

LINEAR CONTROL SYSTEM ANALYSIS AND DESIGN WITH MATLAB

Fifth Edition, Revised and Expanded

John J. D'Azzo and Constantine H. Houppis

*Air Force Institute of Technology
Wright-Patterson Air Force Base, Ohio, U.S.A.*

Stuart N. Sheldon

*U.S. Nuclear Regulatory Commission
Lisle, Illinois, U.S.A.*



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Professor

Applied Control Engineering

University of Manchester Institute of Science and Technology

Manchester, United Kingdom

FRANK L. LEWIS, PH.D.

Moncrief-O'Donnell Endowed Chair

and Associate Director of Research

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Series Introduction

Many textbooks have been written on control engineering, describing new techniques for controlling systems, or new and better ways of mathematically formulating existing methods to solve the ever-increasing complex problems faced by practicing engineers. However, few of these books fully address the applications aspects of control engineering. It is the intention of this new series to redress this situation.

The series will stress applications issues, and not just the mathematics of control engineering. It will provide texts that present not only both new and well-established techniques, but also detailed examples of the application of these methods to the solution of real-world problems. The authors will be drawn from both the academic world and the relevant applications sectors.

There are already many exciting examples of the application of control techniques in the established fields of electrical, mechanical (including aerospace), and chemical engineering. We have only to look around in today's highly automated society to see the use of advanced robotics techniques in the manufacturing industries; the use of automated control and navigation systems in air and surface transport systems; the increasing use of intelligent control systems in the many artifacts available to the domestic consumer market; and the reliable supply of water, gas, and electrical power to the domestic consumer and to industry. However, there are currently many challenging problems that could benefit from wider exposure to the applicability of control methodologies, and the systematic systems-oriented basis inherent in the application of control techniques.

This series presents books that draw on expertise from both the academic world and the applications domains, and will be useful not only as academically recommended course texts but also as handbooks for practitioners in many applications domains. *Linear Control System Analysis and Design with MATLAB* is another outstanding entry in Dekker's Control Engineering series.

Neil Munro

Preface

The countless technological advances of the twentieth century require that future engineering education emphasize *bridging the gap* between theory and the real world. This edition has been prepared with particular attention to the needs of undergraduates, especially those who seek a solid foundation in control theory as well as an ability to bridge the gap between control theory and its real-world applications. To help the reader achieve this goal, computer-aided design accuracy checks (CADAC) are used throughout the text to encourage good habits of computer literacy. Each CADAC uses fundamental concepts to ensure the viability of a computer solution.

This edition has been enhanced as a solid undergraduate and first-year graduate text; it emphasizes applying control theory fundamentals to both analog and sampled-data single-input single-output (SISO) feedback control systems. At the same time, the coverage of digital control systems is greatly expanded. Extensive reference is made to computer-aided design (CAD) packages to simplify the design process. The result is a comprehensive presentation of control theory and design—one that has been thoroughly class-tested, ensuring its value for classroom and self-study use.

This book features extensive use of explanations, diagrams, calculations, tables, and symbols. Such mathematical rigor is necessary for design applications and advanced control work. A solid foundation is built on concepts of modern control theory as well as those elements of conventional control theory that are relevant in analysis and design of control systems. The presentation of various techniques helps the reader understand what A. T. Fuller has called

“the enigmatic control system.” To provide a coherent development of the subject, we eschew formal proofs and lemmas, instead using an organization that draws the perceptive student steadily and surely to the demanding theory of multivariable control systems. Design examples are included throughout each chapter to reinforce the student’s understanding of the material. A student who has reached this point is fully equipped to undertake the challenges of more advanced control theories, as presented in advanced control theory textbooks.

Chapter 2 sets forth the appropriate differential equations to describe the performance of physical systems, networks, and devices. The block diagram, the transfer function, and the state space (the essential concept of modern control theory) are also introduced. The approach used for the state space is the simultaneous derivation of the state-vector differential equation with the SISO differential equation for a chosen physical system. The chapter also shows how to derive the mathematical description of a physical system using LaGrange equations.

Chapter 3 presents the classical method of solving differential equations. Once the state-variable equation has been introduced, a careful explanation of its solution is provided. The relationship of the transfer function to the state equation of the system is presented in **Chapter 14**. The importance of the state transition matrix is described, and the state transition equation is derived. The idea of eigenvalues is explained next; this theory is used with the Cayley–Hamilton and Sylvester theorems to evaluate the state transition matrix.

The early part of **Chapter 4** presents a comprehensive description of Laplace transform methods and pole-zero maps. Some further aspects of matrix algebra are introduced as background for solving the state equation using Laplace transforms. Finally, the evaluation of transfer matrices is clearly explained.

Chapter 5 begins with system representation by the conventional block-diagram approach. This is followed by a discussion of simulation diagrams and the determination of the state transition equation using signal flow graphs. The chapter also explains how to derive parallel state diagrams from system transfer functions, establishing the advantages of having the state equation in uncoupled form.

Chapter 6 introduces basic feedback system characteristics. This includes the relationship between system type and the ability of the system to follow or track polynomial inputs.

Chapter 7 presents the details of the root-locus method. **Chapters 8 and 9** describe the frequency-response method using both log and polar plots. These chapters address the following topics: the Nyquist stability criterion; the correlation between the s -plane, frequency domain, and time domain; and gain setting to achieve a desired output response peak value while tracking polynomial command inputs. **Chapters 10 and 11** describe the methods for improving

system performance, including examples of the techniques for applying cascade and feedback compensators. Both the root-locus and frequency-response methods of designing compensators are covered.

Chapter 12 develops the concept of modeling a desired control ratio with figures of merit to satisfy system performance specifications. The system inputs generally fall into two categories: (1) desired input that the system output is to track (a tracking system) and (2) an external disturbance input for which the system output is to be minimal (a disturbance-rejection system). For both types of systems, the desired control ratio is synthesized by the proper placement of its poles and inclusion of zeros, if required. Chapter 12 also introduces the Guillemin-Truxal design procedure, which is used for designing a tracking control system and a design procedure emphasizing disturbance rejection.

Chapter 13 explains how to achieve desired system characteristics using complete state-variable feedback. Two important concepts of modern control theory—controllability and observability—are treated in a simple and straightforward manner.

Chapter 14 presents the sensitivity concepts of Bode, as used in variation of system parameters. Other tools include the method of using feedback transfer functions to form estimates of inaccessible states for use in state feedback, and a technique for linearizing a nonlinear system about its equilibrium points.

Chapter 15 presents the fundamentals of sampled data (S-D) control systems. **Chapter 16** describes the design of digital control systems, demonstrating, for example, the effectiveness of digital compensation. The concept of a pseudo-continuous-time (PCT) model of a digital system permits the use of continuous-time methods for the design of digital control systems.

The text has been prepared so that it can be used for self-study by engineers in various areas of practice (electrical, aeronautical, mechanical, etc.). To make it valuable to all engineers, we use various examples of feedback control systems and unify the treatment of physical control systems by using mathematical and block-diagram models common to all.

There are many computer-aided design (CAD) packages (e.g., MATLAB[®] [see [App. C](#)], Simulink, and TOTAL-PC) available to help students and practicing engineers analyze, design, and simulate control systems. The use of MATLAB is emphasized throughout the book, and many MATLAB m-files are presented as examples.

We thank the students who have used this book in its previous editions and the instructors who have reviewed this edition for their helpful comments and recommendations. We thank especially Dr. R. E. Fontana, Professor Emeritus of Electrical Engineering, Air Force Institute of Technology, for the encouragement he provided for the previous editions. This edition is dedicated to the memory of Dr. T. J. Higgins, Professor Emeritus of Electrical Engineering, University of Wisconsin, for his thorough review of the earlier manuscripts.

We also express our appreciation to Professor Emeritus Donald McLean of the University of Southampton, England, formerly a visiting professor at the Air Force Institute of Technology. Our association with him has been an enlightening and refreshing experience. The personal relationship with him has been a source of inspiration and deep respect.

*John J. D'Azzo
Constantine H. Houpis
Stuart N. Sheldon*

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