Thou shalt construct in a modular way

By J.A.J. van Leunen.

Retired physicist

26 August 2016

Abstract

Look around and you become easily convinced from the fact that all discrete objects are either modules or modular systems. With other words, the creator of this universe must be a modular designer. His motto is "Construct in a modular way". However, also non-discrete items exist. Universe contains continuums and these continuums appear to relate to the discrete objects. Further, we as observers of these facts, want to place everything into an appropriate model, such that we can comprehend our environment. The paper applies a quaternionic Hilbert space in order to construct a simple dynamic model of a mathematical universe. The Hilbert space stores all historic, current and future dynamic geometric data that describe this mathematical universe. This model acts as a powerful test model for the toolkit of model designers.

> If you think, then think twice.
> In any case, think frankly.

1 Observation

The foundation of physical reality must necessarily be very simple and as a consequence its structure must be easily comprehensible by skilled scientists. So, quite probable the structure was long ago discovered and added as a part to mathematics. As a consequence the best way to investigate the foundation of reality is to use mathematical test models. The rediscovery of this structure as a foundation of reality is a complicated task because extending this foundation must automatically lead to a higher level of the structure of physical reality that shows more features that can be recognized as features of physical reality. In addition, the lower levels of the structure of physical reality must leave some signatures that are visible in many facets of the universe.

Several of such signatures exist. For example the fact that all discrete items in universe are either modules or they are modular systems is probably a signature of the foundation or of the lower levels of the structure of physical reality. Considering this observation as a signature requires the investigation of the peculiarities of modular design and modular construction. That analysis learns that relations between modules and relations that are relevant inside modules or modular systems play a major role. Especially the relations that determine that an object is a module or is part of a module are important. It is quite probable that the foundation of physical reality is a relational structure. A relational structure is a set in which the relations that can exist between the elements are restricted in a well-defined way.

About eighty years ago a relational structure was discovered, which was thought to play a significant role in the description of physical reality by physical theories. The discoverers of the relational structure called it "quantum logic". The mathematicians used a more technical name and called it "orthomodular lattice". In their introductory paper the discoverers showed that the set of closed subspaces of the somewhat earlier discovered Hilbert space has exactly the relational structure of an orthomodular lattice. With other words this Hilbert space is a realization of the orthomodular lattice. The question that arises now is whether this Hilbert space is also a realization of modules and modular systems. This question has a positive answer but the argumentation requires a deep dive into the concept of modularization and into advanced mathematics.

More signatures exist but in this paper we first focus on this one.

2   Modular construction

Diving deep into the fundamental structure of physical reality requires a deep dive into advanced mathematics. Usually this goes together with formulas or other descriptions that are incomprehensible to most people. The nice thing about this situation is that the deepest foundation of reality must be rather simple and as a consequence it can be described in a simple way and without any formulas. For example, if the observed signature characterizes physical reality, then the most fundamental and most influential law of physical reality can be formulated in the form of a commandment:

"THOU SHALT CONSTRUCT IN A MODULAR WAY"

This law is the direct or nearly direct consequence of the structure of the deepest foundation. That foundation restricts the types of relations that may play a role in physical reality. That structure does not yet contain numbers. Therefore it does not yet contain notions such as location and time. Modular construction acts very economic with its resources and the law thus includes an important lesson. "DO NOT SPOIL RESOURCES!"

2.1   Modular design

Understanding that the above statements indeed concern the deepest foundation of physics requires deep mathematical insight. Alternatively it requests belief from those that cannot (yet) understand this methodology. On the other hand intuition quickly leads to trust and acceptance that the above major law must rule our existence! Modular design has the intention to keep the relational structure of the target system as simple as is possible.

Modular design is a complicated concept. Successful modular construction involves the standardization of module types and it involves the encapsulation of modules such that internal relations are hidden from the outside. Systems become complicated when many relations and many types of relations exist inside that system, which must be reckoned when the system is analyzed, configured, operated or changed. The reduction in relational complexity plays a significant role during system configuration. The ability to configure modular systems relies heavily on the ability to couple modules and on the capability to let these modules operate in concordance.

The modular design method becomes very powerful when modules can be constructed from lower level modules. The standardization of modules enables reuse and may generate type communities. The success of a type community may depend on other type communities.

An important category of modules are the elementary modules. This are modules, which are themselves not constructed from other modules. These modules must be generated by a mechanism that constructs these elementary modules. Each elementary module type owns a private generation mechanism.

Another category are modular systems. Modular systems and modular subsystems are conglomerates of connected modules. The constituting modules are bonded. Often the modules are coupled via interfaces that channel the information paths that are used by the relations. Modular subsystems can act as modules and often they can also act as independent modular systems.

The hiding of internal relations inside a module eases the configuration of modular (sub)systems. In complicated systems, modular system generation can be several orders of magnitude more efficient than the generation of equivalent monoliths. This means that stochastic modular system generation gets a winning chance against monolithic system construction.

The generation of modules and the configuration of modular (sub)systems can be performed in a stochastic or in an intelligent way. Stochastic (sub)system generation takes more resources and requires more trials than intelligent (sub)system generation. An inexperienced modular designer must first learn to discern which relations are relevant and which relations can be neglected. Predesigned interfaces that combine provide relations and require relations can save much resources.

If all discrete objects are either modules or modular systems, then intelligent (sub)system generation must wait for the arrival of intelligent modular systems.
Intelligent species can take care of the success of their own type. This includes the care about the welfare of the types on which its type depends. Thus for intelligent modular systems, modularization also includes the lesson "TAKE CARE OF THE TYPES ON WHICH YOU DEPEND".

In reality the elementary modules appear to be generated by mechanisms that apply stochastic processes. In most cases system configuration occurs in a trial and error fashion. Only when intelligent species are present that can control system configuration will intelligent design occasionally manage the system configuration and binding process. Thus in the first phase, stochastic evolution will represent the modular system configuration drive. Due to restricted speed of information transfer, intelligent design will only occur at isolated locations. On those locations intelligent species must be present.

3   Mathematical model

Now we treat some aspects that involve advanced mathematics. We mainly do that in a descriptive way. However, if they really elucidate, then we will apply formulas. In this model we give **new names** to items that we want to discuss in detail. This eases the discussion.

In a modular system, relations play a major role. The success of the described modular construction methodology depends on a particular relational structure that characterizes modular systems.

In 1936 the discoverers of the orthomodular lattice published their discovery in a paper in which they called that lattice "quantum logic". Garret Birkhoff was an expert in lattice theory and John von Neumann was a broadly oriented scientist that was especially interested in quantum physics. "quantum logic" is a strange name because in the same paper the duo showed that the set of closed subspaces of a separable Hilbert spaces has exactly the relational structure of this orthomodular lattice. The name "quantum logic" is only comprehensible, because the relational structure of the orthomodular lattice is quite similar to the relational structure of classical logic and the elements of classical logic are logical propositions. It is not likely that the elements of the orthomodular lattice can be represented by logical propositions, but this immediately creates the question what kind of other objects the elements of the orthomodular lattice represent. The answer is that they represent storage locations of dynamic geometric data. Representations of modules are gathered in a subset of the lattice. This will be elucidated later.

3.1   Separable Hilbert space

The orthomodular lattice extends naturally into a separable Hilbert space. Separable Hilbert spaces are mathematical constructs that act as storage media for dynamic geometric data. Quantum physicists use Hilbert spaces as a base model in which they perform their quantum physical modelling. Each separable Hilbert space is a realization of the orthomodular lattice.

Hilbert spaces are linear vector spaces and each pair of Hilbert vectors owns an **inner product** that represents a number, which is a member of a division ring. Hilbert spaces can only cope with number systems that are division rings. The inner product of two perpendicular vectors equals zero.

3.2 Division rings

For a number system, being a division ring means that every non-zero element of that number system owns a unique inverse. Only three suitable continuum division rings exist. These are the real numbers, the complex numbers and the quaternions. Their rational subsets form discrete division rings. The quaternions form the most elaborate division ring and comprise the other division rings. Number systems exist in several versions that differ in the way that they are ordered. For example multidimensional number systems can be ordered by a selected Cartesian coordinate system and subsequently the result can be ordered by a polar coordinate system. The imaginary part of the quaternionic number system represents a three dimensional space that can be ordered in eight independent ways by a Cartesian coordinate system. The ordering affects the arithmetic properties of the quaternions. Left handed multiplying quaternions exist and right handed multiplying quaternions exist.

3.2.1 Representation of quaternions

Quaternions will be represented by a scalar part that represents the real part of the quaternion and a three dimensional vector part that represents the imaginary part of the quaternion. Bold type face is used for the imaginary parts. The real parts get a suffix $_0$. In many applications the real part represents progression, while the imaginary part represents a spatial location. This representation concerns dynamic geometric data. Quaternions can represent other subjects, but in this paper the representation of dynamic geometric data plays a major role.

3.2.2 Quaternionic multiplication

The quaternionic multiplication rule now follows from:

$$h = f\ g = (f_0 + \boldsymbol{f})\ (g_0 + \boldsymbol{g}) \tag{1}$$

$$h_0 = f_0\ g_0 - \langle \boldsymbol{f}, \boldsymbol{g} \rangle \tag{2}$$

$$\boldsymbol{h} = f_0\ \boldsymbol{g} + \boldsymbol{g}\ f_0 \pm \boldsymbol{f} \times \boldsymbol{g} \tag{3}$$

The$\pm$ sign reflects that left handed and right handed quaternions exist.

3.2.3 Construction and dismantling of numbers

Both Cayley and Conway produced formulas for **constructing** members of number systems from lower dimensional number systems. The dimension increases with a factor two. The reverse process is also possible. The procedure **dismantles** the numbers into two numbers that have a lower dimension. The dimension diminishes with a factor two.

See: http://scorevoting.net/WarrenSmithPages/homepage/nce2.pdf

These procedures can be applied inside a quaternionic Hilbert space. There the procedure helps in constructing complex number based subspaces from two real number based subspaces or the construction of quaternion based subspaces from complex number based subspaces. The road back is also possible. These procedures appear to support pair creation and pair annihilation processes.

3.3 Operators

Hilbert space **operators** describe how Hilbert spaces map into other Hilbert spaces and can describe how Hilbert spaces map onto themselves. In the latter case, the inner product describes the relation

between a Hilbert vector and its map. If the vector is mapped onto itself then the inner product adds an *eigenvalue* to that vector and the vector is called an *eigenvector*. Thus eigenvalues of normal operators must be members of a division ring. If two eigenvalues differ, then their eigenvectors are perpendicular and the inner product of the two eigenvectors equals zero.

3.4   The real number based separable Hilbert space

Inside the real number based separable Hilbert space only operators that feature. We can construct such operator by starting from an orthonormal base that spans this Hilbert space. Next we take all rational numbers and use them to enumerate the base vectors. The corresponding Hermitian operator connects the enumerator with the base vector and in this way they become eigenvalue and eigenvector. This real number based separable Hilbert space can be embedded into a complex number based Hilbert space or in a quaternionic number based Hilbert space. In that case the eigenspace of the specified Hermitian operator can be used as a model wide clock.

3.5   Reference operators in a quaternionic Hilbert space

Number systems that are division rings can be used to define a category of operators that we will call *reference operators*. The rational values of the number system are used to enumerate the members of an orthonormal base of the Hilbert space. The reference operator connects the enumerator with the base vector and in this way they become eigenvalue and eigenvector. Each reference operator implements a *parameter space* that is defined by its eigenspace.

Reference operators are normal operators and normal operators can be split into a Hermitian operator that has an eigenspace, which is formed by all rational real numbers and an anti-Hermitian operator that has an eigenspace, which is formed by the imaginary parts of the eigenvalues of the normal operator.

The Hilbert space can harbor multiple reference operators and in that way it can harbor multiple parameter spaces.

A subset of the reference operators can be considered to be part of the *household* of the Hilbert space. These reference operators all share the Hermitian operator that has been assigned the task to act as a model wide clock. These reference operators form a *family*. A special member of this family plays the role of the generator of the *background parameter space*. Other parameter spaces can *float* with respect to the background parameter space and they can float with respect to each other. The clock relates the kinematics of the geometric centers of these floating parameter spaces.

The anti-Hermitian part of the reference operators treats the spatial part of the reference operator. The Hermitian part indirectly describes via its eigenvectors how the geometric center of the parameter space floats with respect to the background parameter space.

It must be realized that the clock operator for each of its real eigenvalues corresponds with a one-dimensional subspace and that in relation with a member of the family these one-dimensional subspaces define a *hopping path* rather than a full parameter space. This hopping path is defined relative to the parameter space of the considered reference operator.

In the separable quaternionic Hilbert space, each reference operator that is a family member represents a *hopping path* of the geometric center of the eigenspace of the anti-Hermitian operator through the eigenspace of the anti-Hermitian operator that corresponds to the background reference operator. These two hopping paths can be related.

3.6   The scanning vane

If the family of a certain reference operator, such as the background reference operator, is singled out, then a special subspace of the Hilbert space can be specified that represents *the current static status quo* of the Hilbert space. In the subspace all eigenvalues of the selected reference operator share the same real part. This specification divides the Hilbert space into three subspaces. The first subspace represents the *past history*. The third part represents the *future*. If the selected real value, which represents progression is allowed to increase, then the second subspace represents a *vane* that scans over the Hilbert space. This simple model represents a very powerful *mathematical test model*. In the vane-subspace each family member is represented by a one-dimensional subspace. Several processes occur that have a fixed *duration*. This means that the passage of the vane has this duration. The duration defines the *regeneration cycle* of discrete objects. The fixed duration enables the capability to discern *properties* of certain objects and/or enables the definition of their *types*.

3.7   Platforms and symmetry centers

Platforms are eigenspaces of a *selected family* of reference operators. The *background platform* belongs to this family. Platforms represent *floating parameter spaces*. Each platform owns a geometric center. The anti-Hermitian part of the platform operator describes a *symmetry center*. This is the spatial representative of the platform. The *platform operator* represents a *hopping path* through the imaginary part of the quaternionic parameter space that corresponds to the eigenspace of the platform operator. The family's Hermitian operator relates the hopping path of the considered platform operator with the hopping path of the background platform operator.

In the separable Hilbert space, platforms can step relative to the background platform. This can occur with a minimal spatial step size. In the non-separable Hilbert space the corresponding platforms float relative to the background platform.

The symmetry centers can be ordered with the help of a Cartesian coordinate system as well as with the help of a polar coordinate system. *In the platform family the ordering of the symmetry center is defined relative to the ordering of the background platform.* This ordering determines the *symmetry flavor* of the symmetry center. The difference of the symmetry flavor of a selected symmetry center with the symmetry flavor of the background platform determines the symmetry related charge of the selected symmetry center. This charge can be split in an isotropic part, an anisotropic part and a spin.

The short list of isotropic symmetry related charges covers: $-3, -2, -1, 0, +1, +2, +3$. For historical reasons, these numbers must be divided by 3 in order to get the equivalent electric charges. The anisotropic symmetry related charges correspond to color charges and correspond with the three perpendicular directions and the opposite directions in which ordering anisotropy can manifest itself.

Symmetry center s can be rotated by a pair of quaternions that are each other's inverse. Special quaternions exist of which the size of the real part equals the size of the imaginary part. These special quaternions can shift the anisotropy of a symmetry center to another dimension.

Quaternionic platforms can be converted into two complex number based platforms and two complex number based platforms can be converted into a quaternionic platform. At conversion, the quaternionic platforms mirror at the vane.

3.8   Families

Families of reference operators all share the clock operator. In addition their anti-Hermitian parts are ordered such that the Cartesian coordinate axes run in parallel to each other.

3.9   Defined operators

By starting from a selected reference operator, it is possible to define a category of **defined normal operators** that use a **continuous function** in order to replace the parameter value by the function value and connect this value with the corresponding eigenvector of the reference operator. In fact the reference operators are special versions of the defined operators for which the defining functions use the parameter value as the function value. This procedure is very powerful and merges Hilbert space operator technology with function theory.

3.10   Non-separable companion Hilbert space

Each infinite dimensional separable Hilbert space owns a unique companion non-separable Hilbert space that features operators, which have continuum eigenspaces. Such eigenspaces can form flat parameter spaces or dynamic fields. This can easily be comprehended when in the non-separable Hilbert space a similar procedure is used for specifying defined operators as we applied in the separable Hilbert space. In that procedure we specified reference operators and we defined normal operators by using continuous functions. This time we not only use the rational members of the number system, but we also apply the irrational members and we use the same continuous functions. The consequence is that the notion of dimension of the subspaces may lose its sense. The procedure that creates defined operators now links operator technology with function technology, differentiation technology and integration technology.

The separable Hilbert space can be considered to be embedded in its non-separable companion. Platforms that step in the separable Hilbert space will float in the non-separable companion Hilbert space. In a similar way the clock steps in the separable Hilbert space and the clock floats in its non-separable companion.

The scanning vane can be interpreted as a progressive embedding of the separable Hilbert space into its non-separable companion.

3.11   Modules as subspaces

According to the discoverers of the orthomodular lattice, the elements of the lattice can be represented as closed subspaces of a separable Hilbert space. It also has sense to consider a subset of these elements as representatives of modules or modular systems. Thus, not every closed subspace of a separable Hilbert space represents a module or modular system, but every module and every modular system is represented by a closed subspace of the separable Hilbert space. Compared to general closed subspaces of the Hilbert space, will modules and modular systems have extra characteristics.

3.12   Elementary modules

An elementary module cannot be represented by a single Hilbert vector, because that single vector can on the utmost correspond to a static geometric location and from reality we know that modules possess a dynamic geometric location and that fact also holds for the elementary modules. However, elementary modules cannot be split into other modules. Thus the subspaces that represent elementary modules must have multiple dimensions. Still it is possible that at each progression instant each module represents exactly one spatial location. This is a very special condition, but we **postulate** that this special condition is valid for all elementary modules. As a consequence of this postulate the vane contains representatives of elementary modules that are one-dimensional subspaces. They cannot be split into lower level modules.

The postulate forbids that two elementary modules at the same progression instant take the same geometric location.

The vane contains many subspaces that do not contain Hilbert vectors that represent an elementary module. These subspaces are representing **empty space**.

3.13   Germ operators

The elementary modules are represented by a new category of operators that differ from reference operators and that differ from defined operators but that describe the dynamics of elementary

modules. The dynamic location of elementary modules hops as a function of progression. After a while the hops form a **swarm** and both the **hopping path** and the swarm represent the elementary module. The operators will be called **germ operators**. The germ operators are controlled by mechanisms that apply stochastic processes for the generation of the dynamic locations.
The germ operator uses its own private reference operator. This means that the elementary modules reside on their own platform, which applies its own private parameter space that may float with respect to the selected background parameter space. The eigenvalues of the anti-hermitic part of the germ operator correspond exactly with the eigenvalues of the anti-hermitic part of the corresponding platform operator. **The Hermitian parts of these operators are decoupled**. The hop landings act as point-like artifacts!

3.14 Hopping paths and swarms

After generation, the dynamic locations of an elementary module will be ordered with respect to the real value of the quaternions that represents the dynamic location. After ordering of the progression values the elementary module appears to walk along a hopping path and the landing positions form a location swarm. An uncontrolled generation would produce an arbitrary hopping path and a chaotic hop landing location swarm. The mechanism is supposed to ensure that a coherent swarm is generated.

This means that elementary modules are represented by closed subspaces of a Hilbert space that may have a huge dimension. However, at a single progression instant, each elementary module is represented by a subspace that is spanned by a single Hilbert vector. Thus modules and modular systems are represented by a subset of the orthomodular lattice. Within that subset the elementary modules are represented by elements that act as atoms of the subset.
If the swarm contains a large number of hop landing locations, then its geometric center will move in a much less chaotic way. A large number of elements also means an equivalently long regeneration cycle of the swarm. Increasing the number of hop landings will increase the inertness of the swarm.

3.15 Mechanisms

From reality we know that the hopping path is not an arbitrary path and the location swarm is not a chaotic collection. Instead the swarm forms a coherent set of locations that can be characterized by a rather continuous location density distribution. That does not say that the hopping path is not a stochastic path! **The location swarm integrates over the regeneration cycle**. Its characteristics are statistical characteristics.

From physics we know that elementary particles own a wave function and the squared modulus of that wave function forms a continuous probability density distribution, which can be interpreted as a location density distribution of a point-like object. The location density distribution owns a **Fourier transform** and as a consequence the swarm owns a **displacement generator**. This means that at first approximation the swarm can be considered to **move as one unit**. Thus the swarm is a coherent, rather smoothly moving object, which represents the violent stochastic hopping of a point-like object. For a large part this is due to the fact that the swarm contains a huge number of locations that is refreshed in a cyclic fashion.

The fact that at every progression instant the swarm owns a Fourier transform means that at every progression instant the swarm can be interpreted as a wave package. Wave packages can represent interference patterns, thus they can simulate wave behavior. The problem is that moving wave packages tend to disperse. The swarm does not suffer that problem because at every progression instant the wave package is regenerated. The result is that the elementary module shows wave behavior and at the same time it shows particle behavior. When it is detected it is caught at the precise location (the exact swarm element) where it was at this progression instant.

The Hilbert space is nothing more and nothing less than a structured storage medium for dynamic geometric data. It does its storage task in a very precise way, thus without any uncertainty! Neither the separable Hilbert space nor its non-separable companion does contain functionality that ensures the coherent dynamic behavior of the location swarms. Dedicated mechanisms, which do not belong to the household of the Hilbert spaces fill the eigenspaces of the stochastic germ operators that control the elementary modules. The hopping path only stops when the elementary module is "detected" and the controlling mechanism changes to a different mode of operation.

3.16 Dynamic model

We did construct a vane that splits the Hilbert space such that all elementary module eigenvalues that have a selected real value have the corresponding eigenvector inside the vane. The vane splits the Hilbert space in an historic part, the vane itself and a future part. The vane then represents a static status quo that corresponds to the current state of the universe.

This represents an interesting possibility. The Hilbert space can be seen as a storage medium that contains a repository of all historic, present and future data. Or it can be interpreted as a scene that is **observed by modular systems** that travel with the vane. These observers might know part of the stored history, but have no notion of the future. Depending on their capabilities, the observers reflect only a part of their history. Information that inside the vane is generated at a distance has still to travel through an information carrying field that acts as the living space for the elementary modules in order to reach the observer. The encounter will take place in the future. Information that reaches the observer arrives from the past. Those kinds of information travel via information carriers.

The vane forms a subspace of the Hilbert space and for each elementary module that subspace contains a single Hilbert vector that plays as eigenvector for the corresponding geometric location. This location is the landing point of a hop rather than the geometric center of the location swarm.

Quaternionic platforms can be converted into two complex number based platforms and two complex number based platforms can be converted into a quaternionic platform. At conversion, the quaternionic platforms mirror at the vane. The mirrors carry a particle at one side and an anti-particle at the other side. The conversion takes a fixed duration. The processes are supported by number construction procedures or number dismantling procedures.

3.17 Defining fields

Fields are eigenspaces of defined operators that reside in the companion non-separable Hilbert space and that have continuum eigenspaces. This enables the treatment of fields independent of their defining functions and the corresponding parameter spaces. However, if the dynamic behavior of fields must be investigated, then the quaternionic differential calculus must be applied in order to formulate corresponding defining functions and defined operators.

3.18 Living space

The germ operators have no equivalent inside the non-separable Hilbert space. However, their eigenvalues may be sensed by a field that exists as eigenspace of a defined operator, which resides in the non-separable Hilbert space. This can occur when the separable Hilbert space is **embedded** in its non-separable companion Hilbert space. We will call the mentioned field the **living space** of the modules and modular systems.

The hopping path that represents an elementary module, corresponds to a coherent location swarm, which is characterized by a location density distribution. Via the convolution of the **Green's function** of a field and this location density distribution, the swarm corresponds to a deformed part of the field. In this way the field describes the existing elementary modules. The description of a nearby located elementary particles deforms the field in that region. The convolution means that the

Green's function blurs the location density distribution. This can be interpreted as if the hopping landing locations influence the field, but the alternative interpretation is that the field is a kind of blurred descriptor of the hopping landing locations. Anyway the landing locations and the discussed field are intimately coupled. The deformed field can be interpreted as the living space of the modules and modular systems.

3.19 Green's function

At every progression instant, the hop landing locations cause the emission of a spherical shape keeping front. That front keeps its shape, but the amplitude of that shape diminishes as $1/r$ with distance r from the emission location. The fronts proceed outwards with a fixed speed. The shape keeping front is a solution of the homogeneous second order partial differential equation that describes the dynamic behavior of the affected field. Later we will identify both the hop and the corresponding solution by using the name *clamp* for these phenomena. If this effect is integrated over the regeneration cycle of the swarm, then the Green's function results. The integration turns the homogeneous second order partial differential equation into an inhomogeneous second order partial differential equation. The extra term that makes the equation inhomogeneous concerns the Green's function. The amplitude of the Green's also diminishes as $1/r$ with distance r from the emission location.

3.20 Stochastic processes

The mechanisms that generate the hopping landing location control the dynamics of the model. These mechanisms use stochastic processes. These processes appear to belong to a category which is mathematically known as inhomogeneous spatial Poisson point processes. In more detail these processes probably are similar to modified Thomas processes.

This fact is supported by a signature that is visible in the visual trajectory of a category of living species that are called vertebrates. It appears that the visual trajectory of all vertebrates is optimized for low dose rate imaging. This visual system contains noise filters that block information for which the signal to noise ratio is too low. This signal to noise ratio is typical for information generated by Poisson processes that are attenuated by subsequent binomial processes that are implemented by spatial point spread functions. The mechanisms appear to apply inhomogeneous spatial Poisson point processes. See: "Low Dose Rate Imaging"; http://vixra.org/abs/1606.0329 . Humans are vertebrates and their visual perception at starlight conditions is governed by the described processes.

Physical theories stop at the *wave function* of particles. This exposure of the mechanisms dives deeper and reaches the *characteristic function* of the stochastic process that controls the generation of the landing locations that form the hopping path.

3.21 Self-coherence

It is difficult to belief in a creator that installs separate mechanisms, which ensure the dynamic coherence of the generated modules. It is easier to accept that the relation between the generated location swarms and the field that describes these swarms is based on a mathematically explainable kind of self-coherence. In case of self-coherence, the interaction between the field and the swarm restricts the possible location density distribution. As is indicated earlier, this restriction may be influenced by the number of elements that are contained in the swarm. This fact may explain the existence of *generations* of elementary modules. A larger number of elements increases the inertness of the swarm. However, also the living space field takes a role in the self-coherence of the swarm.

In the relation between the swarm and the field, the Green's function of the field plays an important role. It plays the role of a potential that implements an attracting force. Another factor is the kind of stochastic process that generates the individual locations. This process belongs to the category of the inhomogeneous spatial Poisson point processes. Each hop tries to displace the geometric center of the swarm. This displacement represents an acceleration of the geometric center of the swarm.

Let the Green's function represent a scalar potential. When the platform on which the elementary object resides moves with a uniform speed with respect to the background parameter space, then the scalar potential will in that coordinate system turn into a vector potential. Differential calculus learns that the dynamic change of the vector field goes together with a new field that counteracts the acceleration. This effect is similar to the phenomenon that is known as inertia. It looks as if the center of geometry of the swarm is attracting the accelerating hopping elementary object. This is an effective kind of self-coherence that is installed via the living space field.

In order to elucidate this obscure description, we will try to explain this by applying formulas.

The Green's function $G(q)$ represents a **_scalar potential_**. c is the landing location of the hop. The Green's function is given by:

$$G(q) = m_1/|q - c| \tag{1}$$

If the platform travels with **_uniform speed_** v, then this corresponds with a **_vector potential_** $A(q)$:

$$A(q) = G(q)\, v \tag{2}$$

Acceleration goes together with a **_new vector field_** $E(q)$:

$$E(q) \overset{\text{def}}{=} \dot{A}(q) = G(q)\, \dot{v} \tag{3}$$

This goes together with an attracting **_force_** $F(q)$

$$F(q) = m_2\, E(q) = \frac{m_1 m_2\, \dot{v}}{|q - c|} \tag{4}$$

This attractive force acts between the landing location of the hopping object and the geometric center of the swarm. $\dot{v}$ is the **_acceleration_** of the geometric center of the swarm that is due to the addition of the individual hop. The swarm covers a huge number of landing locations. This is represented by the factor m_2. The added hop landing concerns just a single location. This indicates that with respect to inertia **_the mass of the swarm is proportional to the number of contained hopping locations._**

Later we will see that change (3) involves more terms than are used here, but we consider the other terms negligible.

$$\Phi = \Phi_0 + \boldsymbol{\Phi} = \nabla\, \psi = (\nabla_0 + \boldsymbol{\nabla})(\psi_0 + \boldsymbol{\psi}) = \nabla_0 \psi_0 - \langle \boldsymbol{\nabla}, \boldsymbol{\psi} \rangle + \nabla_0 \boldsymbol{\psi} + \boldsymbol{\nabla}\psi_0 \pm \boldsymbol{\nabla} \times \boldsymbol{\psi} \tag{5}$$

The location density distribution of the swarm is blurred by the Green's function. If the location density distribution has the form of a Gaussian distribution, then the blurred function is the convolution of this location density distribution and the Green's function. The shape of this **_example_** is given by:

$$\chi_n(r) = -\frac{C_n}{4\pi}\frac{ERF\left(r/_{\sigma\sqrt{2}}\right)}{r} \qquad (6)$$

In this function every trace of the singularity of the Green's function has disappeared. It is due to the huge number of participating hop locations. This is just an example. Such extra potentials add a local contribution to the field that acts as the living space of modules and modular systems. The shown extra contribution is due to the local elementary module. The living space is constituted by a myriad of such bumps.

3.22 The symmetry related field

The convolution of the location density distribution of the swarm with the Green's function involves an integration and the local contribution to the integral involves two parameter spaces. One of them is the background parameter space. These parameter spaces may differ in their ordering. In order to cope for this difference the platform on which the elementary object resides must be encapsulated. The integration is an application of the **generalized Stokes theorem**. This theorem converts an integral over a volume into an integral over the boundary of that volume. The boundary must only cross regions of the parameter spaces where the field and the extra potential are both continuous and the amplitude of the extra potential must become negligible. In fact the influences of the ordering characterize the parameter spaces rather than the deformed fields. For the parameter spaces the condition is automatically fulfilled and therefore the shape of the boundary does not matter. For that reason we select a boundary that has the form of a **cube**, whose axes are aligned along the axes of the Cartesian coordinate systems that is used to order the background parameter space. This procedure enables the correct accounting for the differences in the ordering. This accounting process reveals values that we will call **charges** that go together with the difference in ordering. This reveals the short list of electric charges and the color charges that appear in the Standard Model. The charges will be anchored on the geometric centers of the floating platforms. These **symmetry related charges** are the source of a new separate basic field that we will call the **symmetry related field**. This field differs fundamentally from the field that represents the living space of the elementary modules.

The field that is influenced by the clamps and the new symmetry related field are coupled via the geometric centers of the platforms that carry the swarm elements.

3.23 Partial differentiation

In this section we intensively use formulas. These formulas keep the description compact and comprehensive.

We use the quaternionic nabla ∇ in order to provide a compact description of quaternionic partial differentiation.

$$\nabla = \left\{\frac{\partial}{\partial\tau},\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right\} = \frac{\partial}{\partial\tau} + i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} = \nabla_0 + \boldsymbol{\nabla} \qquad (1)$$

$$\nabla f = \sum_{\mu=0}^{3}\frac{\partial f}{\partial q_\mu}e_\mu \qquad (2)$$

This form of the partial differential equation highlights the fact that in first order and second order partial differential equations **_the nabla operator and some of the related differential operators can be applied as a multiplier_**. This means that we can apply the quaternionic multiplication rule. As a consequence these partial differential operators can be used to define corresponding fields and their operators.

$$\Phi_0 = \nabla_0 \psi_0 - \langle \boldsymbol{\nabla}, \boldsymbol{\psi} \rangle \tag{3}$$

$$\boldsymbol{\Phi} = \nabla_0 \boldsymbol{\psi} + \boldsymbol{\nabla}\psi_0 \pm \boldsymbol{\nabla} \times \boldsymbol{\psi} \tag{4}$$

These equations invite the definition of derived vector fields:

$$\mathfrak{E} \stackrel{\text{def}}{=} -\nabla_0 \boldsymbol{\psi} - \boldsymbol{\nabla}\psi_0 \tag{5}$$

$$\mathfrak{B} \stackrel{\text{def}}{=} \boldsymbol{\nabla} \times \boldsymbol{\psi} \tag{6}$$

The $\pm$ sign indicates that the nabla operator is also afflicted by symmetry properties of the applied quaternionic number system. The above equations represent only low order partial differential equations. Thus, these partial differential equations represent approximations rather than precise descriptions of the considered change. In this form the equations can still describe point-like disruptions of the continuity of the field. We can take the conjugate:

$$\Phi^* = (\nabla\psi)^* = \nabla^*\psi^* \mp 2\,\boldsymbol{\nabla} \times \boldsymbol{\psi} \tag{7}$$

$$\nabla^*(\nabla^*\psi^*)^* = \nabla^*\Phi = \nabla^*\nabla\psi \tag{8}$$

Two different non-homogeneous second order partial differential equations exist that offer different views on the embedding process. The equation that is based upon the double quaternionic nabla $\nabla\nabla^*$ cannot show wave behavior. However, the equation that is based on d'Alembert's operator $\mathfrak{D}$ acts as a wave equation, which offers waves as part of its set of solutions. Both second order partial differential operators can be applied as multipliers.

$$\nabla\nabla^* = \nabla_0\nabla_0 + \langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle \tag{9}$$

$$\mathfrak{D} \stackrel{\text{def}}{=} -\nabla_0\nabla_0 + \langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle \tag{10}$$

In isotropic conditions the homogeneous equations look like:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \pm \frac{\partial^2 \psi}{\partial \tau^2} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) \pm \frac{\partial^2 \psi}{\partial \tau^2} = 0 \tag{11}$$

These equations have special solutions in odd numbers of participating dimensions in the form of **_shape keeping fronts_**.

The d'Alembert's equation offers solutions in the form of waves. That does not hold for the double nabla operator that is defined in (9). That equation can be split into two first order partial differential equations:

$$\Phi = \nabla \psi \tag{12}$$

$$\rho = \nabla^* \Phi = \nabla^* \nabla \psi \tag{13}$$

The similarity to Maxwell equations is not accidental. Equation (3) has no equivalent in Maxwell equations. In physics this lack is compensated by special gauge equations.

3.24 Elementary behavior

Fields act on point-like artifacts in an elementary way. The d'Alembert's operator offers plane and spherical waves and both second order equations offer shape keeping fronts as elementary solutions.

3.24.1 Waves

Waves are solutions of the wave equation that uses d'Alembert's operator $\mathfrak{D}$:

$$\mathfrak{D}\, f = \left(-\nabla_0 \nabla_0 + \langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle\right) f = 0 \tag{1}$$

$$\nabla_0 \nabla_0 f = \langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle f = -\omega^2 f \tag{2}$$

For Cartesian symmetry conditions this leads to:

$$f(\tau, \boldsymbol{x}) = a \exp\left(\unicode{x1D7D9}\, \omega(c\tau - |\boldsymbol{x} - \boldsymbol{x}'|)\right);\ c = \pm 1 \tag{3}$$

In spherical symmetric conditions, equation (2) leads to a category of solutions that are known as solutions of the Helmholtz equation. However, here proper time τ replaces coordinate time.

3.24.2 One dimensional fronts

These solutions proceed in one spatial dimension, but they may act in a three dimensional spatial setting. Thus :

$$\frac{\partial^2 \psi}{\partial z^2} \pm \frac{\partial^2 \psi}{\partial \tau^2} = 0 \tag{1}$$

We try a solution in the form $\varphi = f(\alpha z + \beta \tau)$:

$$\frac{\partial f}{\partial z} = \alpha f'; \frac{\partial^2 f}{\partial z} = \alpha \frac{\partial f'}{\partial z} = \alpha^2 f'' \tag{2}$$

$$\frac{\partial f}{\partial \tau} = \beta f'; \frac{\partial^2 f}{\partial \tau^2} = \beta \frac{\partial f'}{\partial \tau} = \beta^2 f'' \tag{3}$$

$$\alpha^2 f'' \pm \beta^2 f'' = 0 \tag{4}$$

This is solved when $\alpha^2 = \mp \beta^2$.

For the first kind of the second order partial differential equation this means: $\beta = \pm \alpha \, \boldsymbol{i}$, where $\boldsymbol{i}$ is a normalized imaginary quaternion. With $g(z) = f(\beta \, z)$ follows:

$$\varphi = g(z \, \boldsymbol{i} \pm \tau) \tag{5}$$

The function g represents a **shape keeping front**. It also **keeps its amplitude**. It is not a wave.

The imaginary $\boldsymbol{i}$ represents the base vector in the x, y plane. Its orientation θ may be a function of z.

For the second kind of the second order partial differential equation this means: $\beta = \pm \alpha$. With $g(z) = f(\beta \, z)$ follows:

$$\varphi = g(z \pm \tau) \tag{6}$$

The corresponding hop landing that represents the continuity disturbing artifact will be called **warp**. *A warp corresponds to the hop of the geometric center of the platform on which the corresponding elementary particle resides*. Thus the location of the hop is defined relative to a geometric location on the carrier field that is defined with respect to the background parameter space. The platform carries its own private parameter space. Subsequent warps occur in "linear" **strings** that follow the deformation of the carrier field. Such strings will be called **messengers**. Warps can also occur as single hops. Warp hops shift the geometric centers of parameter spaces relative to the background parameter space. As solutions of the second order partial differential equation the warp shifts the

map of the geometric center of the platform onto the carrying field to a subsequent location on that carrying field.

Combined in strings the warps can only shift empty platforms. Any clamp in the platform would conflict with the warp speed.

3.24.3 Spherical fronts

Next we focus on the three dimensional spherical symmetric condition. In that case the equations can be separated by writing $\psi = r \, \varphi(r, \tau)$

$$\frac{\partial^2 \varphi}{\partial r^2} + \frac{2}{r}\frac{\partial \varphi}{\partial r} \pm \frac{\partial^2 \varphi}{\partial \tau^2} = 0 \implies \frac{\partial^2 \psi}{\partial r^2} \pm \frac{\partial^2 \psi}{\partial \tau^2} = 0 \tag{1}$$

With other words ψ fulfills the conditions of the one-dimensional case. Thus solutions in the form $\varphi = f(\alpha r + \beta \tau)/r$ will fit.

For the first kind of the second order partial differential equation this means: $\beta = \pm \alpha \, \boldsymbol{i}$, where $\boldsymbol{i}$ is a normalized imaginary quaternion. With $g(x) = f(\beta \, x)$ follows:

$$\varphi = g(r \, \boldsymbol{i} \pm \tau)/r \tag{2}$$

$\boldsymbol{i}$ represents a base vector in radial direction.

The corresponding hop landing that represents the continuity disturbing artifact will be called a **clamp**. The clamp corresponds to a hop relative to the geometric center of the platform on which the elementary particle resides. Thus the location of the hop is defined relative to this geometric location. The description uses the parameter space that is private to the platform and the elementary particle. This description is mapped onto the field that represents the living space of the elementary modules. Clamps occur in coherent swarms. All swarm elements share the same platform. Thus the swarm moves as a single unit.

For the second kind of the second order partial differential equation this means: $\beta = \pm \alpha$. With $g(x) = f(\beta \, x)$ follows:

$$\varphi = g(x \pm \tau)/r \tag{3}$$

These solutions feature a fixed speed and a fixed shape. However, their amplitude diminishes as $1/r$ with distance r from the sources. When integrated over a long enough period of progression the result takes the form of the fields Green's function.

3.24.4 Sets of clamps, sets of warps and regeneration cycles

The homogeneous second order partial differential equation offers two kinds of solutions that represent shape keeping fronts. One kind concerns the spherical shape keeping fronts. The second kind acts in one-dimension and not only keeps its shape, it also keeps its amplitude.

3.24.4.1 Swarms of clamps

Hops that correspond to solutions, which represent spherical shape keeping fronts will be called clamp hops or clamps. The same name is used for this type of solutions of the homogeneous second order differential equations. Clamps occur in swarms and reside on a platform that as long as the elementary module exists is private to that elementary module. During that episode, the symmetry center carries an ordered parameter space. Each elementary module type exists in a set of generations and each of these generations shows a cyclic regeneration period. The swarms have a corresponding number of elements.

3.24.4.2 Strings of warps

Hops that correspond to solutions, which represent one-dimensional shape keeping fronts will be called warp hops or warps. The same name is used for this type of solutions of the homogeneous second order differential equations. Warps correspond to hops of platforms on which elementary modules **may** reside. Warps occur in isolation or equidistant in strings. The platforms that correspond to warp strings have lost their symmetry center. As a consequence these platforms do not own a symmetry related charge. The warps do not deform their carrying field, which is the living space field.

The warp strings feature a **spatial and a temporal frequency**. We postulate that locally and in the same progression period the warp strings will feature a **fixed regeneration cycle** and a **fixed spatial path length** that are **the same** for all warp strings. This makes it possible to distinguish the individual warp strings via their frequency. This frequency determines the **information capacity** of the string. Each string member carries a **unit of information**. The path of the warp string is located inside and with respect to the carrying field. Thus the path follows the deformation of the field. This field represents the living space of the elementary modules.

The homogeneous second order partial differential equation of the carrying field describes the corresponding warp solutions. These solutions feature a fixed speed. The path length determines the **passage duration** of an information messenger. That duration equals the regeneration cycle of the string.

The behavior of the warp strings invites their interpretation as **information messengers**. The path length postulate only holds locally. Taken over huge ranges of the carrying field or over a long period, the path length may vary in a smooth way. This phenomenon is the subject of the equivalent of Hubble's law.

3.25 Messenger redistribution and messenger redirection

Some types of modular (sub)systems, which we will call **atomic modular systems** are capable of **splitting** information messengers into a set of new information messengers. Further they can **absorb** information messengers and **emit** information messengers. During the split action the hops are **redistributed** over the resulting parts, such that each part has again the correct path length. The emission can occur in a direction that is independent of the direction from which the messenger was absorbed. The duration of the absorption processes and the duration of the emission processes must be in concordance with the local passage duration of the information messengers.

If the absorption takes place at a location that is a huge distance away from the emission location, then a **difference between absorption spectra and emission spectra** can occur. In physics this phenomenon is known as **cosmological redshift**.

The absorption and emission processes must obey spectral rules that determine the absorption and emission spectra.

3.26 The symmetric pair production and annihilation process

The pair creation and pair annihilation processes appear to be supported be procedures that construct quaternions from two complex numbers or that dismantle quaternions into complex numbers.

At symmetric pair annihilation a pair of elementary modules that are each other's antiparticle convert into a pair of linear messengers that leave in opposite directions. It is the **simplest pair annihilation process**. In this process each clamp element of the hopping path of the elementary (anti)particle converts into a warp element of a linear hop string of a leaving messenger. The messengers leave in a direction that is perpendicular to the direction into which the elementary modules were approaching each other. The chance that the geometric centers of the elementary modules will meet head-on is very low. The direction of the difference will further determine the direction in which the messengers leave. The process unravels the two clamp swarms into two corresponding linear warp strings. The size of the displacement of the geometric centers is supposed to determine the polarization of the messenger string.

The leaving messengers are strings of warps that transport empty and thus massless platforms. The lengths of the leaving strings reflect the number of clamps in the annihilated elementary modules. The path length of the information messengers determines the **duration** of the annihilation process.

If each warp in the string carries a fixed bit of energy, then this process explains the **mass-energy equivalence**.

During the process the symmetry centers of the platforms are annihilated and as a consequence the **symmetry related charges vanish**.

3.27 Pair creation

At pair creation the reverse process takes place. Two strings of warps that have sufficient length that enter from opposite direction combine in order to generate two swarms of clamps that constitute a particle-antiparticle pair. During the creation process the symmetry centers of the platforms are created. As a consequence the symmetry related charges will emerge.

3.28 Interpreting the pair creation/annihilation process

The creation 'event' and the annihilation 'event' occur in the neighborhood of the vane. These processes are not occurring instantaneously. They take a fixed duration. However, each conversion of a clamp into a warp can take a single instant. In reverse, each conversion of a warp into a clamp can also take a single instant. Similarly the emission and absorption processes of atomic modular systems take the same duration. Thus the surround of the vane is reserved for these processes.

Generations of elementary particles involve different numbers of swarm elements, but if no observable difference exist between the duration of the passage of the involved warp strings, then the active region around the vane can be subdivided in multiple step numbers. These subdivisions correspond to elementary module types and elementary module generations.

3.29 Moving elementary modules

On average, clamp swarms will not move with respect to the geometric center of its platform. The mechanism that ensures coherence of the swarm will ensure that the geometric center of the swarm will on average stay on the geometric center of the platform. The regeneration process can at the utmost generate some **jitter** of the geometric center of the swarm.

Isolated warps and strings of warps may cause the hopping of the platform with respect to the background parameter space. As a consequence the platform hops with respect to the field that represents the living space of the elementary modules. Thus a mixture of clamps and warps may cause the movement of the swarm relative to the geometry of this carrying field.

It is not yet clear what causes the extra insertions of warps, however, a uniform movement of a platform already requires the regular insertion of *isolated warps*. This insinuates that isolated wraps can be generated due to the action of something that generates the displacement. These isolated wraps concern the hop of the platform as well as the hop landing location and the corresponding solution of the second order partial differential equation.

4   The particle zoo

This paper does not touch all members of the elementary particle zoo that is described by the standard model. It does not treat spin and as a consequence the paper does not differentiate between fermions and bosons. The symmetry flavors distinguish between electrons, quarks and neutrinos. The paper does not treat the difference between particles and anti-particles and we did not really discuss generations. We did not even touch the very important Pauli principle and we did not treat gluons and color confinement in sufficient detail. Binding of elementary modules into higher order modules is not treated. The targeted Hilbert Book Model does not need the existence of a separate Higgs field.

5   More detail

Those that possess sufficient knowledge of mathematics might be interested in the paper "The Hilbert Book Test Model". This pure mathematical model exploits the above view and extends it a bit. It introduces and describes new mathematical methods. See: http://vixra.org/abs/1603.0021 . This e-book is targeted to become a kind of self-consistent textbook. It will contain all related subjects that are not easily found in literature.

The text in this "Thou Shalt Construct in a Modular Way" is more up to date than the text in "The Hilbert Book Test Model";. In a later phase the updated text will be merged with the older text.