INTERFERENTIAL CONTACT LEVER EXPERIMENTS RELATING TO THE ELASTICS OF SMALL BODIES[1]

By Carl Barus

Department of Physics, Brown University

1. Introductory.—In a preceding paper I communicated a series of experiments on the traction modulus of small bodies, using an interferometer design, which worked admirably so far as the optic measurements were concerned. The mechanical part of the contrivance showed an apparent yield, the nature of which I was unable to detect, but which seemed in some way, to be associated with the flexure of parts of this massive apparatus. In fact pulleys and weights were used for imparting stress. It may be argued that any contrivance of this kind, however convenient in other respects, is dangerous because of the force couples introduced, even when the rigid parts of the apparatus are nearly 2 inches thick, as in the case in question.

In the present apparatus all this is completely avoided by the use of pushing springs to impart stress, and the interferential contact lever to measure strain. True, friction enters into functioning of such an apparatus to a menacing degree. It thus becomes an experimental question to determine in how far it can also be eliminated by judicious tapping, etc. Cf. § 6.

2. Apparatus.—The simplest of the apparatus designed is shown in figure 1. The rod to be tested, 1 to 3 cm. long, is at r held in a brass sheath s, loosely fitting it. See figure 2. This is screwed into the middle of the massive brass cross piece A. A little disc of glass has been attached at a, and the end, e, of the *contact lever* touches it to indicate the small elongations. The longitudinal displacements Δx of the pin e are observed by the interferometer, as explained in the preceding paper.[2]

B is a cast iron brick, about 10 inches high, 2 inches thick, and 3.5 inches broad, provided with 2 horizontal $\frac{1}{4}$ inch perforations, parallel to each other and normal to the large face. Through these pass the $\frac{1}{4}$ inch brass rods bb and cc loosely, rigidly connecting the cross piece A with the similarly massive cross piece C (screws and nuts m, n). The rectangle AC is thus free to slide in B, except so far as it is limited by the contact of the rod r with the smooth face of the brick B.

To apply stress, the system d, w, S, f, has been provided, consisting of the stiff open spring S encircling the brass rod df, firmly screwed into the brick B and d, but passing loosely through a perforation in the middle of C. The end near d of the rod df is threaded (20 threads to the inch), so as to admit of the compression of the spring S, by aid of the thumb nut w. S was a precision spring, taken from an indicator apparatus and provided as usual with two end brass collars. It is essential that the sliding parts of the apparatus
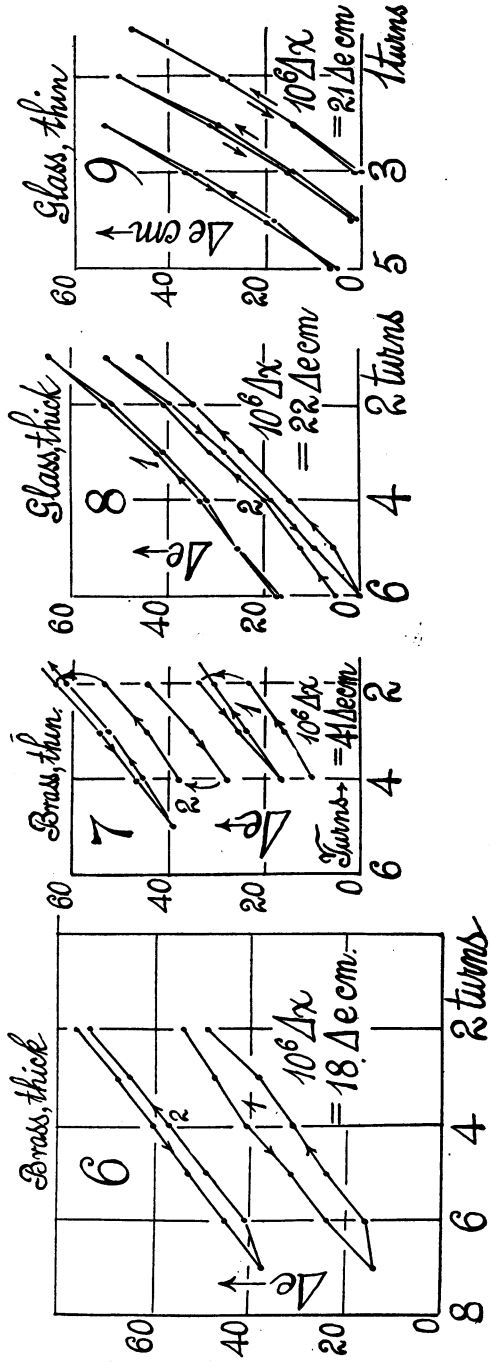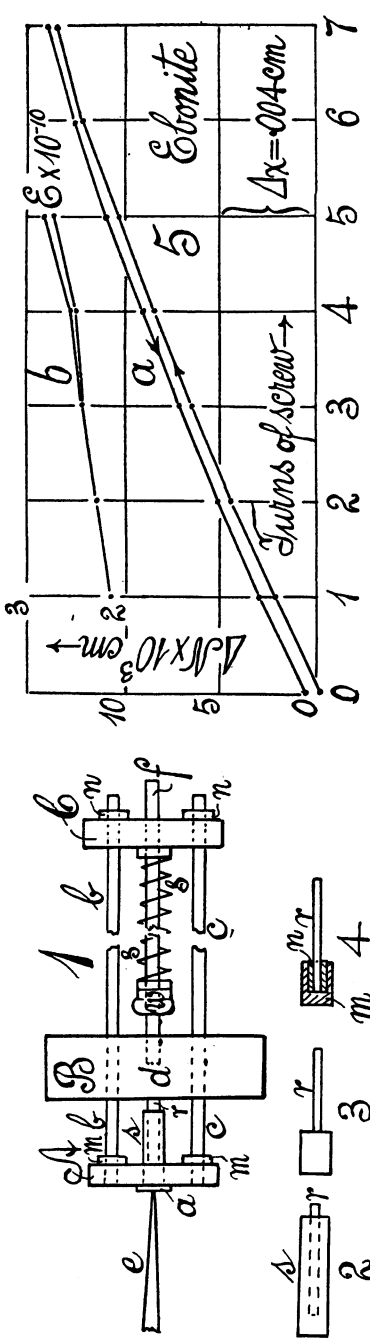
work smoothly and with a minimum of friction. Such as exists may be eliminated by tapping *b* and *c* before each observation. Thrusts up to 15 to 20 kg. may be easily applied by the thumb nut *w*. These stresses act in the direction *ar* and *fd*, colinearly and there are no couples endangering the accuracy of the elastic displacements of *r*. The stress is standardized in terms of the observed rotation of the thumb nut *w*.

Figures 2, 3, 4 are details, showing the methods of clutching the rod *r*, sheathed in figure 2, shouldered in figure 3, soldered in a small cap *m* with fusible metal *n*, in figure 4. The thick ends are threaded to be received by *A* figure 1.

3. Observations. Hard rubber.—As in the preceding paper (l. c.) if Δx is the longitudinal compression of the rod *r* in *s*,

$$\Delta x = (r \cos i / b) \Delta N \tag{1}$$

where ΔN is the displacement of the micrometer, at $i = 45°$ to the rays, *b* the breadth of the ray parallelogram, and *r* the effective length of the contact lever. Furthermore since the modulus *E* for the length of rod *L* and section *A* is $E = (F / A) / (\Delta x / L)$, *F* being the thrust,

$$E = F L b / A r \cos i. \Delta N \tag{2}$$

The ocular micrometer if used is to be standardized in terms of ΔN by direct comparison; i.e., if the former datum is Δe arbitrary scale parts, $\Delta e / \Delta N$ must be known.

To graduate the spring *S*, the apparatus *ABC*, figure 1, was detached from the interferometer and the brick *B* fastened near the edge of a strong flat table, with its large face toward *A* lowermost and horizontal. The rectangle *AC* was thus vertical, *A* below *C*, just clearing the edge of the table. Weights from 1 to 9 kg. were now hung from *A*, compressing the spring *S* by measureable amounts. In this way it was found that the stretch 0.7 mm. corresponded to 1 kg. Since the threads of *w* were 1.275 mm. apart, it follows that 1 rotation of the thumb screw *w* corresponds to 1.82 kg. or to 1.78×10^6 dynes. In the interferometer, $b = 9.3$ cm., $r = 11.0$ cm. were directly measured.

The test rod *r* was here of hard rubber of length $L = 2.47$ cm., diameter $= 0.377$ cm., and area $A = 0.112$ cm.^{2} Hence for *n* turns of the screw *w*, (equation (2), ΔN in cm.)

$$E = 4.69 \times 10^7 n / \Delta N; \tag{3}$$

or if we express ΔN in 10^{-3} cm.,

$$E = 4.69 \times 10^{10} \times n / \Delta N \tag{4}$$

The fringes, found without difficulty, were small though here more than adequate for the purpose. Measurements were made in *cycles*, care being

taken to repeatedly tap the movable parts of the apparatus before each reading, and these came out remarkably smoothly at once.

An example of experiments made after some improvements of apparatus and carried out with the rod specified through a range of about 15 kg. is given in figure 5, *a*. The rod stood the stress well except at the end where it showed slow viscous shrinkage. The data (ordinates) contain the running displacement ΔN of the micrometer screw in terms of the successive turns of the forcing screw, *w*, figure 1. Stress increasing or decreasing is indicated by arrows.

The values of E were computed from 3 turns (5.46 kg.) of the forcing screw. For loads up to 5 additional turns (total 11 kg.), the data for E are practically identical, both in the outgoing and return series. See figure 5, *b*. At 6 turns (total 13 kg.) the rod apparently yields; but at 7 turns it again stiffens in both cases. As a whole the data are quite as good as the reading of the micrometer screw admits. To interpret this apparent increase of E it would be necessary to use a thinner rod, as the following experiments with brass and glass suggest. Again, only in case of more rigid rods, where ΔN fails, is it necessary to use the ocular micrometer (Δe).

4. *The same. Brass.*—By way of contrast, a thick solid brass rod, $L = 2.34$ cm. long, 0.376 cm. in diameter, area $A = .111$ cm.² was now put into the sheath, *s*, figure 1, and tested, the aim being to redetermine the limit of measurement. Here $n / \Delta e$ or the ocular micrometer is the essential datum and $\Delta e / \Delta N$ must be known.

The interferometer was modified to guard against displacements due to tremor, large fringes were installed and readings were made several times before and after tapping. There was but little difference. An example of such results is given in figure 6, (1) and (2), where $\Delta e / \Delta N \times 10^3 = 43$, and therefore per turn of screw ($n = 1$), $E = 10^{11} \times 19.3 \Delta e$.

In the first series Δe per turn was 8.0 and hence $E \times 10^{-11} = 2.4$; in the second $\Delta e = 7.8$ and $E \times 10^{-11} = 2.5$. Seeing that a scale part of Δe in figure 6 is but 23×10^{-6} cm., these results are experimentally very good; but their absolute values, as given by E, is nevertheless very low. The rates for the outgoing and return series are identical. The backlash, as it were, on passing from one to the other is probably in the apparatus.

In triplet observations, naturally, higher values of E will be found; for instance, between 3 and 4 turns of the screw, $\Delta e = 5.5$ per turn, appeared in successive independent experiments. Thus $E \times 10^{-11} = 3.5$.

An example may now be adduced of experiments made with a brass rod, thin and shouldered as indicated in figure 3, the large end ($\frac{1}{4}$ inch in diameter) being threaded and screwed into the cross-piece A. The dimensions of the thin part were, length $L = 1.8$ cm., diameter $= .22$ cm., $A = .038$ cm.² The fringe factor was $\Delta e / \Delta N \times 10^3 = 29$. The mean rate per turn was found to be $\Delta e = 4.9$ and $10 \times E^{-11} = 29.2 / 4.9 = 6.0$.

It seemed therefore, worth while to further decrease the section. This was done, the dimensions being, length $L = 1.8$ cm., diameter 0.175 cm., $A = 0.0199$ cm.^{2} The results gave $\Delta e / \Delta N \times 10^8 = 26.0$ and therefore $E = 5 \times 10^{11} / \Delta e$. The rates per turn lie between $\Delta e = 5.6$ (returning) and $\Delta e = 6.1$ (outgoing), so that $E \times 10^{-11}$ is between 8.9 and 8.2, respectively. This is so near the normal value for brass that a further decrease of section of the rod figure 3, was undertaken. The final dimensions were $L = 1.8$, diameter $= 0.138$, $A = 0.015$ cm.^{2} The results are given in figure 7, care having been taken not to overstrain the thin rod. Here $\Delta e / \Delta N \times 10^8 = 25.8$, $E \times 10^{-11} = 66.4 / \Delta e$ and Δe per turn lies between 6.7 (outgoing) and 8.1 (returning). Hence $E \times 10^{-11} = 9.9$ and 8.2, respectively, so that the normal modulus of brass has actually been reached. In fig. 7 the cycles have been spaced as shown by the arrows, to show the separate observations.

5. *Glass.*—The glass rod first tested was $L = 2.33$ cm. long, 0.37 cm. in diameter, so that $A = 0.107$ cm.^{2}

With a robust interferometer the outgoing and return data were nearly coincident; but the graphs were not as a rule straight. The mean rate per turn was found to be $\Delta e = 8.6$. The fringes were of moderate size $(\Delta e / \Delta N \times 10^8 = 27.5)$, so that $10^{-11} E = 12.75 / \Delta e = 1.5$, a very low result. Larger fringes were now installed, giving $\Delta e / \Delta N \times 10^8 = 34.8$. The results after regrinding the contact face are shown in figure 8 and are again nearly coincident, but lie on curved loci. In the first series the larger rate is $\Delta e = 8.4$ per turn; in the second series $\Delta e = 10.0$ per turn. Hence

$$10^{-11} E = 16.1 / \Delta e = 1.9 \text{ and } 1.6, \text{ respectively,}$$

larger than the preceding; but this is still only about one-third of the normal modulus of glass.

The endeavor was now made to proceed as in the case of brass above, with a shouldered rod and thinner sections. With this, in view, the glass rod was fixed in a small hollow cup, figure 4, with fusible metal. The cup being threaded was thereupon screwed into the cross-piece A, figure 1. With a glass rod $L = 1.9$ cm., $A = 0.28$ cm.,^{2} moduli as high as $E \times 10^{-11} = 3$ to 4 were obtained. On taking the rod out however, it became clear that there had been gradual yielding of the fusible metal clutch. Hence I returned finally to the sheath method (fig. 2) using a thin glass rod, $L = 2.54$, cm. long, 0.185 cm. in diameter, $A = 0.026$ cm.^{2} The results are given in figure 9. The graphs are nearly coincident but curved. The mean rates for the higher loads are per turn of the screw, $\Delta e = 10.4$ (incoming) and $\Delta e = 8.6$ (outgoing). $\Delta e / \Delta N \times 10^8 = 28.8$, being the fringe factor, $E \times 10^{-11} = 57.86 / \Delta e = 5.5$ and 6.8, respectively. Hence here also, as in the case of the brass rod above, the normal value of the modulus has been reached; i.e., one may expect the data for E to be correct on their absolute values, if the ratio of length of rod to diameter is of the order of 10 to 1.

6. Conclusion.—The present experiments made with a totally dissimilar apparatus and in a different manner, are nevertheless (notwithstanding the relative simplicity of the present design) not markedly superior to the earlier experiments (l. c.), as a whole. The misgiving which I felt (see § 1) regarding the force couples entering into the earlier method was not therefore justified. Both apparatus function admirably so far as the optics of the method are concerned. This is particularly noteworthy when one considers the admissibility of the rather rough treatment needed in work of the present kind. Both apparatus are liable to give misleading results from the same cause; i.e., from an insufficiently uniform and continuous contact of the two ends of the rod with the abutments. From this results appreciably unequal distribution of stress in the sections of the rod and possibly flexure. There seems to have been no serious yield in the abutments, etc., of either apparatus.

The values of the modules E as a consequence come out too small. There can therefore (tapping admitted) have been no serious discrepancy from friction in the application of stress; for this would have made E too large. Moreover all slight dislocations within the interferometer as the result of any reasonable jar were finally eliminated, so that the cycles practically closed or merely gave evidence of a difference of slope in the outgoing and return series. Such an effect would be expected from viscosity and hysteresis.

I was at first inclined to regard the small values of the modulus E as an actual or trustworthy result, in keeping with the peculiar crushing stress applied. But inasmuch as E may be increased to the normal value by successively decreasing the diameter of the rod, in the case of glass and even of brass, the small values of E must be associated with the lack of contact at the abutments of the rod. Rods about 1 to 2 cm. in length should not be thicker than 1 or 2 millimeters (ratio about 10 to 1), if results are to be assured in their absolute values. And here again a thin rod, r, with *two* thick ends, if both ends are firmly clutched without strain, is the ultimate desideratum. Figures 3, 4, 2 (sheath, s), are admissible expedients, the latter being particularly convenient. The relative results are almost always smooth and admirable to a fraction of a wave length; but for relatively large sections they can not be interpreted owing to the sectional discrepancy in question. This also is relative in its character; at least for moduli markedly above 10^{10}. Thus it is as difficult to obtain the true modulus for a glass rod as for a brass rod, although the latter body is far more rigid.

It is not easy to interpret the apparant hysteresis in many of the above graphs; for this is always associated with possible changes in a complicated train of apparatus. Similarly the different rates in the outgoing and the return series may be variously explained. If the measurements are made in triplets between definite steps of pressure, this difference soon vanishes. Hence this procedure is to be preferred.

[1] Abridged from a forthcoming report to the Carnegie Institution of Washington, D. C.

[2] These PROCEEDINGS, **3**, 1917, (693–696).