A Variable Step Size Based Method for Online Secondary Path Modeling in Active Noise Control System

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***Abstract*—Online secondary path modeling with higher modeling accuracy and faster convergence is desirable in active noise control systems. This paper proposes a method based on variable step size LMS algorithm for modeling filter and FxLMS algorithm for noise control filter. The proposed idea uses a novel three stage approach for step size selection of modeling filter in order to improve the performance of secondary path modeling. Consequently, a faster initial convergence and reduced steady state error are achieved in comparison with existing online secondary path modeling methods. The computational complexity of the proposed method is comparable with the existing methods which uses only two adaptive filters for ANC with online secondary path modeling. The performance of the proposed method is verified through computer simulation.**

***Index Terms*—Active noise control, Online secondary path modeling, Variable step size least mean square (VSS-LMS) algorithm, FxLMS algorithm.**

I. Introduction

With the invention of high speed digital hardware the field of active noise control (ANC) has found a great attention of researchers during the last two decades. The basic crux of ANC is the principle of superposition in which the acoustic waves from the primary path and the ANC controller interfere with each other destructively [1]. In practical applications the secondary path (between the controller and the error microphone) may be time varying and therefore online secondary path modeling is required when an ANC system is in operation. There are two different approaches for online secondary path modeling in an ANC system. The first approach models the secondary path from the output of the ANC controller. The second method involves injection of additional random noise into the output of noise control filter $W(z)$ and utilizes a system identification method to model the secondary path. The comparison of these two techniques for online secondary path modeling can be found in [2] where it is concluded that the second approach is superior to the first one when compared in terms of convergence rate, speed of response to changes in primary noise and computational complexity. In this paper, the use of second approach is considered for online secondary path modeling.

The basic technique for online secondary path modeling is given by Eriksson [3], and its structure is shown in Fig. 1. Here, $P(z)$ is the primary path transfer function between the reference microphone and the error microphone. $S(z)$ is the secondary path transfer function between noise control filter $W(z)$ and the error microphone. The reference noise signal $x(n)$ is picked up by the reference microphone and processed by noise control filter $W(z)$ to generate a signal $y(n)$. An uncorrelated zero mean white gaussian noise $v(n)$ is added with the output $y(n)$ of noise control filter and the composite signal is then processed by $S(z)$ giving the output $y'(n) - v'(n)$. The output of $S(z)$ is subtracted from $d(n)$ giving $e(n)$ which is sensed by an error microphone. Here, $d(n)$ is the output of primary path transfer function $P(z)$. The basic problem with Eriksson's method is that the signal $e(n)$ is used, not only, as the desired response for the modeling filter $\hat{S}(z)$ but is also given to ANC system for weight updating of noise control filter $W(z)$. Consequently, the term $v'(n)$ acts as interference for noise control filter and $d(n) - y'(n)$ acts as an interference for secondary path modeling filter. Consequently, if a larger step size is used then the ANC system may become unstable. This problem can be partially solved by Bao's method, the detail of which can be found in [5]. In Bao's method, one additional adaptive filter is used to remove the interference from the desired response of the modeling filter. However the term $v'(n)$ still acts as interference for noise control filter $W(z)$, thereby degrading the convergence of ANC system. This problem is solved by Zhang's method [6] which will be briefly discussed in the next section. Another three adaptive filter based method for online secondary path modeling is proposed by Kuo [7]. This method, however, works only for predictable or narrow band reference inputs.

The drawback of using one additional adaptive filter in [5], [6] and [7] is removed by Akhtar's method [8] where a variable step size is used for online secondary path modeling filter. The limitation of Akhtar's method is that the step-size remains large even in the steady state therefore the performance of Akhtar's method (in terms of modeling error in steady state) is inferior in comparison with Zhang's method. In [9] optimal step size is used for modeling and noise control filters and power scheduling is done in order to reduce, in steady state,

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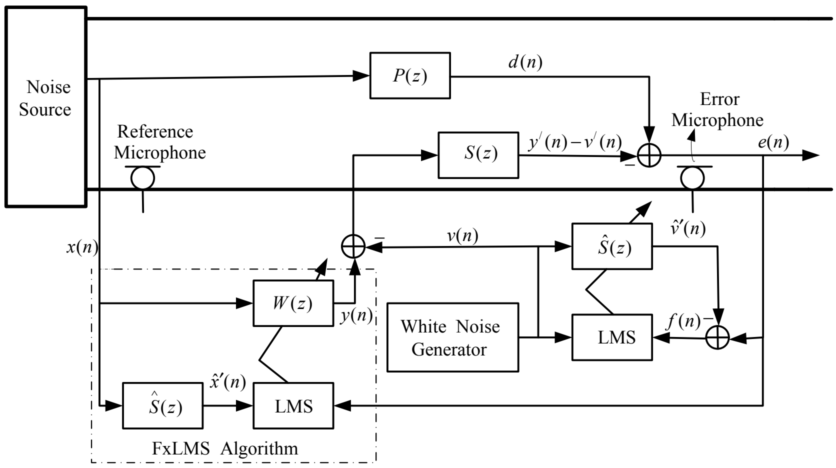

Fig. 1. Eriksson's method for online secondary path modeling in ANC system.

the power of the auxiliary noise in the error signal $e(n)$. In [9] if the optimal step size parameters are used without noise scheduling then a greater estimation accuracy is obtained while the residual error remain same as in [8]. This improvement in accuracy of estimation is achieved at the expense of increase in computational complexity. The proposed method improves the estimation accuracy of Akhtar's method without noise scheduling and using variable step size approach for modeling filter. The computational complexity of proposed method is comparable to Akhtar's method.

The rest of the paper is organized as follows. Section II describes two existing methods for online secondary path modeling in ANC systems. Section III explains the proposed method. Section IV contains simulation results and finally Section V gives the concluding remarks.

II. Existing Methods for Online Secondary path Modeling in ANC Systems

A. Zhang's Method

The block diagram for Zhang's method is shown in Fig. 2. In Zhang's method three cross updated adaptive filters are used to reduce the perturbation effect of the primary noise on the modeling of the secondary path. This method also reduces the perturbation effect of injected random noise from the error signal of noise control filter and desired response of third adaptive filter. Instead of using $e(n)$ directly as an error signal for $W(z)$ and as desired response of the third adaptive filter the three adaptive filters are arranged so that a perturbation free error signals for all the three adaptive filters are obtained.

B. Akhtar's Method

The main feature of Akhtar's method [8] is that only two adaptive filters are used to remove the interference effects on the convergence performance of adaptive filters in ANC system. The proposed structure of Akhtar's method is shown in Fig. 3. A variable step size LMS algorithm is used for online secondary path modeling. The computational complexity of Akhtar's method is less than Zhang's method since only two adaptive filters are used. In addition to the advantage of computational complexity, Akhtar's method provides fast initial convergence of the modeling filter towards its optimum value. The limitation with Akhtar's method is that the step size for online secondary path modeling filter stays high in the steady state, thereby degrading the relative modeling error in steady state. This sets the motivation for proposed method.

III. Proposed Method

A. Method Description

The structure of the proposed method is exactly the same as that of Akhtar's method shown in Fig. 3. The proposed method, however, uses a new variable step size technique for online secondary path modeling filter. The weight update equation for secondary path modeling filter is given by

$$\hat{\mathbf{s}}(n+1) = \hat{\mathbf{s}}(n) + \mu_s(n) f(n) \mathbf{v}(n), \tag{1}$$

where $\mu_s(n)$ is variable step size for $\hat{S}(z)$, $\hat{\mathbf{s}}(n) = [\hat{s}_0(n), \hat{s}_1(n), \ldots, \hat{s}_{M-1}(n)]$ is a tap weight vector of length M for $\hat{S}(z)$ and $\mathbf{v}(n) = [v(n), v(n-1), \ldots, v(n-M+1)]$ is the modeling signal vector. In Akhtar's method the variable step size $\mu_s(n)$ is calculated as

$$\mu_s(n) = \rho(n)\mu_{s_{\min}} + (1-\rho(n))\mu_{s_{\max}}, \tag{2}$$

where $\mu_{s_{\min}}$ and $\mu_{s_{\max}}$ are minimum and maximum values of the step size $\mu_s(n)$ respectively, that are found experimentally and $\rho(n)$ is a parameter calculated as

$$\rho(n) = \frac{P_f(n)}{P_e(n)} = \frac{P_{[d(n)-y'(n)]} + P_{[v'(n)-\hat{v}'(n)]}}{P_{[d(n)-y'(n)]} + P_{v'(n)}}, \tag{3}$$

where $P_f(n)$ and $P_e(n)$ are estimates of power of error signals $f(n)$ and $e(n)$, respectively and are given by

$$P_f(n) = \lambda P_f(n-1) + (1-\lambda) f^2(n)), \tag{4}$$

$$P_e(n) = \lambda P_e(n-1) + (1-\lambda) e^2(n)), \tag{5}$$

where λ is a forgetting factor $(0.9 < \lambda < 1)$. It can be shown that $\rho(0) \approx 1$ and $\rho(n) \to 0$ as $n \to \infty$ Thus Akhtar's

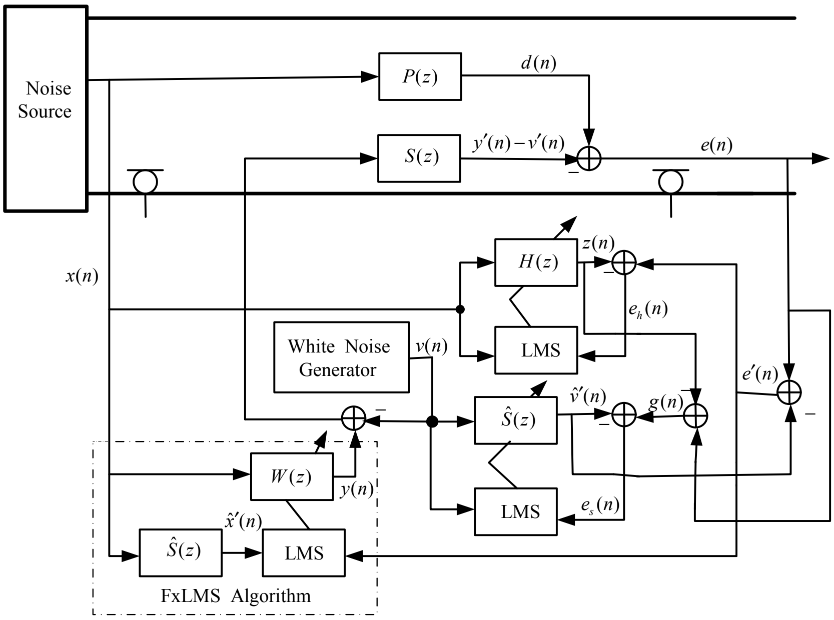

Fig. 2. ANC system based on FxLMS algorithm for online secondary path modeling (Zhang's method).

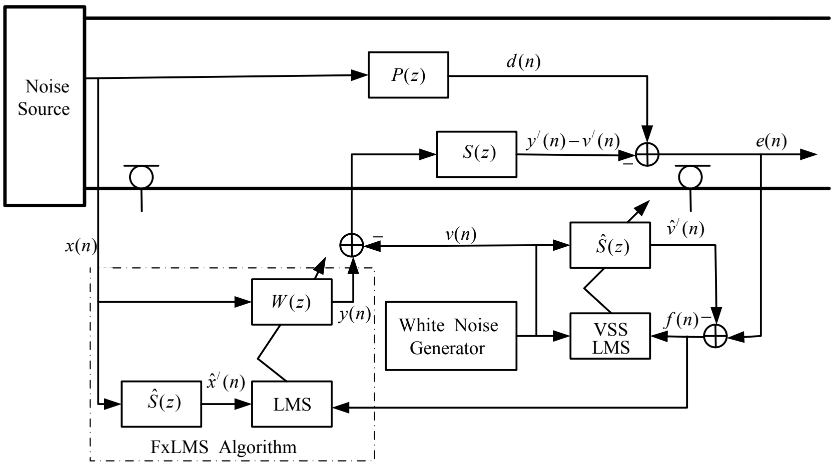

Fig. 3. Structure of Proposed and Akhtar's method for online secondary path modeling in ANC system.

method initially selects small value for step size, $\mu_{\min}$, and later increases step size to a maximum value $\mu_{\max}$. In this way although a fast initial convergence is achieved, however the steady state performance is degraded due to the larger value of step size. In this paper, we solve this problem and proposed following procedure to vary step size of modeling filter.

The variable step size $\mu_s(n)$ in the proposed method is computed as

$$\mu_s(n) = (1 - \rho(n))\mu(n), \tag{6}$$

where $\rho(n)$ is same as given in (3) for Akhtar's method, and $\mu(n)$ is another variable step size [10] and is computed as

$$\mu(n) = \begin{cases} \mu_{\min}; & \beta_{\text{vss}}(n) < \mu_{\min} \\ \beta_{\text{vss}}; & \mu_{\min} \le \beta_{\text{vss}}(n) \le \mu_{\max} \\ \mu_{\max}; & \beta_{\text{vss}}(n) > \mu_{\max} \end{cases} \tag{7}$$

where $\mu_{\min}$ and $\mu_{\max}$ are limits set to guarantee tracking capability and stability, respectively, and $\beta_{\text{vss}}(n)$ is recursively determined from

$$\beta_{\text{vss}}(n+1) = \alpha\beta_{\text{vss}}(n) + (1-\alpha)f^2(n), \tag{8}$$

where α is a control parameter. The step size $\mu(n)$ in [10] is bounded by $\mu_{\min}$ and $\mu_{\max}$ and in between this limit step size $\mu(n)$ is proportional to $f^2(n)$. So at the start the larger value of the disturbance term $d(n) - y'(n)$ in the error signal $f(n)$ of the modeling filter $\hat{S}(z)$ causes the value of step size $\mu(n)$ to be large and thus may cause the ANC system to become unstable. So at the start small value of step size for modeling filter $\hat{S}(z)$ is required as in Akhtar's method. In addition to that, larger value of the step size $\mu_s(n)$ in Akhtar's method in steady state will not allow the weights of the modeling filter to

TABLE I
THE COMPUTATIONAL COMPLEXITY COMPARISON (NUMBER OF COMPUTATIONS PER ITERATION)

Method	Multiplications	Additions	$L = M = N$	
			Multiplications	Additions
Eriksson's Method	$2L + 3M + 2$	$2L + 3M - 1$	$5L + 2$	$5L - 1$
Zhang's Method	$2L + 3M + 2N + 3$	$2L + 3M + 2N + 1$	$7L + 3$	$7L + 1$
Akhtar's Method	$2L + 3M + 10$	$2L + 3M + 5$	$5L + 10$	$5L + 5$
Proposed Method	$2L + 3M + 13$	$2L + 3M + 7$	$5L + 13$	$5L + 7$

TABLE II
THE STEP-SIZE PARAMETERS FOR VARIOUS ADAPTIVE FILTERS

Method	ANC filter $W(z)$	Modeling filter $\hat{S}(z)$	$H(z)$ (in Zhang's method)
Eriksson's Method	$\mu_w = 5 \times 10^{-4}$	$\mu_s = 1 \times 10^{-2}$	
Zhang's Method	$\mu_w = 5 \times 10^{-4}$	$\mu_s = 1 \times 10^{-2}$	$\mu_h = 1 \times 10^{-2}$
Akhtar's Method	$\mu_w = 5 \times 10^{-4}$	$\mu_{s_{\min}} = 7.5 \times 10^{-3}$, $\mu_{s_{\max}} = 2.5 \times 10^{-2}$	
Proposed Method	$\mu_w = 5 \times 10^{-4}$	$\mu_{\min} = 7.5 \times 10^{-3}$, $\mu_{\max} = 5 \times 10^{-2}$	

converge to their optimum value so we want the step size to be small in the steady state as in [10]. The proposed method for step size variation given in (6) combines the desired feature of having smaller value of step size both at the start, as in Akhtar's method, and in the steady state as in [10].

The three stage step size variation for modeling filter in the proposed method is explained as follows

1) As discussed in [8], at the start $P_f(n) = P_e(n)$ so that $\rho(n) = 1$. Therefore, it is clear from (6) that $\mu_s(n)$ is initially close to zero.
2) The error signal $f(n)$ of the modeling filter is decreasing in nature. As the time passes on, the value of $P_f(n)$ decreases rapidly relative to the value of $P_e(n)$ so as a result the value of $\rho(n) \to 0$ very quickly and therefore according to (6) $\mu_s(n) \approx \mu(n)$. It is clear from (7) and (8) that $\mu(n)$ is proportional to $P_f(n)$ which is small relative to $P_e(n)$ but at this stage its value is sufficient enough to make the value of $\mu_s(n)$ to increase from its initially very small value. So, the value of step size increases towards its maximum value.
3) Finally, since $\mu_s(n) \approx \mu(n)$ is proportional to $f^2(n)$ and the power of $P_f(n)$ is decreasing in nature, so with time the value of step size $\mu_s(n)$ again decreases towards its minimum value.

The weight update equation of noise control filter is given by

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu_w f(n) \hat{\boldsymbol{x}}'(n), \tag{9}$$

where μ_w is the step size parameter for noise control filter, $W(z)$, $\boldsymbol{w}(n) = [w_0(n), w_1(n), \ldots, w_{L-1}(n)]$ is tap-weight vector for FIR filter $W(z)$, L is tap-weight length of $W(z)$, $\hat{\boldsymbol{x}}'(n) = [\hat{x}'(n), \hat{x}'(n-1), \ldots, \hat{x}'(n-L+1)]$ is vector for filtered-reference signal $\hat{x}'(n)$, and where $\hat{x}'(n)$ is computed as

$$\hat{x}'(n) = \hat{\boldsymbol{s}}(n)\boldsymbol{x}(n)^T, \tag{10}$$

where $\boldsymbol{x}(n) = [x(n), x(n-1), \ldots, x(n-M+1)]$ is vector for reference signal and $\hat{\boldsymbol{s}}(n) = [\hat{s}_0(n), \hat{s}_1(n), \ldots, \hat{s}_{M-1}(n)]$ is a tap weight vector of length M for $\hat{S}(z)$.

B. Comments on Computational Complexity

The computational complexity is calculated as the number of computations required per iteration of an algorithm. The computational complexity comparison of proposed method with existing methods is given in Table I. Here, L and M are tap-weight lengths of noise control filter $W(z)$ and modeling filter $\hat{S}(z)$, respectively, and N is the length of third adaptive filter in Zhang's method. It can be observed from Table. I that the computational complexity of proposed method is less than Zhang's methods and is comparable with Akhtar's method.

IV. SIMULATION RESULTS

Computer simulations have been conducted in order to evaluate the performance of the proposed method. The performance of proposed method is compared with Eriksson's [3], Zhang's [6] and Akhtar's methods [8]. The performance comparison is done on basis of residual error signal and relative modeling error ΔS(dB). The relative modeling error is defined as

$$\Delta S(\text{dB}) = 10 \log_{10} \left(\frac{\sum_{i=0}^{M-1} [s_i(n) - \hat{s}_i(n)]^2}{\sum_{i=0}^{M-1} [s_i(n)]^2} \right). \tag{11}$$

The primary path $P(z)$ and secondary path $S(z)$ are FIR filters of tap-weight lengths 48 and 16, respectively, where the data is taken from [1]. The control filter $W(z)$ and modeling filter $S(z)$ are FIR filters of tap-weight lengths $L = 32$ and $M = 16$, respectively. The third adaptive filter in Zhang's method is an FIR filter of tap-weight length $N = 16$. The control filters in all methods and the third adaptive filter in Zhang's method are initialized with null vectors.

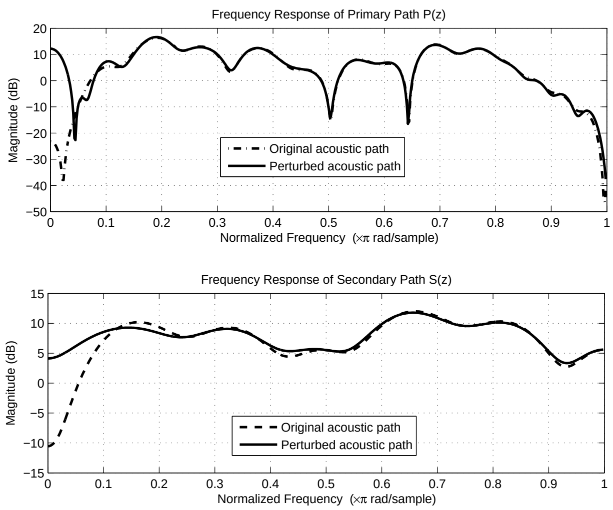

Fig. 4. Frequency response of acoustic paths

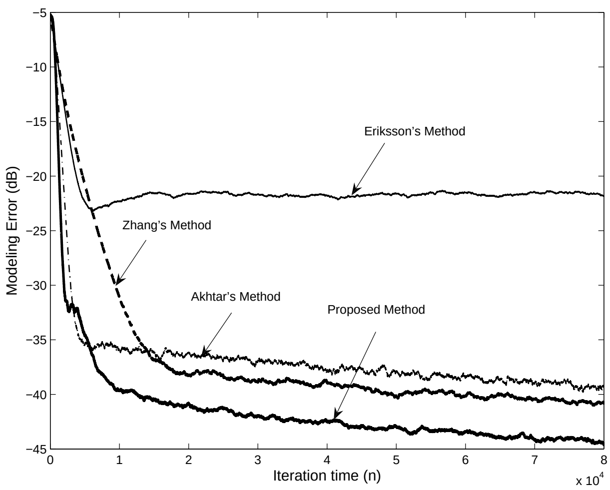

Fig. 5. Relative modeling error ΔS(dB) versus iteration time n

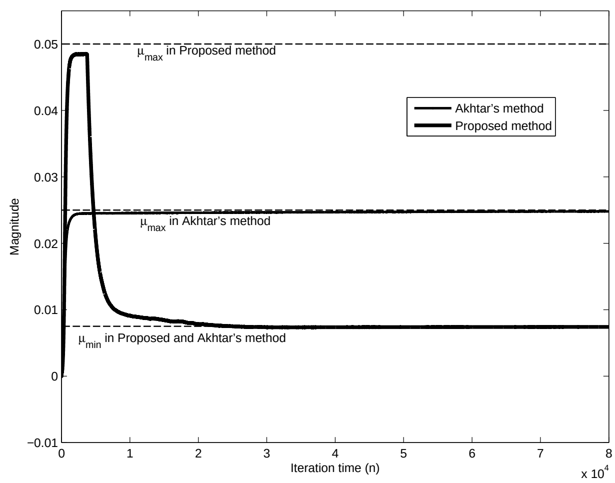

Fig. 6. Variation of modeling filter step size $\mu_s(n)$ with iteration n

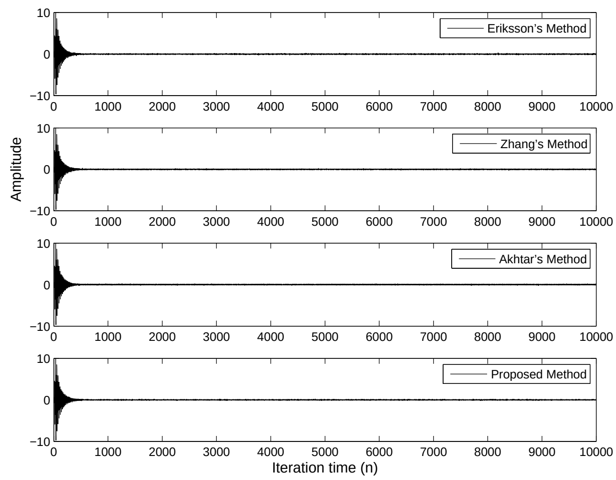

Fig. 7. Residual error signal $e(n)$ for various methods discussed in paper

Offline modeling is used to initialize the modeling filter, as offline measurements are integral part of ANC system. For all methods the step size parameters are adjusted for fast convergence and are given in Table II. The forgetting factor λ and the control parameter α are chosen as 0.99 and 0.999, respectively. A zero mean white Gaussian noise with variance of 0.05 is used for online secondary path modeling. The reference signal $x(n)$ with variance 2 is a narrow band signal made up of 100, 200, 300 and 400Hz frequencies and a zero mean white Gaussian noise with SNR of 30 dB is added to it. The sampling frequency is 2 kHz. The frequency response of original and perturbed (shown dotted) acoustic paths $P(z)$ and $S(z)$ are shown in Fig. 4. The simulation results are averaged over 100 realizations.

Fig. 5 shows relative modeling error for different methods. It is clear that the proposed method not only gives fast initial convergence as in Akhtar's method but it also gives much lower steady state relative modeling error in comparison with other methods. As mentioned before such an improved performance is achieved due to three stage step size variation of the modeling filter as depicted in Fig. 6. In the proposed method a smaller step size is used at the start because of larger disturbance and then step size is increased accordingly so that it converges fast and finally the step size is brought back to minimum value, so that modeling error can be reduced significantly. This will lead to faster convergence at the start as in Akhtar's method and also provides better performance in steady state. The step size for modeling filter in case of Akhtar's method is small at the start but it remains high in the steady state as shown in Fig. 6. Fig. 7 shows the residual error for various methods discussed in the paper. It is observed that when the estimation of secondary-path modeling reaches some level, further improvement of accuracy does not necessarily contribute to the further reduction of noise power in a significant way. All the methods discussed in this paper reduce the modeling error below 20 dB so the performance of all the methods in terms of residual

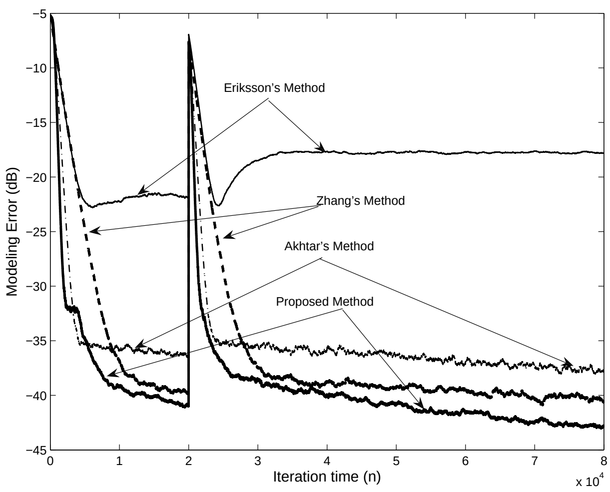

Fig. 8. Relative modeling error ΔS(dB) versus iteration time n with a sudden change in acoustic paths

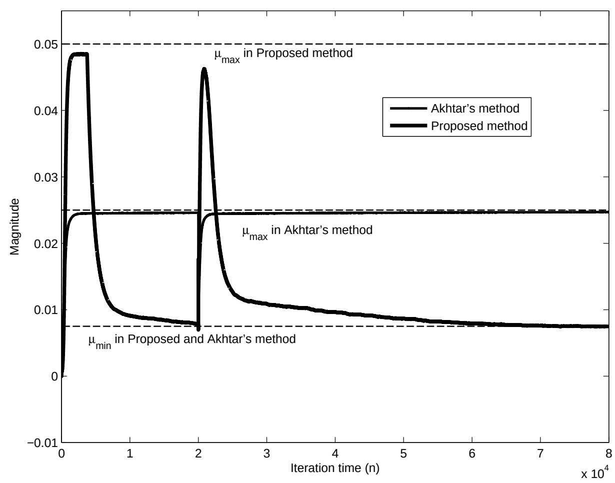

Fig. 9. Variation of modeling filter step size $\mu_s(n)$ with iteration n with a sudden change in acoustic paths

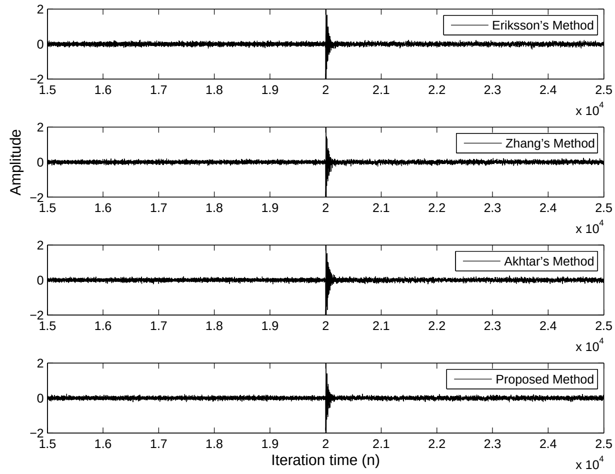

Fig. 10. Residual error signal $e(n)$, with a sudden change in acoustic paths, for various methods discussed in paper

error is almost same. The proposed method achieves as good noise reduction performance as existing approaches, and also ensures improved secondary path modeling.

Fig. 8 shows the relative modeling error for different methods when acoustic paths are perturbed at $n = 20000$. It is clear from Fig. 8 that proposed method performs well before and after the perturbation. Fig. 9 shows the modeling filter step size variation. In Akhtar's method the perturbation causes the step size to decrease and then again it increases towards its maximum value. In proposed method the step size is already at its minimum value when acoustic paths are perturbed so the step size increases from its minimum value and then finally comes back to its minimum value in the steady state. Similarly the plot for residual error when the acoustic paths are perturbed is shown in Fig. 10. It is clear form Fig. 10 that a sudden change of the acoustic paths results in increase in the residual error.

V. Concluding Remarks

The proposed method uses two adaptive filters with new variable step size LMS algorithm for online secondary path modeling filter and FxLMS algorithm for noise control filter. The proposed method overall require less computations than Zhang's methods and provides faster convergence and better steady state performance. The future tasks is to compare the results of the proposed method with [9]. In addition to this the hardware implementation and the extension of this idea for multi-channel online secondary path modeling are part of future work.

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