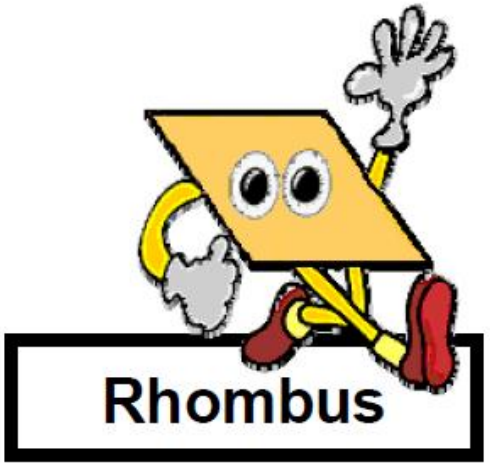
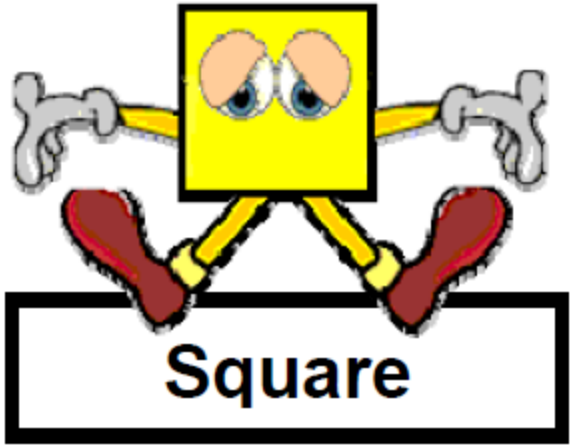
Proving Rhombi and Squares using Coordinate Geometry





SWBAT: Proving Rhombi and Squares Using Coordinate Geometry

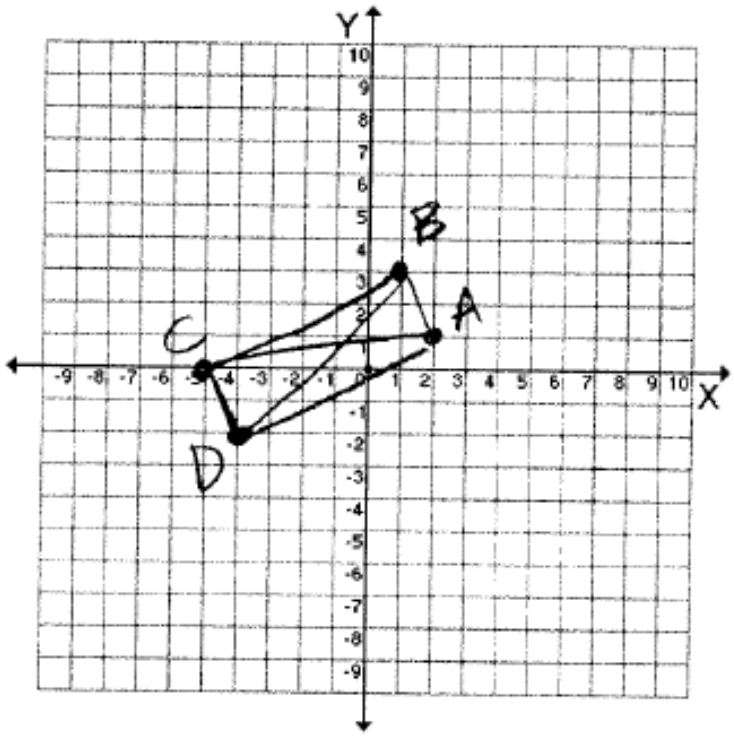
HW REVIEW

Prove that quadrilateral ABCD with the vertices A(2,1), B(1,3), C(-5,0), and D(-4,-2) is a rectangle.

Question: Is this a ▱ with ≅ diags?

Formula: Midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

distance $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$



Work:

Step 1: Prove ▱ first

Mp of CA $= \left(\frac{2+-5}{2}, \frac{1+0}{2}\right) = \left(\frac{-3}{2}, \frac{1}{2}\right)$

Mp of BD $= \left(\frac{1+-4}{2}, \frac{3+-2}{2}\right) = \left(\frac{-3}{2}, \frac{1}{2}\right)$

Step2: Prove ≅ diagonals

$CA = \sqrt{(1)^2 + (7)^2} = \sqrt{1+49} = \sqrt{50}$

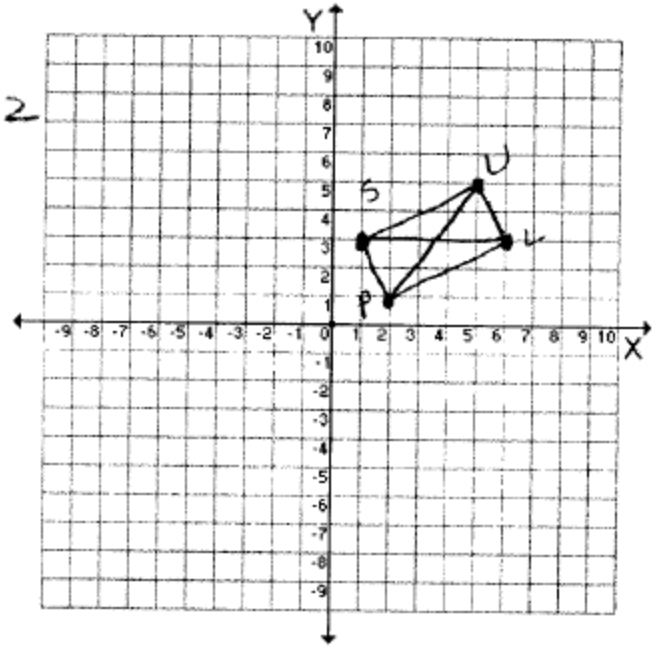
$BD = \sqrt{(5)^2 + (5)^2} = \sqrt{25+25} = \sqrt{50}$

Statement:
ABCD is a rectangle b/c it's a ▱ with ≅ diagonals.

SWBAT: Proving Rhombi and Squares Using Coordinate Geometry

HW REVIEW

Prove that quadrilateral PLUS with the vertices P(2,1), L(6,3), U(5,5), and S(1,3) is a rectangle.



Question: Is this a ▱ with ≅ diags?

Formula: Midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

distance $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

work:

Step 1: Prove ▱ first

M_p of $SL = \left(\frac{6+1}{2}, \frac{3+3}{2}\right) = (7/2, 3)$

M_p of $UP = \left(\frac{2+5}{2}, \frac{1+5}{2}\right) = \left(\frac{7}{2}, 3\right)$

Step 2: Prove ≅ diagonals

$SL = \sqrt{(0)^2+(5)^2} = \sqrt{25}$

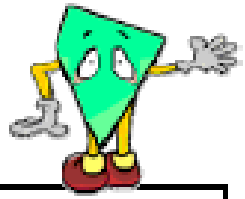
$PU = \sqrt{(4)^2+(3)^2} = \sqrt{16+9} = \sqrt{25}$

Statement:

PLUS is a rectangle b/c its a ▱ with ≅ diagonals.

SWBAT: Proving Rhombi and Squares Using Coordinate Geometry

The Quadrilateral Family



Quadrilateral

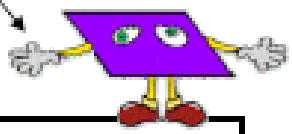
1. four sided polygon
2. sum of interior angles is 360°

Trapezoid

1. quad with only one pair of parallel sides
2. median is parallel to bases and ½ sum of bases

Isosceles Trapezoid

1. trapezoid
2. non-parallel sides (legs) ≅
3. base angles ≅
4. diagonals ≅
5. opposite angles supplementary

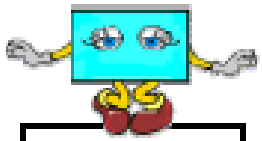


Parallelogram

SIDES:
1. opposite sides parallel
2. opposite sides ≅

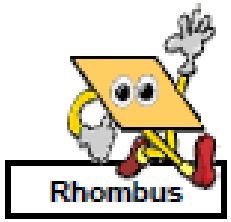
ANGLES:
3. opposite angles ≅
4. consecutive angles supplementary

DIAGONALS:
5. diagonals bisect each other
6. diagonals form ≅ triangles



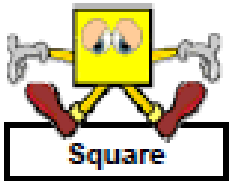
Rectangle

1. parallelogram
2. 4 right angles
3. diagonals ≅



Rhombus

1. parallelogram
2. 4 ≅ sides
3. diagonals bisect angles
4. diagonals perpendicular



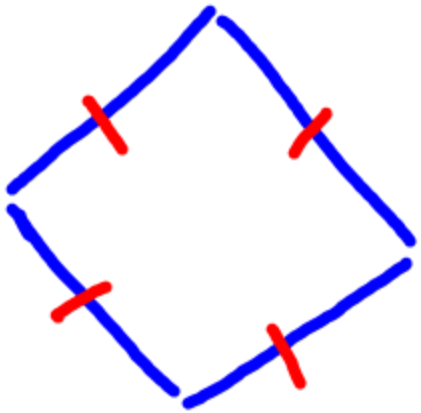
Square

1. parallelogram
2. rectangle
3. rhombus

SWBAT: Proving Rhombi and Squares Using Coordinate Geometry

Proving a Quadrilateral is a Rhombus

Method: Prove that all four sides are equal.



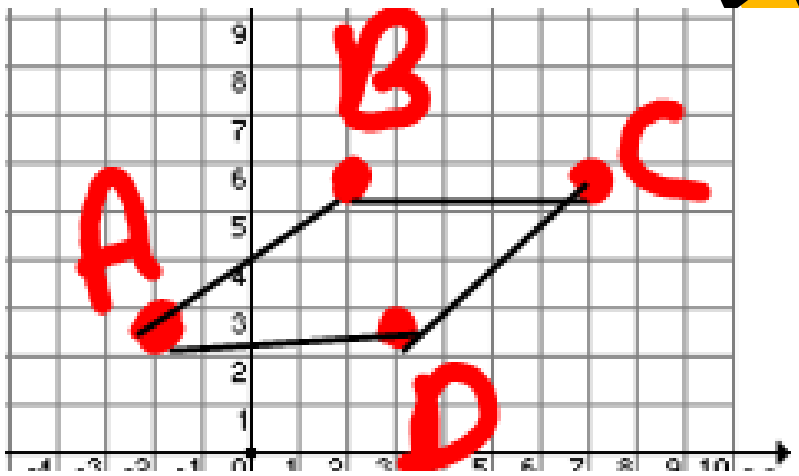
SWBAT: Proving Rhombi and Squares Using Coordinate Geometry

Example Model Problem

Prove that a quadrilateral with the vertices A(-2,3), B(2,6), C(7,6) and D(3,3) is a rhombus.

Question: Are all sides ≅ ?

Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



Work:

$\underline{AB} = \sqrt{(4)^2 + (3)^2} = \sqrt{25}$

$\underline{BC} = \sqrt{(5)^2 + (0)^2} = \sqrt{25}$

$\underline{CD} = \sqrt{(4)^2 + (3)^2} = \sqrt{25}$

$\underline{AD} = \sqrt{(5)^2 + (0)^2} = \sqrt{25}$

Statement:

∴ Quad ABCD is a rhombus b/c all the sides are ≅.

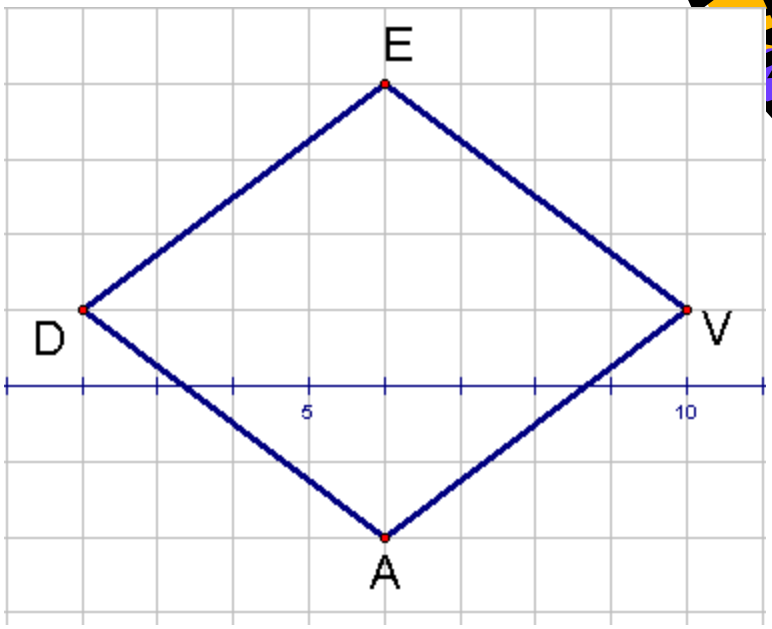
SWBAT: Proving Rhombi and Squares Using Coordinate Geometry

Practice

1. Prove that the quadrilateral with the vertices D(2,1), A(6,-2), V(10,1) and E(6,4) is a rhombus.

Question: Are all sides ≅ ?

Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



Work:

DE = $\sqrt{(4)^2 + (3)^2} = \sqrt{25}$

EV = $\sqrt{(-4)^2 + (3)^2} = \sqrt{25}$

VA = $\sqrt{(4)^2 + (3)^2} = \sqrt{25}$

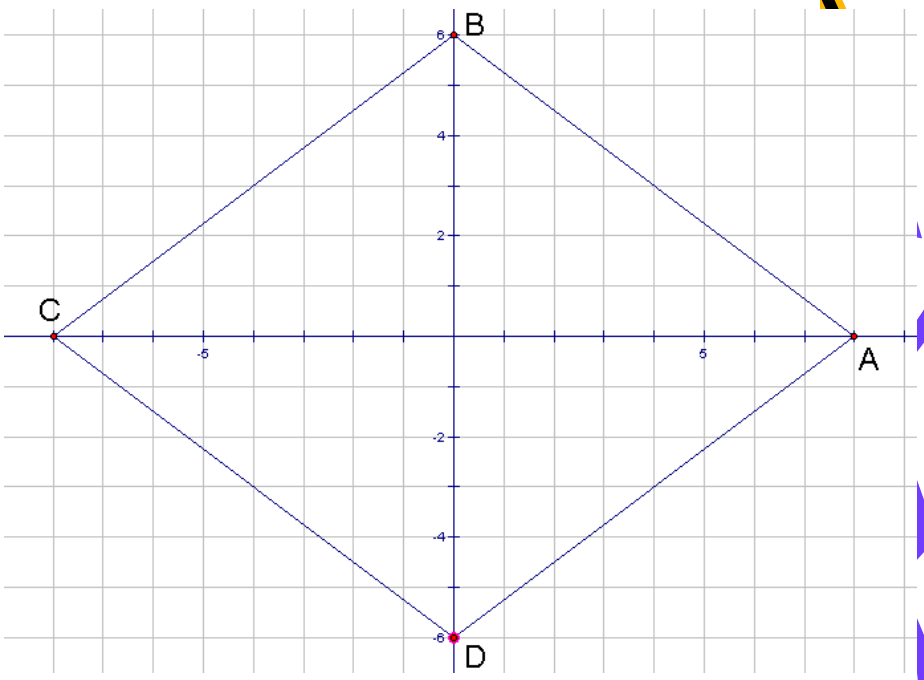
AD = $\sqrt{(-4)^2 + (3)^2} = \sqrt{25}$

Statement:

∴Quad DAVE is a rhombus b/c all the sides are ≅.

SWBAT: Proving Rhombi and Squares Using Coordinate Geometry

Prove that quadrilateral ABCD with the vertices A(8,0), B(0,6), C(-8, 0), and D(0, -6) is a rhombus.



Question: Are all sides ≅ ?

Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Work:

AB $= \sqrt{(-8)^2 + (6)^2} = \sqrt{100}$

BC $= \sqrt{(8)^2 + (6)^2} = \sqrt{100}$

CD $= \sqrt{(-8)^2 + (6)^2} = \sqrt{100}$

DA $= \sqrt{(8)^2 + (6)^2} = \sqrt{100}$

Statement:

∴ Quad ABCD is a rhombus b/c all the sides are ≅.

SWBAT: Proving Rhombi and Squares Using Coordinate Geometry

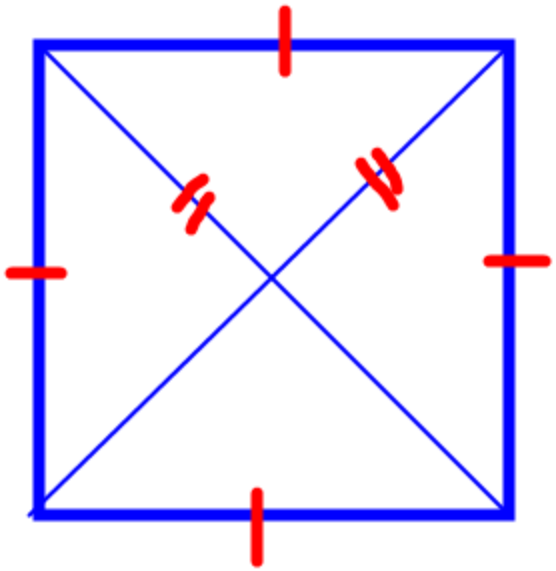
Summary

* Use the distance formula 4x to show all sides ≅.

SWBAT: Proving Rhombi and Squares Using Coordinate Geometry

Proving that a Quadrilateral is a Square

There are many ways to do this. Prove that the quadrilateral is a rectangle and a rhombus.



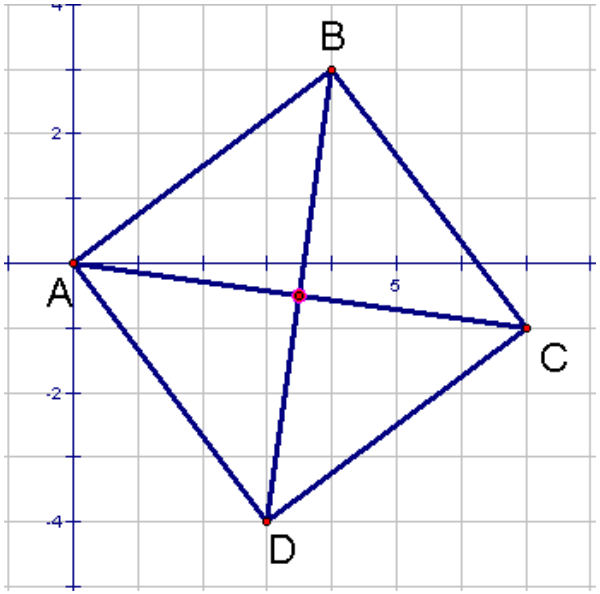
SWBAT: Proving Rhombi and Squares Using Coordinate Geometry

Example Model Problem

1. Prove that the quadrilateral with vertices A(0,0), B(4,3), C(7,-1) and D(3,-4) is a square.

Question: Is this a rhombus and a rectangle?

Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



Work: **Step 1: Prove Rhombus**

$\underline{AB} = \sqrt{(4)^2 + (3)^2} = \sqrt{25}$

$\underline{BC} = \sqrt{(-3)^2 + (4)^2} = \sqrt{25}$

$\underline{CD} = \sqrt{(4)^2 + (3)^2} = \sqrt{25}$

$\underline{DA} = \sqrt{(-3)^2 + (4)^2} = \sqrt{25}$

Step 2: Prove Rectangle

$\underline{AC} = \sqrt{(-7)^2 + (1)^2} = \sqrt{50}$

$\underline{BD} = \sqrt{(1)^2 + (7)^2} = \sqrt{50}$

Statement:

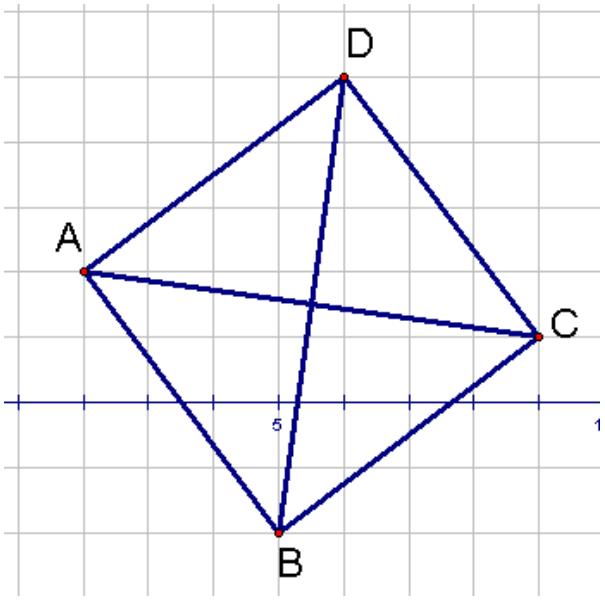
∴ Quad ABCD is a square b/c it's a rhombus and a rectangle.

SWBAT: Proving Rhombi and Squares Using Coordinate Geometry

1. Prove that the quadrilateral with vertices A(2,2), B(5,-2), C(9,1) and D(6,5) is a square.

Question: Is this a rhombus and a rectangle?

Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



Work: **Step 1: Prove Rhombus**

$\underline{AD} = \sqrt{(4)^2 + (3)^2} = \sqrt{25}$

$\underline{DC} = \sqrt{(-3)^2 + (4)^2} = \sqrt{25}$

$\underline{CB} = \sqrt{(4)^2 + (3)^2} = \sqrt{25}$

$\underline{BA} = \sqrt{(-3)^2 + (4)^2} = \sqrt{25}$

Step 2: Prove Rectangle

$\underline{AC} = \sqrt{(-7)^2 + (1)^2} = \sqrt{50}$

$\underline{BD} = \sqrt{(1)^2 + (7)^2} = \sqrt{50}$

Statement:

∴ Quad ABCD is a square b/c it's a rhombus and a rectangle.

SWBAT: Proving Rhombi and Squares Using Coordinate Geometry

Summary

→ **Prove Rhombus 1st**

Calculate distance 4x to show all sides ≅

→ **Prove Rectangle**

Calculate distance 2x on diags to show diags ≅.