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Neutrosophic Groups and Subgroups

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Abstract: This paper is devoted to the study of neutrosophic groups and neutrosophic subgroups. Some properties of neutrosophic groups and neutrosophic subgroups are presented. It is shown that the product of a neutrosophic subgroup and a pseudo neutrosophic subgroup of a commutative neutrosophic group is a neutrosophic subgroup and their union is also a neutrosophic subgroup even if neither is contained in the other. It is also shown that all neutrosophic groups generated by the neutrosophic element I and any group isomorphic to Klein 4-group are Lagrange neutrosophic groups. The partitioning of neutrosophic groups is also presented.

Key Words: Neutrosophy, neutrosophic, neutrosophic logic, fuzzy logic, neutrosophic group, neutrosophic subgroup, pseudo neutrosophic subgroup, Lagrange neutrosophic group, Lagrange neutrosophic subgroup, pseudo Lagrange neutrosophic subgroup, weak Lagrange neutrosophic group, free Lagrange neutrosophic group, weak pseudo Lagrange neutrosophic group, free pseudo Lagrange neutrosophic group, smooth left coset, rough left coset, smooth index.

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§1. Introduction

In 1980, Florentin Smarandache introduced the notion of neutrosophy as a new branch of philosophy. Neutrosophy is the base of neutrosophic logic which is an extension of the fuzzy logic in which indeterminancy is included. In the neutrosophic logic, each proposition is estimated to have the percentage of truth in a subset T, the percentage of indeterminancy in a subset I, and the percentage of falsity in a subset F. Since the world is full of indeterminancy, several real world problems involving indeterminancy arising from law, medicine, sociology, psychology, politics, engineering, industry, economics, management and decision making, finance, stocks and share, meteorology, artificial intelligence, IT, communication etc can be solved by neutrosophic logic.

Using Neutrosophic theory, Vasantha Kandasamy and Florentin Smarandache introduced the concept of neutrosophic algebraic structures in [1,2]. Some of the neutrosophic algebraic

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structures introduced and studied include neutrosophic fields, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups, neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, neutrosophic N-semigroup, neutrosophic loops, neutrosophic biloops, neutrosophic N-loop, neutrosophic groupoids, neutrosophic bigroupoids and so on. In [5], Agboola et al studied the structure of neutrosophic polynomial. It was shown that Division Algorithm is generally not true for neutrosophic polynomial rings and it was also shown that a neutrosophic polynomial ring $\langle R \cup I \rangle [x]$ cannot be an Integral Domain even if R is an Integral Domain. Also in [5], it was shown that $\langle R \cup I \rangle [x]$ cannot be a Unique Factorization Domain even if R is a unique factorization domain and it was also shown that every non-zero neutrosophic principal ideal in a neutrosophic polynomial ring is not a neutrosophic prime ideal. In [6], Agboola et al studied ideals of neutrosophic rings. Neutrosophic quotient rings were also studied. In the present paper, we study neutrosophic group and neutrosophic subgroup. It is shown that the product of a neutrosophic subgroup and a pseudo neutrosophic subgroup of a commutative neutrosophic group is a neutrosophic subgroup and their union is also a neutrosophic subgroup even if neither is contained in the other. It is also shown that all neutrosophic groups generated by I and any group isomorphic to Klein 4-group are Lagrange neutrosophic groups. The partitioning of neutrosophic groups is also studied. It is shown that the set of distinct smooth left cosets of a Lagrange neutrosophic subgroup (resp. pseudo Lagrange neutrosophic subgroup) of a finite neutrosophic group (resp. finite Lagrange neutrosophic group) is a partition of the neutrosophic group (resp. Lagrange neutrosophic group).

§2. Main Results

Definition 2.1 *Let $(G, *)$ be any group and let $\langle G \cup I \rangle = \{a + bI : a, b \in G\}$. $N(G) = (\langle G \cup I \rangle, *)$ is called a neutrosophic group generated by G and I under the binary operation $*$. I is called the neutrosophic element with the property $I^2 = I$. For an integer n, n+I, and nI are neutrosophic elements and $0.I = 0$. I^{-1}, the inverse of I is not defined and hence does not exist.*

N(G) is said to be commutative if $ab = ba$ for all $a, b \in N(G)$.

Theorem 2.2 *Let $N(G)$ be a neutrosophic group.*

(i) $N(G)$ in general is not a group;
(ii) $N(G)$ always contain a group.

Proof (i) Suppose that $N(G)$ is in general a group. Let $x \in N(G)$ be arbitrary. If x is a neutrosophic element then $x^{-1} \notin N(G)$ and consequently N(G) is not a group, a contradiction.

(ii) Since a group G and an indeterminate I generate $N(G)$, it follows that $G \subset N(G)$ and N(G) always contain a group. □

Definition 2.3 *Let $N(G)$ be a neutrosophic group.*

(i) A proper subset $N(H)$ of $N(G)$ is said to be a neutrosophic subgroup of $N(G)$ if $N(H)$ is a neutrosophic group such that is $N(H)$ contains a proper subset which is a group;

(ii) $N(H)$ is said to be a pseudo neutrosophic subgroup if it does not contain a proper subset which is a group.

Example 2.4 (i) $(N(\mathcal{Z}),+)$, $(N(\mathcal{Q}),+)$ $(N(\mathcal{R}),+)$ and $(N(\mathcal{C}),+)$ are neutrosophic groups of integer, rational, real and complex numbers respectively.
(ii) $(\langle\{\mathcal{Q}-\{0\}\}\cup I\rangle,.)$, $(\langle\{\mathcal{R}-\{0\}\}\cup I\rangle,.)$ and $(\langle\{\mathcal{C}-\{0\}\}\cup I\rangle,.)$ are neutrosophic groups of rational, real and complex numbers respectively.

Example 2.5 Let $N(G)=\{e,a,b,c,I,aI,bI,cI\}$ be a set where $a^2=b^2=c^2=e$, $bc=cb=a, ac=ca=b, ab=ba=c$, then N(G) is a commutative neutrosophic group under multiplication since $\{e,a,b,c\}$ is a Klein 4-group. $N(H)=\{e,a,I,aI\}$, $N(K)=\{e,b,I,bI\}$ and $N(P)=\{e,c,I,cI\}$ are neutrosophic subgroups of N(G).

Theorem 2.6 *Let N(H)be a nonempty proper subset of a neutrosophic group $(N(G),\star)$. N(H)is a neutrosophic subgroup of N(G) if and only if the following conditions hold:*

(i) $a,b\in N(H)$ implies that $a\star b\in N(H)\ \forall\ a,b\in N(H)$;

(ii) there exists a proper subset A of N(H) such that $(A,\star)$ is a group.

Proof Suppose that N(H) is a neutrosophic subgroup of $((N(G),\star)$. Then $(N(G),\star)$ is a neutrosophic group and consequently, conditions (i) and (ii) hold.

Conversely, suppose that conditions (i) and (ii) hold. Then $N(H)=\langle A\cup I\rangle$ is a neutrosophic group under $\star$. The required result follows. □

Theorem 2.7 *Let N(H) be a nonempty proper subset of a neutrosophic group (N(G),*). N(H) is a pseudo neutrosophic subgroup of N(G) if and only if the following conditions hold:*

(i) $a,b\in N(H)$ implies that $a*b\in N(H)\ \forall\ a,b\in N(H)$;

(ii) N(H) does not contain a proper subset A such that (A,) is a group.*

Definition 2.8 *Let N(H) and N(K) be any two neutrosophic subgroups (resp. pseudo neutrosophic subgroups) of a neutrosophic group N(G). The product of N(H) and N(K) denoted by N(H).N(K) is the set $N(H).N(K)=\{hk:h\in N(H),k\in N(K)\}$.*

Theorem 2.9 *Let N(H) and N(K) be any two neutrosophic subgroups of a commutative neutrosophic group N(G). Then:*

(i) $N(H)\cap N(K)$ is a neutrosophic subgroup of N(G);

(ii) $N(H).N(K)$ is a neutrosophic subgroup of N(G);

(iii) $N(H)\cup N(K)$ is a neutrosophic subgroup of N(G) if and only if $N(H)\subset N(K)$ or $N(K)\subset N(H)$.

Proof The proof is the same as the classical case. □

Theorem 2.10 *Let N(H) be a neutrosophic subgroup and let N(K) be a pseudo neutrosophic subgroup of a commutative neutrosophic group N(G). Then:*

(i) $N(H).N(K)$ is a neutrosophic subgroup of N(G);

(ii) $N(H)\cap N(K)$ is a pseudo neutrosophic subgroup of N(G);

(iii) $N(H)\cup N(K)$ is a neutrosophic subgroup of N(G) even if $N(H)\not\subseteq N(K)$ or $N(K)\not\subseteq N(H)$.

Proof (*i*) Suppose that N(H) and N(K) are neutrosophic subgroup and pseudo neutrosophic subgroup of N(G) respectively. Let $x, y \in N(H).N(K)$. Then $xy \in N(H).N(K)$. Since $N(H) \subset N(H).N(K)$ and $N(K) \subset N(H).N(K)$, it follows that N(H).N(K) contains a proper subset which is a group. Hence N(H).N(K) is a neutrosophic of N(G).

(*ii*) Let $x, y \in N(H) \cap N(K)$. Since N(H) and N(K) are neutrosophic subgroup and pseudo neutrosophic of N(G) respectively, it follows that $xy \in N(H) \cap N(K)$ and also since $N(H)\cap N(K) \subset N(H)$ and $N(H)\cap N(K) \subset N(K)$, it follows that $N(H)\cap N(K)$ cannot contain a proper subset which is a group. Therefore, $N(H) \cap N(K)$ is a pseudo neutrosophic subgroup of N(G).

(*iii*) Suppose that N(H) and N(K) are neutrosophic subgroup and pseudo neutrosophic subgroup of N(G) respectively such that $N(H) \not\subseteq N(K)$ or $N(K) \not\subseteq N(H)$. Let $x, y \in N(H) \cup N(K)$. Then $xy \in N(H)\cup N(K)$. But then $N(H) \subset N(H)\cup N(K)$ and $N(K) \subset N(H)\cup N(K)$ so that $N(H) \cup N(K)$ contains a proper subset which is a group. Thus $N(H) \cup N(K)$ is a neutrosophic subgroup of N(G). This is different from what is obtainable in classical group theory. □

Example 2.11 $N(G) = \langle \mathcal{Z}_{10} \cup I\rangle = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, I, 2I, 3I, 4I, 5I, 6I, 7I, 8I, 9I, 1 + I, 2 + I, 3 + I, 4 + I, 5 + I, 6 + I, 7 + I, 8 + I, 9 + I, \cdots, 9 + 9I\}$ is a neutrosophic group under multiplication modulo 10. $N(H) = \{1, 3, 7, 9, I, 3I, 7I, 9I\}$ and $N(K) = \{1, 9, I, 9I\}$ are neutrosophic subgroups of N(G) and $N(P) = \{1, I, 3I, 7I, 9I\}$ is a pseudo neutrosophic subgroup of N(G). It is easy to see that $N(H) \cap N(K)$, $N(H) \cup N(K)$, $N(H).N(K)$, $N(P) \cup N(H)$, $N(P) \cup N(K)$, $N(P).N(H)$ and $N(P).N(K)$ are neutrosophic subgroups of N(G) while $N(P) \cap N(H)$ and $N(P) \cup N(K)$ are pseudo neutrosophic subgroups of N(G).

Definition 2.12 *Let N(G) be a neutrosophic group. The center of N(G) denoted by Z(N(G)) is the set $Z(N(G)) = \{g \in N(G) : gx = xg\ \forall\ x \in N(G)\}$.*

Definition 2.13 *Let g be a fixed element of a neutrosophic group N(G). The normalizer of g in N(G) denoted by N(g) is the set $N(g) = \{x \in N(G) : gx = xg\}$.*

Theorem 2.14 *Let N(G) be a neutrosophic group. Then*

(i) $Z(N(G))$ is a neutrosophic subgroup of N(G);

(ii) $N(g)$ is a neutrosophic subgroup of N(G);

Proof (*i*) Suppose that $Z(N(G))$ is the neutrosophic center of N(G). If $x, y \in Z(N(G))$, then $xy \in Z(N(G))$. Since $Z(G)$, the center of the group G is a proper subset of $Z(N(G))$, it follows that $Z(N(G))$ contains a proper subset which is a group. Hence $Z(N(G))$ is a neutrosophic subgroup of N(G).

(*ii*) The proof is the same as (*i*). □

Theorem 2.15 *Let N(G) be a neutrosophic group and let $Z(N(G))$ be the center of N(G) and $N(x)$ the normalizer of x in N(G). Then*

(i) N(G) is commutative if and only if $Z(N(G)) = N(G)$;
(ii) $x \in Z(N(G))$ if and only if $N(x) = N(G)$.

Definition 2.16 *Let N(G) be a neutrosophic group. Its order denoted by $o(N(G))$ or $\mid N(G) \mid$ is the number of distinct elements in $N(G)$. $N(G)$ is called a finite neutrosophic group if $o(N(G))$ is finite and infinite neutrosophic group if otherwise.*

Theorem 2.17 *Let N(H)and N(K)be two neutrosophic subgroups (resp. pseudo neutrosophic subgroups) of a finite neutrosophic group N(G). Then $o(N(H).N(K)) = \frac{o(N(H)).o(N(K))}{o(N(H) \cap N(K))}$.*

Definition 2.18 *Let N(G)and N(H)be any two neutrosophic groups. The direct product of N(G) and N(H) denoted by $N(G) \times N(H)$ is defined by $N(G) \times N(H) = \{(g, h) : g \in N(G), h \in N(H)\}$.*

Theorem 2.19 *If $(N(G), *_1)$ and $(N(H), *_2)$ are neutrosophic groups, then $(N(G) \times N(H), *)$ is a neutrosophic group if $(g_1, h_1) * (g_2, h_2) = (g_1 *_1 g_2, h_1 *_2 h_2)\ \forall\ (g_1, h_1), (g_2, h_2) \in N(G) \times N(H)$.*

Theorem 2.20 *Let N(G)be a neutrosophic group and let H be a classical group. Then $N(G) \times H$ is a neutrosophic group.*

Definition 2.21 *Let $N(G)$ be a finite neutrosophic group and let $N(H)$ be a neutrosophic subgroup of $N(G)$.*

(i) $N(H)$ is called a Lagrange neutrosophic subgroup of $N(G)$ if $o(N(H)) \mid o(N(G))$;

(ii) $N(G)$ is called a Lagrange neutrosophic group if all neutrosophic subgroups of $N(G)$ are Lagrange neutrosophic subgroups;

(iii) $N(G)$ is called a weak Lagrange neutrosophic group if $N(G)$ has at least one Lagrange neutrosophic subgroup;

(iv) $N(G)$ is called a free Lagrange neutrosophic group if it has no Lagrange neutrosophic subgroup.

Definition 2.22 *Let $N(G)$ be a finite neutrosophic group and let $N(H)$ be a pseudo neutrosophic subgroup of $N(G)$.*

(i) $N(H)$ is called a pseudo Lagrange neutrosophic subgroup of $N(G)$ if $o(N(H)) \mid o(N(G))$;

(ii) $N(G)$ is called a pseudo Lagrange neutrosophic group if all pseudo neutrosophic subgroups of $N(G)$ are pseudo Lagrange neutrosophic subgroups;

(iii) $N(G)$ is called a weak pseudo Lagrange neutrosophic group if $N(G)$ has at least one pseudo Lagrange neutrosophic subgroup;

(iv) $N(G)$ is called a free pseudo Lagrange neutrosophic group if it has no pseudo Lagrange neutrosophic subgroup.

Example 2.23 *(i)* Let N(G) be the neutrosophic group of Example 2.5. The only neutrosophic subgroups of N(G)are $N(H)=\{e,a,I,aI\}$, $N(K)=\{e,b,I,bI\}$ and $N(P)=\{e,c,I,cI\}$. Since $o(N(G))=8$ and $o(N(H))=o(N(K))=o(N(P))=4$ and $4\mid 8$, it follows that N(H), N(K)and N(P)are Lagrange neutrosophic subgroups and N(G) is a Lagrange neutrosophic group.

(ii) Let $N(G)=\{1,3,5,7,I,3I,5I,7I\}$ be a neutrosophic group under multiplication modulo 8. The neutrosophic subgroups $N(H)=\{1,3,I,3I\}$, $N(K)=\{1,5,I,5I\}$ and $N(P)=\{1,7,I,7I\}$ are all Lagrange neutrosophic subgroups. Hence N(G) is a Lagrange neutrosophic group.

(iii) $N(G)=N(\mathcal{Z}_2)\times N(\mathcal{Z}_2)=\{(0,0),(0,1),(1,0),(1,1),(0,1+I),(1,I),\cdots,(1+I,1+I)\}$ is a neutrosophic group under addition modulo 2. N(G) is a Lagrange neutrosophic group since all its neutrosophic subgroups are Lagrange neutrosophic subgroups.

(iv) Let $N(G)=\{e,g,g^2,g^3,I,gI,g^2I,g^3I\}$ be a neutrosophic group under multiplication where $g^4=e$. $N(H)=\{e,g^2,I,g^2I\}$ and $N(K)=\{e,I,g^2I\}$ are neutrosophic subgroups of N(G). Since $o(N(H))\mid o(N(G))$ but $o(N(K))$ does not divide $o(N(G))$ it shows that N(G) is a weak Lagrange neutrosophic group.

(v) Let $N(G)=\{e,g,g^2,I,gI,g^2I\}$ be a neutrosophic group under multiplication where $g^3=e$. N(G) is a free Lagrange neutrosophic group.

Theorem 2.24 *All neutrosophic groups generated by I and any group isomorphic to Klein 4-group are Lagrange neutrosophic groups.*

Definition 2.25 *Let $N(H)$ be a neutrosophic subgroup (resp. pseudo neutrosophic subgroup) of a neutrosophic group $N(G)$. For a $g\in N(G)$, the set $gN(H)=\{gh:h\in N(H)\}$ is called a left coset (resp. pseudo left coset) of N(H) in N(G). Similarly, for a $g\in N(G)$, the set $N(H)g=\{hg:h\in N(H)\}$ is called a right coset (resp. pseudo right coset) of N(H) in N(G). If N(G) is commutative, a left coset (resp. pseudo left coset) and a right coset (resp. pseudo right coset) coincide.*

Definition 2.26 *Let N(H)be a Lagrange neutrosophic subgroup (resp. pseudo Lagrange neutrosophic subgroup) of a finite neutrosophic group N(G). A left coset xN(H)of N(H) in N(G) determined by x is called a smooth left coset if $\mid xN(H)\mid=\mid N(H)\mid$. Otherwise, xN(H) is called a rough left coset of N(H) in N(G).*

Definition 2.27 *Let N(H)be a neutrosophic subgroup (resp. pseudo neutrosophic subgroup) of a finite neutrosophic group N(G). The number of distinct left cosets of N(H) in N(G)denoted by [N(G):N(H)] is called the index of N(H) in N(G).*

Definition 2.28 *Let N(H)be a Lagrange neutrosophic subgroup (resp. pseudo Lagrange neutrosophic subgroup) of a finite neutrosophic group N(G). The number of distinct smooth left cosets of N(H) in N(G) denoted by [N(H):N(G)] is called the smooth index of N(H) in N(G).*

Theorem 2.29 *Let X be the set of distinct smooth left cosets of a Lagrange neutrosophic subgroup (resp. pseudo Lagrange neutrosophic subgroup) of a finite neutrosophic group (resp. finite Lagrange neutrosophic group) N(G). Then X is a partition of N(G).*

Proof Suppose that $X=\{X_i\}_{i=1}^n$ is the set of distinct smooth left cosets of a Lagrange neutrosophic subgroup (resp. pseudo Lagrange neutrosophic subgroup) of a finite neutrosophic group (resp. finite Lagrange neutrosophic group) N(G). Since $o(N(H)) \mid o(N(G))$ and $\mid xN(H) \mid = \mid N(H) \mid \forall\, x \in N(G)$, it follows that X is not empty and every member of N(G) belongs to one and only one member of X. Hence $\cap_{i=1}^{n} X_i = \emptyset$ and $\cup_{i=1}^{n} X_i = N(G)$. Consequently, X is a partition of N(G). □

Corollary 2.30 *Let $[N(H) : N(G)]$ be the smooth index of a Lagrange neutrosophic subgroup in a finite neutrosophic group (resp. finite Lagrange neutrosophic group) N(G). Then $\mid N(G) \mid = \mid N(H) \mid [N(H) : N(G)]$.*

Proof The proof follows directly from Theorem 2.29. □

Theorem 2.31 *Let X be the set of distinct left cosets of a neutrosophic subgroup (resp. pseudo neutrosophic subgroup) of a finite neutrosophic group N(G). Then X is not a partition of N(G).*

Proof Suppose that $X = \{X_i\}_{i=1}^{n}$ is the set of distinct left cosets of a neutrosophic subgroup (resp. pseudo neutrosophic subgroup) of a finite neutrosophic group N(G). Since N(H) is a non-Lagrange pseudo neutrosophic subgroup, it follows that $o(N(H))$ is not a divisor of $o(N(G))$ and $\mid xN(H) \mid \neq \mid N(H) \mid \forall\, x \in N(G)$. Clearly, X is not empty and every member of N(G) can not belongs to one and only one member of X. Consequently, $\cap_{i=1}^{n} X_i \neq \emptyset$ and $\cup_{i=1}^{n} X_i \neq N(G)$ and thus X is not a partition of N(G). □

Corollary 2.32 *Let $[N(G) : N(H)]$ be the index of a neutrosophic subgroup (resp. pseudo neutrosophic subgroup) in a finite neutrosophic group N(G). Then $\mid N(G) \mid \neq \mid N(H) \mid [N(G) : N(H)]$.*

Proof The proof follows directly from Theorem 2.31. □

Example 2.33 Let N(G)be a neutrosophic group of Example 2.23(iv).

(a) Distinct left cosets of the Lagrange neutrosophic subgroup $N(H) = \{e, g^2, I, g^2I\}$ are: $X_1 = \{e, g^2, I, g^2I\}$, $X_2 = \{g, g^3, gI, g^3I\}$, $X_3 = \{I, g^2I\}$, $X_4 = \{gI, g^3I\}$. X_1, X_2 are smooth cosets while X_3, X_4 are rough cosets and therefore $[N(G) : N(H)] = 4$, $[N(H) : N(G)] = 2$.
$\mid N(H) \mid [N(G) : N(H)] = 4 \times 4 \neq \mid N(G) \mid$ and $\mid N(H) \mid [N(H) : N(G)] = 4 \times 2 = \mid N(G) \mid$.
$X_1 \cap X_2 = \emptyset$ and $X_1 \cup X_2 = N(G)$ and hence the set $X = \{X_1, X_2\}$ is a partition of N(G).

(b) Distinct left cosets of the pseudo non-Lagrange neutrosophic subgroup $N(H) = \{e, I, g^2I\}$ are: $X_1 = \{e, I, g^2I\}$, $X_2 = \{g, gI, g^3I\}$, $X_3 = \{g^2, I, g^2I\}$, $X_4 = \{g^3, gI, g^3I\}$, $X_5 = \{I, g^2I\}$, $X_6 = \{gI, g^3I\}$. X_1, X_2, X_3, X_4 are smooth cosets while X_5, X_6 are rough cosets. $[N(G) : N(H)] = 6$, $[N(H) : N(G)] = 4$, $\mid N(H) \mid [N(G) : N(H)] = 3 \times 6 \neq \mid N(G) \mid$ and $\mid N(H) \mid [N(H) : N(G)] = 3 \times 4 \neq \mid N(G) \mid$. Members of the set $X = \{X_1, X_2, X_3, X_4\}$ are not mutually disjoint and hence do not form a partition of N(G).

Example 2.34 Let $N(G) = \{1, 2, 3, 4, I, 2I, 3I, 4I\}$ be a neutrosophic group under multiplication modulo 5. Distinct left cosets of the non-Lagrange neutrosophic subgroup $N(H) = \{1, 4, I, 2I, 3I, 4I\}$ are $X_1 = \{1, 4, I, 2I, 3I, 4I\}$, $X_2 = \{2, 3, I, 2I, 3I, 4I\}$, $X_3 = \{I, 2I, 3I, 4I\}$. X_1, X_2 are smooth cosets while X_3 is a rough coset and therefore $[N(G) : N(H)] = 3$,

$[N(H) : N(G)] = 2$, $| N(H) | [N(G) : N(H)] = 6 \times 3 \neq | N(G) |$ and $| N(H) | [N(H) : N(G)] = 6 \times 2 \neq | N(G) |$. Members of the set $X = \{X_1, X_2\}$ are not mutually disjoint and hence do not form a partition of N(G).

Example 2.35 Let N(G)be the Lagrange neutrosophic group of Example 2.5. Distinct left cosets of the Lagrange neutrosophic subgroup $N(H) = \{e, a, I, aI\}$ are: $X_1 = \{e, a, I, aI\}$, $X_2 = \{b, c, bI, cI\}$, $X_3 = \{I, aI\}$, $X_4 = \{bI, cI\}$. X_1, X_2 are smooth cosets while X_3, X_4 are rough cosets and thus $[N(G) : N(H)] = 4$, $[N(H) : N(G)] = 2$, $| N(H) | [N(G) : N(H)] = 4 \times 4 = 16 \neq | N(G) |$ and $| N(H) | [N(H) : N(G)] = 4 \times 2 = 8 = | N(G) |$. Members of the set $X = \{X_1, X_2\}$ are mutually disjoint and $N(G) = X_1 \cup X_2$. Hence X is a partition of N(G).

Example 2.36 Let N(G)be the Lagrange neutrosophic group of Example 2.23(iii).

(a) Distinct left cosets of the Lagrange neutrosophic subgroup $N(H) = \{(0, 0), (0, 1), (0, I), (0, 1+I)\}$ are respectively $X_1 = \{(0, 0), (0, 1), (0, I), (0, 1+I)\}$, $X_2 = \{(1, 0), (1, 1), (1, I), (1, 1+I)\}$, $X_3 = \{(I, 0), (I, 1), (I, I), (I, 1 + I)\}$, $X_4 = \{(I + I, 0), (1 + I, 1), (1 + I, I), (1 + I, 1 + I)\}$, $X_5 = \{(1+I, 0), (1+I, 1), (1+I, 1+I)\}$. X_1, X_2, X_3, X_4 are smooth cosets while X_5 is a rough coset. Thus, $[N(G) : N(H)] = 5$, $[N(H) : N(G)] = 4$, $| N(H) | [N(G) : N(H)] = 4 \times 5 = 20 \neq | N(G) |= 16$ and $| N(H) | [N(H) : N(G)] = 4 \times 4 = 16 = | N(G) |$. Members of the set $X = \{X_1, X_2, X_3, X_4\}$ are mutually disjoint and $N(G) = X_1 \cup X_2 \cup X_3 \cup X_4$ so that X is a partition of N(G).

(b) Distinct left cosets of the pseudo Lagrange neutrosophic subgroup $N(H) = \{(0, 0), (0, I), (I, 0), (I, I)\}$ are respectively $X_1 = \{(0, 0), (0, I), (I, 0), (I, 1)\}$, $X_2 = \{(0, 1), (0, 1 + I), (I, 1), (I, 1+I)\}$, $X_3 = \{(1, 0), (1, I), (1+I, 0), (1+I, I)\}$, $X_4 = \{(1, 1), (1, 1+I), (1+I, 1), (1+I, 1+I)\}$. X_1, X_2, X_3, X_4 are smooth cosets and $[N(G) : N(H)] = [N(H) : N(G)] = 4$. Consequently, $| N(H) | [N(G) : N(H)] =| N(H) | [N(H) : N(G)] = 4 \times 4 = 16 =| N(G) |$. Members of the set $X = \{X_1, X_2, X_3, X_4\}$ are mutually disjoint, $N(G) = X_1 \cup X_2 \cup X_3 \cup X_4$ and hence X is a partition of N(G).

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