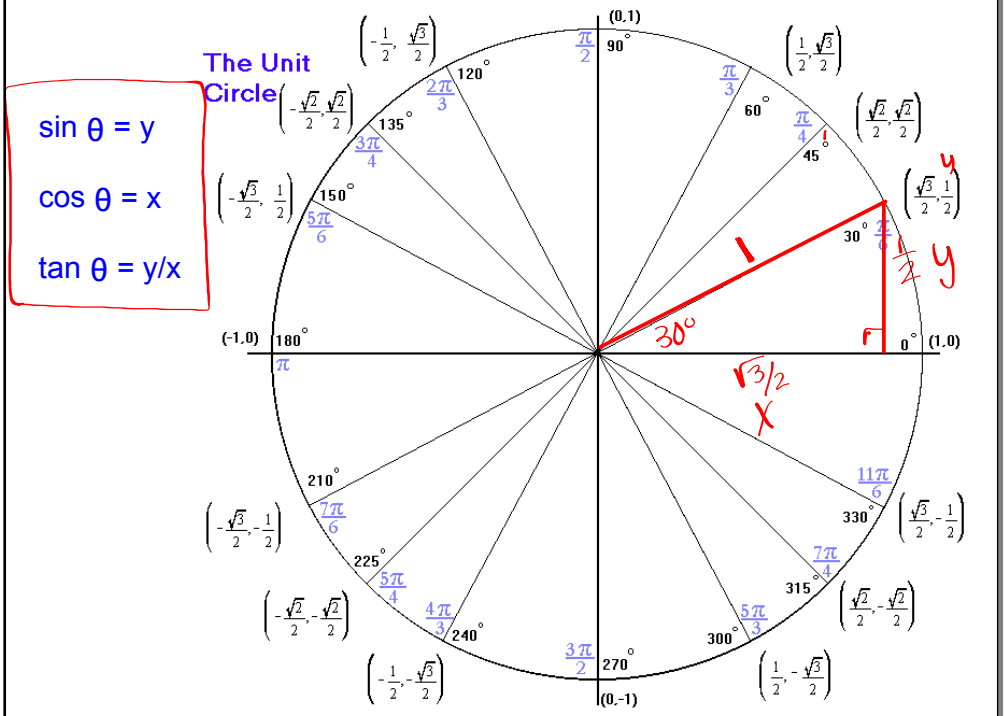
Section 7.3: Angle Sum & Difference Identities



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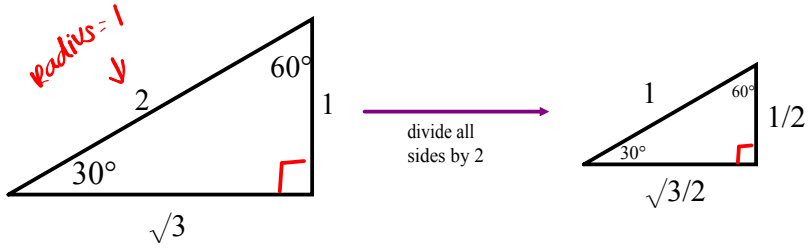
Section 7.3: Angle Sum & Difference Identities

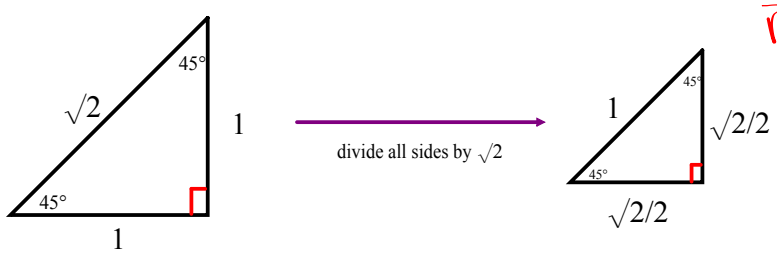
12/11/14

How the unit circle works:

Take both special right triangles and transform them so that the hypotenuse lengths are one



$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$



The unit circle then places the horizontal and vertical components of each triangle and the (x, y) coordinates on the circle.

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Use the unit circle to break each angle into a sum or difference using values equivalent to 30°, 60° or 45°

Examples:

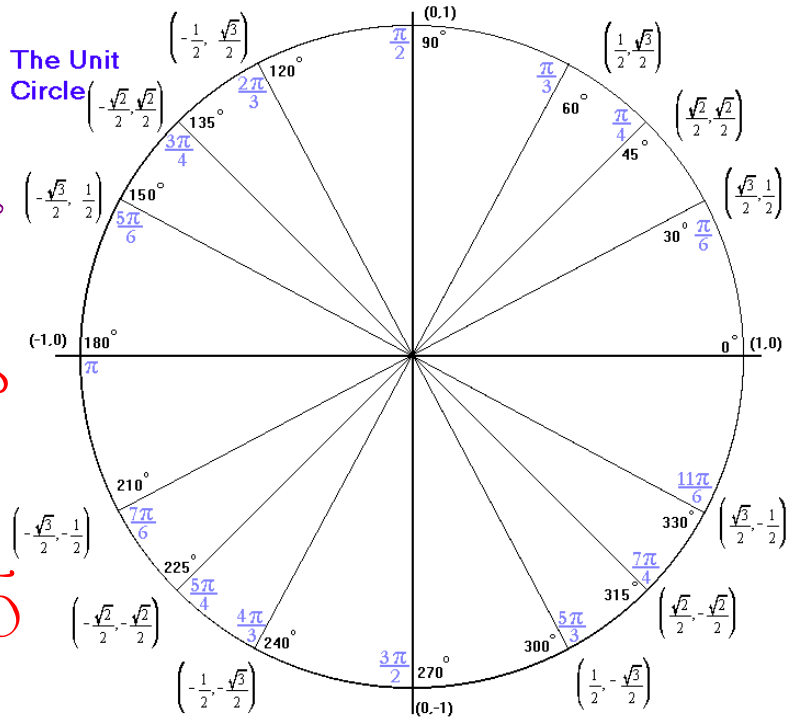
15° = 45°-30°
OR 60°-45°

105° = 60°+45°
OR 150°-45°

165° = 120+45

255° = 210+45

345° = 300+45



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Before Solving.
Figure out what two numbers add to make the original degree.
Draw out the triangles in the correct quadrant/look at unit circle.

Then use the sum & difference identities from yesterday

Solve for the exact value.

1.) cos 105°

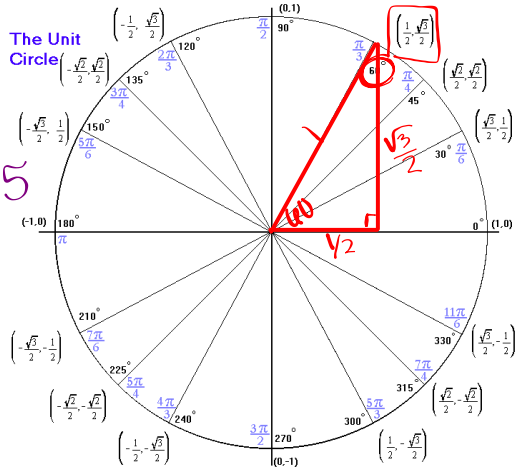
$\cos(60+45)$

$\cos 60 \cdot \cos 45 - \sin 60 \cdot \sin 45$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

$= \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$



sin = y cos = x

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Solve for the exact value.

2.) sin 735°

-360

$\sin 375$

-360

$= \sin 15° = \sin(45 - 30)$

$\sin 45 \cdot \cos 30 - \cos 45 \cdot \sin 30$

$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$

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Solve for the exact value.

3.) tan 285°

$\tan(225 + 60)$

$\frac{\tan 225 + \tan 60}{1 - \tan 225 \cdot \tan 60}$

$= \frac{1 + \sqrt{3}}{1 - 1 \cdot \sqrt{3}} = \frac{1+\sqrt{3}}{1-\sqrt{3}}$

$= \frac{1+\sqrt{3}}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}}$

multiply by conjugate to get rid $\sqrt{6}$ radical in denominator

$= \frac{1 + \sqrt{3} + \sqrt{3} + 3}{1 + \sqrt{3} - \sqrt{3} - 3} = \frac{4 + 2\sqrt{3}}{-2} = \boxed{-2 - \sqrt{3}}$

(F O I L)

If possible, try to avoid using tangent values that have fractional trig values.

i.e. find a value that uses just $\sqrt{3}$ rather than $\frac{\sqrt{3}}{3}$

*Helpful hint: stay away from 30° reference angles (30, 150, 210, 330)

$\tan 225 = \frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = 1$

$\tan 60 = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} \quad \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$

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