Mathematica® for Theoretical Physics

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for Theoretical Physics

Classical Mechanics
and Nonlinear Dynamics

Second Edition

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CD-ROM Included

Springer

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To Carin,
for her love, support, and encuragement.

Preface

As physicists, mathematicians or engineers, we are all involved with mathematical calculations in our everyday work. Most of the laborious, complicated, and time-consuming calculations have to be done over and over again if we want to check the validity of our assumptions and derive new phenomena from changing models. Even in the age of computers, we often use paper and pencil to do our calculations. However, computer programs like *Mathematica* have revolutionized our working methods. *Mathematica* not only supports popular numerical calculations but also enables us to do exact analytical calculations by computer. Once we know the analytical representations of physical phenomena, we are able to use *Mathematica* to create graphical representations of these relations. Days of calculations by hand have shrunk to minutes by using *Mathematica*. Results can be verified within a few seconds, a task that took hours if not days in the past.

The present text uses *Mathematica* as a tool to discuss and to solve examples from physics. The intention of this book is to demonstrate the usefulness of *Mathematica* in everyday applications. We will not give a complete description of its syntax but demonstrate by examples the use of its language. In particular, we show how this modern tool is used to solve classical problems.

This second edition of *Mathematica in Theoretical Physics* seeks to prevent the objectives and emphasis of the previous edition. It is extended to include a full course in classical mechanics, new examples in quantum mechanics, and measurement methods for fractals. In addition, there is an extension of the fractal's chapter by a fractional calculus. The additional material and examples enlarged the text so much that we decided to divide the book in two volumes. The first volume covers classical mechanics and nonlinear dynamics. The second volume starts with electrodynamics, adds quantum mechanics and general relativity, and ends with fractals. Because of the inclusion of new materials, it was necessary to restructure the text. The main differences are concerned with the chapter on nonlinear dynamics. This chapter discusses mainly classical field theory and, thus, it was appropriate to locate it in line with the classical mechanics chapter.

The text contains a large number of examples that are solvable using *Mathematica*. The defined functions and packages are available on CD accompanying each of the two volumes. The names of the files on the CD carry the names of their respective chapters. Chapter 1 comments on the basic properties of *Mathematica* using examples from different fields of physics. Chapter 2 demonstrates the use of *Mathematica* in a step-by-step procedure applied to mechanical problems. Chapter 2 contains a one-term lecture in mechanics. It starts with the basic definitions, goes on with Newton's mechanics, discusses the Lagrange and Hamilton representation of mechanics, and ends with the rigid body motion. We show how *Mathematica* is used to simplify our work and to support and derive solutions for specific problems. In Chapter 3, we examine nonlinear phenomena of the Korteweg–de Vries equation. We demonstrate that *Mathematica* is an appropriate tool to derive numerical and analytical solutions even for nonlinear equations of motion. The second volume starts with Chapter 4, discussing problems of electrostatics and the motion of ions in an electromagnetic field. We further introduce *Mathematica* functions that are closely related to the theoretical considerations of the selected problems. In Chapter 5, we discuss problems of quantum mechanics. We examine the dynamics of a free particle by the example of the time-dependent Schrödinger equation and study one-dimensional eigenvalue problems using the analytic and numeric capabilities of *Mathematica*. Problems of general relativity are discussed in Chapter 6. Most standard books on Einstein's theory discuss the phenomena of general relativity by using approximations. With *Mathematica*, general relativity effects like the shift of the perihelion can be tracked with precision. Finally, the last chapter, Chapter 7, uses computer algebra to represent fractals and gives an introduction to the spatial renormalization theory. In addition, we present the basics of fractional calculus approaching fractals from the analytic side. This approach is supported by a package, FractionalCalculus, which is not included in this project. The package is available by request from the author. Exercises with which *Mathematica* can be used for modified applications. Chapters 2–7 include at the end some exercises allowing the reader to carry out his own experiments with the book.

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Gerd Baumann

Contents

Volume I

Volume II