## Area of a Triangle

The area of a <u>polygon</u> is the number of square units inside that polygon. Area is 2-dimensional like a carpet or an area rug. A **triangle** is a three-sided polygon. We will look at several types of triangles in this lesson.





To find the area of a triangle, multiply the base by the height, and then divide by 2. The division by 2 comes from the fact that a <u>parallelogram</u> can be divided into 2 triangles. For example, in the diagram to the left, the area of each triangle is equal to one-half the area of the parallelogram.

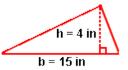
Since the area of a parallelogram is  $A = b \cdot h$ , the area of a triangle must be one-half the area of a parallelogram. Thus, the formula for the area of a triangle is:

$$A = \frac{1}{2} \cdot b \cdot h$$
 or  $A = \frac{b \cdot h}{2}$ 

where h is the base, h is the height and  $\cdot$  means multiply.

The base and height of a triangle must be <u>perpendicular</u> to each other. In each of the examples below, the base is a side of the triangle. However, depending on the triangle, the height may or may not be a side of the triangle. For example, in the right triangle in Example 2, the height is a side of the triangle since it is perpendicular to the base. In the triangles in Examples 1 and 3, the lateral sides are not perpendicular to the base, so a dotted line is drawn to represent the height.

Example 1: Find the area of an <u>acute triangle</u> with a base of 15 inches and a height of 4 inches.



$$A = \frac{1}{2} \cdot b \cdot h$$

$$A = \frac{1}{2} \cdot (15 \text{ in}) \cdot (4 \text{ in})$$

$$A = \frac{1}{2} \cdot (60 \text{ in}^2)$$

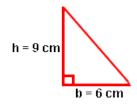
$$A = 30 \text{ in}^2$$

Example 2: Find the area of a <u>right triangle</u> with a base of 6 centimeters and a height of 9 centimeters.

$$A = \frac{1}{2} \cdot b \cdot h$$

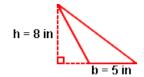
$$A = \frac{1}{2} \cdot (6 \text{ cm}) \cdot (9 \text{ cm})$$

$$A = \frac{1}{2} \cdot (54 \text{ cm}^2)$$



$$A = 27 \text{ cm}^2$$

Example 3: Find the area of an <u>obtuse triangle</u> with a base of 5 inches and a height of 8 inches.



Solution:

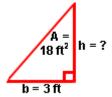
$$A = \frac{1}{2} \cdot b \cdot h$$

$$A = \frac{1}{2} \cdot (5 \text{ in}) \cdot (8 \text{ in})$$

$$A = \frac{1}{2} \cdot (40 \text{ in}^2)$$

$$A = 20 \text{ in}^2$$

Example 4: The area of a triangular-shaped mat is 18 square feet and the base is 3 feet. Find the height. (*Note: The triangle in the illustration to the right is NOT drawn to scale.*)



Solution:

In this example, we are given the area of a triangle and one dimension, and we are asked to work backwards to find the other dimension.

$$A = \frac{1}{2} \cdot b \cdot h$$

$$18 \text{ ft}^2 = \frac{1}{2} \cdot (3 \text{ ft}) \cdot h$$

Multiplying both sides of the equation by 2, we get:

36 
$$ft^2 = (3 ft) \cdot h$$

Dividing both sides of the equation by 3 ft, we get:

$$12 \text{ ft} = h$$

Commuting this equation, we get:

$$h = 12 \text{ ft}$$

Summary:

Given the base and the height of a triangle, we can find the area. Given the area and either the base or the height of a triangle, we can find the other dimension. The formula for area of a triangle is:



$$A = \frac{1}{2} \cdot b \cdot h$$
 or  $A = \frac{b \cdot h}{2}$  where b is the base, h is the height