MODERN PHYSICS

Third edition

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JOHN WILEY & SONS, INC

Chapter 3

THE PARTICLELIKE PROPERTIES OF ELECTROMAGNETIC RADIATION

Thermal emission, the radiation emitted by all objects due to their temperatures, laid the groundwork for the development of quantum mechanics around the beginning of the 20th century. Today we use thermography for many applications, including the study of heat loss by buildings, medical diagnostics, night vision and other surveillance, and monitoring potential volcanoes.

We now turn to a discussion of *wave mechanics,* the second theory on which modern physics is based. One consequence of wave mechanics is the breakdown of the classical distinction between particles and waves. In this chapter we consider the three early experiments that provided evidence that light, which we usually regard as a wave phenomenon, has properties that we normally associate with particles. Instead of spreading its energy smoothly over a wave front, the energy is delivered in concentrated bundles like particles; a discrete bundle (*quantum*) of electromagnetic energy is known as a *photon*.

Before we begin to discuss the experimental evidence that supports the existence of the photon and the particlelike properties of light, we first review some of the properties of electromagnetic waves.

3.1 REVIEW OF ELECTROMAGNETIC WAVES

An electromagnetic field is characterized by its electric field $\vec{\mathbf{E}}$ and magnetic field $\vec{\mathbf{B}}$. For example, the electric field at a distance r from a point charge q at the origin is

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}} \tag{3.1}$$

where $\hat{\mathbf{r}}$ is a unit vector in the radial direction. The magnetic field at a distance r from a long, straight, current-carrying wire along the z axis is

$$\vec{\mathbf{B}} = \frac{\mu_0 i}{2\pi r}\hat{\boldsymbol{\phi}} \tag{3.2}$$

where $\hat{\boldsymbol{\phi}}$ is the unit vector in the azimuthal direction (in the xy plane) in cylindrical coordinates.

If the charges are accelerated, or if the current varies with time, an electromagnetic wave is produced, in which $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ vary not only with $\vec{\mathbf{r}}$ but also with t. The mathematical expression that describes such a wave may have many different forms, depending on the properties of the source of the wave and of the medium through which the wave travels. One special form is the *plane wave,* in which the wave fronts are planes. (A point source, on the other hand, produces spherical waves, in which the wave fronts are spheres.) A plane electromagnetic wave traveling in the positive z direction is described by the expressions

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_0 \sin(kz - \omega t), \qquad \vec{\mathbf{B}} = \vec{\mathbf{B}}_0 \sin(kz - \omega t) \tag{3.3}$$

where the *wave number* k is found from the wavelength λ ($k = 2\pi/\lambda$) and the *angular frequency* ω is found from the frequency f ($\omega = 2\pi f$). Because λ and f are related by $c = \lambda f$, k and ω are also related by $c = \omega/k$.

The polarization of the wave is represented by the vector $\vec{\mathbf{E}}_0$; the plane of polarization is determined by the direction of $\vec{\mathbf{E}}_0$ and the direction of propagation, the z axis in this case. Once we specify the direction of travel and the polarization $\vec{\mathbf{E}}_0$, the direction of $\vec{\mathbf{B}}_0$ is fixed by the requirements that $\vec{\mathbf{B}}$ must be perpendicular to both $\vec{\mathbf{E}}$ and the direction of travel, and that the vector product $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$ point in the direction of travel. For example if $\vec{\mathbf{E}}_0$ is in the x direction ($\vec{\mathbf{E}}_0 = E_0\hat{\mathbf{i}}$, where $\hat{\mathbf{i}}$

is a unit vector in the x direction), then $\vec{\mathbf{B}}_0$ must be in the y direction ($\vec{\mathbf{B}}_0 = B_0\hat{\mathbf{j}}$). Moreover, the magnitude of $\vec{\mathbf{B}}_0$ is determined by

$$B_0 = \frac{E_0}{c} \tag{3.4}$$

where c is the speed of light.

An electromagnetic wave transmits energy from one place to another; the energy flux is specified by the *Poynting vector* $\vec{\mathbf{S}}$:

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}} \tag{3.5}$$

For the plane wave, this reduces to

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}E_0 B_0 \sin^2(kz - \omega t)\hat{\mathbf{k}} \tag{3.6}$$

where $\hat{\mathbf{k}}$ is a unit vector in the z direction. The Poynting vector has dimensions of power (energy per unit time) per unit area—for example, J/s/m^2 or W/m^2. Figure 3.1 shows the orientation of the vectors $\vec{\mathbf{E}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{S}}$ for this special case.

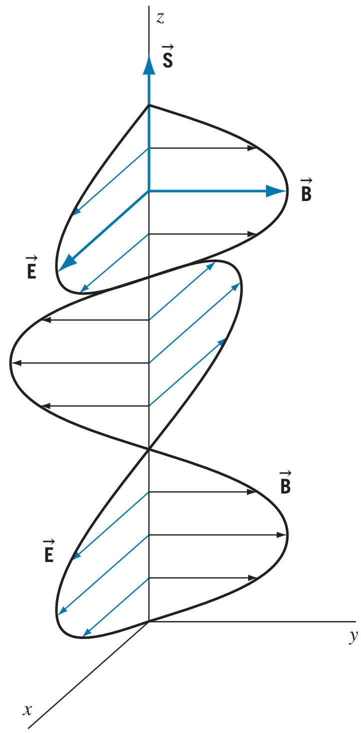


FIGURE 3.1 An electromagnetic wave traveling in the z direction. The electric field $\vec{\mathbf{E}}$ lies in the xz plane and the magnetic field $\vec{\mathbf{B}}$ lies in the yz plane.

Let us imagine the following experiment. We place a detector of electromagnetic radiation (a radio receiver or a human eye) at some point on the z axis, and we determine the electromagnetic power that this plane wave delivers to the receiver. The receiver is oriented with its sensitive area A perpendicular to the z axis, so that the maximum signal is received; we can therefore drop the vector representation of $\vec{\mathbf{S}}$ and work only with its magnitude S. The power P entering the receiver is then

$$P = SA = \frac{1}{\mu_0}E_0 B_0 A \sin^2(kz - \omega t) \tag{3.7}$$

which we can rewrite using Eq. 3.4 as

$$P = \frac{1}{\mu_0 c}E_0^2 A \sin^2(kz - \omega t) \tag{3.8}$$

There are two important features of this expression that you should recognize:

1. The intensity (the average power per unit area) is proportional to E_0^2. This is a general property of waves: *the intensity is proportional to the square of the amplitude*. We will see later that this same property also characterizes the waves that describe the behavior of material particles.

2. The intensity fluctuates with time, with the frequency $2f = 2(\omega/2\pi)$. We don't usually observe this rapid fluctuation—visible light, for example, has a frequency of about 10^{15} oscillations per second, and because our eye doesn't respond that quickly, we observe the time average of many (perhaps 10^{13}) cycles. If T is the observation time (perhaps 10^{-2} s in the case of the eye) then the average power is

$$P_{\text{av}} = \frac{1}{T}\int_0^T P\,dt \tag{3.9}$$

and using Eq. 3.8 we obtain the intensity I:

$$I = \frac{P_{\text{av}}}{A} = \frac{1}{2\mu_0 c}E_0^2 \tag{3.10}$$

because the average value of $\sin^2\theta$ is $^1/_2$.

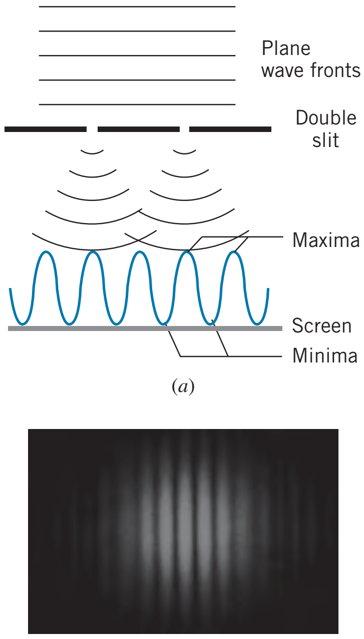


FIGURE 3.2 (*a*) Young's double-slit experiment. A plane wave front passes through both slits; the wave is diffracted at the slits, and interference occurs where the diffracted waves overlap on the screen. (*b*) The interference fringes observed on the screen.

Interference and Diffraction

The property that makes waves a unique physical phenomenon is the *principle of superposition,* which, for example, allows two waves to meet at a point, to cause a combined disturbance at the point that might be greater or less than the disturbance produced by either wave alone, and finally to emerge from the point of "collision" with all of the properties of each wave totally unchanged by the collision. To appreciate this important distinction between material objects and waves, imagine trying that trick with two automobiles!

This special property of waves leads to the phenomena of *interference* and *diffraction.* The simplest and best-known example of interference is *Young's double-slit experiment,* in which a monochromatic plane wave is incident on a barrier in which two narrow slits have been cut. (This experiment was first done with light waves, but in fact any wave will do as well, not only other electromagnetic waves, such as microwaves, but also mechanical waves, such as water waves or sound waves. We assume that the experiment is being done with light waves.)

Figure 3.2 illustrates this experimental arrangement. The plane wave is *diffracted* by each of the slits, so that the light passing through each slit covers a much larger area on the screen than the geometric shadow of the slit. This causes the light from the two slits to overlap on the screen, producing the interference. If we move away from the center of the screen just the right distance, we reach a point at which a wave crest passing through one slit arrives at exactly the same time as the previous wave crest that passed through the other slit. When this occurs, the intensity is a maximum, and a bright region appears on the screen. This is *constructive interference,* and it occurs continually at the point on the screen that is exactly one wavelength further from one slit than from the other. That is, if X_1 and X_2 are the distances from the point on the screen to the two slits, then a condition for maximum constructive interference is $|X_1 - X_2| = \lambda$. Constructive interference occurs when any wave crest from one slit arrives simultaneously with another from the other slit, whether it is the next, or the fourth, or the forty-seventh. The general condition for complete constructive interference is that the difference between X_1 and X_2 be an integral number of wavelengths:

$$|X_1 - X_2| = n\lambda \qquad n = 0, 1, 2, \ldots \tag{3.11}$$

It is also possible for the crest of the wave from one slit to arrive at a point on the screen simultaneously with the trough (valley) of the wave from the other slit. When this happens, the two waves cancel, giving a dark region on the screen. This is known as *destructive interference.* (The existence of destructive interference at intensity minima immediately shows that we must add the electric field vectors $\vec{\mathbf{E}}$ of the waves from the two slits, and not their powers P, because P can never be negative.) Destructive interference occurs whenever the distances X_1 and X_2 are such that the phase of one wave differs from the other by one-half cycle, or by one and one-half cycles, two and one-half cycles, and so forth:

$$|X_1 - X_2| = \tfrac{1}{2}\lambda, \tfrac{3}{2}\lambda, \tfrac{5}{2}\lambda, \ldots = (n + \tfrac{1}{2})\lambda \qquad n = 0, 1, 2, \ldots \tag{3.12}$$

We can find the locations on the screen where the interference maxima occur in the following way. Let d be the separation of the slits, and let D be the distance from the slits to the screen. If y_n is the distance from the center of the screen to the nth maximum, then from the geometry of Figure 3.3 we find (assuming $X_1 > X_2$)

$$X_1^2 = D^2 + \left(\frac{d}{2} + y_n\right)^2 \quad \text{and} \quad X_2^2 = D^2 + \left(\frac{d}{2} - y_n\right)^2 \tag{3.13}$$

Subtracting these equations and solving for y_n, we obtain

$$y_n = \frac{X_1^2 - X_2^2}{2d} = \frac{(X_1 + X_2)(X_1 - X_2)}{2d} \tag{3.14}$$

In experiments with light, D is of order 1 m, and y_n and d are typically at most 1 mm; thus $X_1 \cong D$ and $X_2 \cong D$, so $X_1 + X_2 \cong 2D$, and to a good approximation

$$y_n = (X_1 - X_2)\frac{D}{d} \tag{3.15}$$

Using Eq. 3.11 for the values of $(X_1 - X_2)$ at the maxima, we find

$$y_n = n\frac{\lambda D}{d} \tag{3.16}$$

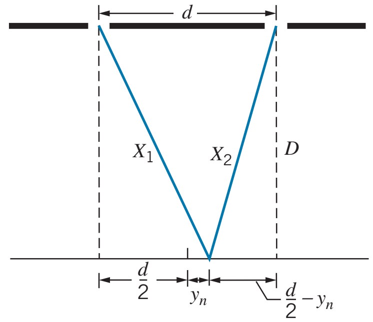


FIGURE 3.3 The geometry of the double-slit experiment.

Crystal Diffraction of X Rays

Another device for observing the interference of light waves is the *diffraction grating,* in which the wave fronts pass through a barrier that has *many* slits (often thousands or tens of thousands) and then recombine. The operation of this device is illustrated in Figure 3.4; interference maxima corresponding to different wavelengths appear at different angles θ, according to

$$d \sin\theta = n\lambda \tag{3.17}$$

where d is the slit spacing and n is the order number of the maximum ($n = 1, 2, 3, \ldots$).

The advantage of the diffraction grating is its superior resolution—it enables us to get very good separation of wavelengths that are close to one another, and thus it is a very useful device for measuring wavelengths. Notice, however, that in order to get reasonable values of the angle θ—for example, $\sin\theta$ in the range of 0.3 to 0.5—we must have d of the order of a few times the wavelength. For visible light this is not particularly difficult, but for radiations of very short wavelength, mechanical construction of a grating is not possible. For example, for X rays with a wavelength of the order of 0.1 nm, we would need to construct a grating in which the slits were less than 1 nm apart, which is roughly the same as the spacing between the atoms of most materials.

The solution to this problem has been known since the pioneering experiments of Laue and Bragg:* use the atoms themselves as a diffraction grating! A beam of X rays sees the regular spacings of the atoms in a crystal as a sort of three-dimensional diffraction grating.

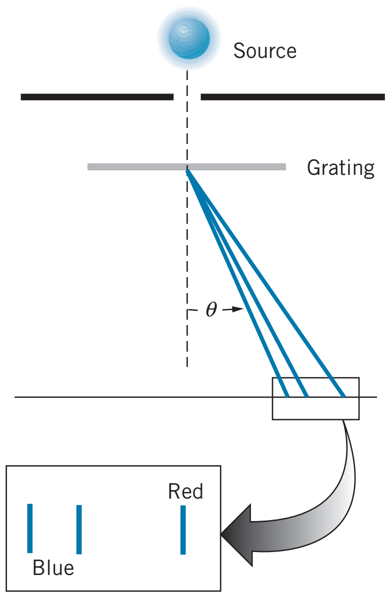


FIGURE 3.4 The use of a diffraction grating to analyze light into its constituent wavelengths.

*Max von Laue (1879–1960, Germany) developed the method of X-ray diffraction for the study of crystal structures, for which he received the 1914 Nobel Prize. Lawrence Bragg (1890–1971, England) developed the Bragg law for X-ray diffraction while he was a student at Cambridge University. He shared the 1915 Nobel Prize with his father, William Bragg, for their research on the use of X rays to determine crystal structures.

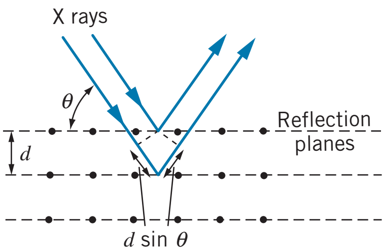


FIGURE 3.5 A beam of X rays reflected from a set of crystal planes of spacing d. The beam reflected from the second plane travels a distance $2d \sin\theta$ greater than the beam reflected from the first plane.

Consider the set of atoms shown in Figure 3.5, which represents a small portion of a two-dimensional slice of the crystal. The X rays are reflected from individual atoms in all directions, but in only one direction will the scattered "wavelets" constructively interfere to produce a reflected beam, and in this case we can regard the reflection as occurring from a plane drawn through the row of atoms. (This situation is identical with the reflection of light from a mirror—only in one direction will there be a beam of reflected light, and in that direction we can regard the reflection as occurring on a plane with the angle of incidence equal to the angle of reflection.)

Suppose the rows of atoms are a distance d apart in the crystal. Then a portion of the beam is reflected from the front plane, and a portion is reflected from the second plane, and so forth. The wave fronts of the beam reflected from the second plane lag behind those reflected from the front plane, because the wave reflected from the second plane must travel an additional distance of $2d \sin\theta$, where θ is the angle of incidence as *measured from the face of the crystal*. (Note that this is different from the usual procedure in optics, in which angles are defined with respect to the *normal* to the surface.) If this path difference is a whole number of wavelengths, the reflected beams interfere constructively and give an intensity maximum; thus the basic expression for the interference maxima in X-ray diffraction from a crystal is

$$2d \sin\theta = n\lambda \qquad n = 1, 2, 3, \ldots \tag{3.18}$$

This result is known as *Bragg's law* for X-ray diffraction. Notice the factor of 2 that appears in Eq. 3.18 but does *not* appear in the otherwise similar expression of Eq. 3.17 for the ordinary diffraction grating.

Example 3.1

A single crystal of table salt (NaCl) is irradiated with a beam of X rays of wavelength 0.250 nm, and the first Bragg reflection is observed at an angle of 26.3°. What is the atomic spacing of NaCl?

Solution

Solving Bragg's law for the spacing d, we have

$$d = \frac{n\lambda}{2\sin\theta} = \frac{0.250\,\text{nm}}{2\sin 26.3^\circ} = 0.282\,\text{nm}$$

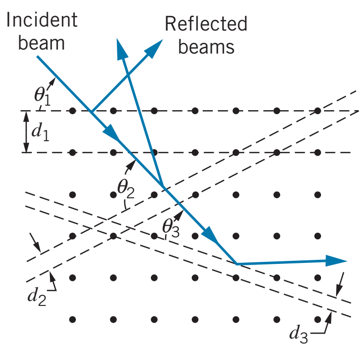


FIGURE 3.6 An incident beam of X rays can be reflected from many different crystal planes.

Our drawing of Figure 3.5 was very arbitrary—we had no basis for choosing which set of atoms to draw the reflecting planes through. Figure 3.6 shows a larger section of the crystal. As you can see, there are many possible reflecting planes, each with a different value of θ and d. (Of course, d_i and θ_i are related and cannot be varied independently.) If we used a beam of X rays of a single wavelength, it might be difficult to find the proper angle and set of planes to observe the interference. However, if we use a beam of X rays of a continuous range of wavelengths, for each d_i and θ_i interference will occur for a certain wavelength λ_i, and so there will be a pattern of interference maxima appearing at different angles of reflection as shown in Figure 3.6. The pattern of interference maxima depends on the spacing and the type of arrangement of the atoms in the crystal.

Figure 3.7 shows sample patterns (called *Laue patterns*) that are obtained from X-ray scattering from two different crystals. The bright dots correspond to interference maxima for wavelengths from the range of incident wavelengths that happen to satisfy Eq. 3.18. The three-dimensional pattern is more complicated

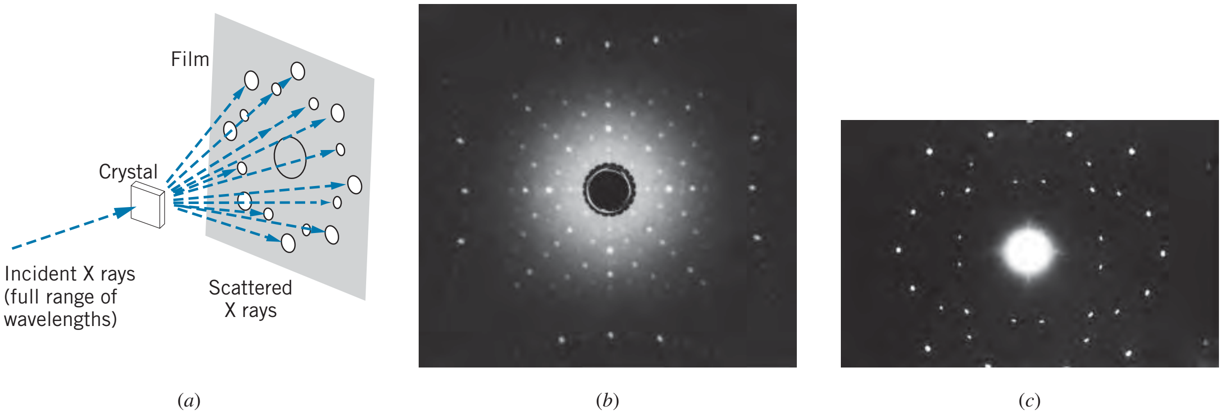


FIGURE 3.7 (*a*) Apparatus for observing X-ray scattering by a crystal. An interference maximum (dot) appears on the film whenever a set of crystal planes happens to satisfy the Bragg condition for a particular wavelength. (*b*) Laue pattern of TiO_2 crystal. (*c*) Laue pattern of a polyethylene crystal. The differences between the two Laue patterns are due to the differences in the geometric structure of the two crystals.

than our two-dimensional drawings, but the individual dots have the same interpretation. Figure 3.8 shows the pattern obtained from a sample that consists of many tiny crystals, rather than one single crystal. (It looks like Figure 3.7*b* or 3.7*c* rotated rapidly about its center.) From such pictures it is also possible to deduce crystal structures and lattice spacing.

All of the examples we have discussed in this section depend on the wave properties of electromagnetic radiation. However, as we now begin to discuss, there are other experiments that cannot be explained if we regard electromagnetic radiation as waves.

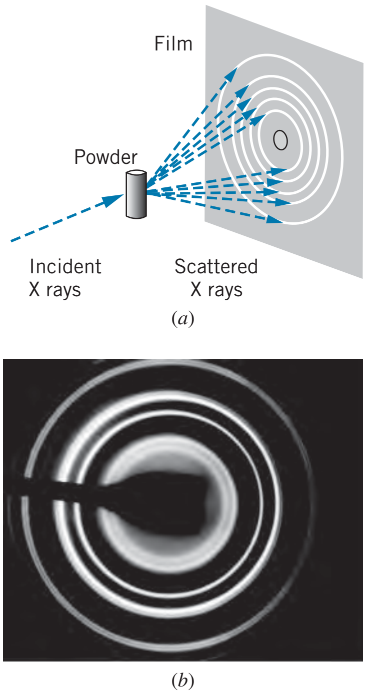


FIGURE 3.8 (*a*) Apparatus for observing X-ray scattering from a powdered or polycrystalline sample. Because the individual crystals have many different orientations, each scattered ray of Figure 3.7 becomes a cone which forms a circle on the film. (*b*) Diffraction pattern (known as *Debye-Scherrer* pattern) of polycrystalline gold.

3.2 THE PHOTOELECTRIC EFFECT

We'll now turn to our discussion of the first of three experiments that cannot be explained by the wave theory of light. When a metal surface is illuminated with light, electrons can be emitted from the surface. This phenomenon, known as the *photoelectric effect,* was discovered by Heinrich Hertz in 1887 in the process of his research into electromagnetic radiation. The emitted electrons are called *photoelectrons.*

A sample experimental arrangement for observing the photoelectric effect is illustrated in Figure 3.9. Light falling on a metal surface (the emitter) can release electrons, which travel to the collector. The experiment must be done in an evacuated tube, so that the electrons do not lose energy in collisions with molecules of the air. Among the properties that can be measured are the rate of electron emission and the maximum kinetic energy of the photoelectrons.*

The rate of electron emission can be measured as an electric current i by an ammeter in the external circuit. The maximum kinetic energy of the electrons

*The electrons can be emitted with many different kinetic energies, depending on how tightly bound they are to the metal. Here we are concerned only with the *maximum* kinetic energy, which depends on the energy needed to remove the least tightly bound electron from the surface of the metal.

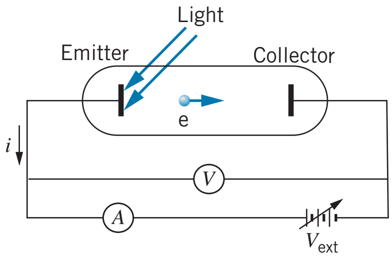


FIGURE 3.9 Apparatus for observing the photoelectric effect. The flow of electrons from the emitter to the collector is measured by the ammeter A as a current i in the external circuit. A variable voltage source V_{ext} establishes a potential difference between the emitter and collector, which is measured by the voltmeter V.

can be measured by applying a negative potential to the collector that is just enough to repel the most energetic electrons, which then do not have enough energy to "climb" the potential energy hill. That is, if the potential difference between the emitter and the collector is ΔV (a negative quantity), then electrons traveling from the emitter to the collector would gain a potential energy of $\Delta U = q\,\Delta V = -e\,\Delta V$ (a positive quantity) and would lose the same amount of kinetic energy. Electrons leaving the emitter with a kinetic energy smaller than this ΔU cannot reach the collector and are pushed back toward the emitter.

As the magnitude of the potential difference is increased, at some point even the most energetic electrons do not have enough kinetic energy to reach the collector. This potential, called the *stopping potential* V_s, is determined by increasing the magnitude of the voltage until the ammeter current drops to zero. At this point the maximum kinetic energy K_{max} of the electrons as they leave the emitter is just equal to the kinetic energy eV_s lost by the electrons in "climbing" the hill:

$$K_{max} = eV_s \tag{3.19}$$

where e is the magnitude of the electric charge of the electron. Typical values of V_s are a few volts.*

In the classical picture, the surface of the metal is illuminated by an electromagnetic wave of intensity I. The surface absorbs energy from the wave until the energy exceeds the binding energy of the electron to the metal, at which point the electron is released. The minimum quantity of energy needed to remove an electron is called the *work function* ϕ of the material. Table 3.1 lists some values of the work function of different materials. You can see that the values are typically a few electron-volts.

TABLE 3.1 Some Photoelectric Work Functions

Material	ϕ (eV)
Na	2.28
Al	4.08
Co	3.90
Cu	4.70
Zn	4.31
Ag	4.73
Pt	6.35
Pb	4.14

The Classical Theory of the Photoelectric Effect

What does the classical wave theory predict about the properties of the emitted photoelectrons?

1. *The maximum kinetic energy of the electrons should be proportional to the intensity of the radiation.* As the brightness of the light source is increased, more energy is delivered to the surface (the electric field is greater) and the electrons should be released with greater kinetic energies. Equivalently, increasing the intensity of the light source increases the electric field $\vec{\mathbf{E}}$ of the wave, which also increases the force $\vec{\mathbf{F}} = -e\vec{\mathbf{E}}$ on the electron and its kinetic energy when it eventually leaves the surface.
2. *The photoelectric effect should occur for light of any frequency or wavelength.* According to the wave theory, as long as the light is intense enough to release electrons, the photoelectric effect should occur no matter what the frequency or wavelength.
3. *The first electrons should be emitted in a time interval of the order of seconds after the radiation begins to strike the surface.* In the wave theory, the energy of the wave is uniformly distributed over the wave front. If the electron absorbs energy directly from the wave, the amount of energy delivered to any

*The potential difference ΔV read by the voltmeter is not equal to the stopping potential when the emitter and collector are made of different materials. In that case a correction must be applied to account for the *contact potential difference* between the emitter and collector.

electron is determined by how much radiant energy is incident on the surface area in which the electron is confined. Assuming this area is about the size of an atom, a rough calculation leads to an estimate that the time lag between turning on the light and observing the first photoelectrons should be of the order of seconds (see Example 3.2).

Example 3.2

A laser beam with an intensity of $120\,\text{W/m}^2$ (roughly that of a small helium-neon laser) is incident on a surface of sodium. It takes a minimum energy of 2.3 eV to release an electron from sodium (the work function ϕ of sodium). Assuming the electron to be confined to an area of radius equal to that of a sodium atom (0.10 nm), how long will it take for the surface to absorb enough energy to release an electron?

Solution

The average power P_{av} delivered by the wave of intensity I to an area A is IA. An atom on the surface displays a "target area" of $A = \pi r^2 = \pi(0.10 \times 10^{-9}\,\text{m})^2 = 3.1 \times 10^{-20}\,\text{m}^2$. If the entire electromagnetic power is delivered to the electron, energy is absorbed at the rate $\Delta E/\Delta t = P_{\text{av}}$. The time interval Δt necessary to absorb an energy $\Delta E = \phi$ can be expressed as

$$\Delta t = \frac{\Delta E}{P_{\text{av}}} = \frac{\phi}{IA}$$

$$= \frac{(2.3\,\text{eV})(1.6 \times 10^{-19}\,\text{J/eV})}{(120\,\text{W/m}^2)(3.1 \times 10^{-20}\,\text{m}^2)} = 0.10\,\text{s}$$

In reality, electrons in metals are not always bound to individual atoms but instead can be free to roam throughout the metal. However, no matter what reasonable estimate we make for the area over which the energy is absorbed, the characteristic time for photoelectron emission is estimated to have a magnitude of the order of seconds, in a range easily accessible to measurement.

The experimental characteristics of the photoelectric effect were well known by the year 1902. How do the predictions of the classical theory compare with the experimental results?

1. *For a fixed value of the wavelength or frequency of the light source, the maximum kinetic energy of the emitted photoelectrons (determined from the stopping potential) is totally independent of the intensity of the light source*. Figure 3.10 shows a representation of the experimental results. Doubling the intensity of the source leaves the stopping potential unchanged, indicating no change in the maximum kinetic energy of the electrons. This experimental result disagrees with the wave theory, which predicts that the maximum kinetic energy should depend on the intensity of the light.
2. *The photoelectric effect does not occur at all if the frequency of the light source is below a certain value*. This value, which is characteristic of the kind of metal surface used in the experiment, is called the *cutoff frequency* f_c. Above f_c, any light source, no matter how weak, will cause the emission of photoelectrons; below f_c, no light source, no matter how strong, will cause the emission of photoelectrons. This experimental result also disagrees with the predictions of the wave theory.
3. *The first photoelectrons are emitted virtually instantaneously (within 10^{-9} s) after the light source is turned on*. The wave theory predicts a measurable time delay, so this result also disagrees with the wave theory.

These three experimental results all suggest the complete failure of the wave theory to account for the photoelectric effect.

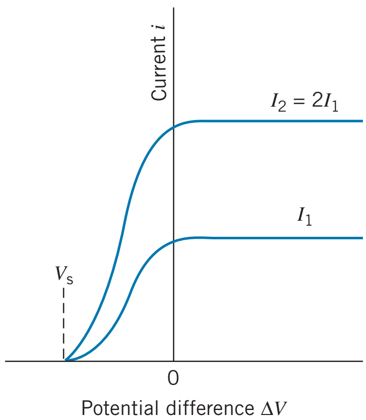


FIGURE 3.10 The photoelectric current i as a function of the potential difference ΔV for two different values of the intensity of the light. When the intensity I is doubled, the current is doubled (twice as many photoelectrons are emitted), but the stopping potential V_s remains the same.

The Quantum Theory of the Photoelectric Effect

A successful theory of the photoelectric effect was developed in 1905 by Albert Einstein. Five years earlier, in 1900, the German physicist Max Planck had developed a theory to explain the wavelength distribution of light emitted by hot, glowing objects (called *thermal radiation*, which is discussed in the next section of this chapter). Based partly on Planck's ideas, Einstein proposed that the energy of electromagnetic radiation is not continuously distributed over the wave front, but instead is concentrated in localized bundles or *quanta* (also known as *photons*). The energy of a photon associated with an electromagnetic wave of frequency f is

$$E = hf \tag{3.20}$$

where h is a proportionality constant known as *Planck's constant.* The photon energy can also be related to the wavelength of the electromagnetic wave by substituting $f = c/\lambda$, which gives

$$E = \frac{hc}{\lambda} \tag{3.21}$$

We often speak about photons as if they were particles, and as concentrated bundles of energy they have particlelike properties. Like the electromagnetic waves, photons travel at the speed of light, and so they must obey the relativistic relationship $p = E/c$. Combining this with Eq. 3.21, we obtain

$$p = \frac{h}{\lambda} \tag{3.22}$$

Photons carry linear momentum as well as energy, and thus they share this characteristic property of particles.

Because a photon travels at the speed of light, it must have zero mass. Otherwise its energy and momentum would be infinite. Similarly, a photon's rest energy $E_0 = mc^2$ must also be zero.

In Einstein's interpretation, a photoelectron is released as a result of an encounter with a *single photon.* The entire energy of the photon is delivered instantaneously to a *single photoelectron.* If the photon energy hf is greater than the work function ϕ of the material, the photoelectron will be released. If the photon energy is smaller than the work function, the photoelectric effect will not occur. This explanation thus accounts for two of the failures of the wave theory: the existence of the cutoff frequency and the lack of any measurable time delay.

If the photon energy hf exceeds the work function, the excess energy appears as the kinetic energy of the electron:

$$K_{\text{max}} = hf - \phi \tag{3.23}$$

The intensity of the light source does not appear in this expression! For a fixed frequency, doubling the intensity of the light means that twice as many photons strike the surface and twice as many photoelectrons are released, but they all have precisely the same maximum kinetic energy.

You can think of Eq. 3.23 as giving a relationship between energy quantities in analogy to making a purchase at a store. The quantity hf represents the payment you hand to the cashier, the quantity ϕ represents the cost of the object, and K_{max} represents the change you receive. In the photoelectric effect, hf is the amount of energy that is available to "purchase" an electron from the surface, the work function ϕ is the "cost" of removing the least tightly bound electron from the surface, and the difference between the available energy and the removal cost is the leftover energy that appears as the kinetic energy of the emitted electron. (The more tightly bound electrons have a greater "cost" and so emerge with smaller kinetic energies.)

A photon that supplies an energy equal to ϕ, exactly the minimum amount needed to remove an electron, corresponds to light of frequency equal to the cutoff frequency f_c. At this frequency, there is no excess energy for kinetic energy, so Eq. 3.23 becomes $hf_c = \phi$, or

$$f_c = \frac{\phi}{h} \tag{3.24}$$

The corresponding cutoff wavelength $\lambda_c = c/f_c$ is

$$\lambda_c = \frac{hc}{\phi} \tag{3.25}$$

The cutoff wavelength represents the *largest* wavelength for which the photoelectric effect can be observed for a surface with the work function ϕ.

The photon theory appears to explain all of the observed features of the photoelectric effect. The most detailed test of the theory was done by Robert Millikan in 1915. Millikan measured the maximum kinetic energy (stopping potential) for different frequencies of the light and obtained a plot of Eq. 3.23. A sample of his results is shown in Figure 3.11. From the slope of the line, Millikan obtained a value for Planck's constant of

$$h = 6.57 \times 10^{-34}\ \text{J}\cdot\text{s}$$

In part for his detailed experiments on the photoelectric effect, Millikan was awarded the 1923 Nobel Prize in physics. Einstein was awarded the 1921 Nobel Prize for his photon theory as applied to the photoelectric effect.

As we discuss in the next section, the wavelength distribution of thermal radiation also yields a value for Planck's constant, which is in good agreement with Millikan's value derived from the photoelectric effect. Planck's constant is one of the fundamental constants of nature; just as c is the characteristic constant of relativity, h is the characteristic constant of quantum mechanics. The value of Planck's constant has been measured to great precision in a variety of experiments. The presently accepted value is

$$h = 6.6260696 \times 10^{-34}\ \text{J}\cdot\text{s}$$

This is an experimentally determined value, with a relative uncertainty of about 5×10^{-8} (± 3 units in the last digit).

Robert A. Millikan (1868–1953, United States). Perhaps the best experimentalist of his era, his work included the precise determination of Planck's constant using the photoelectric effect (for which he received the 1923 Nobel Prize) and the measurement of the charge of the electron (using his famous "oil-drop" apparatus).

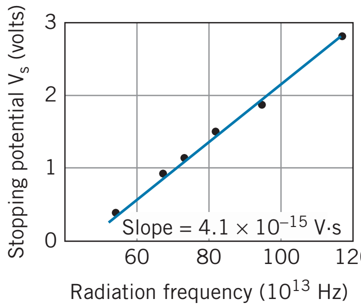


FIGURE 3.11 Millikan's results for the photoelectric effect in sodium. The slope of the line is h/e; the experimental determination of the slope gives a way of determining Planck's constant. The intercept should give the cutoff frequency; however, in Millikan's time the contact potentials of the electrodes were not known precisely and so the vertical scale is displaced by a few tenths of a volt. The slope not affected by this correction.

Example 3.3

(*a*) What are the energy and momentum of a photon of red light of wavelength 650 nm? (*b*) What is the wavelength of a photon of energy 2.40 eV?

Solution

(*a*) Using Eq. 3.21 we obtain

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^{8}\ \text{m/s})}{650 \times 10^{-9}\ \text{m}} = 3.06 \times 10^{-19}\ \text{J}$$

Converting to electron-volts, we have

$$E = \frac{3.06 \times 10^{-19}\ \text{J}}{1.60 \times 10^{-19}\ \text{J/eV}} = 1.91\ \text{eV}$$

This type of problem can be simplified if we express the combination hc in units of eV · nm:

$$E = \frac{hc}{\lambda} = \frac{1240\ \text{eV}\cdot\text{nm}}{650\ \text{nm}} = 1.91\ \text{eV}$$

The momentum is found in a similar way, using Eq. 3.22

$$p = \frac{h}{\lambda} = \frac{1}{c}\frac{hc}{\lambda} = \frac{1}{c}\left(\frac{1240\ \text{eV}\cdot\text{nm}}{650\ \text{nm}}\right) = 1.91\ \text{eV}/c$$

The momentum could also be found directly from the energy:

$$p = \frac{E}{c} = \frac{1.91\ \text{eV}}{c} = 1.91\ \text{eV}/c$$

(It may be helpful to review the discussion in Example 2.11 about these units of momentum.)

(*b*) Solving Eq. 3.21 for λ, we find

$$\lambda = \frac{hc}{E} = \frac{1240\ \text{eV}\cdot\text{nm}}{2.40\ \text{eV}} = 517\ \text{nm}$$

Example 3.4

The work function for tungsten metal is 4.52 eV. (*a*) What is the cutoff wavelength λ_c for tungsten? (*b*) What is the maximum kinetic energy of the electrons when radiation of wavelength 198 nm is used? (*c*) What is the stopping potential in this case?

Solution

(*a*) Equation 3.25 gives

$$\lambda_c = \frac{hc}{\phi} = \frac{1240\ \text{eV}\cdot\text{nm}}{4.52\ \text{eV}} = 274\ \text{nm}$$

in the ultraviolet region.

(*b*) At the shorter wavelength,

$$K_{\text{max}} = hf - \phi = \frac{hc}{\lambda} - \phi = \frac{1240\ \text{eV}\cdot\text{nm}}{198\ \text{nm}} - 4.52\ \text{eV} = 1.74\ \text{eV}$$

(*c*) The stopping potential is the voltage corresponding to K_{max}:

$$V_s = \frac{K_{\text{max}}}{e} = \frac{1.74\ \text{eV}}{e} = 1.74\ \text{V}$$

3.3 THERMAL RADIATION

The second type of experiment we discuss that cannot be explained by the classical wave theory is *thermal radiation*, which is the electromagnetic radiation emitted by all objects because of their temperature. At room temperature the thermal radiation is mostly in the infrared region of the spectrum, where our eyes are not sensitive. As we heat objects to higher temperatures, they may emit visible light.

A typical experimental arrangement is shown in Figure 3.12. An object is maintained at a temperature T_1. The radiation emitted by the object is detected by an apparatus that is sensitive to the wavelength of the radiation. For example, a dispersive medium such as a prism can be used so that different wavelengths appear at different angles θ. By moving the radiation detector to different angles θ we can measure the intensity* of the radiation at a specific wavelength. The detector is not a geometrical point (hardly an efficient detector!) but instead subtends a small range of angles $\Delta\theta$, so what we really measure is the amount of radiation in some range $\Delta\theta$ at θ, or, equivalently, in some range $\Delta\lambda$ at λ.

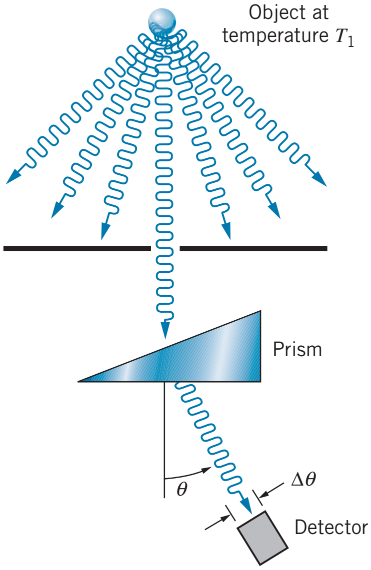


FIGURE 3.12 Measurement of the spectrum of thermal radiation. A device such as a prism is used to separate the wavelengths emitted by the object.

Many experiments were done in the late 19th century to study the wavelength spectrum of thermal radiation. These experiments, as we shall see, gave results that totally disagreed with the predictions of the classical theories of thermodynamics and electromagnetism; instead, the successful analysis of the experiments provided the first evidence of the quantization of energy, which would eventually be seen as the basis for the new quantum theory.

Let's first review the experimental results. The goal of these experiments was to measure the intensity of the radiation emitted by the object as a function of wavelength. Figure 3.13 shows a typical set of experimental results when the object is at a temperature $T_1 = 1000\,\text{K}$. If we now change the temperature of the object to a different value T_2, we obtain a different curve, as shown in Figure 3.13 for $T_2 = 1250\,\text{K}$. If we repeat the measurement for many different temperatures, we obtain systematic results for the radiation intensity that reveal two important characteristics:

1. The total intensity radiated over all wavelengths (that is, the area under each curve) increases as the temperature is increased. This is not a surprising result: we commonly observe that a glowing object glows brighter and thus radiates more energy as we increase its temperature. From careful measurement, we find that the total intensity increases as the fourth power of the absolute or kelvin temperature:

$$I = \sigma T^4 \tag{3.26}$$

   where we have introduced the proportionality constant σ. Equation 3.26 is called *Stefan's law* and the constant σ is called the *Stefan-Boltzmann constant.* Its value can be determined from experimental results such as those illustrated in Figure 3.13:

$$\sigma = 5.67037 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4$$

2. The wavelength λ_{max} at which the emitted intensity reaches its maximum value decreases as the temperature is increased, in inverse proportion to the temperature: $\lambda_{\text{max}} \propto 1/T$. From results such as those of Figure 3.13, we can determine the proportionality constant, so that

$$\lambda_{\text{max}} T = 2.8978 \times 10^{-3}\ \text{m} \cdot \text{K} \tag{3.27}$$

   This result is known as *Wien's displacement law*; the term "displacement" refers to the way the peak is moved or displaced as the temperature is

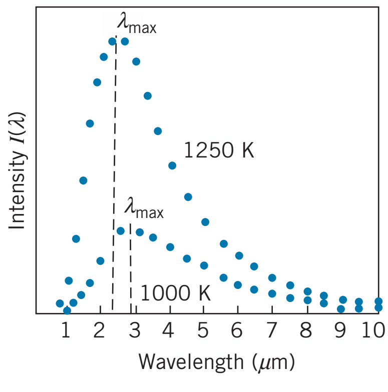


FIGURE 3.13 A possible result of the measurement of the radiation intensity over many different wavelengths. Each different temperature of the emitting body gives a different peak λ_{max}.

*As always, intensity means energy per unit time per unit area (or power per unit area), as in Eq. 3.10. Previously, "unit area" referred to the wave front, such as would be measured if we recorded the waves with an antenna of a certain area. Here, "unit area" indicates the electromagnetic radiation emitted from each unit area of the surface of the object whose thermal emissions are being observed.

varied. Wien's law is qualitatively consistent with our common observation that heated objects first begin to glow with a red color, and at higher temperatures the color becomes more yellow. As the temperature is increased, the wavelength at which most of the radiation is emitted moves from the longer-wavelength (red) part of the visible region toward medium wavelengths. The term "white hot" refers to an object that is hot enough to produce the mixture of all wavelengths in the visible region to make white light.

Example 3.5

(*a*) At what wavelength does a room-temperature ($T = 20^\circ$C) object emit the maximum thermal radiation? (*b*) To what temperature must we heat it until its peak thermal radiation is in the red region of the spectrum ($\lambda = 650$ nm)? (*c*) How many times as much thermal radiation does it emit at the higher temperature?

Solution

(*a*) Using the absolute temperature, $T_1 = 273 + 20 = 293$ K, Wien's displacement law gives

$$\lambda_{max} = \frac{2.8978 \times 10^{-3}\ \text{m}\cdot\text{K}}{T_1} = \frac{2.8978 \times 10^{-3}\ \text{m}\cdot\text{K}}{293\ \text{K}} = 9.89\ \mu\text{m}$$

This is in the infrared region of the electromagnetic spectrum.

(*b*) For $\lambda_{max} = 650$ nm, we again use Wien's displacement law to find the new temperature T_2 :

$$T_2 = \frac{2.8978 \times 10^{-3}\ \text{m}\cdot\text{K}}{\lambda_{max}} = \frac{2.8978 \times 10^{-3}\ \text{m}\cdot\text{K}}{650 \times 10^{-9}\ \text{m}} = 4460\ \text{K}$$

(*c*) The total intensity of radiation is proportional to T^4, so the ratio of the total thermal emissions will be

$$\frac{I_2}{I_1} = \frac{\sigma T_2^4}{\sigma T_1^4} = \frac{(4460\ \text{K})^4}{(293\ \text{K})^4} = 5.37 \times 10^4$$

Be sure to notice the use of absolute (kelvin) temperatures in this example.

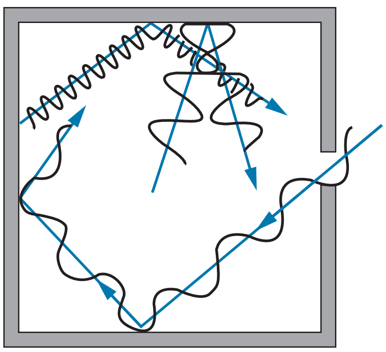


FIGURE 3.14 A cavity filled with electromagnetic radiation in thermal equilibrium with its walls at temperature T. Some radiation escapes through the hole, which represents an ideal blackbody.

The theoretical analysis of the emission of thermal radiation from an arbitrary object is extremely complicated. It depends on details of the surface properties of the object, and it also depends on how much radiation the object reflects from its surroundings. To simplify our analysis, we consider a special type of object called a *blackbody*, which absorbs all radiation incident on it and reflects none of the incident radiation.

To simplify further, we consider a special type of blackbody: a hole in a hollow metal box whose walls are in thermal equilibrium at temperature T. The box is filled with electromagnetic radiation that is emitted and reflected by the walls. A small hole in one wall of the box allows some of the radiation to escape (Figure 3.14). *It is the hole, and not the box itself, that is the blackbody.* Radiation from outside that is incident on the hole gets lost inside the box and has a negligible chance of reemerging from the hole; thus no reflections occur from the blackbody (the hole). The radiation that emerges from the hole is just a sample of the radiation inside the box, so understanding the nature of the radiation inside the box allows us to understand the radiation that leaves through the hole.

Let's consider the radiation inside the box. It has an energy density (energy per unit volume) per unit wavelength interval $u(\lambda)$. That is, if we could look into the interior of the box and measure the energy density of the electromagnetic radiation with wavelengths between λ and $\lambda + d\lambda$ in a small volume element, the result would be $u(\lambda)d\lambda$. For the radiation in this wavelength interval, what is the corresponding intensity (power per unit area) emerging from the hole? At any particular instant, half of the radiation in the box will be moving away from the hole. The other half of the radiation is moving toward the hole at velocity of magnitude c but directed over a range of angles. Averaging over this range of angles to evaluate the energy flowing perpendicular to the surface of the hole introduces another factor of $^1/_2$, so the contribution of the radiation in this small wavelength interval to the intensity passing through the hole is

$$I(\lambda) = \frac{c}{4}u(\lambda) \tag{3.28}$$

The quantity $I(\lambda)d\lambda$ is the radiant intensity in the small interval $d\lambda$ at the wavelength λ. This is the quantity whose measurement gives the results displayed in Figure 3.13. Each data point represents a measurement of the intensity in a small wavelength interval. The goal of the theoretical analysis is to find a mathematical function $I(\lambda)$ that gives a smooth fit through the data points of Figure 3.13.

If we wish to find the total intensity emitted in the region between wavelengths λ_1 and λ_2, we divide the region into narrow intervals $d\lambda$ and add the intensities in each interval, which is equivalent to the integral between those limits:

$$I(\lambda_1{:}\lambda_2) = \int_{\lambda_1}^{\lambda_2} I(\lambda)\, d\lambda \tag{3.29}$$

This is similar to Eq. 1.27 for determining the number of molecules with energies between two limits. The total emitted intensity can be found by integrating over all wavelengths:

$$I = \int_0^{\infty} I(\lambda)\, d\lambda \tag{3.30}$$

This total intensity should work out to be proportional to the 4th power of the temperature, as required by Stefan's law (Eq. 3.26).

Classical Theory of Thermal Radiation

Before discussing the quantum theory of thermal radiation, let's see what the classical theories of electromagnetism and thermodynamics can tell us about the dependence of I on λ. The complete derivation is not given here, only a brief outline of the theory.* The derivation involves first computing the amount of radiation (number of waves) at each wavelength and then finding the contribution of each wave to the total energy in the box.

*For a more complete derivation, see R. Eisberg and R. Resnick, *Quantum Theory of Atoms, Molecules, Solids, Nuclei, and Particles*, 2nd edition (Wiley, 1985), pp. 9–13.

1. *The box is filled with electromagnetic standing waves.* If the walls of the box are metal, radiation is reflected back and forth with a node of the electric field at each wall (the electric field must vanish inside a conductor). This is the same condition that applies to other standing waves, like those on a stretched string or a column of air in an organ pipe.
2. *The number of standing waves with wavelengths between λ and $\lambda + d\lambda$ is*

$$N(\lambda)\,d\lambda = \frac{8\pi V}{\lambda^4}\,d\lambda \tag{3.31}$$

where V is the volume of the box. For one-dimensional standing waves, as on a stretched string of length L, the allowed wavelength are $\lambda = 2L/n$, $(n = 1, 2, 3, \ldots)$. The number of possible standing waves with wavelengths between λ_1 and λ_2 is $n_2 - n_1 = 2L(1/\lambda_2 - 1/\lambda_1)$. In the small interval from λ to $\lambda + d\lambda$, the number of standing waves is $N(\lambda)d\lambda = |dn/d\lambda|d\lambda = (2L/\lambda^2)d\lambda$. Equation 3.31 can be obtained by extending this approach to three dimensions.
3. *Each individual wave contributes an average energy of kT to the radiation in the box.* This result follows from an analysis similar to that of Section 1.3 for the statistical mechanics of gas molecules. In this case we are interested in the statistics of the oscillating atoms in the walls of the cavity, which are responsible for setting up the standing electromagnetic waves in the cavity. For a one-dimensional oscillator, the energies are distributed according to the Maxwell-Boltzmann distribution:*

$$N(E) = \frac{N}{kT}e^{-E/kT} \tag{3.32}$$

Recall from Section 1.3 that $N(E)$ is defined so that the number of oscillators with energies between E and $E + dE$ is $dN = N(E)dE$, and thus the total number of oscillators at all energies is $\int dN = \int_0^\infty N(E)dE$, which (as you should show) works out to N. The average energy per oscillator is then found in the same way as the average energy of a gas molecule (Eq. 1.25):

$$E_{\text{av}} = \frac{1}{N}\int_0^\infty E\,N(E)\,dE = \frac{1}{kT}\int_0^\infty E\,e^{-E/kT}\,dE \tag{3.33}$$

which does indeed work out to $E_{\text{av}} = kT$.

Putting all these ingredients together, we can find the energy density of radiation in the wavelength interval $d\lambda$ inside the cavity: energy density = (number of standing waves per unit volume) × (average energy per standing wave) or

$$u(\lambda)\,d\lambda = \frac{N(\lambda)\,d\lambda}{V}kT = \frac{8\pi}{\lambda^4}kT\,d\lambda \tag{3.34}$$

The corresponding intensity per unit wavelength interval $d\lambda$ is

$$I(\lambda) = \frac{c}{4}u(\lambda) = \frac{c}{4}\frac{8\pi}{\lambda^4}kT = \frac{2\pi c}{\lambda^4}kT \tag{3.35}$$

This result is known as the *Rayleigh-Jeans formula*; based firmly on the classical theories of electromagnetism and thermodynamics, it represents our best attempt

*The exponential part of this expression is that same as that of Eq. 1.22 for gas molecules, but the rest of the equation is different, because the statistical behavior of one-dimensional oscillators is different from that of gas molecules moving in three dimensions. We'll consider these calculations in greater detail in Chapter 10.

to apply classical physics to understanding the problem of blackbody radiation. In Figure 3.15 the intensity calculated from the Rayleigh-Jeans formula is compared with typical experimental results. The intensity calculated with Eq. 3.35 approaches the data at long wavelengths, but at short wavelengths, the classical theory (which predicts $u \to \infty$ as $\lambda \to 0$) fails miserably. The failure of the Rayleigh-Jeans formula at short wavelengths is known as the *ultraviolet catastrophe* and represents a serious problem for classical physics, because the theories of thermodynamics and electromagnetism on which the Rayleigh-Jeans formula is based have been carefully tested in many other circumstances and found to give extremely good agreement with experiment. It is apparent in the case of blackbody radiation that the classical theories do not work, and that a new kind of physical theory is needed.

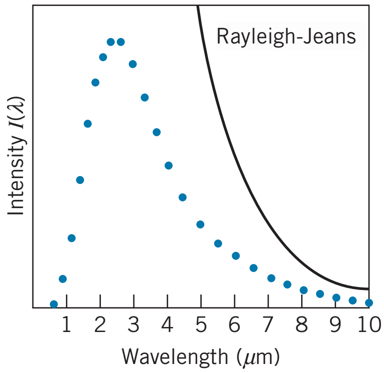


FIGURE 3.15 The failure of the classical Rayleigh-Jeans formula to fit the observed intensity. At long wavelengths the theory approaches the data, but at short wavelengths the classical formula fails miserably.

Quantum Theory of Thermal Radiation

The new physics that gave the correct interpretation of thermal radiation was proposed by the German physicist Max Planck in 1900. The ultraviolet catastrophe occurs because the Rayleigh-Jeans formula predicts too much intensity at short wavelengths (or equivalently at high frequencies). What is needed is a way to make $u \to 0$ as $\lambda \to 0$, or as $f \to \infty$. Again considering the electromagnetic standing waves to result from the oscillations of atoms in the walls of the cavity, Planck tried to find a way to reduce the number of high-frequency standing waves by reducing the number of high-frequency oscillators. He did this by a bold assumption that formed the cornerstone of a new physical theory, *quantum physics*. Associated with this theory is a new version of mechanics, known as *wave mechanics* or *quantum mechanics*. We discuss the methods of wave mechanics in Chapter 5; for now we show how Planck's theory provided the correct interpretation of the emission spectrum of thermal radiation.

Planck suggested that an oscillating atom can absorb or emit energy only in discrete bundles. This bold suggestion was necessary to keep the average energy of a low-frequency (long-wavelength) oscillator equal to kT (in agreement with the Rayleigh-Jeans law at long wavelength), but it also made the average energy of a high-frequency (low-wavelength) oscillator approach zero. Let's see how Planck managed this remarkable feat.

In Planck's theory, each oscillator can emit or absorb energy only in quantities that are integer multiples of a certain basic quantity of energy ε,

$$E_n = n\varepsilon \qquad n = 1, 2, 3, \ldots \tag{3.36}$$

where n is the number of quanta. Furthermore, the energy of each of the quanta is determined by the frequency

$$\varepsilon = hf \tag{3.37}$$

where h is the constant of proportionality, now known as Planck's constant. From the mathematical standpoint, the difference between Planck's calculation and the classical calculation using Maxwell-Boltzmann statistics is that the energy of an oscillator at a certain wavelength or frequency is no longer a continuous variable—it is a discrete variable that takes only the values given by Eq. 3.36. The integrals in the classical calculation are then replaced by sums, and the number of oscillators with energy E_n is then

$$N_n = N(1 - e^{-\varepsilon/kT})e^{-n\varepsilon/kT} \tag{3.38}$$

Max Planck (1858–1947, Germany). His work on the spectral distribution of radiation, which led to the quantum theory, was honored with the 1918 Nobel Prize. In his later years, he wrote extensively on religious and philosophical topics.

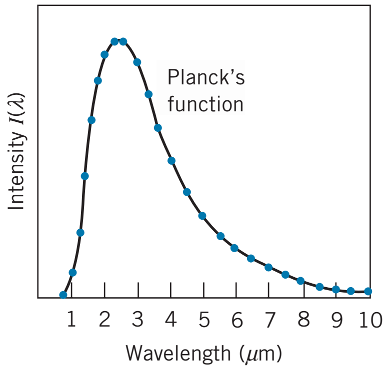


FIGURE 3.16 Planck's function fits the observed data perfectly.

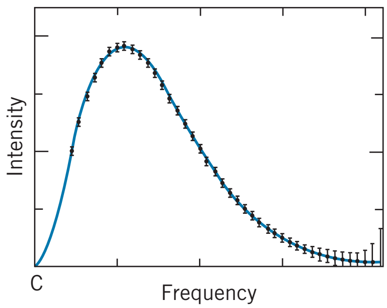


FIGURE 3.17 Data from the COBE satellite, launched in 1989 to determine the temperature of the cosmic microwave background radiation from the early universe. The data points exactly fit the Planck function corresponding to a temperature of 2.725 K. To appreciate the remarkable precision of this experiment, note that the sizes of the error bars have been increased by a factor of 400 to make them visible! (Source: NASA Office of Space Science)

(Compare this result with Eq. 3.32 for the continuous case.) Here N_n represents the number of oscillators with energy E_n, while N is the total number. You should be able to show that $\sum_{n=0}^{\infty} N_n = N$, again giving the total number of oscillators when summed over all possible energies. Planck's calculation then gives the average energy:

$$E_{\mathrm{av}} = \frac{1}{N}\sum_{n=0}^{\infty} N_n E_n = (1 - e^{-\varepsilon/kT})\sum_{n=0}^{\infty}(n\varepsilon)e^{-n\varepsilon/kT} \tag{3.39}$$

which gives (see Problem 14)

$$E_{\mathrm{av}} = \frac{\varepsilon}{e^{\varepsilon/kT} - 1} = \frac{hf}{e^{hf/kT} - 1} = \frac{hc/\lambda}{e^{hc/\lambda kT} - 1} \tag{3.40}$$

Note from this equation that $E_{\mathrm{av}} \cong kT$ at small f (large λ) but that $E_{\mathrm{av}} \to 0$ at large f (small λ). Thus the small-wavelength oscillators carry a vanishingly small energy, and the ultraviolet catastrophe is solved!

Based on Planck's result, the intensity of the radiation then becomes (using Eqs. 3.28 and 3.31):

$$I(\lambda) = \frac{c}{4}\left(\frac{8\pi}{\lambda^4}\right)\left[\frac{hc/\lambda}{e^{hc/\lambda kT} - 1}\right] = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1} \tag{3.41}$$

(An alternative approach to deriving this result is given in Section 10.6.) The perfect agreement between experiment and Planck's formula is illustrated in Figure 3.16.

In Problems 15 and 16 at the end of this chapter you will demonstrate that Planck's formula can be used to deduce Stefan's law and Wien's displacement law. In fact, deducing Stefan's law from Planck's formula results in a relationship between the Stefan-Boltzmann constant and Planck's constant:

$$\sigma = \frac{2\pi^5 k^4}{15c^2h^3} \tag{3.42}$$

By determining the value of the Stefan-Boltzmann constant from the intensity data available in 1900, Planck was able to determine a value of the constant h :

$$h = 6.56 \times 10^{-34}\ \mathrm{J\cdot s}$$

which agrees very well with the value of h that Millikan deduced 15 years later based on the analysis of data from the photoelectric effect. The good agreement of these two values is remarkable, because they are derived from very different kinds of experiments—one involves the *emission* and the other the *absorption* of electromagnetic radiation. This suggests that the quantization property is not an accident arising from the analysis of one particular experiment, but is instead a property of the electromagnetic field itself. Along with many other scientists of his era, Planck was slow to accept this interpretation. However, later experimental evidence (including the Compton effect) proved to be so compelling that it left no doubt about Einstein's photon theory and the particlelike structure of the electromagnetic field.

Planck's formula still finds important applications today in the measurement of temperature. By measuring the intensity of radiation emitted by an object at a particular wavelength (or, as in actual experiments, in a small interval of wavelengths), Eq. 3.41 can be used to deduce the temperature of the object. Note that only one measurement, at any wavelength, is all that is required to obtain the temperature. A *radiometer* is a device for measuring the intensity of thermal radiation at selected wavelengths, enabling a determination of temperature. Radiometers in orbiting satellites are used to measure the temperature of the land and sea areas of the Earth and of the upper surface of clouds. Other orbiting radiometers have been aimed toward "empty space" to measure the temperature of the radiation from the early history of the universe (Figure 3.17).

Example 3.6

You are using a radiometer to observe the thermal radiation from an object that is heated to maintain its temperature at 1278 K. The radiometer records radiation in a wavelength interval of 12.6 nm. By changing the wavelength at which you are measuring, you set the radiometer to record the most intense radiation emission from the object. What is the intensity of the emitted radiation in this interval?

Solution

The wavelength setting for the most intense radiation is determined from Wien's displacement law:

$$\lambda_{max} = \frac{2.8978 \times 10^{-3}\ \text{m} \cdot \text{K}}{T} = \frac{2.8978 \times 10^{-3}\ \text{m} \cdot \text{K}}{1278\ \text{K}}$$
$$= 2.267 \times 10^{-6}\ \text{m} = 2267\ \text{nm}$$

The given temperature corresponds to $kT = (8.6174 \times 10^{-5}\ \text{eV/K})(1278\ \text{K}) = 0.1101\ \text{eV}$. The radiation intensity in this small wavelength interval is

$$I(\lambda)d\lambda = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} d\lambda$$
$$= 2\pi(6.626 \times 10^{-34}\ \text{J} \cdot \text{s})(2.998 \times 10^8\ \text{m/s})^2$$
$$\times (12.6 \times 10^{-9}\ \text{m})(2.267 \times 10^{-6}\ \text{m})^{-5}$$
$$\times (e^{(1240\ \text{eV}\cdot\text{nm})/(2267\ \text{nm})(0.1101\ \text{eV})} - 1)^{-1}$$
$$= 552\ \text{W/m}^2$$

3.4 THE COMPTON EFFECT

Another way for radiation to interact with matter is by means of the Compton effect, in which radiation scatters from loosely bound, nearly free electrons. Part of the energy of the radiation is given to the electron; the remainder of the energy is reradiated as electromagnetic radiation. According to the wave picture, the scattered radiation is less energetic than the incident radiation (the difference going into the kinetic energy of the electron) but has the same wavelength. As we will see, the photon concept leads to a very different prediction for the scattered radiation.

The scattering process is analyzed simply as an interaction (a "collision" in the classical sense of particles) between a single photon and an electron, which

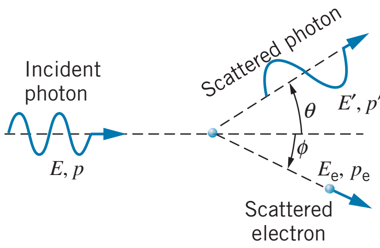


FIGURE 3.18 The geometry of Compton scattering.

we assume to be at rest. Figure 3.18 shows the process. Initially, the photon has energy E and linear momentum p given by

$$E = hf = \frac{hc}{\lambda} \quad \text{and} \quad p = \frac{E}{c} \tag{3.43}$$

The electron, initially at rest, has rest energy $m_e c^2$. After the scattering, the photon has energy $E' = hc/\lambda'$ and momentum $p' = E'/c$, and it moves in a direction at an angle θ with respect to the direction of the incident photon. The electron has total final energy E_e and momentum p_e and moves in a direction at an angle ϕ with respect to the initial photon. (To allow for the possibility of high-energy incident photons giving energetic scattered electrons, we use relativistic kinematics for the electron.) The conservation laws for total relativistic energy and momentum are then applied:

$$E_{\text{initial}} = E_{\text{final}}: \qquad E + m_e c^2 = E' + E_e \tag{3.44a}$$

$$p_{x,\text{initial}} = p_{x,\text{final}}: \qquad p = p_e \cos\phi + p' \cos\theta \tag{3.44b}$$

$$p_{y,\text{initial}} = p_{y,\text{final}}: \qquad 0 = p_e \sin\phi - p' \sin\theta \tag{3.44c}$$

We have three equations with four unknowns (θ, ϕ, E_e, E'; p_e and p' are not independent unknowns) that cannot be solved uniquely, but we can eliminate any two of the four unknowns by solving the equations simultaneously. If we choose to measure the energy and direction of the scattered photon, we eliminate E_e and ϕ. The angle ϕ is eliminated by first rewriting the momentum equations:

$$p_e \cos\phi = p - p' \cos\theta \quad \text{and} \quad p_e \sin\phi = p' \sin\theta \tag{3.45}$$

Squaring these equations and adding the results, we obtain

$$p_e^2 = p^2 - 2pp' \cos\theta + p'^2 \tag{3.46}$$

The relativistic relationship between energy and momentum is, according to Eq. 2.39, $E_e^2 = c^2 p_e^2 + m_e^2 c^4$. Substituting in this equation for E_e from Eq. 3.44*a* and for p_e^2 from Eq. 3.46, we obtain

$$(E + m_e c^2 - E')^2 = c^2(p^2 - 2pp' \cos\theta + p'^2) + m_e^2 c^4 \tag{3.47}$$

and after a bit of algebra, we find

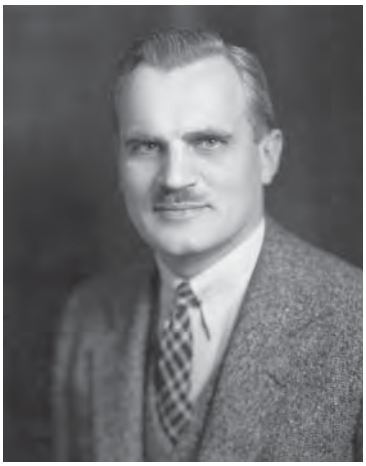
$$\frac{1}{E'} - \frac{1}{E} = \frac{1}{m_e c^2}(1 - \cos\theta) \tag{3.48}$$

In terms of wavelength, this equation can also be written as

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta) \tag{3.49}$$

where λ is the wavelength of the incident photon and λ' is the wavelength of the scattered photon. The quantity $h/m_e c$ is known as the *Compton wavelength of the electron* and has a value of 0.002426 nm; however, keep in mind that it is *not* a true wavelength but rather is a *change* of wavelength.

Equations 3.48 and 3.49 give the change in energy or wavelength of the photon, as a function of the *scattering angle* θ. Because the quantity on the right-hand side is never negative, E' is always less than E, so that the scattered photon has less energy than the original incident photon; the difference $E - E'$ is just the kinetic



Arthur H. Compton (1892–1962, United States). His work on X-ray scattering verified Einstein's photon theory and earned him the 1927 Nobel Prize. He was a pioneer in research with X rays and cosmic rays. During World War II he directed a portion of the U.S. atomic bomb research.

energy given to the electron, $E_e - m_e c^2$. Similarly, λ' is greater than λ, meaning the scattered photon always has a longer wavelength than the incident photon; the change in wavelength ranges from 0 at $\theta = 0^\circ$ to twice the Compton wavelength at $\theta = 180^\circ$. Of course the descriptions in terms of energy and wavelength are equivalent, and the choice of which to use is merely a matter of convenience.

Using $E_e = K_e + m_e c^2$, where K_e is the kinetic energy of the electron, conservation of energy (Eq. 3.44*a*) can also be written as $E + m_e c^2 = E' + K_e + m_e c^2$. Solving for K_e, we obtain

$$K_e = E - E' \tag{3.50}$$

That is, the kinetic energy acquired by the electron is equal to the difference between the initial and final photon energies.

We can also find the direction of the electron's motion by dividing the two momentum relationships in Equation 3.45:

$$\tan\phi = \frac{p_e \sin\phi}{p_e \cos\phi} = \frac{p' \sin\theta}{p - p'\cos\theta} = \frac{E' \sin\theta}{E - E'\cos\theta} \tag{3.51}$$

where the last result comes from using $p = E/c$ and $p' = E'/c$.

Example 3.7

X rays of wavelength 0.2400 nm are Compton-scattered, and the scattered beam is observed at an angle of 60.0° relative to the incident beam. Find: (*a*) the wavelength of the scattered X rays, (*b*) the energy of the scattered X-ray photons, (*c*) the kinetic energy of the scattered electrons, and (*d*) the direction of travel of the scattered electrons.

Solution

(*a*) λ' can be found immediately from Eq. 3.49:

$$\begin{aligned}\lambda' &= \lambda + \frac{h}{m_e c}(1 - \cos\theta) \\ &= 0.2400 + (0.00243\,\text{nm})(1 - \cos 60^\circ) \\ &= 0.2412\,\text{nm}\end{aligned}$$

(*b*) The energy E' can be found directly from λ':

$$E' = \frac{hc}{\lambda'} = \frac{1240\,\text{eV}\cdot\text{nm}}{0.2412\,\text{nm}} = 5141\,\text{eV}$$

(*c*) The initial photon energy E is $hc/\lambda = 5167\,\text{eV}$, so

$$K_e = E - E' = 5167\,\text{eV} - 5141\,\text{eV} = 26\,\text{eV}$$

(*d*) From Eq. 3.51,

$$\begin{aligned}\phi &= \tan^{-1}\frac{E'\sin\theta}{E - E'\cos\theta} \\ &= \tan^{-1}\frac{(5141\,\text{eV})(\sin 60^\circ)}{(5167\,\text{eV}) - (5141\,\text{eV})(\cos 60^\circ)} \\ &= 59.7^\circ\end{aligned}$$

The first experimental demonstration of this type of scattering was done by Arthur Compton in 1923. A diagram of his experimental arrangement is shown in Figure 3.19. A beam of X rays of a single wavelength λ is incident on a scattering target, for which Compton used carbon. (Although no scattering target contains actual "free" electrons, the outer or valence electrons in many materials are very weakly attached to the atom and behave like nearly free electrons. The binding energies of these electrons in the atom are so small compared with the energies of the incident X-ray photons that they can be regarded as nearly "free" electrons.) A movable detector measured the energy of the scattered X rays at various angles θ.

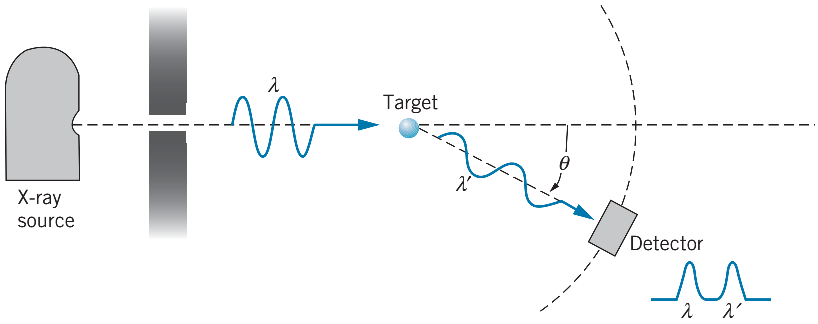


FIGURE 3.19 Schematic diagram of Compton-scattering apparatus. The wavelength λ' of the scattered X rays is measured by the detector, which can be moved to different positions θ. The wavelength difference $\lambda' - \lambda$ varies with θ.

Compton's original results are illustrated in Figure 3.20. At each angle, two peaks appear, corresponding to scattered X-ray photons with two different energies or wavelengths. The wavelength of one peak does not change as the angle is varied; this peak corresponds to scattering that involves "inner" electrons of the atom, which are more tightly bound to the atom so that the photon can scatter with no loss of energy. The wavelength of the other peak, however, varies strongly with angle; as can be seen from Figure 3.21, this variation is exactly as the Compton formula predicts.

Similar results can be obtained for the scattering of gamma rays, which are higher-energy (shorter wavelength) photons emitted in various radioactive decays. Compton also measured the variation in wavelength of scattered gamma rays, as illustrated in Figure 3.22. The change in wavelength in the

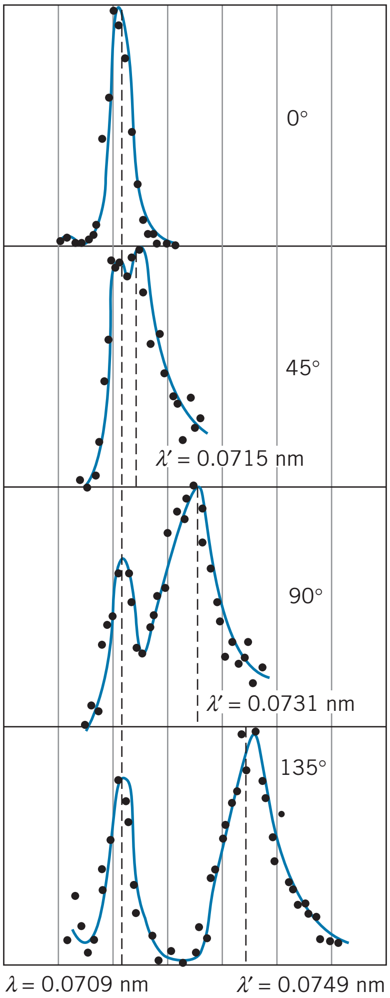


FIGURE 3.20 Compton's original results for X-ray scattering.

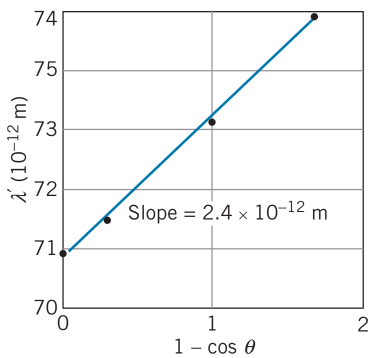


FIGURE 3.21 The scattered X-ray wavelengths λ', from Figure 3.20, for different scattering angles. The expected slope is 2.43×10^{-12} m, in agreement with the measured slope of Compton's data points.

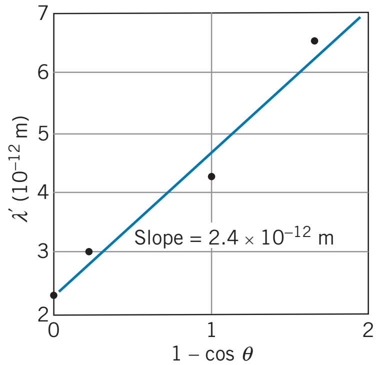


FIGURE 3.22 Compton's results for gamma-ray scattering. The wavelengths are much smaller than for X-rays, but the slope is the same as in Figure 3.21, which the Compton formula, Eq. 3.49, predicts.

gamma-ray measurements is identical with the change in wavelength in the X-ray measurements, as Eq. 3.49 predicts—the change in wavelength does not depend on the incident wavelength.

3.5 OTHER PHOTON PROCESSES

Although thermal radiation, the photoelectric effect, and Compton scattering provided the earliest experimental evidence in support of the quantization (particlelike behavior) of electromagnetic radiation, there are numerous other experiments that can also be interpreted correctly only if we assume the existence of photons as discrete quanta of electromagnetic radiation. In this section we discuss some of these processes, which cannot be understood if we consider only the wave nature of electromagnetic radiation. As you study the descriptions of these processes, note how photons interact with atoms or electrons by delivering energy in discrete bundles, in contrast to the wave interpretation in which the energy can be regarded as arriving continuously.

Interactions of Photons with Atoms

The emission of electromagnetic radiation from atoms takes place in discrete amounts characterized by one or more photons. When an atom emits a photon of energy E, the atom loses an equivalent amount of energy. Consider an atom at rest that has an initial energy E_i. The atom emits a photon of energy E. After the emission, the atom is left with a final energy E_f, which we will take as the energy associated with the internal structure of the atom. Because of conservation of momentum, the final atom must have a momentum that is equal and opposite to the momentum of the emitted photon, so the atom must also have a "recoil" kinetic energy K. (Normally this kinetic energy is very small.) Conservation of energy then gives

$$E_\mathrm{i} = E_\mathrm{f} + K + E \quad \text{or} \quad E = (E_\mathrm{i} - E_\mathrm{f}) - K \tag{3.52}$$

The energy of the emitted photon is equal to the net energy lost by the atom, minus a negligibly small contribution to the recoil kinetic energy of the atom.

In the reverse process, an atom can *absorb* a photon of energy E. If the atom is initially at rest, it must again acquire a small recoil kinetic energy in order to conserve momentum. Now conservation of energy gives

$$E_\mathrm{i} + E = E_\mathrm{f} + K \quad \text{or} \quad E_\mathrm{f} - E_\mathrm{i} = E - K \tag{3.53}$$

The energy available to add to the atom's internal supply of energy is the photon energy, less a recoil kinetic energy that is usually negligible.

Photon emission and absorption experiments are among the most important techniques for acquiring information about the internal structure of atoms, as we discuss in Chapter 6.

Bremsstrahlung and X-ray Production

When an electric charge, such as an electron, is accelerated or decelerated, it radiates electromagnetic energy; according to the quantum interpretation, we would say that it emits photons. Suppose we have a beam of electrons, which has been accelerated through a potential difference ΔV, so that the electrons experience a loss in potential energy of $-e\,\Delta V$ and thus acquire a kinetic energy of $K = e\,\Delta V$ (Figure 3.23). When the electrons strike a target they are slowed down and eventually come to rest, because they make collisions with the atoms of the target material. In such a collision, momentum is transferred to the atom, the electron slows down, and photons are emitted. The recoil kinetic energy of the atom is small (because the atom is so massive) and can safely be neglected. If the electron has a kinetic energy K before the encounter and if it leaves after the collision with a smaller kinetic energy K', then the photon energy $hf = hc/\lambda$ is

$$hf = \frac{hc}{\lambda} = K - K' \tag{3.54}$$

The amount of energy lost, and therefore the energy and wavelength of the emitted photon, are not uniquely determined, because K is the only known energy in Eq. 3.54. An electron usually will make many collisions, and therefore emit many different photons, before it is brought to rest; the photons then will range all the way from very small energies (large wavelengths) corresponding to small energy losses, up to a maximum photon energy hf_{max} equal to K, corresponding to an electron that loses all of its kinetic energy K in a single encounter (that is, when $K' = 0$). The smallest emitted wavelength λ_{min} is therefore determined by the maximum possible energy loss,

$$\lambda_{\text{min}} = \frac{hc}{K} = \frac{hc}{e\,\Delta V} \tag{3.55}$$

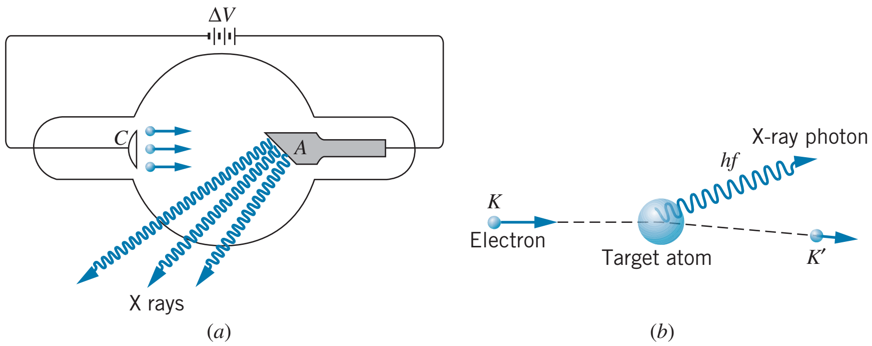


FIGURE 3.23 (*a*) Apparatus for producing bremsstrahlung. Electrons from a cathode C are accelerated to the anode A through the potential difference ΔV. When an electron encounters a target atom of the anode, it can lose energy, with the accompanying emission of an X-ray photon. (*b*) A schematic representation of the bremsstrahlung process.

For typical accelerating voltages in the range of 10,000 V, λ_{min} is in the range of a few tenths of nm, which corresponds to the X-ray region of the spectrum. This *continuous* distribution of X rays (which is very different from the *discrete* X-ray energies that are emitted in atomic transitions; more about these in Chapter 8) is called *bremsstrahlung*, which is German for braking, or decelerating, radiation. Some sample bremsstrahlung spectra are illustrated in Figure 3.24.

Symbolically we can write the bremsstrahlung process as

$$\text{electron} \rightarrow \text{electron} + \text{photon}$$

This is just the reverse process of the photoelectric effect, which is

$$\text{electron} + \text{photon} \rightarrow \text{electron}$$

However, neither process occurs for free electrons. In both cases there must be a heavy atom in the neighborhood to take care of the recoil momentum.

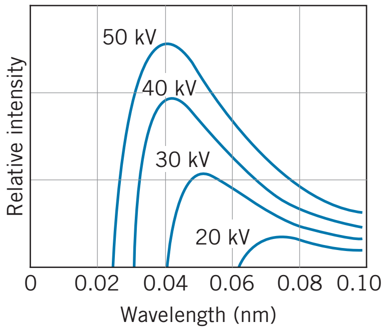


FIGURE 3.24 Some typical bremsstrahlung spectra. Each spectrum is labeled with the value of the accelerating voltage ΔV.

Pair Production and Annihilation

Another process that can occur when photons encounter atoms is *pair production*, in which the photon loses all its energy and in the process two particles are created: an electron and a positron. (A positron is a particle that is identical in mass to the electron but has a positive electric charge; more about *antiparticles* in Chapter 14.) Here we have an example of the creation of rest energy. The electron did not exist before the encounter of the photon with the atom (it was *not* an electron that was part of the atom). The photon energy hf is converted into the relativistic total energies E_+ and E_- of the positron and electron:

$$hf = E_+ + E_- = (m_e c^2 + K_+) + (m_e c^2 + K_-) \tag{3.56}$$

Because K_+ and K_- are always positive, the photon must have an energy of at least $2m_e c^2 = 1.02\,\text{MeV}$ in order for this process to occur; such high-energy photons are in the region of *nuclear gamma rays*. Symbolically,

$$\text{photon} \rightarrow \text{electron} + \text{positron}$$

This process, like bremsstrahlung, will not occur unless there is an atom nearby to supply the necessary recoil momentum. The reverse process,

$$\text{electron} + \text{positron} \rightarrow \text{photon}$$

also occurs; this process is known as *electron-positron annihilation* and can occur for free electrons and positrons as long as at least two photons are created. In this process the electron and positron disappear and are replaced by two photons. Conservation of energy requires that

$$(m_e c^2 + K_+) + (m_e c^2 + K_-) = E_1 + E_2 \tag{3.57}$$

where E_1 and E_2 are the photon energies. Usually the kinetic energies K_+ and K_- are negligibly small, so we can assume the positron and electron to be essentially at rest. Momentum conservation then requires the two photons to have equal and opposite momenta and thus equal energies. The two annihilation photons have equal energies of 0.511 MeV ($= m_e c^2$) and move in exactly opposite directions.

3.6 WHAT IS A PHOTON?

We can describe photons by giving a few of their basic properties:

- like an electromagnetic wave, photons move with the speed of light;
- they have zero mass and rest energy;
- they carry energy and momentum, which are related to the frequency and wavelength of the electromagnetic wave by $E = hf$ and $p = h/\lambda$;
- they can be created or destroyed when radiation is emitted or absorbed;
- they can have particlelike collisions with other particles such as electrons.

In this chapter we have described some experiments that favor the photon interpretation of electromagnetic radiation, according to which the energy of the radiation is concentrated in small bundles. Other experiments, such as interference and diffraction, favor the wave interpretation, according to which the energy of the radiation is spread over its entire wavefront. For example, the explanation of the double-slit interference experiment requires that the wavefront be divided so that some of its intensity can pass through each slit. A particle must choose to go through one slit or the other; only a wave can go through both.

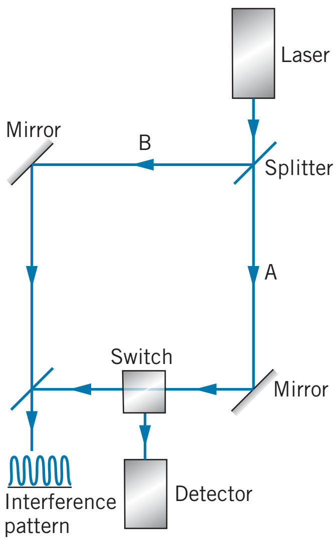


FIGURE 3.25 Apparatus for delayed choice experiment. Photons from the laser strike the beam splitter and can then travel paths A or B. The switch in path A can deflect the beam into a detector. If the switch is off, the beam on path A recombines with the beam on path B to form an interference pattern. [*Source:* A. Shimony, "The Reality of the Quantum World," *Scientific American* **258**, 46 (January 1988)].

If we regard the wave and particle pictures as valid but exclusive alternatives, we must assume that the light emitted by a source must travel *either* as waves *or* as particles. How does the source know what kind of light (particles or waves) to emit? Suppose we place a double-slit apparatus on one side of the source and a photoelectric cell on the other side. Light emitted toward the double slit behaves like a wave and light emitted toward the photocell behaves like particles. How did the source know in which direction to aim the waves and in which direction to aim the particles?

Perhaps nature has a sort of "secret code" in which the kind of experiment we are doing is signaled back to the source so that it knows whether to emit particles or waves. Let us repeat our dual experiment with light from a distant galaxy, light that has been traveling toward us for a time roughly equal to the age of the universe (13×10^9 years). Surely the kind of experiment we are doing could not be signaled back to the limits of the known universe in the time it takes us to remove the double-slit apparatus from the laboratory table and replace it with the photoelectric apparatus. Yet we find that the starlight can produce both the double-slit interference and also the photoelectric effect.

Figure 3.25 shows a recent experiment that was designed to test whether this dual nature is an intrinsic property of light or of our apparatus. A light beam from a laser goes through a beam splitter, which separates the beam into two components (A and B). The mirrors reflect the two component beams so that they can recombine to form an interference pattern. In path A there is a switch that can deflect the beam into a detector. If the switch is off, beam A is not deflected and will combine with beam B to produce the interference pattern. If the switch is on, beam A is deflected and observed in the detector, indicating that the light traveled a definite path, as would be characteristic of a particle. To put this another way, if the switch is off, the light beam is observed as a wave; if it is on, the light beam is observed as particles.

If light behaves like *particles,* the beam splitter sends it along *either* path A or path B; either path can be randomly chosen for the particle, but each particle can travel only one path. If light behaves like a *wave,* on the other hand, the beam splitter sends it along *both* paths, dividing its intensity between the two. Perhaps the beam splitter can somehow sense whether the switch is open or closed, so that it knows whether we are doing a particle-type or a wave-type experiment. If this were true, then the beam splitter would "know" whether to send all of the intensity down one path (so that we would observe a particle) or to split the intensity between the paths (so that we would observe a wave). However, in this experiment the experimenters used a very fast optical switch whose response time was shorter than the time it takes for light to travel through the apparatus to the switch. That is, the state of the switch could be changed *after* the light had already passed through the beam splitter, and so it was impossible for the beam splitter to "know" how the switch was set and thus whether a particle-type or a wave-type experiment was being done. This kind of experiment is called a "delayed choice" experiment, because the experimenter makes the choice of what kind of experiment to do after the light is already traveling on its way to the observation apparatus.

In this experiment, the investigators discovered that whenever they had the switch off, they observed the interference pattern characteristic of waves. When they had the switch on, they observed particles in the detector and no interference pattern. That is, whenever they did a wave-type experiment they observed waves, and whenever they did a particle-type experiment they observed particles. The wave and particle natures are both present simultaneously in the light, and this dual nature is clearly associated with the light and is not characteristic of the apparatus.

Many other experiments of this type have been done, and they all produce similar results. We are therefore trapped into an uncomfortable conclusion: Light is not *either* particles *or* waves; it is somehow *both* particles *and* waves, and only shows one or the other aspect, depending on the kind of experiment we are doing. A particle-type experiment shows the particle nature, while a wave-type experiment shows the wave nature. Our failure to classify light as *either* particle *or* wave is not so much a failure to understand the nature of light as it is a failure of our limited vocabulary (based on experiences with ordinary particles and waves) to describe a phenomenon that is more elegant and mysterious than either simple particles or waves.

Wave-Particle Duality

The dilemma of the dual particle+wave nature of light, which is called *wave-particle duality*, cannot be resolved with a simple explanation; physicists and philosophers have struggled with this problem ever since the quantum theory was introduced. The best we can do is to say that neither the wave nor the particle picture is wholly correct all of the time, that both are needed for a complete description of physical phenomena, and that in fact the two are *complementary* to one another.

Suppose we use a photographic film to observe the double-slit interference pattern. The film responds to individual photons. When a single photon is absorbed by the film, a single grain of the photographic emulsion is darkened; a complete picture requires a large number of grains to be darkened.

Let us imagine for the moment that we could see individual grains of the film as they absorbed photons and darkened, and let us do the double-slit experiment with a light source that is so weak that there is a relatively long time interval between photons. We would see first one grain darken, then another, and so forth, until after a large number of photons we would see the interference pattern begin to emerge. Some areas of the film (the interference maxima) show evidence for the arrival of a large number of photons, while in other areas (the interference minima) few photons arrive.

Alternatively, the wave picture of the double-slit experiment suggests that we could find the net electric field of the wave that strikes the screen by superimposing the electric fields of the portions of the incident wave fronts that pass through the two slits; the intensity or power in that combined wave could then be found by a procedure similar to Eqs. 3.7 through 3.10, and we would expect that the resultant intensity should show maxima and minima just like the observed double-slit interference pattern.

In summary, the correct explanation of the origin and appearance of the interference pattern comes from the wave picture, and the correct interpretation of the evolution of the pattern on the film comes from the photon picture; the two explanations, which according to our limited vocabulary and common-sense experience cannot simultaneously be correct, must somehow be taken together to give a complete description of the properties of electromagnetic radiation.

Keep in mind that "photon" and "wave" represent descriptions of the behavior of electromagnetic radiation when it encounters material objects. It is not correct to think of light as being "composed" of photons, just as we don't think of light as being "composed" of waves. The explanation in terms of photons applies to some interactions of radiation with matter, while the explanation in terms of waves applies to other interactions. For example, when we say that an atom "emits" a photon, we don't mean that there is a supply of photons stored within the atom; instead, we mean that the atom has given up a quantity of its internal energy to create an equivalent amount of energy in the form of electromagnetic radiation.

In the case of the double-slit experiment, we might reason as follows: the interaction between a "source" of radiation and the electromagnetic field is quantized, so that we can think of the emission of radiation by the atoms of the source in terms of individual photons. The interaction at the opposite end of the experiment, the photographic film, is also quantized, and we have the similarly useful view of atoms absorbing radiation as individual photons. In between, the electromagnetic radiation propagates smoothly and continuously as a wave and can show wave-type behavior (interference or diffraction) when it encounters the double slit.

Where the wave has large intensity, the film reveals the presence of many photons; where the wave has small intensity, few photons are observed. Recalling that the intensity of the wave is proportional to the square of its amplitude, we then have

$$\text{probability to observe photons} \propto |\text{electric field amplitude}|^2$$

It is this expression that provides the ultimate connection between the wave behavior and the particle behavior, and we will see in the next two chapters that a similar expression connects the wave and the particle aspects of those objects, such as electrons, which have been previously considered to behave as classical particles.

Chapter Summary

		Section
Double-slit maxima	$y_n = n\dfrac{\lambda D}{d} \quad n = 0, 1, 2, 3, \ldots$	3.1
Bragg's law for X-ray diffraction	$2d \sin\theta = n\lambda \quad n = 1, 2, 3, \cdots$	3.1
Energy of photon	$E = hf = hc/\lambda$	3.2
Maximum kinetic energy of photoelectrons	$K_{\max} = eV_s = hf - \phi$	3.2
Cutoff wavelength	$\lambda_c = hc/\phi$	3.2
Stefan's law	$I = \sigma T^4$	3.3
Wien's displacement law	$\lambda_{\max} T = 2.8978 \times 10^{-3}\ \text{m} \cdot \text{K}$	3.3

		Section
Rayleigh-Jeans formula	$I(\lambda) = \dfrac{2\pi c}{\lambda^4} kT$	3.3
Planck's blackbody distribution	$I(\lambda) = \dfrac{2\pi hc^2}{\lambda^5} \dfrac{1}{e^{hc/\lambda kT} - 1}$	3.3
Compton scattering	$\dfrac{1}{E'} - \dfrac{1}{E} = \dfrac{1}{m_e c^2}(1 - \cos\theta)$, $\lambda' - \lambda = \dfrac{h}{m_e c}(1 - \cos\theta)$	3.4
Bremsstrahlung	$\lambda_{\min} = hc/K = hc/e\Delta V$	3.5
Pair production	$hf = E_+ + E_- = (m_e c^2 + K_+) + (m_e c^2 + K_-)$	3.5
Electron-positron annihilation	$(m_e c^2 + K_+) + (m_e c^2 + K_-) = E_1 + E_2$	3.5

Questions

1. The diameter of an atomic nucleus is about 10×10^{-15} m. Suppose you wanted to study the diffraction of photons by nuclei. What energy of photons would you choose? Why?
2. How is the wave nature of light unable to account for the observed properties of the photoelectric effect?
3. In the photoelectric effect, why do some electrons have kinetic energies smaller than $K_{\max}$?
4. Why doesn't the photoelectric effect work for free electrons?
5. What does the work function tell us about the properties of a metal? Of the metals listed in Table 3.1, which has the least tightly bound electrons? Which has the most tightly bound?
6. Electric current is charge flowing per unit time. If we increase the kinetic energy of the photoelectrons (by increasing the energy of the incident photons), shouldn't the current increase, because the charge flows more rapidly? Why doesn't it?
7. What might be the effects on a photoelectric effect experiment if we were to double the frequency of the incident light? If we were to double the wavelength? If we were to double the intensity?
8. In the photoelectric effect, how can a photon moving in one direction eject an electron moving in a different direction? What happens to conservation of momentum?
9. In Figure 3.10, why does the photoelectric current rise slowly to its saturation value instead of rapidly, when the potential difference is greater than V_s ? What does this figure indicate about the experimental difficulties that might arise from trying to determine V_s in this way?
10. Suppose that the frequency of a certain light source is just above the cutoff frequency of the emitter, so that the photoelectric effect occurs. To an observer in relative motion, the frequency might be Doppler shifted to a lower value that is below the cutoff frequency. Would this moving observer conclude that the photoelectric effect does not occur? Explain.
11. Why do cavities that form in a wood fire seem to glow brighter than the burning wood itself? Is the temperature in such cavities hotter than the surface temperature of the exposed burning wood?
12. What are the fields of classical physics on which the classical theory of blackbody radiation is based? Why don't

we believe that the "ultraviolet catastrophe" suggests that something is wrong with one of those classical theories?

13. In what region of the electromagnetic spectrum do room-temperature objects radiate? What problems would we have if our eyes were sensitive in that region?
14. How does the total intensity of thermal radiation vary when the temperature of an object is doubled?
15. Compton-scattered photons of wavelength λ' are observed at 90°. In terms of λ', what is the scattered wavelength observed at 180° ?
16. The Compton-scattering formula suggests that objects viewed from different angles should show scattered light of different wavelengths. Why don't we observe a change in color of objects as we change the viewing angle?
17. You have a monoenergetic source of X rays of energy 84 keV, but for an experiment you need 70 keV X rays. How would you convert the X-ray energy from 84 to 70 keV?
18. TV sets with picture tubes can be significant emitters of X rays. What is the origin of these X rays? Estimate their wavelengths.
19. The X-ray peaks of Figure 3.20 are not sharp but are spread over a range of wavelengths. What reasons might account for that spreading?
20. A beam of photons passes through a block of matter. What are the three ways discussed in this chapter that the photons can lose energy in interacting with the material?
21. Of the photon processes discussed in this chapter (photoelectric effect, thermal radiation, Compton scattering, bremsstrahlung, pair production, electron-positron annihilation), which conserve momentum? Energy? Mass? Number of photons? Number of electrons? Number of electrons minus number of positrons?

Problems

3.1 Review of Electromagnetic Waves

1. A double-slit experiment is performed with sodium light ($\lambda = 589.0$ nm). The slits are separated by 1.05 mm, and the screen is 2.357 m from the slits. Find the separation between adjacent maxima on the screen.
2. In Example 3.1, what angle of incidence will produce the second-order Bragg peak?
3. Monochromatic X rays are incident on a crystal in the geometry of Figure 3.5. The first-order Bragg peak is observed when the angle of incidence is 34.0°. The crystal spacing is known to be 0.347 nm. (*a*) What is the wavelength of the X rays? (*b*) Now consider a set of crystal planes that makes an angle of 45° with the surface of the crystal (as in Figure 3.6). For X rays of the same wavelength, find the angle of incidence measured from the surface of the crystal that produces the first-order Bragg peak. At what angle from the surface does the emerging beam appear in this case?
4. A certain device for analyzing electromagnetic radiation is based on the Bragg scattering of the radiation from a crystal. For radiation of wavelength 0.149 nm, the first-order Bragg peak appears centered at an angle of 15.15°. The aperture of the analyzer passes radiation in the angular range of 0.015°. What is the corresponding range of wavelengths passing through the analyzer?

3.2 The Photoelectric Effect

5. Find the momentum of (*a*) a 10.0-MeV gamma ray; (*b*) a 25-keV X ray; (*c*) a 1.0-μm infrared photon; (*d*) a 150-MHz radio-wave photon. Express the momentum in kg · m/s and eV/c.
6. Radio waves have a frequency of the order of 1 to 100 MHz. What is the range of energies of these photons? Our bodies are continuously bombarded by these photons. Why are they not dangerous to us?
7. (*a*) What is the wavelength of an X-ray photon of energy 10.0 keV? (*b*) What is the wavelength of a gamma-ray photon of energy 1.00 MeV? (*c*) What is the range of energies of photons of visible light with wavelengths 350 to 700 nm?
8. What is the cutoff wavelength for the photoelectric effect using an aluminum surface?
9. A metal surface has a photoelectric cutoff wavelength of 325.6 nm. It is illuminated with light of wavelength 259.8 nm. What is the stopping potential?
10. When light of wavelength λ illuminates a copper surface, the stopping potential is V. In terms of V, what will be the stopping potential if the same wavelength is used to illuminate a sodium surface?
11. The cutoff wavelength for the photoelectric effect in a certain metal is 254 nm. (*a*) What is the work function for that metal? (*b*) Will the photoelectric effect be observed for $\lambda > 254$ nm or for $\lambda < 254$ nm?
12. A surface of zinc is illuminated and photoelectrons are observed. (*a*) What is the largest wavelength that will cause photoelectrons to be emitted? (*b*) What is the stopping potential when light of wavelength 220.0 nm is used?

3.3 Blackbody Radiation

13. (*a*) Show that in the classical result for the energy distribution of the cavity wall oscillators (Eq. 3.32), the total number of oscillators at all energies is N. (*b*) Show that $E_{\text{av}} = kT$ for the classical oscillators.
14. (*a*) Writing the discrete Maxwell-Boltzmann distribution for Planck's cavity wall oscillators as $N_n = Ae^{-E_n/kT}$ (where A is a constant to be determined), show that the

condition $\sum_{n=0}^{\infty} N_n = N$ gives $A = N(1 - e^{-\varepsilon/kT})$ as in Eq. 3.38. [*Hint:* Use $\sum_{n=0}^{\infty} e^{nx} = (1 - e^x)^{-1}$]. (*b*) By taking the derivative with respect to x of the equation given in the hint, show that $\sum_{n=0}^{\infty} ne^{nx} = e^x/(1 - e^x)^2$. (*c*) Use this result to derive Eq. 3.40 from Eq. 3.39. (*d*) Show that $E_{av} \cong kT$ at large λ and $E_{av} \to 0$ for small λ.

15. By differentiating Eq. 3.41 show that $I(\lambda)$ has its maximum as expected according to Wien's displacement law, Eq. 3.27.
16. Integrate Eq. 3.41 to obtain Eq. 3.26. Use the definite integral $\int_0^\infty x^3 dx/(e^x - 1) = \pi^4/15$ to obtain Eq. 3.42 relating the Stefan-Boltzmann constant to Planck's constant.
17. Use the numerical value of the Stefan-Boltzmann constant to find the numerical value of Planck's constant from Eq. 3.42.
18. The surface of the Sun has a temperature of about 6000 K. At what wavelength does the Sun emit its peak intensity? How does this compare with the peak sensitivity of the human eye?
19. The universe is filled with thermal radiation, which has a blackbody spectrum at an effective temperature of 2.7 K (see Chapter 15). What is the peak wavelength of this radiation? What is the energy (in eV) of quanta at the peak wavelength? In what region of the electromagnetic spectrum is this peak wavelength?
20. (*a*) Assuming the human body (skin temperature 34°C) to behave like an ideal thermal radiator, find the wavelength where the intensity from the body is a maximum. In what region of the electromagnetic spectrum is radiation with this wavelength? (*b*) Making whatever (reasonable) assumptions you may need, estimate the power radiated by a typical person isolated from the surroundings. (*c*) Estimate the radiation power *absorbed* by a person in a room in which the temperature is 20°C.
21. A cavity is maintained at a temperature of 1650 K. At what rate does energy escape from the interior of the cavity through a hole in its wall of diameter 1.00 mm?
22. An analyzer for thermal radiation is set to accept wavelengths in an interval of 1.55 nm. What is the intensity of the radiation in that interval at a wavelength of 875 nm emitted from a glowing object whose temperature is 1675 K?
23. (*a*) Assuming the Sun to radiate like an ideal thermal source at a temperature of 6000 K, what is the intensity of the solar radiation emitted in the range 550.0 nm to 552.0 nm? (*b*) What fraction of the total solar radiation does this represent?

3.4 The Compton Effect

24. Show how Eq. 3.48 follows from Eq. 3.47.
25. Incident photons of energy 10.39 keV are Compton scattered, and the scattered beam is observed at 45.00° relative to the incident beam. (*a*) What is the energy of the scattered photons at that angle? (*b*) What is the kinetic energy of the scattered electrons?
26. X-ray photons of wavelength 0.02480 nm are incident on a target and the Compton-scattered photons are observed at 90.0°. (*a*) What is the wavelength of the scattered photons? (*b*) What is the momentum of the incident photons? Of the scattered photons? (*c*) What is the kinetic energy of the scattered electrons? (*d*) What is the momentum (magnitude and direction) of the scattered electrons?
27. High-energy gamma rays can reach a radiation detector by Compton scattering from the surroundings, as shown in Figure 3.26. This effect is known as *back-scattering*. Show that, when $E \gg m_e c^2$, the back-scattered photon has an energy of approximately 0.25 MeV, independent of the energy of the original photon, when the scattering angle is nearly 180°.

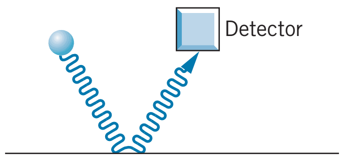


FIGURE 3.26 Problem 27.

28. Gamma rays of energy 0.662 MeV are Compton scattered. (*a*) What is the energy of the scattered photon observed at a scattering angle of 60.0°? (*b*) What is the kinetic energy of the scattered electrons?

3.5 Other Photon Processes

29. Suppose an atom of iron at rest emits an X-ray photon of energy 6.4 keV. Calculate the "recoil" momentum and kinetic energy of the atom. (*Hint:* Do you expect to need classical or relativistic kinetic energy for the atom? Is the kinetic energy likely to be much smaller than the atom's rest energy?)
30. What is the minimum X-ray wavelength produced in bremsstrahlung by electrons that have been accelerated through 2.50×10^4 V?
31. An atom absorbs a photon of wavelength 375 nm and immediately emits another photon of wavelength 580 nm. What is the net energy absorbed by the atom in this process?

General Problems

32. A certain green light bulb emits at a single wavelength of 550 nm. It consumes 55 W of electrical power and is 75% efficient in converting electrical energy into light. (*a*) How many photons does the bulb emit in one hour? (*b*) Assuming the emitted photons to be distributed uniformly in space, how many photons per second strike a 10 cm by 10 cm paper held facing the bulb at a distance of 1.0 m?
33. When sodium metal is illuminated with light of wavelength 4.20×10^2 nm, the stopping potential is found to be 0.65 V; when the wavelength is changed to 3.10×10^2 nm, the

stopping potential is 1.69 V. Using *only these data* and the values of the speed of light and the electronic charge, find the work function of sodium and a value of Planck's constant.

34. A photon of wavelength 192 nm strikes an aluminum surface along a line perpendicular to the surface and releases a photoelectron traveling in the opposite direction. Assume the recoil momentum is taken up by a single aluminum atom on the surface. Calculate the recoil kinetic energy of the atom. Would this recoil energy significantly affect the kinetic energy of the photoelectron?
35. A certain cavity has a temperature of 1150 K. (*a*) At what wavelength will the intensity of the radiation inside the cavity have its maximum value? (*b*) As a fraction of the maximum intensity, what is the intensity at twice the wavelength found in part (*a*)?
36. In Compton scattering, calculate the maximum kinetic energy given to the scattered electron for a given photon energy.
37. The COBE satellite was launched in 1989 to study the cosmic background radiation and measure its temperature. By measuring at many different wavelengths, researchers were able to show that the background radiation exactly followed the spectral distribution expected for a blackbody. At a wavelength of 0.133 cm, the radiant intensity is 1.440×10^{-7} W/m^2 in a wavelength interval of 0.00833 cm. What is the temperature of the radiation that would be deduced from these data?
38. The WMAP satellite launched in 2001 studied the cosmic microwave background radiation and was able to chart small fluctuations in the temperature of different regions of the background radiation. These fluctuations in temperature correspond to regions of large and small density in the early universe. The satellite was able to measure differences in temperature of 2×10^{-5} K at a temperature of 2.7250 K. At the peak wavelength, what is the difference in the radiation intensity per unit wavelength interval between the "hot" and "cold" regions of the background radiation?
39. You have been hired as an engineer on a NASA project to design a microwave spectrometer for an orbital mission to measure the cosmic background radiation, which has a blackbody spectrum with an effective temperature of 2.725 K. (*a*) The spectrometer is to scan the sky between wavelengths of 0.50 mm and 5.0 mm, and at each wavelength it accepts radiation in a wavelength range of 3.0×10^{-4} mm. What maximum and minimum radiation intensity do you expect to find in this region? (*b*) The photon detector in the spectrometer is in the form of a disk of diameter 0.86 cm. How many photons per second will the spectrometer record at its maximum and minimum intensities?
40. A photon of wavelength 7.52 pm scatters from a free electron at rest. After the interaction, the electron is observed to be moving in the direction of the original photon. Find the momentum of the electron.
41. A hydrogen atom is moving at a speed of 125.0 m/s. It absorbs a photon of wavelength 97 nm that is moving in the opposite direction. By how much does the speed of the atom change as a result of absorbing the photon?
42. Before a positron and an electron annihilate, they form a sort of "atom" in which each orbits about their common center of mass with identical speeds. As a result of this motion, the photons emitted in the annihilation show a small Doppler shift. In one experiment, the Doppler shift in energy of the photons was observed to be 2.41 keV. (*a*) What would be the speed of the electron or positron before the annihilation to produce this Doppler shift? (*b*) The positrons form these atom-like structures with the nearly "free" electrons in a solid. Assuming the positron and electron must have about the same speed to form this structure, find the kinetic energy of the electron. This technique, called "Doppler broadening," is an important method for learning about the energies of electrons in materials.
43. Prove that it is *not* possible to conserve both momentum and total relativistic energy in the following situation: A free electron moving at velocity $\vec{\mathbf{v}}$ emits a photon and then moves at a slower velocity $\vec{\mathbf{v}}'$.
44. A photon of energy E interacts with an electron at rest and undergoes pair production, producing a positive electron (positron) and an electron (in addition to the original electron):

$$\text{photon} + e^- \rightarrow e^+ + e^- + e^-$$

The two electrons and the positron move off with identical momenta in the direction of the initial photon. Find the kinetic energy of the three final particles and find the energy E of the photon. (*Hint:* Conserve momentum and total relativistic energy.)