Newton and Einstein's Gravity in a New Perspective for Planck Masses and Smaller Sized Objects

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Abstract

In a recent paper, Haug [1] has rewritten many of Newton's and Einstein's gravitational results, without changing their output, into a quantized Planck form. However, his results only hold down to the scale of Planck mass size objects. Here we derive similar results for any mass less than or equal to a Planck mass. All of the new formulas presented in this paper give the same numerical output as the traditional formulas. However, they have been rewritten in a way that gives a new perspective on the formulas when working with gravity at the level of the subatomic world. To rewrite the well-known formulas in this way could make it easier to understand strength and weakness in Newton and Einstein gravitation formulas at the subatomic scale, potentially opening them up for new interpretations.

Keywords: Gravitation, gravitational constant, escape velocity, gravitational time dilation, Schwarzschild radius, Planck length, reduced Compton wavelength, bending of light, red-shift, Planck mass.

1 Introduction

Based on dimensional analysis, [2] the Newton gravitational constant [3] can be written as a function of the reduced Planck constant, the Planck length, and the speed of light:

$$G_p = \frac{l_p^2 c^3}{\hbar} \tag{1}$$

Alternatively, this way of writing the gravitational constant can be obtained directly by solving the Planck length formula, [4] with respect to G. One could argue that this leads to a circular argument, since the Planck length is obtained from the gravitational constant. This circular problem is discussed and solved by Haug, [2] where he gives new theoretical insight that strongly indicates one can find l_p independent of any knowledge of G. See also [5]. Further, based on the recent developments in mathematical atomism [6, 7], it is reasonable to think that the Planck length is among the most fundamental constants that could represent the diameter of an indivisible particle. We are not questioning if big G is a universal constant; we are asking if G could be a universal composite constant consisting of even more fundamental constants, and we have reason to think these are c, $\hbar$, and l_p. This paper actually confirms that big G is also valid, and therefore Universal, for any subatomic particles, at least inside the theory offered here.

There is still considerable uncertainty about the exact measurement of the gravitational constant. Experimentally, substantial progress has been made in recent years based on various methods. See, for example, [8, 9, 10, 11, 12]. Also, the relationship between physical constants from the microcosmos (subatomic world) and the macrocosmos (cosmos) plays an important role in physics. A continuous effort is going into improving our measurements and understanding these relationships. See, for example, [13].

The Planck form of the gravitational constant enables us to rewrite the Planck length as

$$l_p = \sqrt{\frac{\hbar G_p}{c^3}} = \sqrt{\frac{\hbar \frac{l_p^2 c^3}{\hbar}}{c^3}} = l_p \tag{2}$$

and the mass of any subatomic particle with mass up or equal to the Planck mass, m_p, can be written as

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$$m = \frac{l_p}{\bar{\lambda}} m_p = \frac{l_p}{\bar{\lambda}} \sqrt{\frac{\hbar c}{G_p}} = \frac{l_p}{\bar{\lambda}} \sqrt{\frac{\hbar c}{\frac{l_p^2 c^3}{\hbar}}} = \frac{\hbar}{\bar{\lambda}} \frac{1}{c} \tag{3}$$

where $\bar{\lambda}$ is the reduced Compton wavelength of the mass in question. In the special case of a Planck mass, we have $\bar{\lambda} = l_p$ and we get

$$m_p = \frac{l_p}{l_p} \sqrt{\frac{\hbar c}{G_p}} = \sqrt{\frac{\hbar c}{\frac{l_p^2 c^3}{\hbar}}} = \frac{\hbar}{l_p} \frac{1}{c} \tag{4}$$

Using the gravitational constant in the Planck form, as well as the mass formula relationship above, we can easily rewrite a series of mathematical end results from Newton and Einstein gravitation without changing their output values. The elements that will change are the input parameters and their mathematical form. Seeing well-known formulas in this new perspective seems to give new and additional insight that we believe can be useful.

2 Newton's Universal Gravitational Force

Newton's gravitational force is given by

$$F_G = G_p \frac{m_1 m_2}{r^2} \tag{5}$$

Using the gravitational constant of the form $G_p = \frac{l_p^2 c^3}{\hbar}$ and two subatomic particles with the same reduced Compton wavelength, we can rewrite Newton's gravitational force for two subatomic particles as

$$\begin{aligned} F &= G_p \frac{m_1 m_2}{r^2} \\ F &= \frac{l_p^2 c^3}{\hbar} \frac{\frac{\hbar}{\bar{\lambda}} \frac{1}{c} \frac{\hbar}{\bar{\lambda}} \frac{1}{c}}{r^2} = \frac{l_p^2}{\bar{\lambda}^2} \frac{\hbar c}{r^2} \end{aligned} \tag{6}$$

In the case where $r = \bar{\lambda}$, we get

$$F = \frac{l_p^2}{\bar{\lambda}^2} \frac{\hbar c}{\bar{\lambda}^2} \tag{7}$$

In the special case of two Planck masses, we have $\bar{\lambda} = l_p$ and we get the well-known Planck force

$$F_p = \frac{\hbar c}{l_p^2} \tag{8}$$

3 Escape Velocity

The traditional Newton escape velocity [14, 15] is given by

$$v_e = \sqrt{\frac{2Gm}{r}} \tag{9}$$

where G is the traditional gravitational constant, m is the mass of the object we are "trying" to escape from, and r is the radius we (for example a particle) are leaving from. Exactly the same escape velocity formula can be derived directly from Einstein's general relativity using the Schwarzschild metric, see [16].

Based on the gravitational constant written in the Planck form and the mass as written on the form given in the introduction, we can derive the escape velocity for a particle with mass equal or less than the Planck mass. We will assume the escape happens at a radius equal to the reduced Compton wavelength of the particle we are escaping from, $r = \bar{\lambda}$, this gives

$$v_e = \sqrt{\frac{2G_p m}{\bar{\lambda}}}$$
$$v_e = \sqrt{\frac{2\frac{l_p^2 c^3}{\hbar}\frac{\hbar}{\bar{\lambda}}\frac{1}{c}}{\bar{\lambda}}}$$
$$v_e = \sqrt{\frac{2\frac{l_p^2}{\bar{\lambda}}c^2}{\bar{\lambda}}}$$
$$v_e = \sqrt{2}\frac{l_p}{\bar{\lambda}}c \quad (10)$$

This also means the escape velocity of a particle is equal to the mass of the particle in question divided by the Planck mass and then multiplied by the speed of light and again multiplied by square root of two

$$v_e = \sqrt{2}\frac{l_p}{\bar{\lambda}}c = \sqrt{2}\frac{m}{m_p}c \quad (11)$$

In the special case where the reduced Compton wavelength is $\sqrt{2}l_p$, we have an escape velocity of c, something that is well-known. This corresponds to a particle with mass

$$m = \frac{\hbar}{\sqrt{2}l_p}\frac{1}{c} = \frac{1}{\sqrt{2}}m_p = \frac{1}{\sqrt{2}}\sqrt{\frac{\hbar c}{G}} \quad (12)$$

This is sometimes called the Planck particle; it is the only particle where the Schwarzschild radius is equal to the reduced Compton wavelength.

Another interesting special case for the escape velocity is related to a particle with half the Planck mass in radius of l_p; this gives an escape velocity of c

$$v_e = \sqrt{\frac{2G_p\frac{1}{2}m_p}{l_p}}$$
$$v_e = \sqrt{\frac{\frac{l_p^2 c^3}{\hbar}\frac{\hbar}{l_p}\frac{1}{c}}{l_p}}$$
$$v_e = \sqrt{\frac{\frac{l_p^2}{l_p}c^2}{l_p}}$$
$$v_e = \frac{l_p}{l_p}c = c \quad (13)$$

Motz and Epstein suggested in 1979 that there likely exists a very fundamental particle with half the Planck mass, [23]. However, they had no good explanation why such a particle was so much heavier than any known observed particle. Haug [6, 7] has recently built a new fundamental theory around an indivisible particle with mass equal to half the Planck mass, where he has speculated on how such a particle could be the fundament of every observable mass. The Haug particle always moves at speed c and is quite different from what modern physics thinks of as particles. The indivisible particle is half a Planck mass when counter-striking (colliding) with another indivisible particle, but only for a Planck second. When non-counter striking it is mass-less and the origin of energy.

4 Orbital Velocity

The orbital velocity is given by

$$v_o \approx \sqrt{\frac{GM}{r}} \quad (14)$$

We can rewrite this in the form of the Planck gravitational constant and the Planck mass as

$$
\begin{aligned}
v_o &\approx \sqrt{\frac{G_p m}{\bar{\lambda}}} \\
v_o &\approx \sqrt{\frac{\frac{l_p^2 c^3}{\hbar}\frac{\hbar}{\bar{\lambda}}\frac{1}{c}}{\bar{\lambda}}} \\
v_o &\approx \frac{l_p}{\bar{\lambda}} c
\end{aligned} \tag{15}
$$

This can also be written as

$$
v_o = \frac{v_e}{\sqrt{2}} = \frac{l_p}{\bar{\lambda}} c \tag{16}
$$

In the special case where $\bar{\lambda} = l_p$, then the orbital velocity is equal to c. The reduced Compton wavelength of a Planck mass is l_p. This also means the orbital velocity of a particle is equal to the mass of the particle in question divided by the Planck mass and then multiplied by the speed of light

$$
v_o = \frac{l_p}{\bar{\lambda}} c = \frac{m}{m_p} c \tag{17}
$$

5 Gravitational Acceleration Parameter

The gravitational acceleration field in modern physics is given by

$$
g = \frac{GM}{r^2} \tag{18}
$$

This can be rewritten in quantized form for particles with a mass less than or equal to a Planck mass as

$$
\begin{aligned}
g &= \frac{G_p m}{\bar{\lambda}^2} \\
g &= \frac{\frac{l_p^2 c^3}{\hbar}\frac{\hbar}{\bar{\lambda}}\frac{1}{c}}{\bar{\lambda}^2} \\
g &= \frac{l_p^2}{\bar{\lambda}^3} c^2
\end{aligned} \tag{19}
$$

In the special case of a Planck mass, $\bar{\lambda} = l_p$ we get the well-known Planck mass acceleration $a_p = \frac{l_p^2}{l_p^3} c = \frac{c^2}{l_p}$; see [24] and [25]. If we have this maximum acceleration for one Planck second, we get to the speed of light:

$$
a_p t_p = \frac{c^2}{l_p}\frac{l_p}{c} = c
$$

This should be interpreted to mean that the Planck acceleration can only last one Planck second, as nothing can move faster than the speed of light. This likely also means that a Planck acceleration field can only last for one Planck second, but more research is needed here.

We can also find big G from the gravitational acceleration parameter of any fundamental particle

$$
G = \frac{g}{m} \bar{\lambda}^2 \tag{20}
$$

For example, we can find find big G from the gravitational acceleration field and the mass of a Planck mass particle

$$
G = \frac{g}{m} \bar{\lambda}^2 = \frac{\frac{c^2}{l_p}}{\frac{\hbar}{l_p}\frac{1}{c}} l_p^2 = \frac{l_p^2 c^3}{\hbar} \tag{21}
$$

Or we can find big G from the gravitational acceleration field and the mass of an electron

$$G=\frac{g_e}{m_e}\bar{\lambda}_e^2=\frac{\frac{l_p^2}{\bar{\lambda}_e^3}c^2}{\frac{\hbar}{\bar{\lambda}_e}\frac{1}{c}}\bar{\lambda}_e^2=\frac{l_p^2c^3}{\hbar}\approx 6.67\times 10^{-11} \tag{22}$$

That is to say that G is always the same no matter the particle; this even holds true for a half-Planck mass particle:

$$G=\frac{g_i}{m_i}\bar{\lambda}_i^2=\frac{\frac{l_p^2}{(2l_p)^3}c^2}{\frac{\hbar}{2l_p}\frac{1}{c}}(2l_p)^2=\frac{l_p^2c^3}{\hbar}\approx 6.67\times 10^{-11} \tag{23}$$

We can conclude that big G is independent of the reduced Compton wavelength of the particle in question. With respect to this concept, we can call big G a Universal constant at the subatomic level as well, since it must be the same for any subatomic particle, at least inside our theoretical model, which is based on Newton.

6 Gravitational Parameter

The standard gravitational parameter is given by

$$\mu = Gm \tag{24}$$

This can be rewritten in quantized form as

$$\begin{aligned}\mu_p &= G_p m\\ \mu_p &= G_p m\\ \mu_p &= \frac{l_p^2c^3}{\hbar}N\frac{\hbar}{\bar{\lambda}}\frac{1}{c}\\ \mu_p &= \frac{l_p^2}{\bar{\lambda}}c^2\end{aligned} \tag{25}$$

where N is the number of particles with mass $m=\frac{\hbar}{\bar{\lambda}}\frac{1}{c}$ that make up the object in question. This can further be rewritten as

$$\mu_p = \frac{l_p^2}{\bar{\lambda}}c^2 = l_pc^2\sqrt{1-\frac{v_{max}^2}{c^2}} \tag{26}$$

where $v_{max}=\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}$ is the maximum velocity for any particle with mass equal to or less than the Planck mass. This maximum velocity has recently been introduced by [2, 5, 7], and it seems to play a central role in understanding the Planck mass and its relation to other particles. See the suggested papers for an in-depth discussion on this topic.

7 Gravitational Time Dilation at the Subatomic Level

Einstein's gravitational time dilation [26] is given by

$$t_0=t_f\sqrt{1-\frac{2Gm}{rc^2}}=t_f\sqrt{1-\frac{v_e^2}{c^2}} \tag{27}$$

where v_e is the traditional escape velocity. This means that the gravitational time-dilation could have been derived from the Newton escape velocity, but to my knowledge Einstein is the first one to mention gravitational time dilation. We can rewrite this in the form of the quantized escape velocity (derived above).

$$
\begin{aligned}
t_o &= t_f\sqrt{1-\frac{v_{e,p}^2}{c^2}} \\
t_o &= t_f\sqrt{1-\frac{\left(\sqrt{2}\frac{l_p}{\bar{\lambda}}c\right)^2}{c^2}} \\
t_o &= t_f\sqrt{1-2\frac{l_p^2}{\bar{\lambda}^2}}
\end{aligned} \tag{28}
$$

Notice that for a reduced Compton wavelength equal to the Planck length $\bar{\lambda} = l_p$, the formula above is invalid, or more precisely the time-dilation becomes imaginary:

$$
t_o = t_f\sqrt{1-2\frac{l_p^2}{\bar{l_p}^2}} = t_f\sqrt{1-2} = t_f\sqrt{-1} = t_f i \tag{29}
$$

However, since v_e for a Planck mass is $> c$, it is more precise to say that Einstein's time dilation formula likely breaks down for a Planck mass.

Interestingly, for a half-Planck mass when at the Schwarzschild radius we get

$$
\begin{aligned}
t_o &= t_f\sqrt{1-\frac{v_e^2}{c^2}} \\
t_o &= t_f\sqrt{1-\frac{c^2}{c^2}} \\
t_o &= t_f\sqrt{1-1} = 0
\end{aligned} \tag{30}
$$

In other words, only for half-Planck mass particles does time stand still. Haug has derived a new theory based on an indivisible particle with potential mass equal to half the Planck mass, and half the Planck rest-mass when colliding. Under atomism time is simply counter-strikes between indivisible particles. For a single indivisible particle when not counter-striking, time also stands still; this is due to the concept that time under atomism is counter-strikes (clock ticks). Based on atomism as well as the Newton corpuscular theory, light consists of indivisible particles traveling one after another. When they travel after each other they cannot counter-strike as they all travel with the same speed, and time stands still as "observed" from these particles.

Circular orbits gravitational time dilation

The time dilation for a clock at circular orbit[1] is given by

$$
t_0 = t_f\sqrt{1-\frac{3}{2}\frac{2Gm}{rc^2}} = \sqrt{1-\frac{3}{2}\frac{v_e^2}{c^2}} \tag{31}
$$

where v_e is the traditional escape velocity. We can rewrite this in the form of the escape velocity for a subatomic particle, where we set the "radius" equal to the reduced Compton wavelength $r = \bar{\lambda}$,

$$
\begin{aligned}
t_o &= t_f\sqrt{1-\frac{3}{2}\frac{v_{e,p}^2}{c^2}} \\
t_o &= t_f\sqrt{1-\frac{3}{2}\frac{\left(c\sqrt{2}\frac{l_p}{\bar{\lambda}}\right)^2}{c^2}} \\
t_o &= t_f\sqrt{1-3\frac{l_p^2}{\bar{\lambda}^2}}
\end{aligned} \tag{32}
$$

[1] At orbital radius larger then $\frac{3}{2}r_s$

8 The Schwarzschild Radius

The so-called Schwarzschild radius [17, 18, 19, 20, 21] of a mass m is given by

$$r_s = \frac{2Gm}{c^2} \tag{33}$$

Rewritten as a function of the Planck units and the reduced Compton wavelength, it is given by

$$\begin{aligned} r_s &= \frac{2G_p m}{c^2} \\ r_s &= \frac{2\frac{l_p^2 c^3}{\hbar}\frac{\hbar}{\bar{\lambda}}\frac{1}{c}}{c^2} \\ r_s &= 2\frac{l_p^2}{\bar{\lambda}} \end{aligned} \tag{34}$$

We think this formula only make sense for sizes down to half-Planck masses and the Planck mass, but not for any mass smaller than half the Planck mass. In 1979, Motz and Epstein [23] introduced a hypothetical particle with half the Planck mass. Haug has derived an extensive theory around such a particle [6, 7]. For the half Planck mass particle, we have a Schwarzschild radius of

$$\begin{aligned} r_s &= \frac{2G_p \frac{1}{2} m_p}{c^2} \\ r_s &= \frac{G_p m_p}{c^2} \\ r_s &= \frac{\frac{l_p^2 c^3}{\hbar}\frac{\hbar}{l_p}\frac{1}{c}}{c^2} \\ r_s &= l_p \end{aligned} \tag{35}$$

This indivisible particle has according to Haug a "rest" mass equal to half the Planck mass and is likely the lowest mass a particle can take while the escape velocity still is c. It is likely meaningless to talk about a Schwarzschild radius for fundamental particles with a mass less than half the Planck mass. An electron, for example, does not have a valid Schwarzschild radius even if formula 34 above could be used to calculate a hypothetical Schwarzschild radius for an electron. The Schwarzschild radius for any "object" with mass less than half the Planck mass would be smaller than l_p. This is likely impossible and may be best understood from the recent renewal of atomism. The Schwarzschild radius of an indivisible particle is simply the diameter of the indivisible particle. Further, the Schwarzschild "radius" of a Planck mass is the length of two indivisible particles lying next to each other. Only for the indivisible particle is the Schwarzschild radius truly a radius in the sense that, at the depth of reality, it has to do with a perfectly spherical shaped particle.

9 Gravitational Light Deflection

In 1884, Soldner derived work based on Newton classical mechanics that predicted the following deflection of light

$$\delta_S = \frac{2Gm}{c^2 r} \tag{36}$$

The angle of deflection in Einstein's general relativity theory [26] is twice that of the Soldner formula (1884)

$$\delta_{GR} = \frac{4Gm}{c^2 r} \tag{37}$$

The solar eclipse experiment of Dyson, Eddington, and Davidson performed in 1919 confirmed [27] the idea that the deflection of light was very close to that predicted by Einstein's general relativity theory. That is 1.75 arcseconds compared to the 0.875 as predicted by the Soldner 1884 formula.[2] This was one of the main reasons general relativity took off and partly replaced Newton gravitation.

Interestingly, Accioly and Ragusa [28] have shown that in semi-classical general relativity, the bending of light is dependent on $\frac{v}{c}$, and that when $v = 0$ one gets the Soldner formula for Newtonian bending of light, and when $v = c$ one gets the Einsteinian bending of light result. See also [37]. Sato and Sato [29] have recently pointed out that it looks like the two factors (double of Newton) in the light deflection likely are due to a unknown property of the photon rather than the bending of space. Further, theoretical and experimental research will hopefully give us deeper insight into the deflection of light.

For a subatomic particle, the Einstein deflection of light can be rewritten as

$$\begin{aligned} \delta &= \frac{4G_p m}{c^2 \bar{\lambda}} \\ \delta &= \frac{4\frac{l_p^2 c^3}{\hbar}\frac{\hbar}{\bar{\lambda}}\frac{1}{c}}{c^2 \bar{\lambda}} \\ \delta &= 4\frac{l_p^2}{\bar{\lambda}^2} \end{aligned} \tag{38}$$

In the special case where we deal with half a Planck mass $\bar{\lambda} = 2l_p$ we get

$$\delta = 4\frac{l_p^2}{(2l_p)^2} = 1 \tag{39}$$

The light deflection in relation to a half Planck mass seems to be 180 degrees. The gravitational deflection formula can also be written for any particle as

$$\delta = 4\frac{l_p^2}{\bar{\lambda}^2} = 4\left(1 - \frac{v_{max}^2}{c^2}\right) \tag{40}$$

Only for a Planck mass we have $v_{max} = 0$ and therefore $\delta = 4$, and for a half-Planck mass we get $\delta = 1$ again. In terms of degrees, this is $4 \times \frac{648000}{\pi 3600} \approx 229.1°$ and for the half-Planck mass $114.59°$. Whether or not Einstein's light deflection formula truly makes sense at such short distances is an open question.

10 Gravitational Redshift

Einstein's gravitational redshift is given by

$$\lim_{r\to+\infty} z(r) = \frac{1}{\sqrt{1 - \frac{\frac{2Gm}{r}}{c^2}}} - 1 = \frac{1}{\sqrt{1 - \frac{v_e^2}{c^2}}} - 1 \tag{41}$$

where r is the distance between the center of the mass of the gravitating body and the point at which the photon is emitted. For a subatomic particle, when we assume $r = \bar{\lambda}$, we can rewrite this as

$$\begin{aligned} \lim_{r\to+\infty} z(r) &= \frac{1}{\sqrt{1 - \frac{\sqrt{\frac{2Gm}{\bar{\lambda}}}^2}{c^2}}} - 1 \\ \lim_{r\to+\infty} z(r) &= \frac{1}{\sqrt{1 - \frac{\frac{2\frac{l_p^2 c^3}{\hbar}\frac{\hbar}{\bar{\lambda}}\frac{1}{c}}{c^2}}{\bar{\lambda}}}} - 1 \\ \lim_{r\to+\infty} z(r) &= \frac{1}{\sqrt{1 - 2\frac{l_p^2}{\bar{\lambda}^2}}} - 1 \end{aligned} \tag{42}$$

[2]Soldner in 1881 calculated the light deflection to be 0.84 arcseconds based on less accurate knowledge of the mass of the sun and speed of light than we have today.

By using a series expansion approximation of $\sqrt{1-2\frac{l_p^2}{\bar{\lambda}^2}}$ when $\bar{\lambda} >> l_p$ we get

$$\sqrt{1-2\frac{l_p^2}{\bar{\lambda}^2}} \approx 1-\frac{1}{2}2\frac{l_p^2}{\bar{\lambda}^2} \tag{43}$$

and this gives

$$\begin{aligned}
\lim_{r\to+\infty} z(r) &= \frac{1}{1-\frac{1}{2}2\frac{l_p^2}{\bar{\lambda}^2}} - 1 \\
\lim_{r\to+\infty} z(r) &= \frac{1}{1-\frac{l_p^2}{\bar{\lambda}^2}} - \frac{1-\frac{l_p^2}{\bar{\lambda}^2}}{1-\frac{l_p^2}{\bar{\lambda}^2}} \\
\lim_{r\to+\infty} z(r) &= \frac{\frac{l_p^2}{\bar{\lambda}^2}}{1-\frac{l_p^2}{\bar{\lambda}^2}} \approx \frac{l_p^2}{\bar{\lambda}^2}
\end{aligned} \tag{44}$$

Several researchers have derived a similar redshift formula with no recourse to the general relativity theory, nor to the principle of equivalence, see [30] and [31]

$$\begin{aligned}
\lim_{r\to+\infty} z(r) &= \frac{Gm}{c^2\bar{\lambda}} \\
\lim_{r\to+\infty} z(r) &= \frac{\frac{l_p^2 c^3}{\hbar}\frac{\hbar}{\bar{\lambda}}\frac{1}{c}}{c^2\bar{\lambda}} \\
\lim_{r\to+\infty} z(r) &= \frac{l_p^2}{\bar{\lambda}^2}
\end{aligned} \tag{45}$$

Formula 45 is often considered as an approximation to general relativity redshift, but one could also argue the other way around, that GR is an approximation to this formula. The prediction difference between the two formulas can only be observed in very high gravitational fields. One of the famous experiments that is claimed to have confirmed general relativity with very high precision is the Pound and Rebka [32] experiment. They measured the gravitational redshift in a tower over a distance of approximately 22.5 meters. This was an excellent experiment that got the same result as predicted by Einstein's general relativity theory. However, this experiment did not provide evidence that the general relativity theory is a complete theory. The experiment was done in a very weak gravitational field, where we know that the formula 45 should work just as well. However, the theory proved that Einstein was correct in the notion that gravity affects time; something that has been confirmed by a long series of experiments since then. To my knowledge, no one had assumed that gravity could affect time before Einstein.

We also have that

$$\begin{aligned}
\lim_{r\to+\infty} z(r) &= \frac{l_p^2}{\bar{\lambda}^2} \\
\lim_{r\to+\infty} z(r) &= \sqrt{1-\frac{v_{max}^2}{c^2}}\sqrt{1-\frac{v_{max}^2}{c^2}} \\
\lim_{r\to+\infty} z(r) &= 1-\frac{v_{max}^2}{c^2}
\end{aligned} \tag{46}$$

This means the redshift of a Planck mass is $\lim_{r\to+\infty} z(r) = \frac{l_p^2}{l_p^2} = 1$, because v_{max} for a Planck mass is zero. Again if GR truly holds all the way down to the Planck length (Planck mass) is an open question.

11 Table Summary

Table 1 summarizes our rewritten versions of several gravitational formulas.

Units:	Newton and Einstein form:	"Wavelength"-form:	Mass form
Gravitational constant	$G \approx 6.67408 \times 10^{-11}$	$G_p = \frac{l_p^2 c^3}{\hbar}$	
Newton's gravitational force	$F = G\frac{m_1 m_2}{r^2}$	$F = \frac{\hbar c}{r^2}\frac{l_p^2}{\bar{\lambda}^2}$	$F = \frac{\hbar c}{r^2}\frac{m^2}{m_p^2}$
Escape velocity	$v_e = \sqrt{\frac{2Gm}{r}}$	$v_e = \frac{l_p}{\bar{\lambda}}c\sqrt{2}$	$v_e = \sqrt{2}\frac{m}{m_p}c$
Orbital velocity	$v_o = \sqrt{\frac{Gm}{r}}$	$v_o = \frac{l_p}{\bar{\lambda}}c$	$v_o = \frac{m}{m_p}c$
Gravitational parameter	$\mu = Gm$	$\mu = \frac{l_p^2}{\bar{\lambda}}c^2$	$\mu = \frac{m}{m_p}l_p c^2$
Gravitational acceleration field	$g = \frac{Gm}{r^2}$	$g = \frac{l_p^2}{\bar{\lambda}^3}c^2$	$g = \frac{m^2}{m_p^2}\frac{c^2}{\bar{\lambda}}$
Gravitational time dilation	$t_0 = t_f\sqrt{1-\frac{2Gm}{rc^2}} = \sqrt{1-\frac{v_e^2}{c^2}}$	$t_o = t_f\sqrt{1-2\frac{l_p^2}{\bar{\lambda}^2}}$	$t_o = t_f\sqrt{1-2\frac{m^2}{m_p^2}}$
Orbital time dilation	$t_0 = t_f\sqrt{1-3\frac{Gm}{rc^2}} = \sqrt{1-\frac{3}{2}\frac{v_e^2}{c^2}}$	$t_o = t_f\sqrt{1-3\frac{l_p^2}{\bar{\lambda}^2}}$	$t_o = t_f\sqrt{1-3\frac{m^2}{m_p^2}}$
Light deflection	$\delta = \frac{4Gm}{c^2 r}$	$\delta = 4\frac{l_p^2}{\bar{\lambda}^2}$	$\delta = 4\frac{m^2}{m_p^2}$
Schwarzschild Radius [a]	$r_s = \frac{2Gm}{c^2}$	$r_s = 2\frac{l_p^2}{\bar{\lambda}}$	$r_s = 2l_p\frac{m}{m_p}$
Gravitational redshift GR	$\lim_{r\to+\infty} z(r) = \frac{1}{\sqrt{1-\frac{\frac{2Gm}{r}}{c^2}}} - 1$	$\lim_{r\to+\infty} z(r) = \frac{1}{\sqrt{1-2\frac{l_p^2}{\bar{\lambda}^2}}} - 1$	$\frac{1}{\sqrt{1-2\frac{m^2}{m_p^2}}} - 1$
Gravitational redshift ABS[b]	$\lim_{r\to+\infty} z(r) = \frac{Gm}{c^2 r}$	$\lim_{r\to+\infty} z(r) = \frac{l_p^2}{\bar{\lambda}^2}$	$\frac{m^2}{m_p^2}$

Table 1: The table shows some of the standard gravitational relationships given by Newton and Einstein and their expression rewritten for Planck masses and beyond. The radius set for subatomic particles are equal to the reduced Compton wavelength of the particle in question.

[a] We think this formula is only valid for particles with a larger or equal to half the Planck mass.
[b] Based on the derivation by Adler Bassin and Shiffer; see [30] that is done totally independently of GR assumptions.

12 Any deeper meaning behind $\frac{l_p}{\bar{\lambda}}$ and $\frac{l_p^2}{\bar{\lambda}^2}$?

The fact that the factors $\frac{l_p}{\bar{\lambda}}$ and $\frac{l_p^2}{\bar{\lambda}^2}$ are showing up in several of the formulas in this paper, including the gravitational deflection of light, the redshift, and even Newton's gravitational force formula is particularly interesting. Haug [7] has recently shown that

$$\frac{l_p}{\bar{\lambda}} = \sqrt{1-\frac{v_{max}^2}{c^2}} \tag{47}$$

where v_{max} is the maximum speed a uniform mass can take. At this maximum speed, a fundamental mass (an electron, for example) will have reached a relativistic mass equal to the Planck mass. This is discussed in detail in the recent papers by Haug [7, 5, 2]. If correct, this means that many of the $\frac{l_p}{\bar{\lambda}}$ and $\frac{l_p^2}{\bar{\lambda}^2}$ factors showing up in the quantified formulas could be quantum relativistic adjustments that we have been aware until now are already there. Haug has shown that

$$F = G\frac{m_e m_e}{r^2} = G\frac{m_p\sqrt{1-\frac{v_{max}^2}{c^2}}\,m_p\sqrt{1-\frac{v_{max}^2}{c^2}}}{r^2} = \frac{\hbar c}{r^2}\left(1-\frac{v_{max}^2}{c^2}\right) = \frac{\hbar c}{r^2}\frac{l_p^2}{\bar{\lambda}^2} \tag{48}$$

where v_{max} in this case is the speed the electron needs to take to reach a relativistic mass equal to the Planck mass, that is $v_{max} = c\sqrt{1-\frac{l_p^2}{\bar{\lambda}_e^2}}$. The term $\frac{l_p^2}{\bar{\lambda}_e^2}$ is the well-known dimensionless gravitational coupling constant; see [33, 34, 35, 36], that more commonly is written in the form $\alpha_G = \frac{m_e^2}{m_p^2}$.

In a earlier paper, [38] when looking at a possible connection between electromagnetism and gravity, Haug has also speculated that $\frac{l_p^2}{\bar{\lambda}^2}$ could a be conditional probability factor, related to the probability of gravity hits. The gravitational coupling constant could be both a quantum relativistic adjustment as well as a conditional probability factor at the same time; this is fully possible and should encourage further investigation.

13 Conclusion

By making the gravitational constant a function form of the reduced Planck constant, one can easily rewrite many of the end results from Newton's and Einstein's gravitation in quantized form. This has been done recently for masses above or equal to the Planck mass. In this paper, this framework has been extended to hold below Planck mass size objects as well. This gives the same numerical end results as those obtained by Newton and Einstein, but now provides new insight about the subatomic world, or at least insight in how well or not so well Newton's and Einstein's gravitational theories fits the subatomic world. How these new formulas, which only rely on the Planck length, the Planck constant, the speed of light, and the reduced Compton wavelength, should be interpreted is open to discussion.

References

[1] E. G. Haug. Planck quantization of Newton and Einstein gravitation. International Journal of Astronomy and Astrophysics, Vol. 6. 2016.

[2] E. G. Haug. The gravitational constant and the Planck units. a simplification of the quantum realm. Forthcoming: Physics Essays Vol 29, No 4, 2016.

[3] I. Newton. Philosophiae Naturalis Principia Mathematics. London, 1686.

[4] M. Planck. The Theory of Radiation. Dover 1959 translation, 1906.

[5] E. G. Haug. A new solution to Einstein's relativistic mass challenge based on maximum frequency. http://vixra.org/abs/1609.0083, 2016.

[6] E. G. Haug. Unified Revolution, New Fundamental Physics. Oslo, E.G.H. Publishing, 2014.

[7] E. G. Haug. The Planck mass particle finally discovered! the true God particle! good bye to the point particle hypothesis! http://vixra.org/pdf/1607.0496v6.pdf, 2016.

[8] G. S. Bisnovatyi-Kogan. Checking the variability of the gravitational constant with binary pulsars. International Journal of Modern Physics D, 15(07), 2006.

[9] B. Fixler, G. T. Foster, J. M. McGuirk, and M. A. Kasevich. Atom interferometer measurement of the Newtonian constant of gravity. Science, 315, 2007.

[10] S. Galli, A. Melchiorri, G. F. Smoot, and O. Zahn. From Cavendish to Planck: Constraining Newton's gravitational constant with CMB temperature and polarization anisotropy. Physical Review D, 80, 2009.

[11] G. Rosi, F. Sorrentino, L. Cacciapuoti, M. Prevedelli, and G. M. Tino. Precision measurement of the Newtonian gravitational constant using cold atoms. Nature, 510, 2014.

[12] S. Schlamminger. A fundamental constants: A cool way to measure big G. Nature, 510, 2014.

[13] P. Collaboration and et al. Planck intermediate results – xxiv. constraints on variations in fundamental constants. Astronomy & Astrophysics, 580, 2015.

[14] B. Schutz. Gravity from the Ground Up. Cambridge University Press, 2003.

[15] P. A. Tipler and R. A. Llewellyn. Modern Physics, Sixth Edition. W. H. Freeman and Company, New York, 2012.

[16] A. T. Augousti and A. Radosz. An observation on the congruence of the escape velocity in classical mechanics and general relativity in a Schwarzschild metric. European Journal of Physics, 376:331–335, 2006.

[17] K. Schwarzschild. über das gravitationsfeld einer kugel aus inkompressibler flussigkeit nach der einsteinschen theorie. Sitzungsberichte der Deutschen Akademie der Wissenschaften zu Berlin, Klasse fur Mathematik, Physik, und Technik, page 424, 1916.

[18] K. Schwarzschild. Über das gravitationsfeld eines massenpunktes nach der einsteinschen theorie. Sitzungsberichte der Deutschen Akademie der Wissenschaften zu Berlin, Klasse fur Mathematik, Physik, und Technik, page 189, 1916.

[19] D. Hilbert. Nachr. ges. wiss. Göttingen, Math. Phys. (Submitted 23 Dec. 1916), 1917.

[20] S. Antoci. David Hilbert and the origin of the "Schwarzschild solution". www.arXiv:physics/0310104.

[21] S. J. Crothers. A brief histroy of black holes. Progress in Physics, 2, 2006.

[22] E. G. Haug. Charged Particle Radiation Power at the Planck Scale. http://vixra.org/pdf/1604.0228v2.pdf, 2016.

[23] L. Motz and J. Epstein. The gravitational charge $1/2\sqrt{\hbar c}$ as a unifying principle in physics. Il Nuovo Cimento, 51(1), 1979.

[24] G. Scarpetta. Letter Nuovo Cimento,, 51, 1984.

[25] E. G. Haug. Is the Schwarzschild radius truly a radius. www.viXra.org, 2016.

[26] A. Einstein. Näherungsweise integration der feldgleichungen der gravitation. Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften Berlin, 1916.

[27] F. Dyson, A. Eddington, and C. Davidson. A determination of the deflection of light by the Sun's gravitational field, from observations made at the total eclipse of may 29, 1919. Philosophical Translation Royal Society, 1920.

[28] A. Accioly and S. Ragusa. Gravitational deflection of massive particles in classical and semiclassical general relativity. Classical and Quantum Gravity, 19(21), 2002.

[29] M. Sato and H. Sato. Light bending by gravity: Back to space and time from spacetime. Physics Essays, 29, 2016.

[30] R. Adler, M. Bazin, and M. Schiffer. Introduction to General Relativity. McGraw-Hill, New York, 1965.

[31] R. Evans and J. Dunning-Davies. The gravitational red-shift. www.arXiv.org 18 March, 2004.

[32] R. V. Pound and G. A. Rebka Jr. Gravitational red-shift in nuclear resonance. Physical Review Letters, 3(9):439–441, 1959.

[33] J. Silk. Cosmogony and the magnitude of the dimensionless gravitational coupling constant. Nature, 265:710–711, 1977.

[34] I. L. Rozental. On the numerical values of the fine-structure constant and the gravitational constant. Soviet Journal of Experimental and Theoretical Physics Letters, 31(9):19–27, 1980.

[35] M. d. O. Neto. Using the dimensionless Newton gravity constant $\bar{\alpha}_g$ to estimate planetary orbits. Chaos, Solitons and Fractals, 24(1):19–27, 2005.

[36] A. S. Burrows and J. P. Ostriker. Astronomical reach of fundamental physics. Proceedings of the National Academy of Sciences of the United States of America, 111(7):31–36, 2013.

[37] B. Patla, R. Nemiroff, D. Hoffmann, and K. Zioutas. Flux enhancement of slow-moving particles by Sun or Jupiter: Can they be detected on earth? The Astrophysical Journal, 780(158), 2014.

[38] E. G. Haug. Unification of gravity and electromagnetism: Gravityelectromagnetism: A probability interpretation of gravity. www.viXra.org, 2016.