Introduction

- By this time we know that an electric motor is the basic component of a modern variable speed drive.
- The motor converts electrical energy into mechanical energy. Energy is the power multiplied by time.
- The mechanical power output of the motor is proportional to the product of torque & speed.
- A part of torque is lost in friction etc. An understanding of the mechanical aspects is necessary to grasp the working of an electric drive. In this chapter we study some of these mechanical aspects.

2.1. Types of Load

- In Electrical term, load means, a component which conserves the electrical energy to do some useful task
- Load can be classified in many ways

(a) According to Electrical Phases

- Single Phase Load
- Three Phase Load

(b) According to Consumer Category

- Industrial Load
- Commercial Load
- Residential Load

(c) According to Electrical Load Unit

- Load in KVA
- Load in KW
- Load in HP

(d) According to Nature of Load

- Resistive Load
- Inductive Load
- Capacitive Load

2.2. Dynamics of Motor-load System

- Motor generally drives a load (mechanical load i.e. machines) through some transmission system.
- While motor always rotates and load may rotate or undergo a translation motion.
- Load speed may be different than the speed of motor. If the load has many parts then speed of their may be different. Some load may undergo through the translation motion & rest may undergo through the rotational motion.
- To derive the fundamental torque equation, consider the diagram shown below.

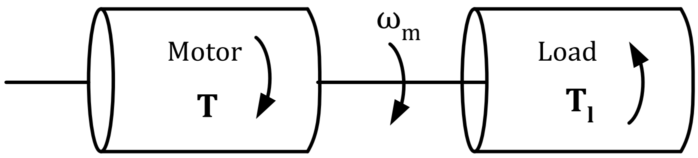

Figure 2 . 1 Equivalent motor load system

- Here,

$$\begin{aligned} J &= \text{moment of inertia } (\text{kg-m2}) \\ \omega_m &= \text{angular velocity } (\text{rad/sec}) \\ T &= \text{developed motor torque } (\text{N-m}) \\ T_l &= \text{load}(\text{resisting}) \text{ torque } (\text{N-m}) \end{aligned}$$

- Here, load torque includes friction & windage torque of motor.
- Fundamental torque equation of motor-load system is given by,

$$T - T_l = \frac{d}{dt}(J\omega_m) = J\frac{d\omega_m}{dt} + \omega_m\frac{dJ}{dt} \quad (2.1)$$

- This equation is applicable to variable inertia drives such as mine winders, reel drives, industrial robots etc. Because, in these application moment of inertia changes with respect to time.
- For the drives with the constant inertia *(dJ/dt) = 0.*

$$T = T_l + J\frac{d\omega_m}{dt} \quad (2.2)$$

- Torque developed by motor is counter balanced by a load torque and a dynamic torque.
- Dynamic torque ($J.d\omega_m/dt$) is only present during the transient operations.
- Acceleration & Deceleration of the drive depends on magnitudes of T & T_l .
- Drive accelerates if T is greater than T_l plus the dynamic torque in order to overcome the drive inertia
- As shown in the figure 2.1, the load torque always opposes the torque developed by the motor.

2.3. Multi-Quadrant Operation of Drive

- A four-quadrant or multiple-quadrant operation is required in industrial as well as commercial applications. These applications require both driving and braking, i.e., motoring and generating capability.
- Some of these applications include electric traction systems, cranes and lifts, cable laying winders, and engine test loading systems.
- In multi-quadrant operation or four quadrant operation, motor accelerates or decelerates depending on whether motor torque is lesser or greater than load torque.
- During motor acceleration, it should supply not only the load torque, but an additional component of load current to overcome the inertia.

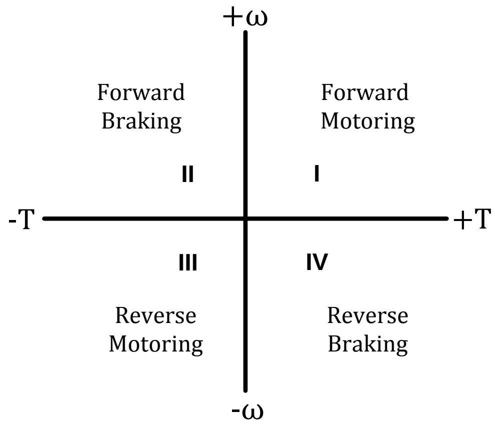

Figure 2 . 2 Multi-quadrant operation

- Motor positive torque produces the acceleration in forward direction. In this, the motor speed is positive when the motor is rotating in forward direction.
- During motor deceleration, the resultant or dynamic torque has a negative sign. This torque assists with motor developed torque and maintains the motion by extracting the energy from stored energy.
- Hence the motor torque is considered as negative if it produces deceleration.
- A motor can be controlled in such a way that it operates in two cases; motor action and braking action.
- Motor action converts the electric energy into mechanical energy and it produces forward motion, hence it called as motoring action, whereas braking action converts mechanical energy to electrical energy which gives forward braking motion, it is termed as generator.
- Similarly, these two actions are performed in case motor operating in reverse direction, i.e., (reverse motoring and reverse braking actions).

2.4. Quadratal Diagram of Speed-Torque Characteristics

- Let us look at the four quadrant operation of a motor driving a hoist load as shown in figure 2.3.
- This hoist consists of a cage with or without any load. A rope, generally made up of a steel wire is wounded on a drum to raise the cage and a balance weight.
- This balance weight or counterweight magnitude is greater than that of empty cage, but less than the loaded cage.
- For each quadrant of operation, direction of rotation, ω, load torque T_L, and motor torque T_m are shown in figure 2.3.
- Consider that the load torque is constant and independent of motor speed.

1st Quadrant Operation

- The hoist in which the loaded cage is moving upwards.
- The direction of rotation of motor, ω will be in anticlockwise direction i.e., positive speed.

- The load torque acts in opposite direction to the direction of motor rotation.
- To raise the hoist to upwards, the motor torque, T_m must act in the same direction of motor speed, ω.
- So both motor speed and motor torque will be positive.
- To make these as positive, the power taken from the supply should be positive. This is called forward motoring.

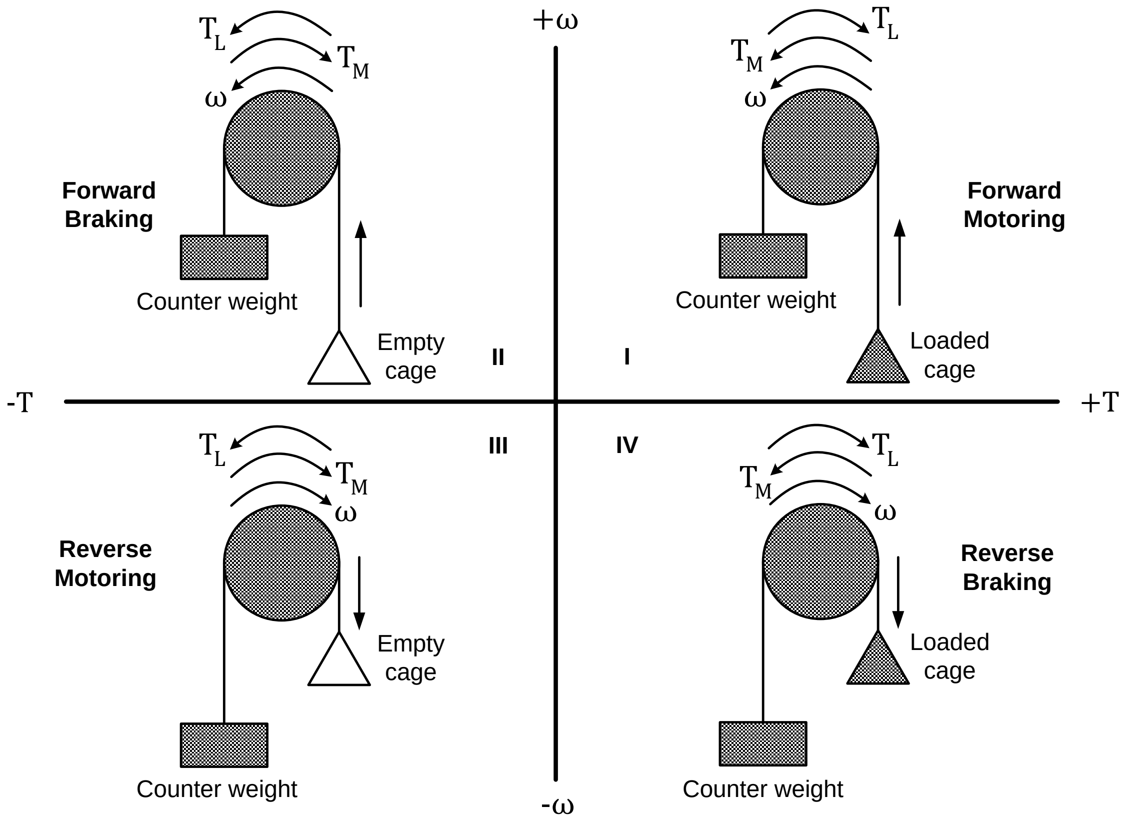

Figure 2 . 3 Four quadrant operation of a motor driving a hoist load

2nd Quadrant Operation

- The hoist in which unloaded cage is moving upwards.
- The counterweight is heavier than the unloaded cage and hence hoist can move upwards at a dangerous speed.
- To prevent this, motor must produce a torque in the opposite direction of motor speed, ω in order to produce brake to the motor.
- Therefore, the motor torque, T_m will be negative and motor speed, ω will be positive.
- This quadrant operation is called forward braking.

3rd Quadrant Operation

- The empty cage is hoisting down.
- The downward journey of empty cage is prevented by the torque exerted by the counterweight.
- So the direction of motor torque, T_m should be in the same direction of motor rotation-ω.

- Due to the downward movement of the cage, the direction of rotation is reversed, i.e., ω is negative and hence T_m is also negative.
- Since the machine acting as motor in reverse direction, it receives the power from the supply and hence power is positive.
- This quadrant operation is called reverse motoring.

4th Quadrant Operation

- Loaded cage is moving downwards.
- The loaded cage is moving downward (of which weight is more than counterweight), the motion takes place without use of any motor.
- There will be a chance to go downward at a dangerous speed because of loaded cage.
- To limit the speed of the cage within a safe range, the electrical machine must act as a brake.
- In this the direction of the motor, ω is negative and hence the motor torque T_m is positive to decrease the speed of the motor.
- Thus, the power is negative that means the electrical machine delivering power to the supply.
- This phenomenon is called as regenerative action. This quadrant operation is called reverse braking.

2.5. Equivalent Values of Drive Parameters

- Different parts of the load may be coupled through different mechanisms, such as V-belts, crankshaft, gears etc.
- These parts may have different speed and different types of motions such as
    - Rotational
    - Translational

(a) Loads with Rotational Motion

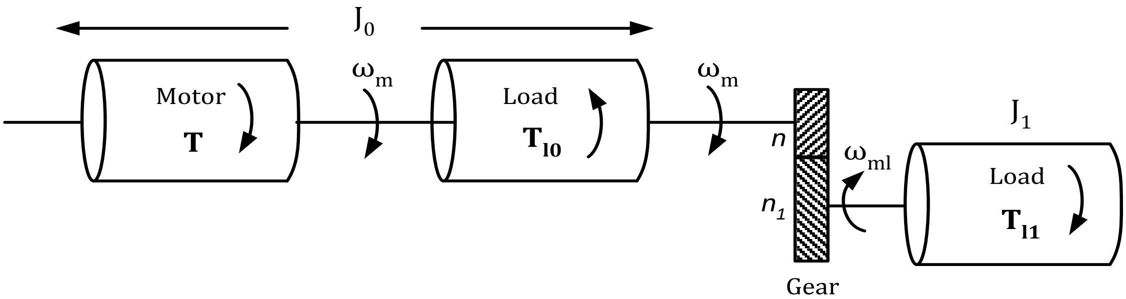

Figure 2 . 4 Rotational load coupled to motor shaft

- Let's consider a motor driving two loads, one coupled directly to the shaft and other through gear with n & n1 teeth

J_0=moment of inertia of motor & load directly coupled to its shaft (kg - m^2)

ω_m=motor speed (rad/sec)

T_{l0}=torque of directly coupled load (N - m)

$$J_1 = \text{moment of inertia of load coupled through a gear (Kg - m}^2\text{)}$$
$$\omega_{ml} = \text{speed of load coupled through a gear (rad/sec)}$$
$$T_{l1} = \text{torque of load coupled through a gear (N - m)}$$

Now,

$$\frac{\omega_{m1}}{\omega_m} = \frac{n}{n_1} = a_1;\ where\ a_1\ is\ gear\ tooth\ ratio \tag{2.3}$$

- If the losses in the transmission are neglected then kinetic energy due to equivalent inertia must be same as kinetic energy of various moving parts,

$$\frac{1}{2}J\omega_m^2 = \frac{1}{2}J_0\omega_m^2 + \frac{1}{2}J_1\omega_{ml}^2 \tag{2.4}$$

- From equations (2.3) and (2.4) we can write

$$J = J_0 + a_1^2 J_1 \tag{2.5}$$

- Power at the motor & load must be same, if transmission efficiency of the gears is η_1 then,

$$T_l\omega_m = T_{l0}\omega_m + \frac{T_{l1}\omega_{ml}}{\eta_1} \tag{2.6}$$

- Where, T_l is the total equivalent torque referred to motor shaft. From equation (2.3) & (2.6)

$$T_l = T_{l0} + \frac{a_1 T_{l1}}{\eta_1} \tag{2.7}$$

- If there are m other loads with moment of inertias $J_1, J_2, \ldots, J_m$ & gear teeth ratios of $a_1, a_2, \ldots, a_m$ then,

$$J = J_0 + a_1^2 J_1 + a_2^2 J_2 + \ldots + a_m^2 J_m \tag{2.8}$$

- If there are m other loads with moment of inertias $J_1, J_2, \ldots, J_m$ & gear teeth ratios of $a_1, a_2, \ldots, a_m$ & transmission efficiencies $\eta_1, \eta_2, \ldots, \eta_m$ then,

$$T_l = T_{l0} + \frac{a_1 T_{l1}}{\eta_1} + \frac{a_2 T_{l2}}{\eta_2} + \ldots + \frac{a_m T_{lm}}{\eta_m} \tag{2.9}$$

(b) Loads with Translation Motion

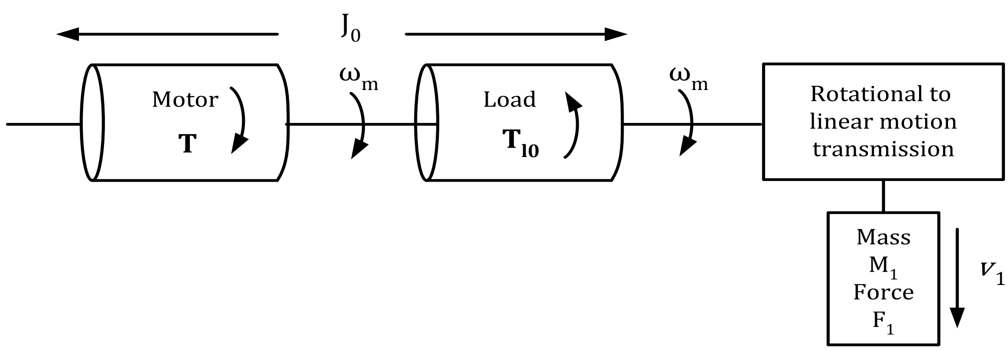

Figure 2 . 5 Loads with translation motion

- Let's consider motor driving two loads, one coupled directly to its shaft & other through a transmission system

J_0=moment of inertia of motor & load directly coupled to its shaft (Kg - m^2)
ω_m=motor speed (rad/sec)
T_{l0}=torque of directly coupled load (N - m)
M_1=mass of load coupled through a transmission system (Kg)
v_1=velocity of load coupled through a transmission system (m/sec)
F_1=force of load coupled through a transmission system (N)

- If the losses in the transmission are neglected then kinetic energy due to equivalent inertia J must be same as kinetic energy of various moving parts.

$$\frac{1}{2}J\omega_m^2 = \frac{1}{2}J_0\omega_m^2 + \frac{1}{2}M_1 v_1^2 \tag{2.9}$$

$$J = J_0 + M_1\left(\frac{v_1}{\omega_m}\right)^2 \tag{2.10}$$

- Similarly, power at the load & motor should be same, thus if efficiency of transmission be η_1 then,

$$T_l\omega_m = T_{l0}\omega_m + \frac{F_1 v_1}{\eta_1} \tag{2.11}$$

$$T_l = T_{l0} + \frac{F_1}{\eta_1}\left(\frac{v_1}{\omega_m}\right) \tag{2.12}$$

- If there are m other loads with translational motion with velocities $v_1, v_2, \ldots, v_m$ & masses $M_1, M_2, \ldots, M_m$ then,

$$J = J_0 + M_1\left(\frac{v_1}{\omega_m}\right)^2 + M_2\left(\frac{v_2}{\omega_m}\right)^2 + \ldots + M_m\left(\frac{v_m}{\omega_m}\right)^2 \tag{2.13}$$

$$T_l = T_{l0} + \frac{F_1}{\eta_1}\left(\frac{v_1}{\omega_m}\right) + \frac{F_2}{\eta_2}\left(\frac{v_2}{\omega_m}\right) + \ldots + \frac{F_m}{\eta_m}\left(\frac{v_m}{\omega_m}\right) \tag{2.14}$$

2.6. Components of Load Torque

- The load torque has the following components
  - *Torque required doing useful mechanical work:* This is the component which is actually doing the work for which the motor is being operated. It may be constant or dependent on speed.
  - *Friction torque:* It opposes the motion and loss of the torque due to friction is called friction torque.
  - *Windage torque:* The resistance of the air results in the loss of torque and is referred as windage torque.

- Generally friction and windage torques are grouped together and can be expressed as $D\omega$ where D is friction constant.
- The magnitude of friction torque depends on the speed. Figure below shows variation in friction torque during rotation in the positive direction and negative direction.

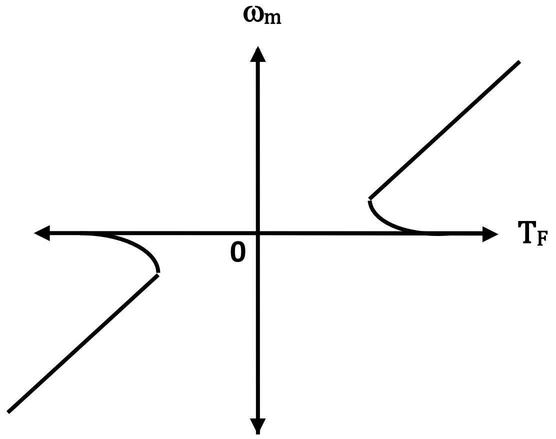

Figure 2 . 6 Variation of friction torque with speed

- This friction at standstill is called static friction. When the motor is to be started the torque developed by the motor must overcome the friction torque. Otherwise, motor will not run.
- Friction torque has three components ***($T_F = T_C + T_V + T_S$)***:
  - *Coulomb friction*: independent of speed. It is constant.
  - *Viscous friction*: varies linearly with speed, $T_V = B\omega_m$. where, ω_m is the speed in rad/sec & B is viscous friction constant.
  - *Stiction or Static friction*: friction at zero speed. For driving, motor torque should at least exceed stiction.

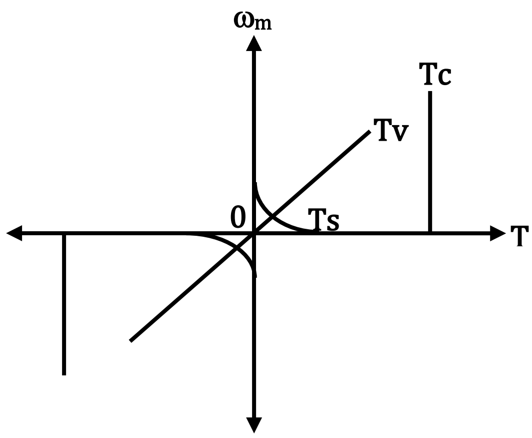

Figure 2 . 7 Friction torque and its components

- Windage torque is required to overcome the resistance offered by the air and it varies with square of the speed and can be written as $T_W = C\,\omega_m^2$
- Therefore, total torque T required at any speed ω_m can be given as

$$T_l = T_L + B\,\omega_m + T_C + C\,\omega_m^2$$

2.7. Nature and Classification of Load Torque

- Load torques can be classified into two categories: active load torque & passive load torque.

(a) Active Load Torque

- It has potential to drive motor under equilibrium condition.
- Such load torque usually retains their sign when the direction of the drive rotation is changed.
- Torque due to gravitational force, tension, compression & torsion come under this category.

(b) Passive Load Torque

- Torque which always opposes the motion is called passive torque.
- Their sign change on the reversal of motion.
- Torque due to friction, windage, cutting etc. fall under this category.

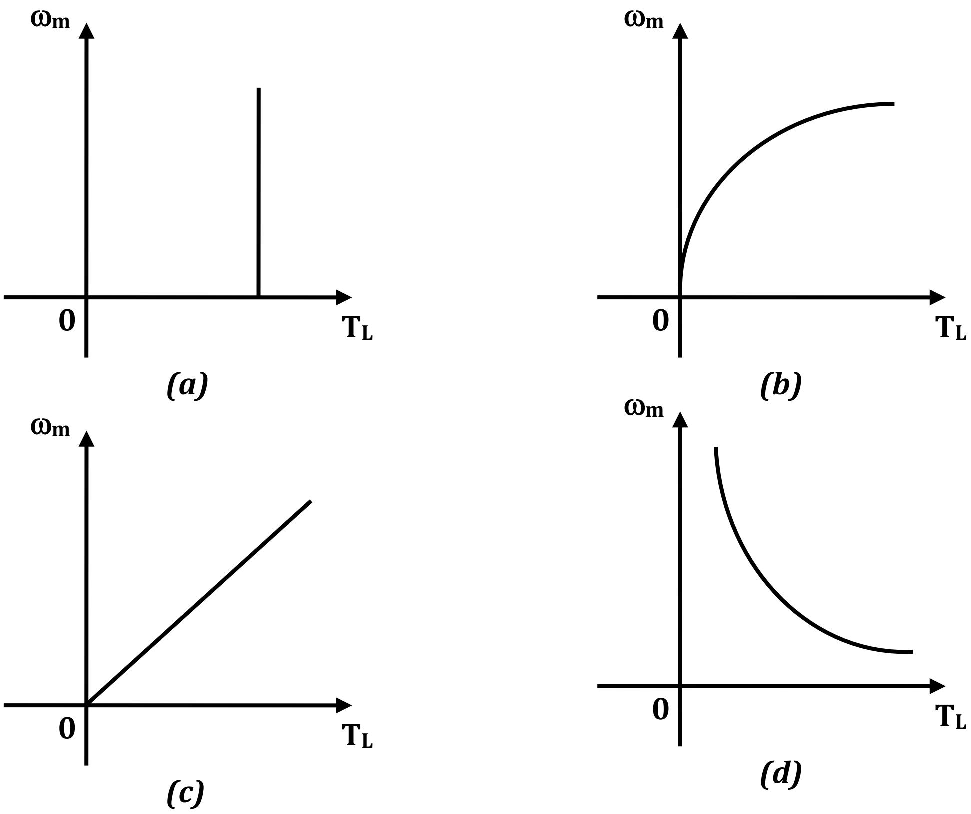

Figure 2 . 1 Nature of load torque (a) T = constant (b) $T \alpha \omega_m^2$ (c) $T \alpha \omega_m$ (d) $T \alpha 1/\omega_m$

- Nature of load torque depends on application: torque is constant (independent of speed) & torque is function of speed

(a) Torque is Constant (Independent of Speed)

- *A low speed hoist*: Torque is constant & independent of speed.
- At low speed windage torque is negligible.
- Net torque is mainly due to gravity that is constant & independent.
- *Paper mill drive*: coulomb friction dominates over other torque components.

(b) Torque is Function of Speed

- *Torque is proportional to square of speed ($T \propto \omega_m^2$)*: It has low starting torque. Examples of such loads are; axial & centrifugal pumps, centrifugal compressors, fans, ship propellers etc.
- *Torque is linearly proportional to speed ($T \propto \omega_m$)*: Examples of such loads are mixers and stirrers.
- *Torque is inversely proportional to speed ($T \propto 1/\omega_m$)*: Here developed power is nearly constant. It is approximately hyperbolic in nature. Examples of such loads are lifts, lathes, wire drawer, winders, reciprocating rolling mills etc.

2.8. Steady State and Transient Stability

- It is quite important to investigate the stability of the electric drive when its equilibrium state is disturbed.
- Before we classify the stability let us understand equilibrium speed and stability terminologies.
- Equilibrium speed: Speed at which motor torque becomes same as load torque is known as equilibrium speed.
- Stability: The stability of motor-load combination is defined as the capacity of the system which enables it to develop forces of such a nature as to restore equilibrium after any small departure therefrom.
- There are two types of disturbances:
    1. Changes from state of equilibrium take place slowly which is related to steady state stability.
    2. Sudden & fast change from the equilibrium state which is related to field of transient stability.

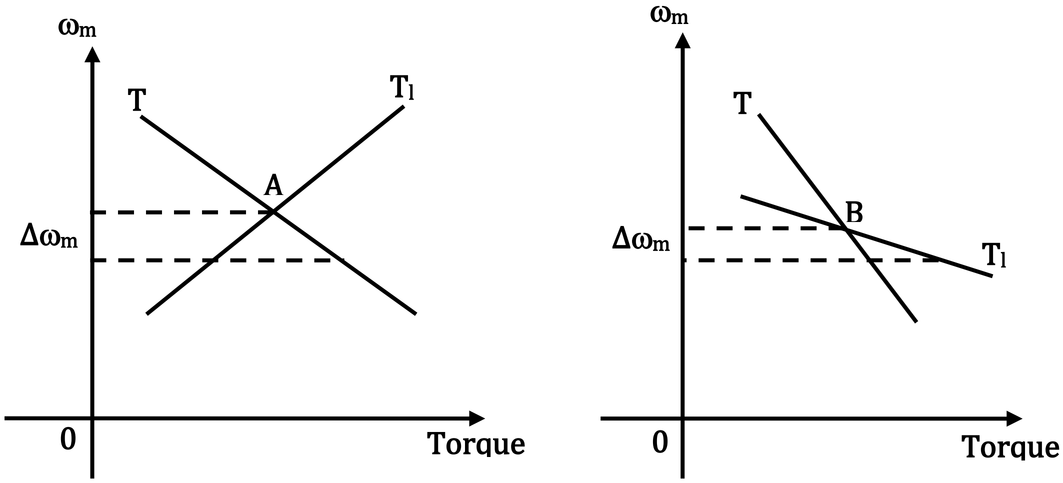

Figure 2 . 2 Steady state stability: point A-stable, point B-unstable

- Let us examine the steady state stability by referring the speed – torque characteristic of a certain load when there is change in the speed caused by disturbances.
- For the first system, let the disturbance causes a reduction of $\Delta\omega_m$ in speed. At new speed motor torque (T) is greater than the load torque (T_l). Therefore, motor will accelerate and operation will be restored to A.

- Similarly, an increase of $\Delta\omega_m$ in speed cased by disturbance will make load torque greater than the motor torque, resulting into deceleration and restoration of operation to point A. Hence, the drive is steady state stable at point A.
- Let us now examine for the second system. A decrease in the speed causes the load torque to become greater than the motor torque, drive decelerates and operating point moves away from B.
- Similarly, when speed increases, the motor torque becomes greater than the load torque that will move the operating point away from B. Thus, B is an unstable point of equilibrium.

2.9. Criteria for Steady-State Stability

- After a small displacement from the equilibrium, the torque equation becomes

$$J\frac{d\omega_m}{dt}+J\frac{d(\Delta\omega_m)}{dt}+T_l+\Delta T_l-T-\Delta T=0 \tag{2.15}$$

Where,

J=moment of inertia (Kg - m^2)
ω_m=motor speed (rad/sec)
T_l=torque developed by Load (N -m)
T=torque developed by motor (N - m)
$\Delta\omega_m$=a small displacement in motor speed from the equilibrium (rad/sec)
ΔT_l=a small displacement in load torque from the equilibrium (N - m)
ΔT=a small displacement in motor torque from the equilibrium (N - m)

- But we know that

$$J\frac{d\omega_m}{dt}+T_l-T=0 \tag{2.16}$$

- Hence,

$$J\frac{d(\Delta\omega_m)}{dt}+\Delta T_l-\Delta T=0 \tag{2.16}$$

$$\Delta T=\frac{dT}{d\omega_m}\Delta\omega_m \tag{2.17}$$

$$\Delta T_l=\frac{dT_l}{d\omega_m}\Delta\omega_m \tag{2.18}$$

- Substitute (2.17) & (2.18) into (2.16),

$$J\frac{d(\Delta\omega_m)}{dt}+\left[\frac{dT_l}{d\omega_m}-\frac{dT}{d\omega_m}\right]\Delta\omega_m=0 \tag{2.19}$$

- Solution of equation (2.19) is

$$\Delta\omega_m=\left(\Delta\omega_m\right)_0 e^{-1/2\left[\frac{dT_l}{d\omega_m}-\frac{dT}{d\omega_m}\right]t} \tag{2.20}$$

- Where, $(\Delta \omega_m)_0$= Initial value of deviation in speed
- Based on the value of exponent there are three cases
    1. Exponent > 0: The speed deviation will increase with time and the system will move away from the equilibrium, results in *unstable system*
    2. Exponent < 0: The speed deviation will decrease with time and the system will move towards the equilibrium, results in *stable system*
    3. Exponent = 0: The equitation is *insufficient* to discuss about stability
- The exponent will always be negative if,

$$\frac{dT_l}{d\omega_m} - \frac{dT}{d\omega_m} > 0 \tag{2.21}$$

2.10. Measurement of Moment of Inertia

- Moment of inertia can be calculated by two ways
    1. *Theoretically*: if dimensions & weight of various parts of load and motor are known
    2. *Practically*: Retardation Test
- In retardation test the drive is run at the speed slightly greater than the rated speed and then supply to it cut off
- Drive continues to run due to stored Kinetic Energy and decelerate due to rotational mechanical losses
- Variation of speed with time is recorded. At any speed ωm, Power P consumed in supplying the rotational losses give by

$$P = Rate\ of\ change\ of\ Kinetic\ energy\ (Watt)$$

$$P = \frac{d}{dt}\left(\frac{1}{2} J \omega_m^2\right) = J \omega_m \frac{d\omega_m}{dt} \tag{2.22}$$

- From retardation test $(d\omega_m)/dt$ at rated speed is obtained. Now drive is reconnect to the supply and run at rated speed and rotational mechanical power input to the drive is measured.
- This is approximately equal to P. Now, J can be calculated from above equation.

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