tion of a general principle. That is, I am not certain whether the accident of Nomadic invasion produced the outstanding Pueblo traits or whether in the absence of such pressure we should have observed the same phenomenon in all its essentials. In short, does or does not the 'age and area' hypothesis, commonly subscribed to by students of organic life, hold true also in the province of human culture? But, passing over that detail, one thing seems fairly established both here in the Southwest and in several of our eastern culture areas. It is that North America north of Mexico, before it became settled by sedentary agricultural tribes who developed many of the traits common to that type of life the world over, was settled by a generally more primitive nomadic type of peoples subsisting mainly by hunting, such as still persist over all of the northern and northwestern portions of the continent. The above summary account is based upon data from the Archer M. Huntington Survey of Southwestern United States conducted by the American Museum of Natural History. The full report upon this survey will be published by the Museum.

AN ADJUSTMENT IN RELATION TO THE FRESNEL COEFFICIENT[1]

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1. Apparatus. One internal reflection.—The specific part of the apparatus is the glass cylinder, G, figure 1, with a carefully polished mantel, capable of rotating around an axis, A, normal to the ray-plane of the interferometer.

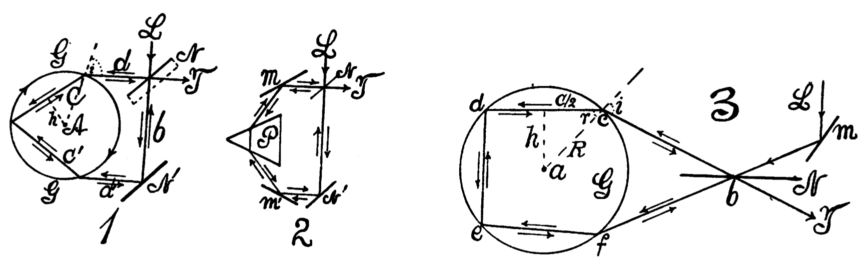
If micrometer facilities are to be dispensed with, and that is permissible in the present experiment,[2] the interferometer may be designed as in figure 1. The white light L from the collimator takes the respective paths $dCC'd'b$ and $bd'C'Cd$, the plate N being half silvered and N' an opaque mirror. The telescope or spectro-telescope is at T. The glass face at N may be turned either way.

Such an interferometer is self adjusting (cf. preceding paper). In the form, figure 1, two reversed spectra will be visible in the telescope, which if superimposed by rotating N or N' on a vertical axis, will show the linear phenomenon at once, in any color at pleasure. The fringes may be enlarged by rotating N or N' on a horizontal axis and they are symmetrically equal in size on the two sides of the adjustment for infinitely large fringes.

If the achromatics are wanted, a prism must be inserted into the rays b (preferably) between N and N', with a prism angle and other conditions selected to counteract the refraction of the cylinder G.

2. Apparatus. Two internal reflections.—As the fringes were found without much difficulty (§5) in case of one internal reflection, it seemed desirable to ascertain whether this would still be feasible in the apparently more favorable, but also more difficult case of two internal reflections. In figure 3, white light arrives from a collimator at L and strikes an auxiliary mirror m, before reaching the half silver N. If m is capable of rotating both on a horizontal and vertical axis as well as sliding right and left in the diagram, it greatly facilitates adjustments of angle and location of rays. The two beams *bcdef* and *bfedc* reunite at b after passing the glass cylinder G (rotating around the axis a) and are observed by the telescope at T. As the spectra (after refraction at c and f) are reflected 3 and 2 times respectively, the fringes of non-reversed spectra will be obtained covering the whole length of spectrum. A glass G of low index of refraction will here usually be preferable.

In case of a half silver mirror at a small glancing angle there are usually two pairs of bright spectrum images, and one fainter pair, apart from very faint ones. One bright and one faint pair carry identical fringes and the spectrum images may be small enough to be separated. In case of clear



glass, however, there is practically but one pair of bright images, and they carry fringes when properly superposed.

3. Equations.—The first question to be elucidated is the nature of the conditions of refraction. From the figure, in view of the symmetry of the arrangement, if b is the breadth of the ray parallelogram and R the radius of the cylinder, μ its index of refraction, h the distance of the chord C from the axis A, i and r the angles of incidence and refraction of the rays dC or $d'C'$:

$$\sin i = \sin 2r = b/2R \qquad (1)$$

$$\sin r = h/R \qquad (2)$$

$$\mu = 2 \cos r = b/2h \qquad (3)$$

The relations remain the same if $b/2r$ is constant. If the (small) value b = 10 cm. is inserted into the equation, the conditions may best be shown by a graph for i and μ. It will then be seen that for diameter $2r$ between 10 and 11 cm., the available indices of refraction of the glass would increase from 1.4 to 1.7 roughly, while the angle i falls from 90° to about 65°. Hence the experiment requires the interfering rays to impinge near the outer limits of the cylinder.

It is next in order to consider the possibly observable conditions of the (apparent) ether drag. The velocity within the refracting medium of index μ is usually written (or follows from the theory[3] of relativity) in the form

$$c/\mu \pm v\,(1 - 1/\mu^2) \tag{4}$$

where v is the velocity of the medium in the direction, or contrary to the direction of the velocity of light c. It remains to determine the average speed of the beam along the chord C of figure 1. From the figure

$$C = \mu\,R \text{ and } b = 2\,\mu\,h \tag{5}$$

whence

$$b = 2\,\mu\,R\,\sqrt{1 - \mu^2/4} \tag{6}$$

In figure 1 let ω be the angular velocity of the cylinder G and dx an element of the chord C at a distance ρ from the axis A. Let the minimum distance of this chord from A be h and θ its angle with ρ.
Then

$$dx = \rho^2\,\omega\,dt/h,$$

if dx is described in the time dt. Hence

$$dx/dt = \omega\,(h^2 + x^2)/h \tag{7}$$

To find the mean speed v along C, we may multiply dx/dt by dx, integrate between 0 and $C/2$ and divide the result by $C/2$. Thus

$$v = \omega\,(h + C^2/12h) \tag{8}$$

Reducing this equation by (1), (2), (5), eventually

$$v = R\omega\,\frac{1 - \mu^2/6}{\sqrt{1 - \mu^2/4}} \tag{9}$$

or the mean speed along C may be expressed in terms of R, ω, μ, while v is naturally proportional to R and ω

The ratio of the speed in equation 9 (seeing that it is respectively + and − for the two interfering rays) to the velocity of light is thus $2v/c$. Since these rays traverse a path $2C$ in the rotating cylinder in opposite directions the path difference resulting will be

$$\Delta P' = (2v/C)\,2C = 4Cv/c = \frac{4\mu\omega R^2}{c}\,\frac{1 - \mu^2/6}{\sqrt{1 - \mu^2/4}} \tag{10}$$

so that the path difference for a given μ and ω increases with the square of the radius, R, of the cylinder or disc.

But the equation (4) introduces another factor $(1 - 1/\mu^2)$ so that finally the path difference is

$$\Delta P = \frac{4\omega R^2}{c} \frac{\mu(1-\mu^2/6)(1-1/\mu^2)}{\sqrt{1-\mu^2/4}} \tag{11}$$

We may now take the above case (b = 10 cm.) from the graph i, μ, for a small cylinder, making 100 turns per second.

$$R = 5.3 \text{ cm}; \mu = 1.63; \omega = 628; i = 70.6°; r = 35.3°; b = 10 \text{ cm}.$$

In accordance with equation (10), therefore, the uncorrected path difference is

$$\Delta P' = 2.95 \times 10^{-6} \text{ cm}.$$

and the corrected path difference finally,

$$\Delta P = 1.84 \times 10^{-6} \text{ cm}.$$

The fringes which appear in the above interferometer are primarily those of reversed spectra. If the yellow parts of the spectra ($\lambda = 60 \times 10^{-6}$) are superposed, 0.031 of a fringe would pass for the given radius of cylinder (R = 5.3 cm.) at 100 turns. A cylinder 30 cm. in diameter (about a foot) would therefore show .28 fringe, and since this may be doubled by reversing the rotation of the cylinder, (by which strains due to centrifugal force are also eliminated) something short of $\frac{2}{3}$ of a fringe should be observed.

With an ocular micrometer divided in 1/10 millimeter, it should be possible to secure fringes as much as 3 mm. apart, so that a displacement of 20 scale parts may be expected, ten for each of the directions of rotation.

4. *Equations. Two reflections.*—The equations for this case are somewhat more involved than the preceding; but it suffices to accept for the angle of incidence i at the cylinder G, figure 3, the value given by the old-fashioned theory of the rainbow; viz.,

$$8 \cos^2 i = \mu^2 - 1 \tag{12}$$

The chord C from c to d, etc., and its distance h from the axis a will be, as before, $C = 2R \cos r$, $h = R \sin r$, where r is the angle of refraction and R the radius of the cylinder. Finally equation (8), for the average speed v along a chord, also applies. Hence with the inclusion of equation (4), the path difference on rotation may be written, c being the velocity of light,

$$3 \times 2\, C\, (v/c)\, (1 - 1/\mu^2) \tag{13}$$

since there are three chords, C, in sequence. This expression may be reduced by the equations for C, h, v, and equation (12), eventually to a form convenient for computation.

$$(9R^2\omega/c)\,(3/\mu^2+1)\,\sqrt{(1-1/\mu^2)/(9/\mu^2-1)}$$

Data similar to the above may now be inserted; viz., for a small cylinder of water (to be used in the experiments below)

$$R = 5 \text{ cm.}; \mu = 1.33; \omega = 628; c = 3 \times 10^{10}$$

whence the path difference 1.82×10^{-6} cm. results.

This, curiously enough, is about the same value which was obtained in case of equation (11), so that identical deductions apply. The conditions are somewhat more favorable for larger values of μ. Thus in the limiting case $\mu^2 = 3$, the path difference would be about doubled.

5. *Experiments.*—To carry out these experiments at the present time is of course out of the question; but a number of contributory observations may be made with advantage. The case of figure 2 is similar to figure 1, where the dispersion of the cylinder *G* in the former case is simulated by the prism *P* and the auxiliary mirrors *m*, *m'*, of the latter. If the slit of the collimator at *L* is not too coarse, two reversed spectra will be seen in the telescope at *T*, which on being superposed by rotating *m* or *N* on a vertical axis, will show a vivid linear phenomenon in the line of symmetry of the two superposed spectra. On rotating *m* or *N* on a horizontal axis, the distance apart of the fringe dots along this line may be given any reasonable value, at pleasure. These displacements are at once referred to the definite wave length in which the linear phenomenon is put. The dispersion of the prism has no bearing on the clearness of the phenomenon: 30° and 60° prism were tested with like results.

To obtain the achromatics and increased luminosity in the spectrum fringes (now to be horizontal bands throughout from red to blue), the rays of the spectrum will have to be reassembled and that may be done by inserting a second prism say *P'*, between *L* and *N'*, in a way to counteract the effect of the first. If the achromatics are to be obtained, the glass paths of the two rays in *P* and *P'*, respectively, must be coincident. Hence, the axis of the collimator at *L* must be inclined to accommodate the angle of minimum deviation of the identical prisms *P*, *P'*; and while *N* and *m* are parallel, *N'* and *m'* normal to each other, *L* and *T* have their axes symmetric to *N*. The adjustment is not difficult as they need not be perfect to secure good achromatics; but if it is not made the fringes are numerous, colored, and unsatisfactory.

The experiments, figure 2, differ from the case figure 1, because the rays are parallel in the former case and condensed to a caustic by the eccentric refraction of the cylinder in the latter. Hence with these a short range telescope with strong objective is necessary; but as has been stated, the lines of the solar spectrum nevertheless come out clearly. Experiments were therefore made by simulating the glass cylinder *GG* by a thin cylindrical glass shell, closed below and above and containing a solution of mercury potassic iodide with an index at pleasure between 1.5 and 1.7. It was not difficult to meet the conditions of figure 1 so far as mere refraction is concerned, and certain incidental results obtained in this work have been given elsewhere.

The active slit in this experiment is the image within the cylinder, of the slit of the collimator and the former is sufficiently fine to show the Fraunhofer lines, even when the latter is a millimeter broad, so that there is no deficiency of light.

But in relation to the detection of the interferences, the two reversed spectra, strongly divergent in their homogeneous rays, introduce certain grave difficulties. For it will appear that the spectrum issuing at d', figure 1, passes over the distance b further than the spectrum issuing at d, before they reach the telescope together. The result is that the apices of two spectra lie in different focal planes, unless the telescope T is very remote. This makes the adjustment difficult.

To obviate this annoyance a symmetrical adjustment, with an additional mirror at d, figure 1, corresponding symmetrically to N and a symmetrically placed cylinder G, is here preferable. In such a case the spectra lie in the same focal plane, and since they have undergone 2 and 3 reflections, respectively, before reaching T, the interferences of non-reversed spectra are obtained without much difficulty. In my experiments, owing to the irregularity of the glass cylinder used, the fringes were correspondingly irregular; but otherwise clear and strong, as a wide slit is admissible.

The case of two internal reflections is complicated by the occurrence of multiple images from N, figure 3, even when one side is half silvered. This is particularly the case when the cylinder G contains water, as in my first experiments; for the glancing angle at b is then but 25°. There is an advantage, however, inasmuch as N may be placed at a correspondingly large distance from G. In spite of the duplicated images, the fringes were found more easily and were less irregular than anticipated. They are liable to be reproduced usually in a different size and orientation in each of the images. They will be found in the colored edge and even in the white glare (caustic) which emanates from the cylindrical surfaces. They could be made quite large, clear and strong, moreover, although the cylinder used was (as above) an ordinary glass shade. As in case of the triangular interferometer, the fringes rotate when N is displaced parallel to itself on the micrometer screw. To control their size N is to be rotated on a horizontal axis.

[1] Advance Report from the Carnegie Publications of Washington, D. C.

[2] With regard to the symmetrical interferometer form, cf. Michelson and Morley, *Amer. J. Sci., New Haven*, **31**, 1886, (377), also Zeeman, below.

[3] The insufficiency of this equation has been shown by Zeeman, *Proc., Amsterdam Acad.*, September 1914, and September 1915. But an estimate only is above in question.