}\zeta$, and $0 < \zeta < 1$. The transient response of this second-order system for various values of the damping ratio ζ is shown in Figure 5.5.

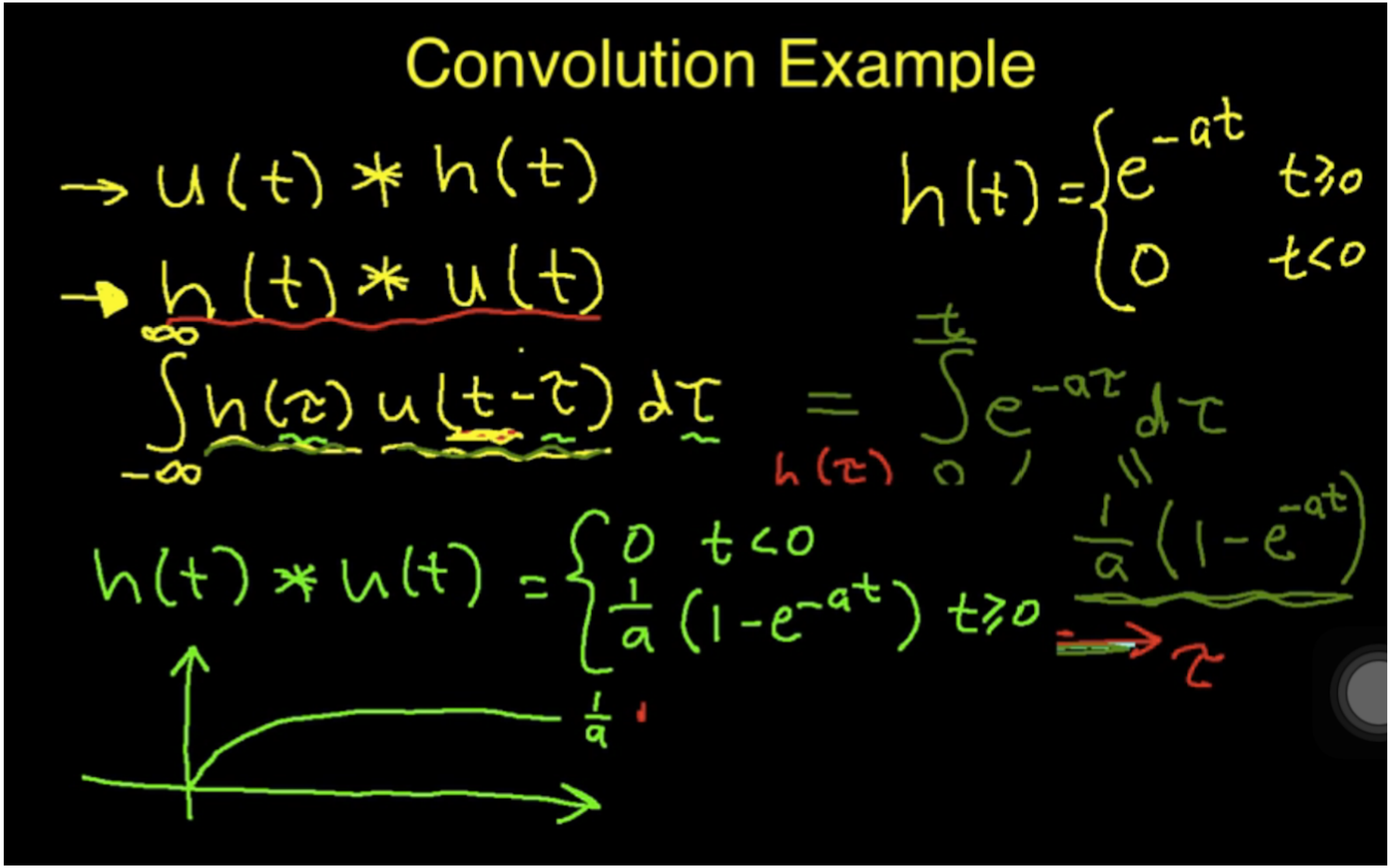


(a)

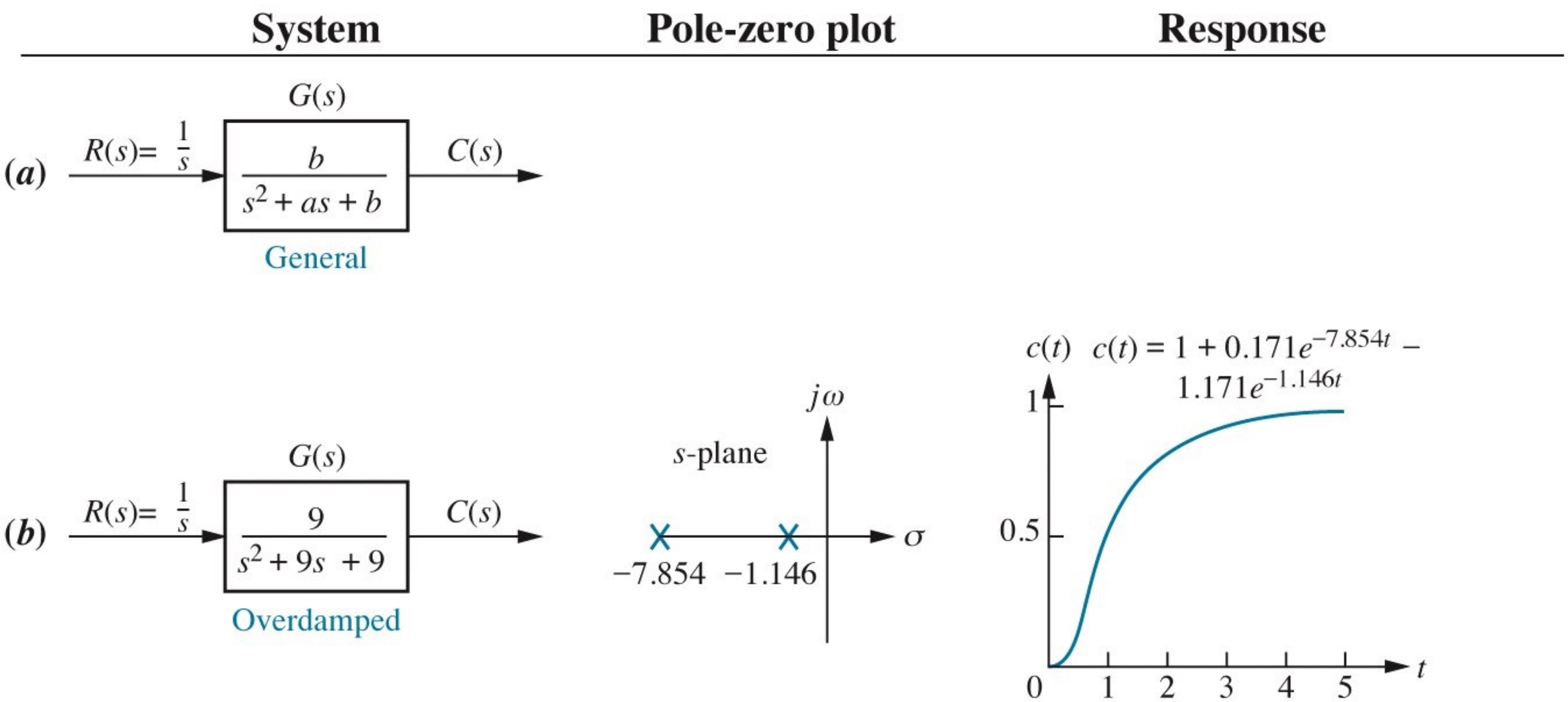


(b)

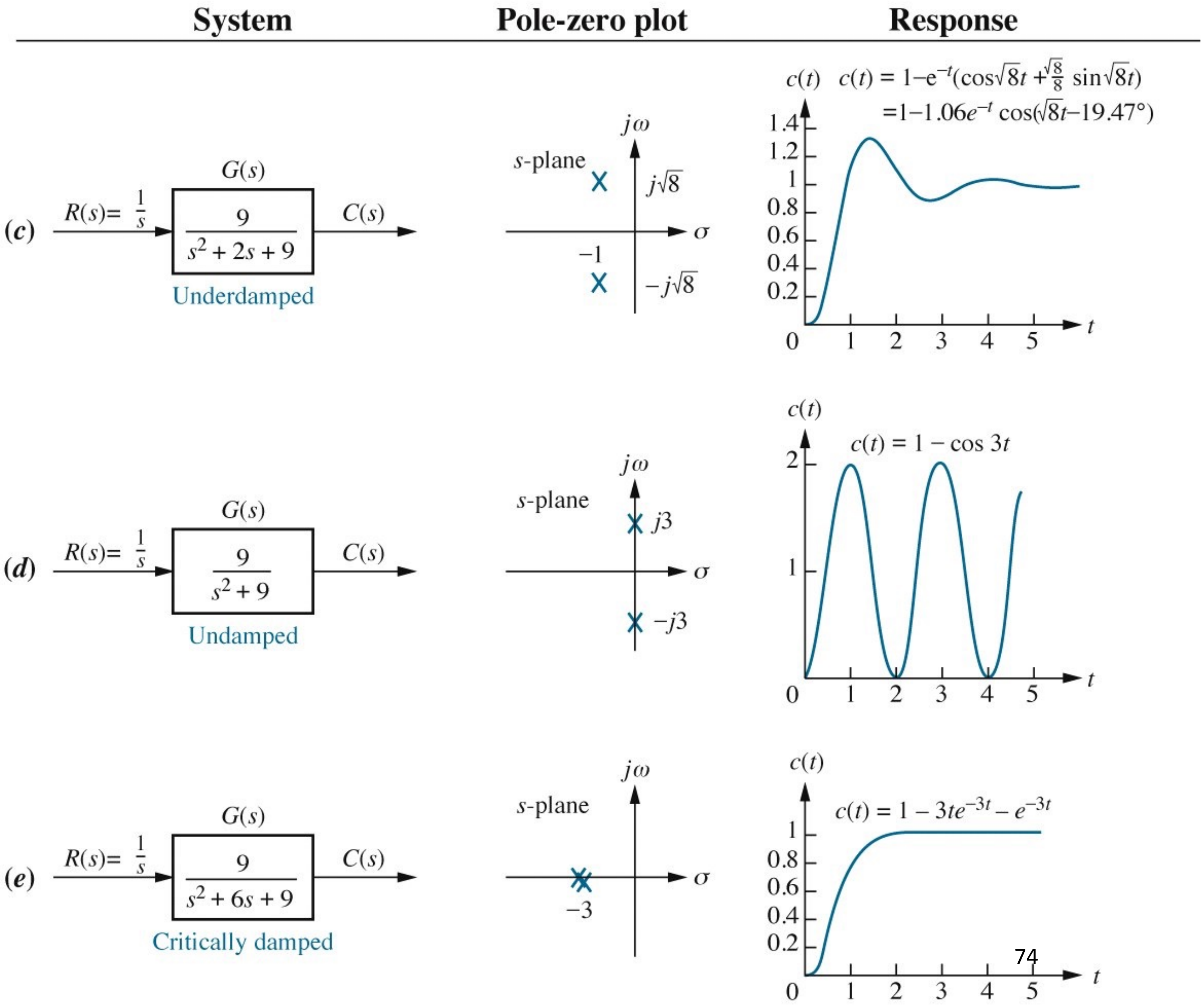
Time domain vs **Frequency** domain: Result will be similar



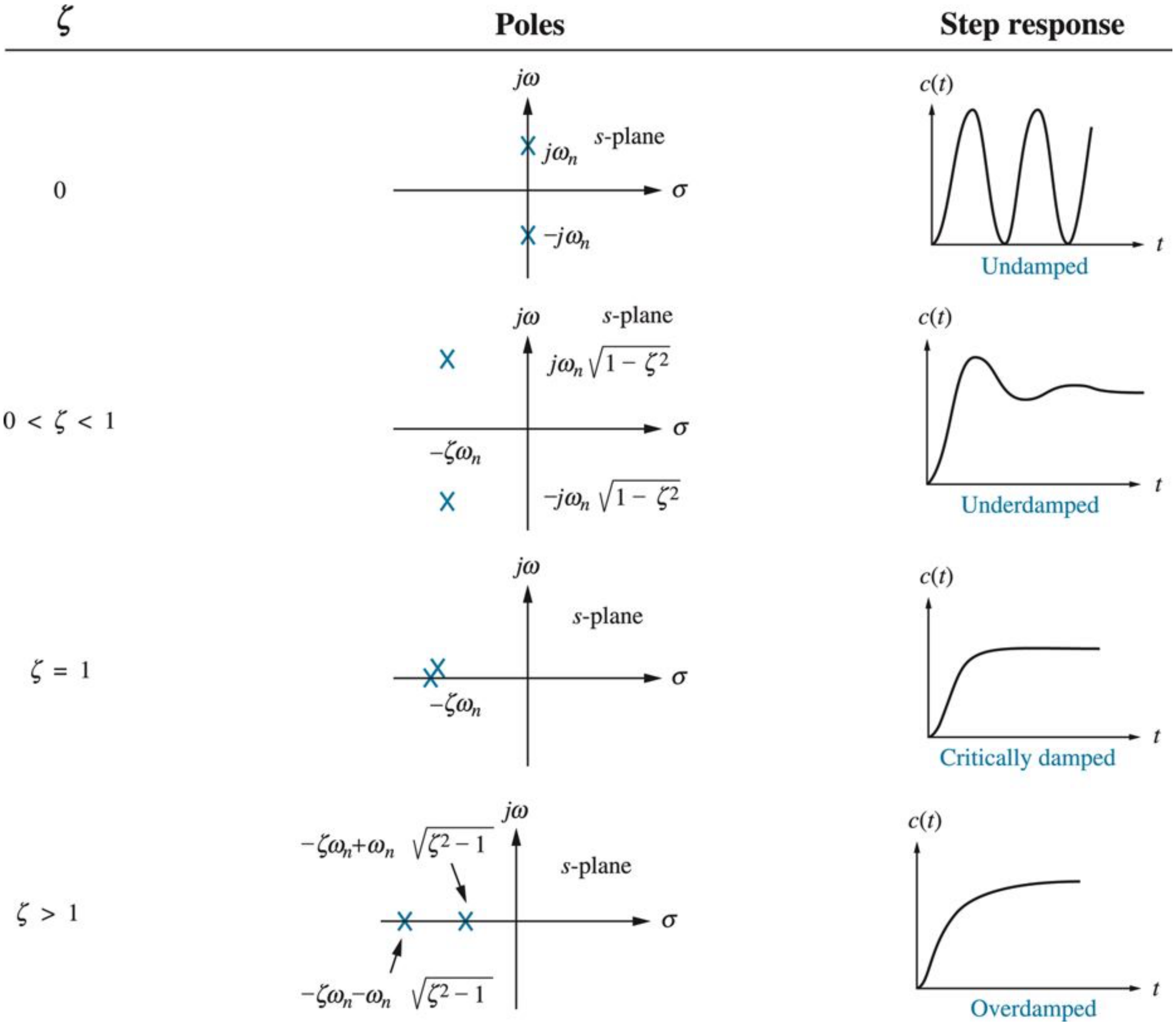
Transient Response of Second-Order Systems- Performance



Second-Order Systems



Second-order response as a function of damping ratio



Summary: Second Order System

1. *Overdamped responses*

Poles: Two real at $-\sigma_1, -\sigma_2$

Natural response: Two exponentials with time constants equal to the reciprocal of the pole locations, or

$$c(t) = K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t}$$

2. *Underdamped responses*

Poles: Two complex at $-\sigma_d \pm j\omega_d$

Natural response: Damped sinusoid with an exponential envelope whose time constant is equal to the reciprocal of the pole's real part. The radian frequency of the sinusoid, the damped frequency of oscillation, is equal to the imaginary part of the poles, or

$$c(t) = Ae^{-\sigma_d t}\cos(\omega_d t - \phi)$$

UniMAP

3. Undamped responses

Poles: Two imaginary at $\pm j\omega_1$

Natural response: Undamped sinusoid with radian frequency equal to the imaginary part of the poles, or

$$c(t) = A\cos(\omega_1 t - \phi)$$

4. Critically damped responses

Poles: Two real at $-\sigma_1$

Natural response: One term is an exponential whose time constant is equal to the reciprocal of the pole location. Another term is the product of time, *t, and an exponential with time constant equal to the reciprocal of the pole location, or*

$$c(t) = K_1 e^{-\sigma_1 t} + K_2 t e^{-\sigma_1 t}$$

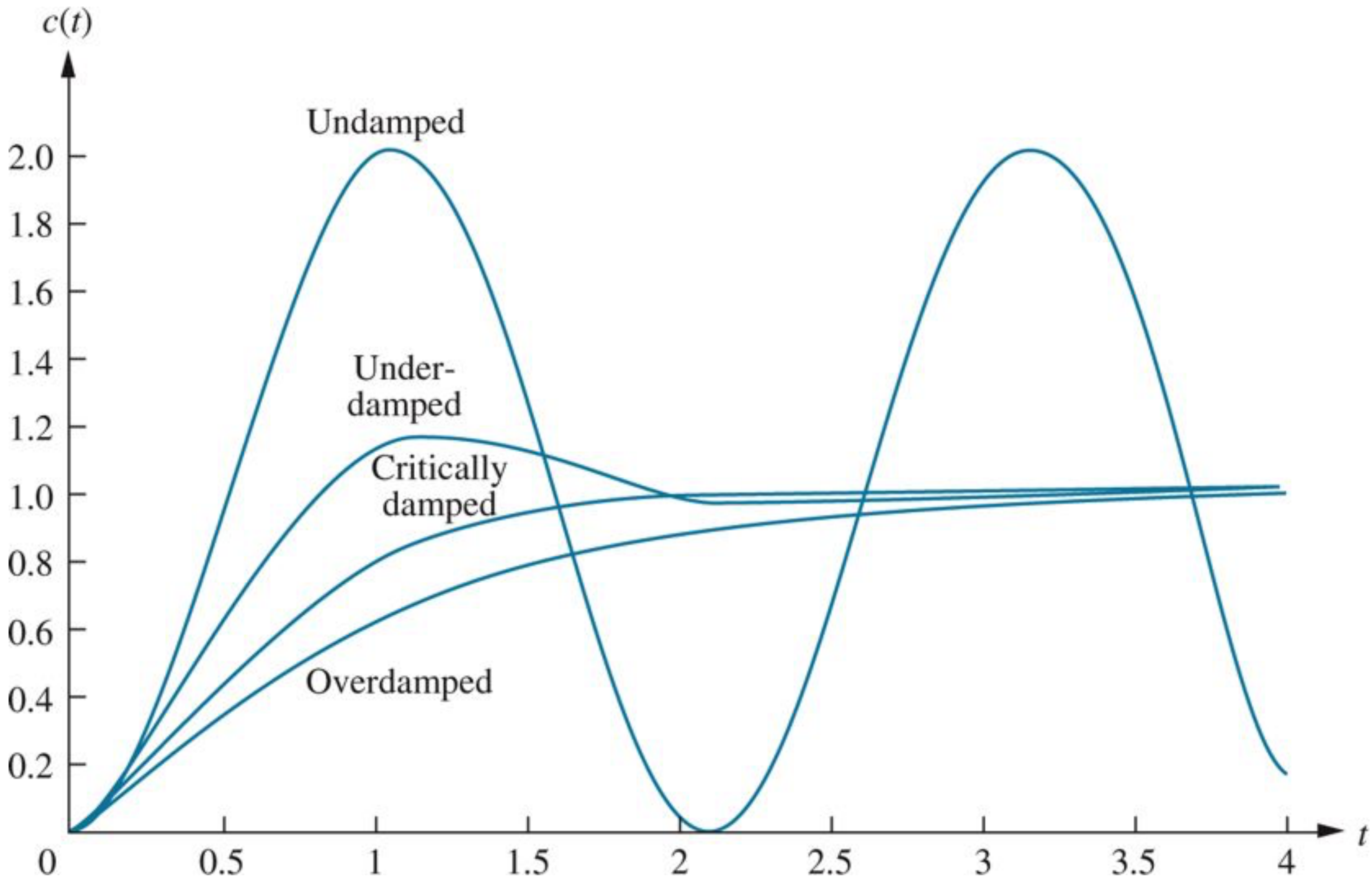


FIGURE 4.10: Step responses for second-order system damping cases

Exercise:

For each of the following transfer functions, write, by inspection, the general form of the step response:

$$G(s) = \frac{400}{s^2 + 12s + 400}$$

$$G(s) = \frac{900}{s^2 + 90s + 900}$$

$$G(s) = \frac{225}{s^2 + 30s + 225}$$

$$G(s) = \frac{625}{s^2 + 625}$$

Section 4.5: The General 2^{nd} Order System

We generalise the discussion and establish quantitative specifications defined in such a way that the response of a second-order system can be described to a designer without the need for **sketching the response**.

In this section, we define **two physically meaningful specifications** for second-order systems. These quantities can be used to describe the **characteristics of the second-order transient response** just as time constants describe the first-order system response.

The two quantities are called **natural frequency** and **damping ratio**.

Natural Frequency, ω_n

The natural frequency of a second-order system is the **frequency of oscillation of the system without damping**.

For example, the frequency of oscillation of a series RLC circuit with the resistance shorted would be the natural frequency.

Damping Ratio, ζ

A viable definition for this quantity is one that compares the **exponential decay frequency of the envelope to the natural frequency**

The *damping ratio*, ***ζ***, *defined to be:*

$$\zeta = \frac{\text{Exponential decay frequency}}{\text{Natural frequency (rad/second)}} = \frac{1}{2\pi}\frac{\text{Natural period (seconds)}}{\text{Exponential time constant}}$$

General 2nd Order Transfer Function

$$G(s) = \frac{b}{s^2 + as + b}$$

↓

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Example 4.3: Finding *ζ and ω_n For a Second-Order System*

PROBLEM: Given the transfer function G(s), find *ζ and ω_n.*

$$G(s) = \frac{36}{s^2 + 4.2s + 36}$$

Now that we have defined ζ and $\omega_{n,}$ let us relate these quantities to the pole location. Solving for the poles of the transfer function in

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

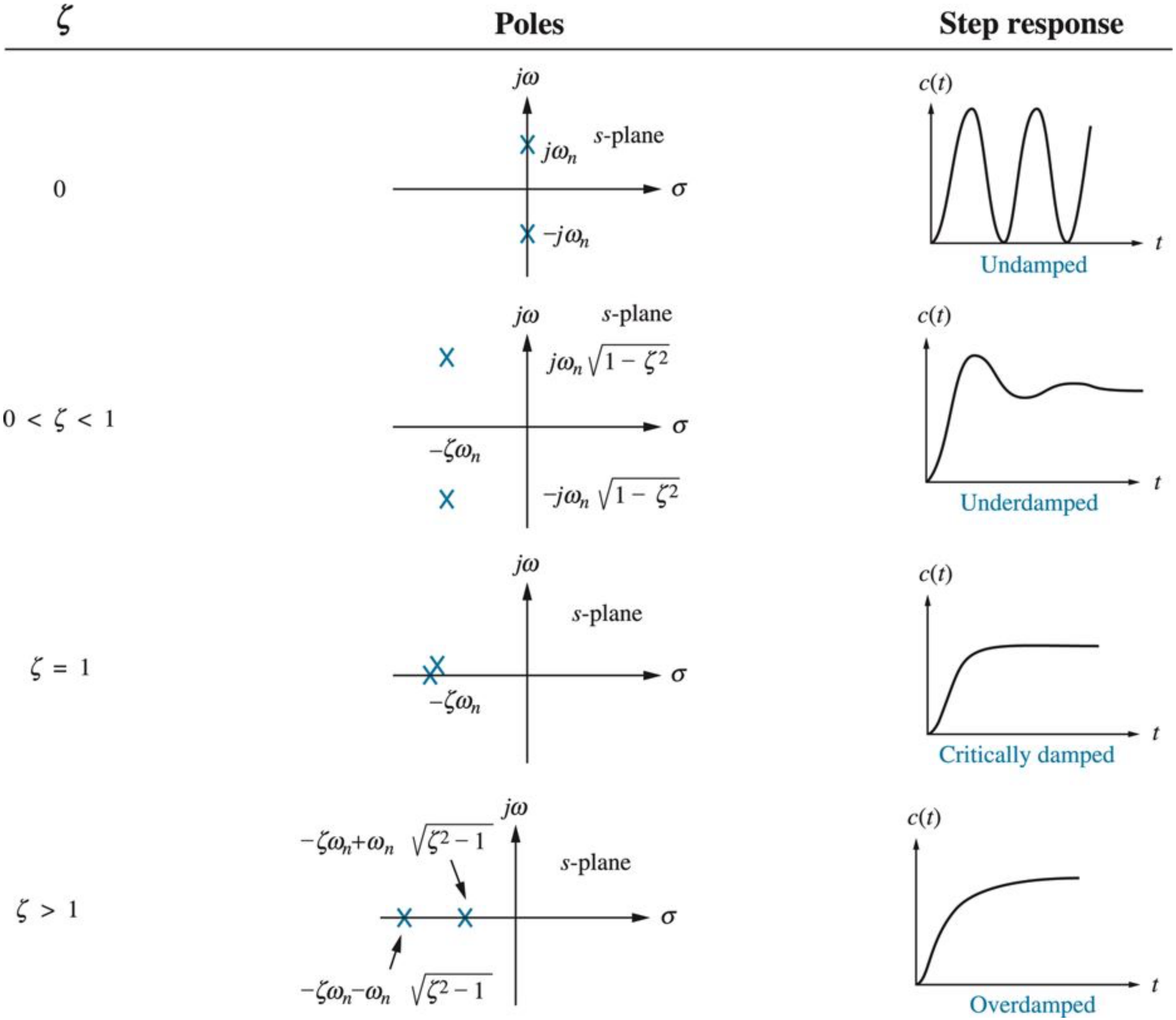


FIGURE 4.11: Second-order response as a function of damping ratio

Example 4.4:

For each of the systems shown in Figure 4.12, find the value of zeta and report the kind of response expected.

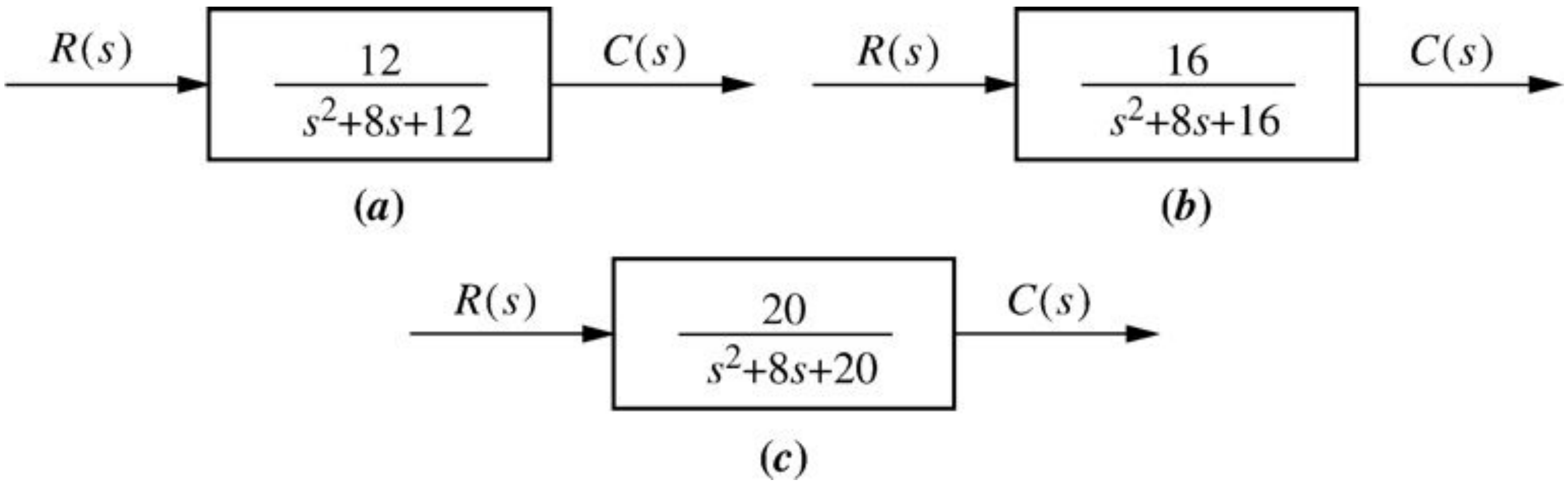
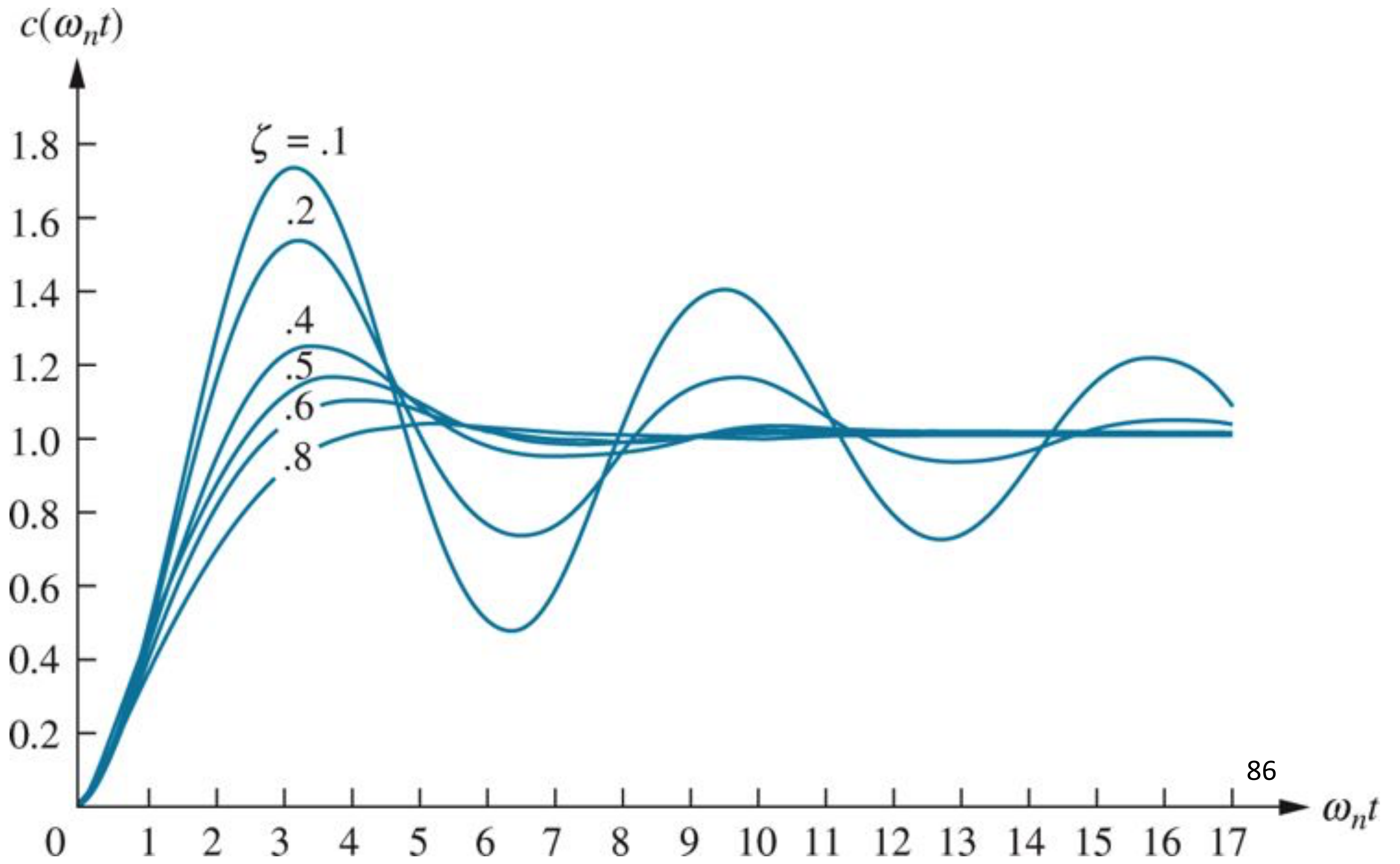


Figure 4.12

4.6: Underdamped Second-Order Systems

$$c(t) = 1 - e^{-\zeta\omega_n t}\left(\cos \omega_n \sqrt{1-\zeta^2}\,t + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin \omega_n \sqrt{1-\zeta^2}\,t\right)$$

$$= 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cos(\omega_n \sqrt{1-\zeta^2}\,t - \phi)$$



1. Rise Time, $\mathbf{T_r}$

- 0.1-0.9 of final value

2. Peak Time, $\mathbf{T_p}$

- Fist maximum peak
-

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$

3. Percent Overshoot, **%OS**

- .

$$\%OS = \frac{c_{\max} - c_{\text{final}}}{c_{\text{final}}} \times 100$$

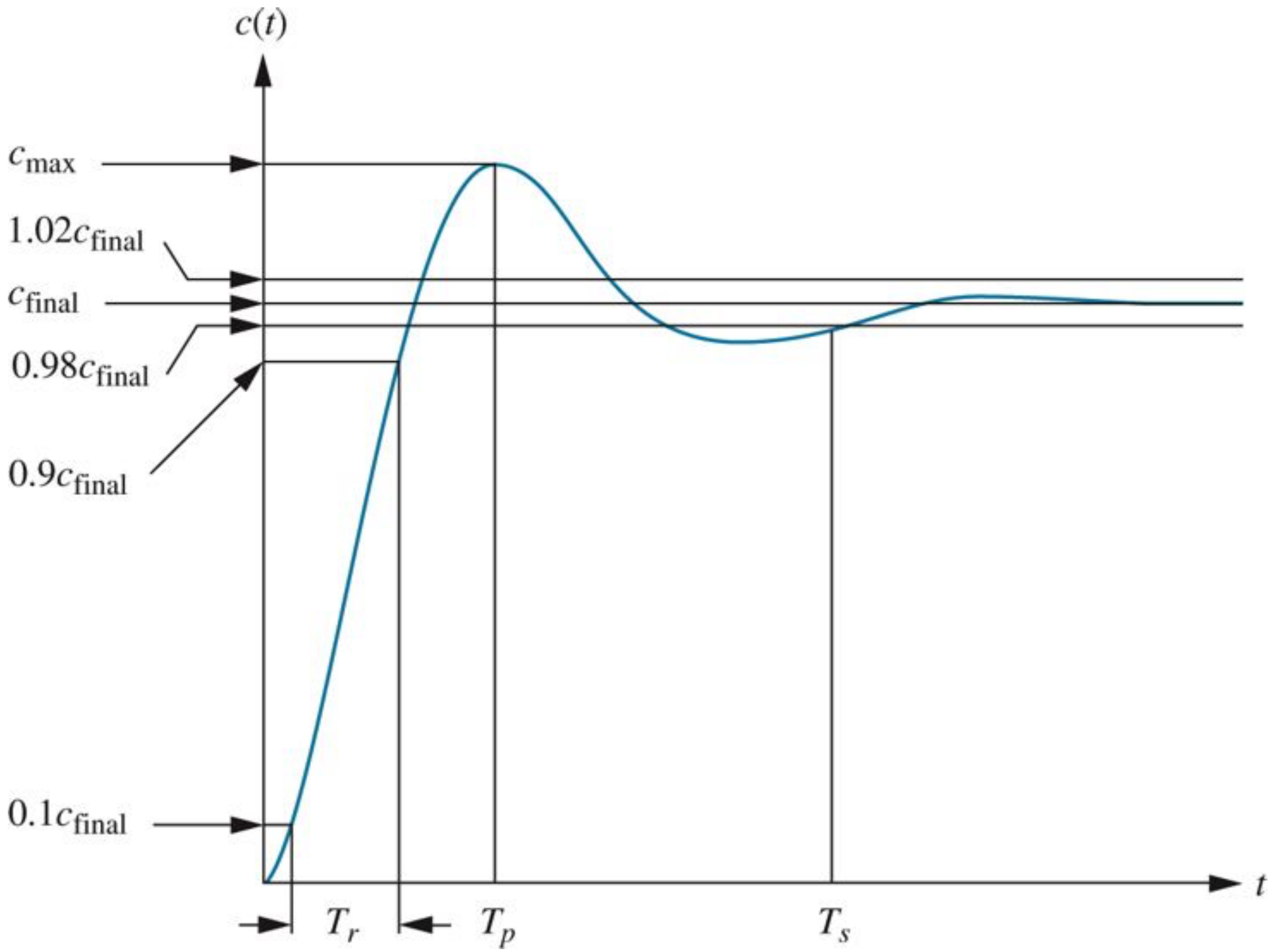
$$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100$$

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}}$$

4. Settling Time, $\mathbf{T_s}$

- .

$$T_s = \frac{4}{\zeta\omega_n}$$



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Example 4.5:

Find $\mathbf{T_p}$, **%OS**, $\mathbf{T_s}$ and $\mathbf{T_r}$ for the given transfer function:

$$G(s) = \frac{100}{s^2 + 15s + 100}$$