INDIVIDUAL MODELING OF COMPOSITE MATERIALS WITH MESH SUPERPOSITION METHOD UNDER PERIODIC BOUNDARY CONDITION

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Abstract

Finite element modeling of textile composite is not so easy because matrix part between winding fiber bundles has to be filled. In this paper, individual modeling in which multiple meshes are used to model the composite structure, has been proposed and applied to textile composites. The matrix such as resin or multi-ply is defined as global mesh, and the reinforcement such as complex fiber bundle is defined as local mesh. The periodicity of unit cell such as even in-plane periodicity is considered by the arrangement of stiffness equation. By the proposed method, the modeling of complicated textile composites such as non-crimp fabric composites, calculation of equivalent properties, stress analysis and damage development simulation has been carried out.

1 Introduction

Fiber Reinforced Plastics have been applied for many structures, because of its superior properties. It is not efficient and not economical procedures to obtain the properties of FRP to design from experiments. Though the finite element method (FEM) is used to analyze and evaluate the mechanical behavior of structures or materials, we have a limitation for total number of elements [1]. The homogenization method [2][3] can not be applied to the complicated composite materials, because of difficulty in making finite element mesh. But it is effective to obtain the equivalent properties. In order to analyze composite materials easily, we must solve two problems. First is to make the finite element mesh for even textile composites which have complicated structure such as plain, satin, twill weave and non-crimp fabrics. Second is to need the much memory in computer to carry out FEM.

In this paper, a new technique for FE modeling and analysis for textile composites has been proposed. In the proposed method, each material in composite materials has been modeled as individual mesh, and all meshes has been combined each other in FE analysis.

2 Modeling of Composite Materials

2.1 Periodic Boundary Condition

Textile composites sometimes have periodic textile structure in mesoscopic level. Because the mechanical properties are depended on the meso-structure, it is important for modeling of textile composite to consider the periodic winding fiber bundles. Periodicity in FEM for structural analysis is defined as keeping the same shape between the corresponding surfaces. The equivalent properties of unit cell with perfect periodicity are sometimes calculated with homogenization method however that of unit cell with imperfect periodicity, for example periodicity in plane, can not be obtained. In this study, the calculation method of FE analysis for unit cell with imperfect periodicity was proposed, which go through the different procedure with the homogenization method. The effect of lamination on equivalent properties can be evaluated by the proposed method.

At first, finite element mesh must have the corresponding surfaces which have the corresponding nodes each other in order to apply periodic boundary conditions except of by the penalty method. An example of mesh with P.B.C. was shown in Fig.1. In this mesh, both surfaces in right and left sides have a periodicity, and there are some periodic couples as nodes a and b, nodes c and e. In this case, displacement at nodes b and e are obtained as the following equations.

$$\boldsymbol{d_b} = \boldsymbol{d_a} + \boldsymbol{d_d} \tag{1}$$

$$\boldsymbol{d_e} = \boldsymbol{d_c} + \boldsymbol{d_d} \tag{2}$$

where, d_d is displacement against the corresponding surface. As the nodes *a* and *c* in Eq. 1 and 2, nodes referred from other nodes are called master node, otherwise, nodes referring other node are called slave node.

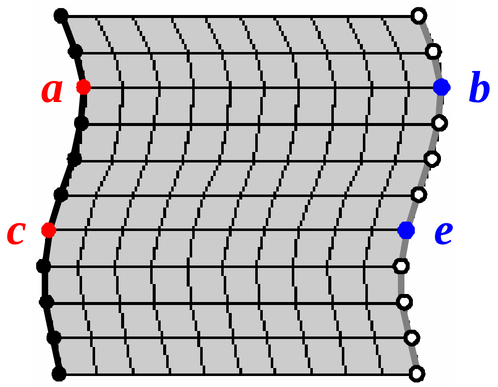


Fig.1 Corresponding nodes of periodicity

Next, descriptions about how to make stiffness equation applied periodic boundary condition will be given as follows. Eq. 3 shows stiffness equation without periodic boundary condition, where there are 6 nodes *a* to *f* as a matter of convenience. When a periodic boundary condition is applied as shown in Fig. 1, corresponding nodes *a* and *b*, nodes *c* and *e* are related by the periodic boundary condition. Then, Eq. 3 is transformed to Eq. 4 from Eq. 1 and 2. Furthermore the elements of column b are moved to column *a* and *d* based on the periodicity and Eq. 4 is transformed to Eq. 5. In the same procedure, column e, row b and e are moved too. Finally stiffness equation becomes as shown in Eq. 6. The rows and columns concerned with slave nodes in stiffness matrix are removed and new row and column concerned with periodicity are added. In case of multi-periodicity, removed and added row and column are more increased.

Although the application of periodicity to structural analysis has been already adopted in finite element method like homogenization method, the periodic boundary condition is treated as one of convergence condition. There are three advantages of modifying stiffness equation. One is that we can adopt direct process to solve uneasy stiffness equation. Another is unnecessary of definition of particular coordinate system, which is depended on the unit cell such as equivalent coordinate system (ECS) by Whitcomb [4]. The other is that we need not to assume perfect periodicity as in homogenization method. These advantages contribute to finite element analysis of textile composite with complex internal structure.

$$\begin{Bmatrix} f_a \\ f_b \\ f_c \\ f_d \\ f_e \\ f_f \end{Bmatrix} = \begin{bmatrix} k_{aa} & k_{ab} & k_{ac} & k_{ad} & k_{af} & k_{af} \\ k_{ba} & k_{bb} & k_{bc} & k_{bd} & k_{be} & k_{bf} \\ k_{ca} & k_{cb} & k_{cc} & k_{cd} & k_{ce} & k_{cf} \\ k_{da} & k_{db} & k_{dc} & k_{dd} & k_{de} & k_{df} \\ k_{ea} & k_{eb} & k_{ec} & k_{fd} & k_{ee} & k_{ef} \\ k_{fa} & k_{fb} & k_{fc} & k_{ed} & k_{fe} & k_{ff} \end{bmatrix} \begin{Bmatrix} d_a \\ d_b \\ d_c \\ d_d \\ d_e \\ d_f \end{Bmatrix} \tag{3}$$

$$\begin{Bmatrix} f_a \\ f_b \\ f_c \\ f_d \\ f_e \\ f_f \end{Bmatrix} = \begin{bmatrix} k_{aa} & k_{ab} & k_{ac} & k_{ad} & k_{ae} & k_{af} \\ k_{ba} & k_{bb} & k_{bc} & k_{bd} & k_{be} & k_{bf} \\ k_{ca} & k_{cb} & k_{cc} & k_{cd} & k_{ce} & k_{cf} \\ k_{da} & k_{db} & k_{dc} & k_{dd} & k_{de} & k_{df} \\ k_{ea} & k_{eb} & k_{ec} & k_{ed} & k_{ee} & k_{ef} \\ k_{fa} & k_{fb} & k_{fc} & k_{fd} & k_{fe} & k_{ff} \end{bmatrix} \begin{Bmatrix} d_a \\ d_a + d_d \\ d_c \\ d_d \\ d_c + d_d \\ d_f \end{Bmatrix} \tag{4}$$

$$\begin{Bmatrix} f_a \\ f_b \\ f_c \\ f_d \\ f_e \\ f_f \end{Bmatrix} = \begin{bmatrix} k_{aa}+k_{ab} & 0 & k_{ac} & k_{ad} & k_{ae} & k_{af} & k_{ab} \\ k_{ba}+k_{bb} & 0 & k_{bc} & k_{bd} & k_{be} & k_{bf} & k_{bb} \\ k_{ca}+k_{cb} & 0 & k_{cc} & k_{cd} & k_{ce} & k_{cf} & k_{cb} \\ k_{da}+k_{db} & 0 & k_{dc} & k_{dd} & k_{de} & k_{df} & k_{db} \\ k_{ea}+k_{eb} & 0 & k_{ec} & k_{ed} & k_{ee} & k_{ef} & k_{eb} \\ k_{fa}+k_{fb} & 0 & k_{fc} & k_{fd} & k_{fe} & k_{ff} & k_{fb} \end{bmatrix} \begin{Bmatrix} d_a \\ d_a \\ d_c \\ d_d \\ d_c + d_d \\ d_f \\ d_d \end{Bmatrix} \tag{5}$$

$$\begin{Bmatrix} f_a + f_b \\ f_c + f_e \\ f_d \\ f_f \\ f_d \end{Bmatrix} = \begin{bmatrix} k_{aa}+k_{ab}+k_{ba}+k_{bb} & k_{ac}+k_{ae}+k_{bc}+k_{be} & k_{ad}+k_{bd} & k_{af}+k_{bf} & k_{ab}+k_{ae}+k_{bb}+k_{be} \\ k_{ca}+k_{cb}+k_{ea}+k_{eb} & k_{cc}+k_{ce}+k_{ec}+k_{ee} & k_{cd}+k_{ed} & k_{cf}+k_{ef} & k_{cb}+k_{ce}+k_{eb}+k_{ee} \\ k_{da}+k_{db} & k_{dc}+k_{de} & k_{dd} & k_{df} & k_{db}+k_{de} \\ k_{fa}+k_{fb} & k_{fc}+k_{fe} & k_{fd} & k_{ff} & k_{fb}+k_{fe} \\ k_{ba}+k_{bb}+k_{ea}+k_{eb} & k_{bc}+k_{be}+k_{ec}+k_{ee} & k_{bd}+k_{ed} & k_{bf}+k_{ef} & k_{bb}+k_{be}+k_{eb}+k_{ee} \end{bmatrix} \begin{Bmatrix} d_a \\ d_c \\ d_d \\ d_f \\ d_d \end{Bmatrix} \tag{6}$$

2.2 Mesh Superposition Method

In order to get the modeling of composite materials easy, we use a numerical technique concerned with FEM. The mesh superposition method is one of the multi-meshes FEM as shown in Fig. 2. In this method, we can use two or more FE meshes in order to model composite structures. Local mesh play role of complementary mesh. In this paper, a new modeling technique with mesh superposition method has been proposed. For example, in case of FRP composite, global mesh consists of matrix, and reinforcement such as fibers inside the matrix is modeled in local mesh.

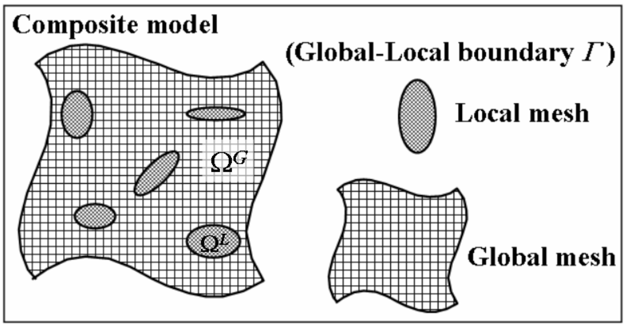


Fig. 2. Individual modeling of composite material

In proposed method, in order to relate these meshes, stiffness matrix equation is defined as Eq. 7.

$$\begin{bmatrix} [K^G] & [K^{GL}] \\ [K^{GL}]^T & [K^L] \end{bmatrix} \begin{Bmatrix} \{d^G\} \\ \{d^L\} \end{Bmatrix} = \begin{Bmatrix} \{F_s^G\} \\ \{F_s^L\} \end{Bmatrix} \tag{7}$$

Where, $[K^G]$ and $[K^L]$ are the stiffness matrices of the global mesh and local mesh, respectively. $[K^{GL}]$ is correlation matrix between the global and local mesh. $\{\boldsymbol{d}^G\}$ and $\{\boldsymbol{d}^L\}$ are displacement vectors in global mesh and local mesh, respectively. $\{\boldsymbol{F}^G\}$ and $\{\boldsymbol{F}^L\}$ are nodal force vectors in global mesh and local mesh, respectively. In this equation, stiffness matrices $[\boldsymbol{K}^G]$, $[\boldsymbol{K}^L]$ and $[\boldsymbol{K}^{GL}]$ are calculated by the following equations.

$$\left[K^G\right] = \int_{W^G} \left[B^G\right]^T \left[D^G\right]\left[B^G\right] dW + \int_{W^L} \left[B^G\right]^T \left[D^L\right]\left[B^G\right] dW \tag{8}$$

$$\left[K^L\right] = \int_{W^L} \left[B^L\right]^T \left[D^L\right]\left[B^L\right] dW \tag{9}$$

$$\left[K^{GL}\right] = \int_{W^L} \left[B^G\right]^T \left[D^L\right]\left[B^L\right] dW = \left[K^{LG}\right]^T \tag{10}$$

Where, $[\boldsymbol{B}]$ and $[\boldsymbol{D}]$ are strain-displacement matrix and stress-strain matrix, respectively. In the method, however displacement is obtained by both meshes, the field of displacement is defined as

$$\{d\} = \begin{cases} \{d^G\} & \text{on } W^G \\ \{d^G\} + \{d^L\} & \text{on } W^L \end{cases} \tag{11}$$

Where, $\{\boldsymbol{d}\}$ is displacement vector of the whole model, W^L is the domain of local mesh. W^G is the domain of global mesh expected W^L. In addition, the fields of strain $\{e\}$ and stress $\{s\}$ are obtained by following equations.

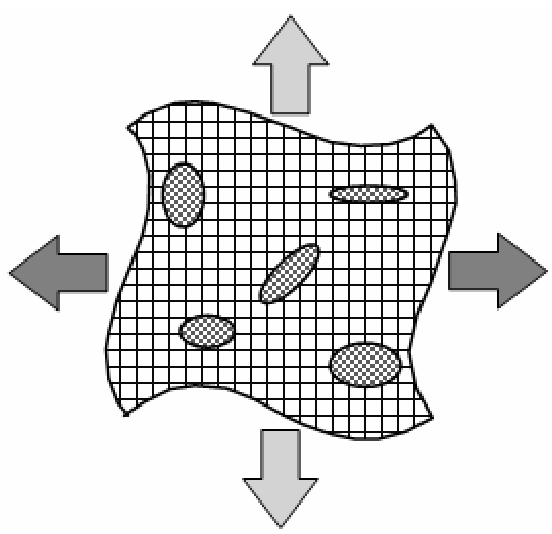
$$\{e\} = \begin{cases} \{e^G\} & \text{on } W^G \\ \{e^G\} + \{e^L\} & \text{on } W^L \end{cases} = \begin{cases} \{B^G\}\{d^G\} & \text{on } W^G \\ \{B^G\}\{d^G\} + \{B^L\}\{d^L\} & \text{on } W^L \end{cases} \tag{12}$$

$$\{s\} = \begin{cases} [D^G]\{e\} & \text{on } W^G \\ [D^L]\{e\} & \text{on } W^L \end{cases} \tag{13}$$

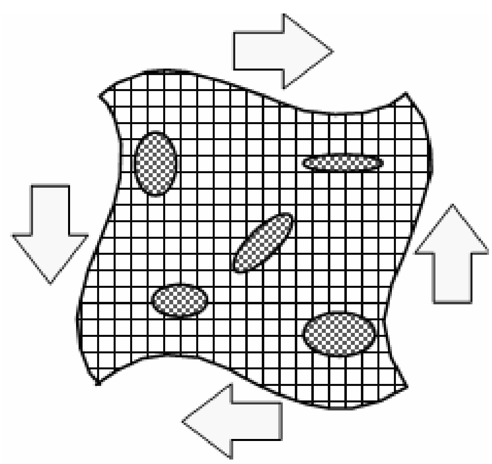
Traditionally in the mesh superposition method, it has been discussed that the size of local mesh is important to guarantee the precision of analysis and the region of local model must be expanded to uniform strain field in global mesh. In proposed method, we can make FE models of reinforcement mesh such as fiber bundle mesh and whole region mesh individually. The fiber bundle mesh can be obtained easily because of no consideration for resin part between fiber bundles. The whole region mesh can be also obtained easily because of no distinction between resin part and fiber bundle part, therefore only grid mesh consisting of resin must be prepared.

2.3 Equivalent property of composite materials

It is not efficient and not economical procedures to obtain the properties of FRP to design from experiments. The homogenization method is effective to obtain the equivalent properties. But it can not be applied to the complicated composite materials, because of the difficulty in making finite element mesh. Some papers discussed about equivalent mechanical properties with periodic boundary condition [5],[6],[7]. Those are calculated by the average stress and the average strain in the unit cell with periodicity. In the proposed method, a calculation method by using above the mesh superposition method and the periodic boundary condition has been proposed [8].The local mesh, which means for example the fiber bundle mesh, is superposed on the global mesh. The equivalent properties of composite model are calculated from FE analysis results in applying normal or shear strain as shown in Fig. 3. The elastic moduli are obtained from the applied strain and reaction force in global mesh with periodicity. The Poison's ratios are obtained from the transverse strain of global mesh.



(a) Normal strain



(b) Shear strain

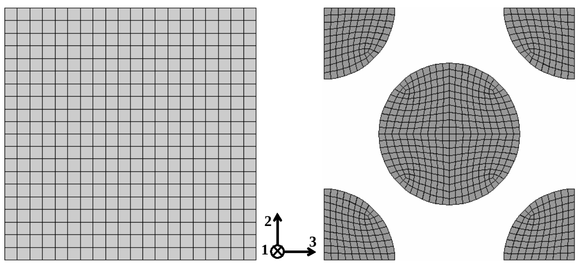
Fig. 3. Applying strain to boundary condition of global mesh

As shown in Fig. 4, we have modeled FRP as resin and unidirectional fibers and calculated the equivalent mechanical properties, and the equivalent properties were calculated by the proposed method. The results in case of several volume fractions are shown in Fig.6. In these figure, the subscript 1 means the fiber direction, 2 and 3 mean transverse direction, respectively. As a verification of the proposed method, the results by ordinary mesh in Fig. 5 and empirical formulae of Uemura [9] and Chamis [10] are shown in the same figure. All results show good agreements except the result of Poison's ratio (ν23). The reason why the differences occur is that the empirical formulae of Uemura's and Chamis's have assumed the equations respectively like as isotropic materials as shown in Eq. 14 and 15.

$$n_{23} = \frac{E_{22}}{2G_{23}} - 1 \qquad (14)$$

$$n_{23} = n_{fT} V_f + n_m (1 - V_f) \qquad (15)$$

Eq. 14, which is used for isotropic materials, has been used in Chamis's, however it can not be really assumed because of dependence of E_{11}. On the other hand, Eq. 15 has been used in Uemura's, however it can not be assumed too.



(a) Resin (Global mesh) (b) Fiber (Local mesh)

Fig.4 Ordinary mesh

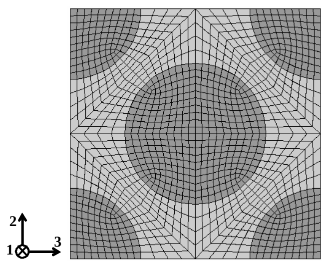
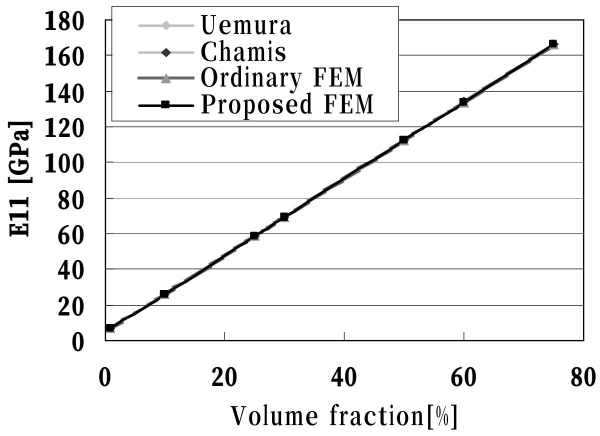
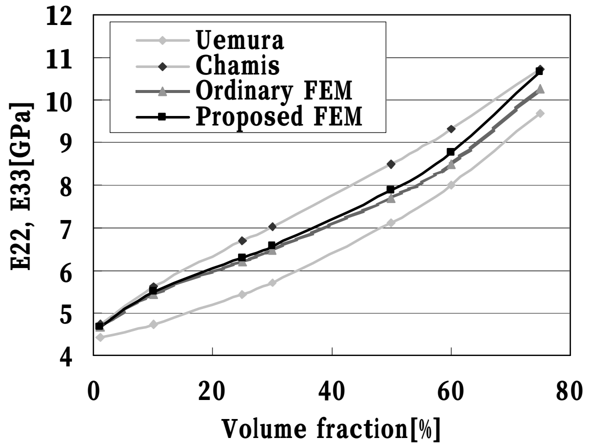


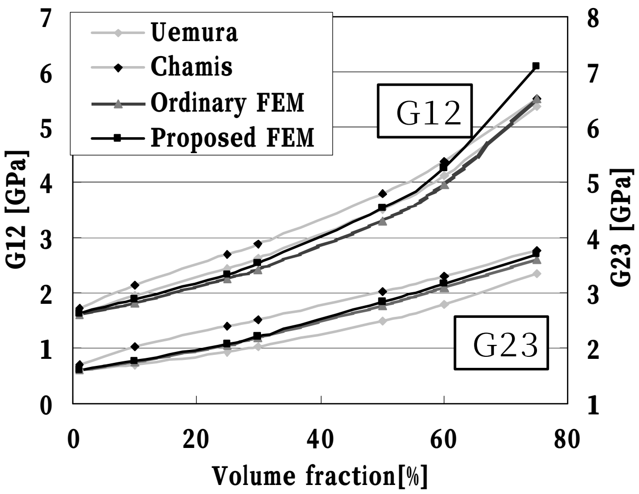
Fig.5 FE meshes of Fiber-Matrix model (V_f = 50%)



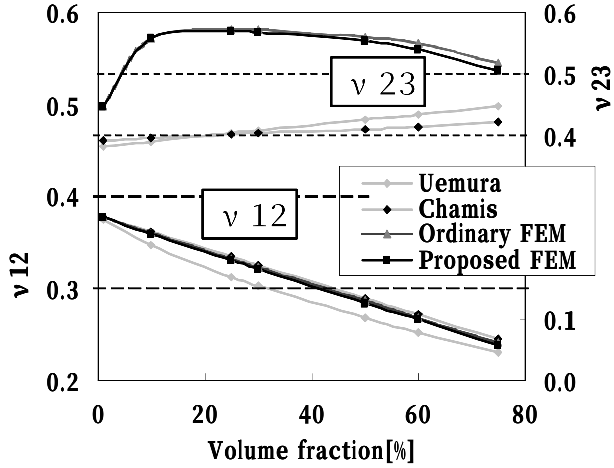
(a) Elastic modulus E11



(b) Elastic modulus E22, E33



(c) Shear modulus



(d) Poison's ratio

Fig.6 Comparison of the equivalent properties of Fiber-Matrix model

Fibers:carbon $E1^{(f)}$=220GPa, $E2^{(f)}$=$E3^{(f)}$=13.8GPa, $G12^{(f)}$=$G31^{(f)}$=9.0GPa, $G23^{(f)}$=4.8GPa, $\nu12^{(f)}$=0.20, $\nu23^{(f)}$=0.47, $\nu31^{(f)}$=0.013,
Resin:epoxy $E^{(m)}$=4.4GPa, $G^{(m)}$=1.6GPa, $\nu^{(m)}$=0.38

2.4 Strength of composite materials

Strength of composite materials is depended on the internal structure, the fiber volume fraction, the properties of materials etc. However fracture modes of unidirectional FRP are matrix cracks except of fiber breakage. So, the strength of unidirectional FRP as shown in Fig. 4 was calculated by stiffness degradation of damaged elements [11][12]. There is no difference; however the calculation can be carried with the ordinary FE model of course as shown in Fig. 5. The uniaxial strain was applied to the unit cell under periodic boundary conditions and the applied strain increased gradually before stresses at matrix part are less than matrix strength. In case the stress is over the strength, the stiffness of the element is decreased at the same applied strain. Fiber breakage was occurred in applied normal strain in fiber direction. The results in cases of applying normal strain 11, 22 and 33, and shear strain 12, 31 and 23 are shown in Fig. 7. Then strength of fiber is 2890MPa, strength of matrix is 70MPa and Mises criterion is applied.

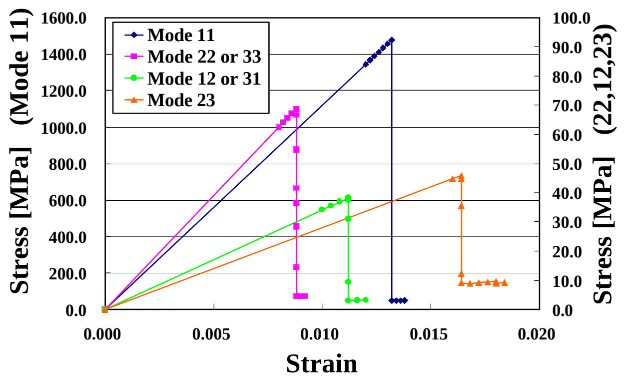


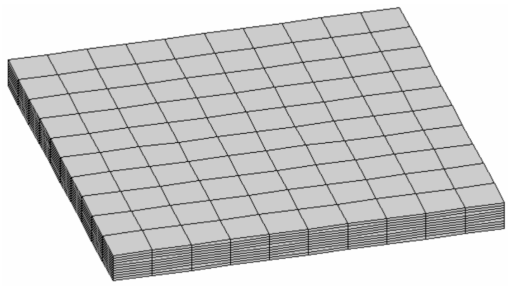
Fig.7 Stress-Strain curve of Fiber-Matrix model (Volume fraction 50%)

From these results, it is recognized that the whole stiffness of unidirectional FRP is extremely decreased. So, the strength of unidirectional FRP can be defined from the maximum stress of stress-strain curve.

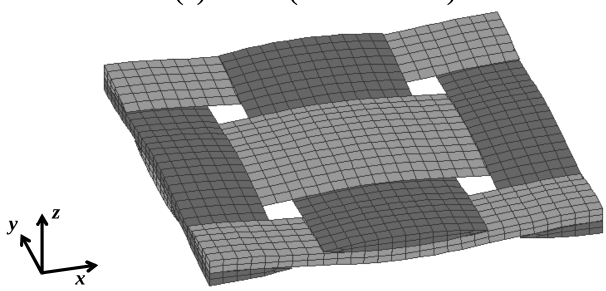
3 Finite Element Analysis of Textile Composite

3.1 Equivalent properties

In the proposed method, plain woven fabric composite is modeled as fiber bundle mesh and whole region mesh. FE meshes of plain woven fabric composite are shown in Fig. 8. It is easy to make FE meshes because it is no need to make elements between fiber bundles. The local mesh consists of fiber bundles which modeled as unidirectional FRP, the mechanical properties were got from the results in the preceding chapter as shown in Table 1. The global mesh consists of only resin. As the same as in case of unidirectional FRP, the ordinary mesh of plain woven fabric composite, which was modeled as single mesh as shown in Fig. 9 was prepared in order to compare the results. This mesh can be obtained by WiseTex and MeshTex software [11][14][15]. The results are shown in Table 2. By the comparison with the ordinary FEM, it is recognized that the equivalent properties by the proposed method are acceptable.



(a) Resin (Global mesh)



(b) Fiber bundles (Local mesh)

Fig.8 Finite element meshes of plain woven fabric composite

Table1 Mechanical properties of fiber bundle and resin

	Fiber bundle (V_f = 50%)	Resin
E11	112 GPa	4.44 GPa
E22	7.70 GPa	
E33	7.70 GPa	
G23	2.77 GPa	1.58 GPa
G31	3.31 GPa	
G12	3.31 GPa	
n23	0.560	0.380
n31	0.020	
n12	0.287	
F11	1479 MPa	70 MPa
F22	68.7 MPa	
F33	68.7 MPa	
F23	45.9 MPa	
F31	38.2 MPa	
F12	38.2 MPa	

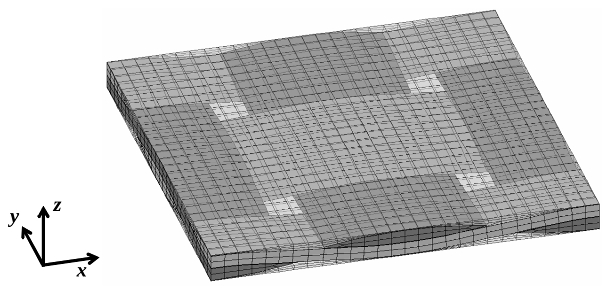


Fig.9 Ordinary mesh of plain woven fabric composite

Table2 Comparison of the equivalent properties of woven fabric composite

Equivalent property	Ordinary FEM	Individual model
Ex	40.304 GPa	40.597 GPa
Ey	40.304 GPa	40.597 GPa
Ez	4.855 GPa	4.920 GPa
Gyz	1.517 GPa	1.640 GPa
Gzx	1.517 GPa	1.640 GPa
Gxy	2.185 GPa	2.236 GPa
ν yz	0.431	0.430
ν zx	0.052	0.052
ν xy	0.108	0.099

3.2 Stress analysis

The stress and strain distributions by the individual modeling in case of applying normal strain are shown in Fig. 10. However as the above description, the load was applied to the global mesh, strain was also generated in the local mesh. Otherwise, the strain distribution by the ordinary FEM was shown in Fig. 11. Both results are good agreement in appearance and quantitatively.

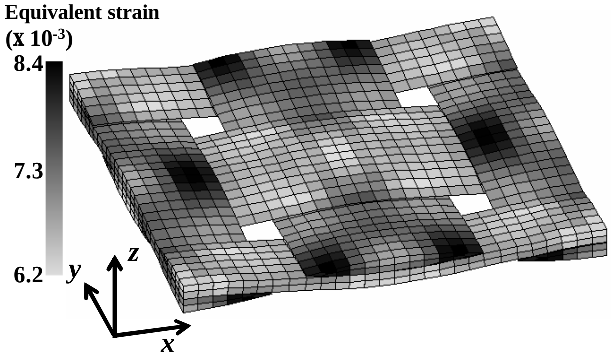


Fig. 10 Strain distribution in Local mesh by the individual modeling in applying average strain 0.74% in *x* direction

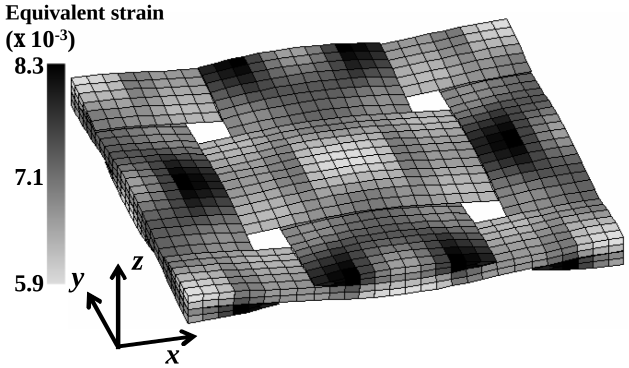


Fig. 11 Strain distribution in fiber bundle part by the ordinary FEM in applying average strain 0.74% in *x* direction

3.3 Damage development

In order to simulate the failure of textile composite, failure development analysis was carried out by the stiffness degradation of failure elements. The stress-strain curves of plain woven fabric composite by the individual model and ordinary FEM are shown in Fig. 12. The curves by the both methods are almost same each other.

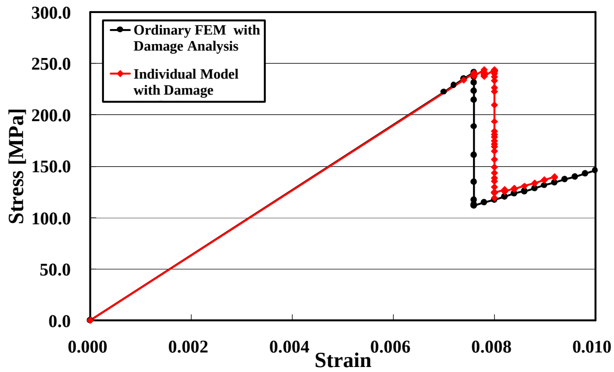


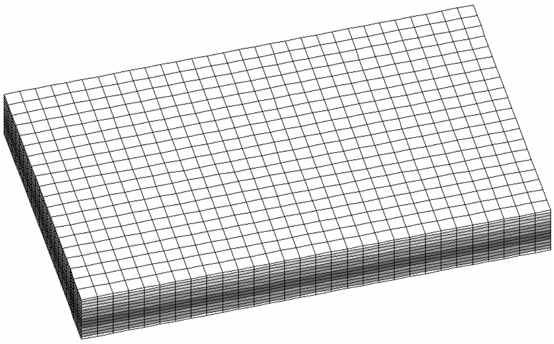
Fig.12 Stress-Strain curve of plain woven fabric composite

3.4 Modeling of complicated textile composites

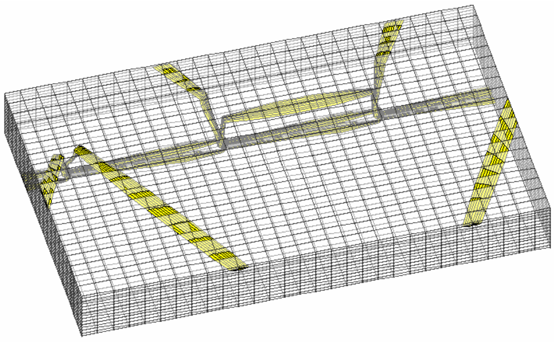
Some textile composites can not be modeled with the ordinary FE, because the textile structure is too complex and it is difficult to fill in the part between fiber bundles with the ordinary single FE mesh. The purpose of this study is to model and evaluate such complicated textile composites.

An example of the individual modeling of a multi-axial multi-ply stitched perform (called "non-crimp fabric") is shown in Fig. 13. In this case, multi-axial multi-ply mesh is used as the global mesh under in-plane periodic boundary condition, and the stitching yarn mesh is used as the local mesh. The strain distribution at applying normal strain 0.5% in *x* direction is shown in Fig. 14. From the result, it is recognized that the larger strain occurred in the vicinity of the stitching yarn.

So, by the individual modeling, complicated textile composites such as non-crimp fabric composite can be analyzed and evaluated.

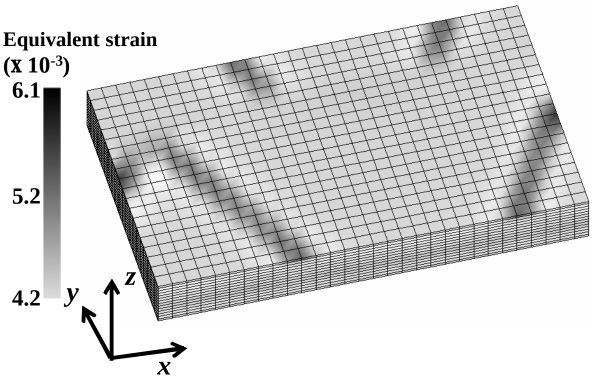


(a) multi-axial multi-ply mesh [0/-45/90/45]

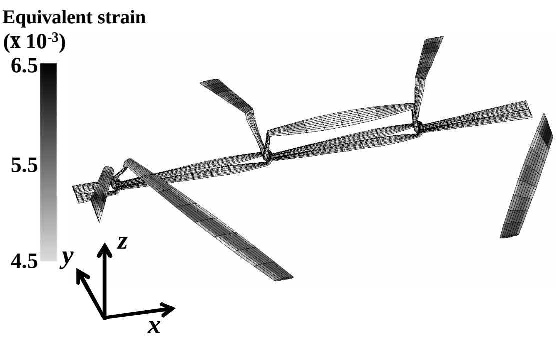


(b) stitching yarn mesh

Fig. 13 Individual model of non-crimp fabric composite



(a) multi-axial multi-ply mesh [0/-45/90/45]



(b) stitching yarn mesh

Fig. 14 Strain distribution of non-crimp fabric composite at applying normal strain 0.5%

4 Conclusions

In order to get modeling of composite materials easy, an individual modeling and a calculation system have been proposed. In the proposed method, the behavior of textile composite with complicated fabric structure such as non-crimp fabrics can be evaluated. Therefore it is convenient technique and efficient to evaluate the macroscopic behavior, local stress and failure development of textile composite.

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