

Vocabulary

hypotenuse, p. 465
 leg, p. 465
 Pythagorean theorem,
 p. 465

The Pythagorean Theorem

BEFORE

You used square roots.

Now

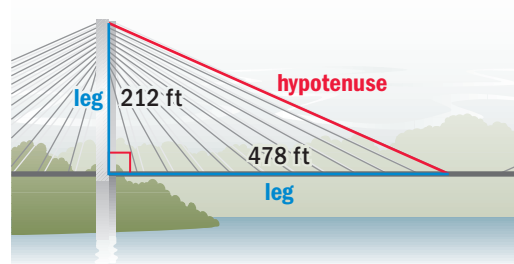
You'll use the Pythagorean theorem to solve problems.

WHY?

So you can find the length of a side of a TV screen, as in Ex. 19.

Bridges The William H. Harsha Bridge is a cable-stayed bridge that spans the Ohio River between Maysville, Kentucky, and Aberdeen, Ohio. About how long is the cable shown in red?

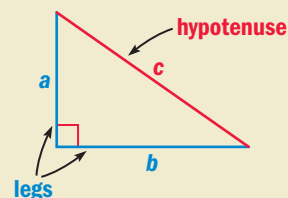
In a right triangle, the **hypotenuse** is the side opposite the right angle. The **legs** are the sides that form the right angle. The lengths of the legs and the length of the hypotenuse of a right triangle are related by the **Pythagorean theorem**.



Pythagorean Theorem

Words For any right triangle, the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse.

Algebra $a^2 + b^2 = c^2$

**Example 1****Finding the Length of a Hypotenuse**

To find the length (to the nearest foot) of the cable shown above, use the right triangle formed by the tower, the bridge surface, and the cable.

$$a^2 + b^2 = c^2$$

Pythagorean theorem

$$212^2 + 478^2 = c^2$$

Substitute 212 for a and 478 for b .

$$44,944 + 228,484 = c^2$$

Evaluate powers.

$$273,428 = c^2$$

Add.

$$\sqrt{273,428} = c$$

Take positive square root of each side.

$$523 \approx c$$

Use a calculator. Round to nearest whole number.

Answer The length of the cable is about 523 feet.



Example 2**Finding the Length of a Leg**

Find the unknown length a in simplest form.

$$a^2 + b^2 = c^2$$

Pythagorean theorem

$$a^2 + 10^2 = 12^2$$

Substitute.

$$a^2 + 100 = 144$$

Evaluate powers.

$$a^2 = 44$$

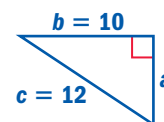
Subtract 100 from each side.

$$a = \sqrt{44}$$

Take positive square root of each side.

$$a = 2\sqrt{11}$$

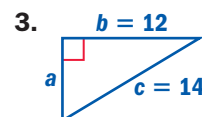
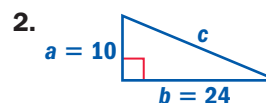
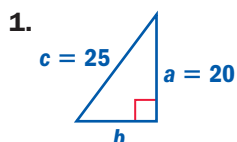
Simplify.



Answer The unknown length a is $2\sqrt{11}$ units.

Checkpoint

Find the unknown length. Write your answer in simplest form.



Reading Algebra

The converse of the theorem “If two integers are odd, then the sum of the integers is even” is “If the sum of two integers is even, then the two integers are odd.” Notice that the converse is *not* true since, for example, the even integer 8 can be written as the sum of the even integers 2 and 6.

Converse of the Pythagorean Theorem The Pythagorean theorem can be written in “if-then” form.

Theorem: If **a triangle is a right triangle**, then $a^2 + b^2 = c^2$.

If you reverse the two parts of the statement, the new statement is called the *converse* of the Pythagorean theorem.

Converse: If $a^2 + b^2 = c^2$, then **the triangle is a right triangle**.

Although not all converses of true statements are true, the converse of the Pythagorean theorem is true. You can use it to determine whether a triangle is a right triangle.

Example 3**Identifying Right Triangles**

Determine whether the triangle with the given side lengths is a right triangle.

a. $a = 3, b = 5, c = 7$

b. $a = 15, b = 8, c = 17$

Solution

a. $a^2 + b^2 = c^2$

$$3^2 + 5^2 \stackrel{?}{=} 7^2$$

$$9 + 25 \stackrel{?}{=} 49$$

$$34 \neq 49$$

Answer Not a right triangle

b. $a^2 + b^2 = c^2$

$$15^2 + 8^2 \stackrel{?}{=} 17^2$$

$$225 + 64 \stackrel{?}{=} 289$$

$$289 = 289 \checkmark$$

Answer A right triangle

Study Strategy

When determining whether three given lengths a , b , and c satisfy the equation $a^2 + b^2 = c^2$, always let c be the greatest length.

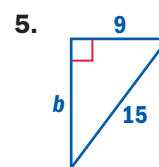
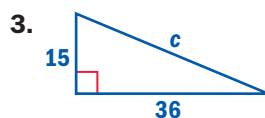
Guided Practice

Vocabulary Check

- Copy and complete: The side opposite the right angle of a right triangle is called the ?.
- The lengths of the sides of a triangle are 6, 8, and 10. Explain how you can determine whether the triangle is a right triangle.

Skill Check

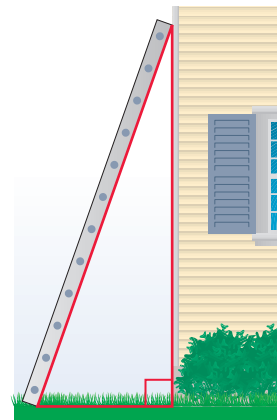
In Exercises 3–5, find the unknown length.



Guided Problem Solving

6. **Ladders** A ladder that is 15 feet long is placed against a wall. The bottom of the ladder is 5 feet from the wall. To the nearest foot, how far up the wall does the ladder reach?

- Copy the diagram. Label the known lengths. Label the unknown length x .
- Use the Pythagorean theorem to write an equation you can use to find the value of x .
- Solve the equation in Step 2 for x .
- Round your answer from Step 3 to the nearest whole number.



Practice and Problem Solving

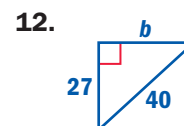
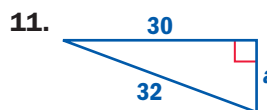
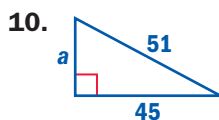
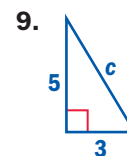
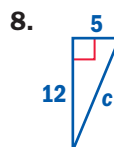
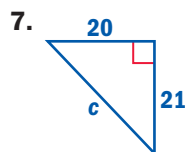
Homework Help

Example	Exercises
1	7–9, 20, 26–29
2	10–12, 19, 22–25
3	13–18, 20–21


 Online Resources
 CLASSZONE.COM

- More Examples
- eTutorial Plus

Find the unknown length. Write your answer in simplest form.



Determine whether the triangle with the given side lengths is a right triangle.

13. 2, 3, 4

14. 12, 35, 37

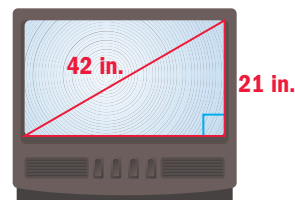
15. 5, 12, 13

16. 8, 16, 18

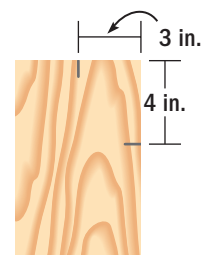
17. 11, 60, 61

18. 8, 9, 12

19. **Television Screen** A television screen is a rectangle, and its size is indicated by the length of a diagonal. A 42 inch television screen is about 21 inches high. How wide is the screen to the nearest inch?



20. **Carpentry** To determine whether the corner of a shelf is actually a right angle, a carpenter uses a ruler and a pencil to make a mark at 3 inches along one side of the shelf and at 4 inches along the other. The carpenter takes a measurement and is satisfied that the corner is a right angle. What distance did the carpenter measure? Why does the carpenter's method work?



21. **Pythagorean Triples** A *Pythagorean triple* is a set of three positive integers a , b , and c such that $a^2 + b^2 = c^2$. That is, the integers a , b , and c are a Pythagorean triple if a triangle with side lengths a , b , and c is a right triangle. You can generate a Pythagorean triple by substituting an integer greater than 1 into each of these expressions: $2n$, $n^2 - 1$, and $n^2 + 1$. For example, the integers 3, 4, and 5 form a Pythagorean triple that is generated by the given expressions when $n = 2$.

- Choose an integer value of n greater than 2 and substitute the given value into each of the given expressions.
- Verify that the numbers you generated form a Pythagorean triple.

The lengths of the legs of a right triangle are given. Tell whether you would use *mental math*, *paper and pencil*, or a *calculator* to find the length of the hypotenuse. Explain your answers.

22. $a = 87$, $b = 136$

23. $a = 1$, $b = 2$

24. $a = 15$, $b = 20$

The lengths of two sides of a right triangle are given. Find the length of the third side.

25. $a = 28$, $c = 53$

26. $a = 48$, $c = 73$

27. $b = 24$, $c = 26$

28. $b = 77$, $c = 85$

29. $a = 84$, $b = 80$

30. $a = 48$, $b = 189$

31. **Synchronized Swimming** At the beginning of a performance, two synchronized swimmers start at opposite corners of a rectangular pool that is 50 meters long and 25 meters wide. They swim toward each other along a diagonal and meet halfway. To the nearest tenth of a meter, how far from their starting points do the swimmers meet?

32. **Writing** One leg of a right triangle has a length of 32 units. The hypotenuse has a length of 68 units. Describe how you would go about finding the area of the triangle.



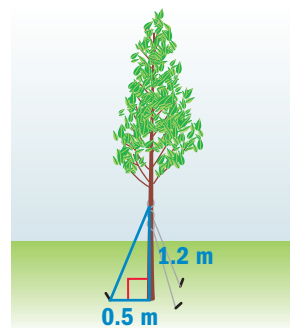
In the Real World

Synchronized Swimming In a pool that meets minimum standards for an Olympic synchronized swimming competition, the bottom of the pool has a sloped surface over which the depth decreases from 3 meters to 2.5 meters. If the horizontal distance over which the depth changes is 8 meters, how long is the sloped edge of the bottom to the nearest tenth of a meter?



33. **Soccer** In a college soccer league, the smallest field allowed is a rectangle 65 yards wide and 110 yards long. The largest field allowed is a rectangle 80 yards wide and 120 yards long. To the nearest yard, how much longer is the diagonal of the largest field than the diagonal of the smallest field?

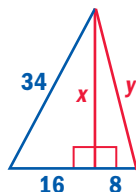
34. **Extended Problem Solving** You plan to support a young tree by attaching three wires to the tree. Your plan is to attach one end of each wire to the tree at a point 1.2 meters above the ground, and the other end of each wire to a stake in the ground at a point 0.5 meter from the tree.



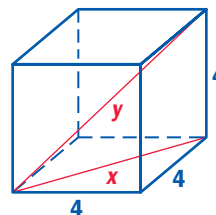
- a. **Calculate** Find the total amount of wire you need. Include an extra 10 centimeters of wire at each attachment point.
- b. **Interpret and Apply** You have only 4.2 meters of wire and plan to attach each wire 0.5 meter out from the tree, allowing extra wire as described in part (a). To the nearest tenth of a meter, how far up the tree can you attach each wire? Explain.

Challenge Find each unknown length in simplest form. In Exercise 36, use the fact that the edges of a cube meet at right angles.

35.



36.



Mixed Review

Write the decimal as a fraction or mixed number. (Lesson 5.1)

37. 0.16 38. -0.45 39. 1.075 40. -3.875

41. **Tree House** You are building a tree house. In a scale drawing of the tree house, the length of one wall is 6 inches. The length of the actual wall is 5 feet. Find the scale of the drawing. (Lesson 6.6)
42. **Eating Out** At a restaurant, you choose a first course, a main dish, and a vegetable for dinner. You can choose fruit cup or soup for the first course. For the main course, you can choose steak, chicken, fish, or lasagna. For a vegetable, you can choose beans, broccoli, or carrots. Find the number of possible meals you can choose. (Lesson 6.8)

Standardized Test Practice

43. **Multiple Choice** Which lengths are *not* side lengths of a right triangle?
 A. 9, 12, 15 B. 13, 84, 85 C. 8, 14, 18 D. 24, 70, 74
44. **Short Response** A ship travels 7 miles due north, then 20 miles due east. The ship then sails directly back to its starting point. Explain how you would find the total distance of the trip. Then find the distance to the nearest mile.

