Inscribed & Circumscribed – September 22, 2014
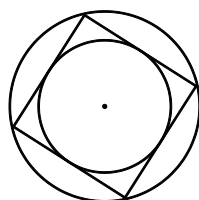
Problems & Solutions

The figure shows a square inscribed in a circle with radius 2 cm. What is the ratio of the area of the circle to the area of the square? Express your answer as a common fraction in terms of π.
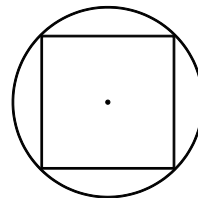
*The area of a circle with radius 2 cm is $π × 2^2 = 4π$ cm². Each diagonal of the square is a diameter of the circle. Therefore, the length of the square's diagonal is 4 cm, and we can use the properties of 45-45-90 right triangles to determine the square's side length s. We have $s\sqrt{2} = 4 \to s = 4/\sqrt{2} × \sqrt{2}/\sqrt{2} = 4\sqrt{2}/2 = 2\sqrt{2}$ cm. The area of a square with side length $2\sqrt{2}$ cm is $(2\sqrt{2})^2 = 8$ cm². The ratio of the areas of the circle and the square, then, is $4π/8 = $ **π/2**.*

In this figure, a square is inscribed in a circle of radius 12 in, and another circle is inscribed in that square. What is the ratio of the area of the smaller circle to the area of the larger circle? Express your answer as a common fraction.

*We are told the larger circle has radius 12 inches, so its area is $π × 12^2 = 144π$ in². The diameter of the larger circle has the same length as a diagonal of the square, and a side of the square has the same length as a diameter of the smaller circle. Using the properties of 45-45-90 right triangles, since a diagonal of the square has length 12 inches, we have $s\sqrt{2} = 12 \to s = 12/\sqrt{2} × \sqrt{2}/\sqrt{2} = 12\sqrt{2}/2 = 6\sqrt{2}$ inches. This also is the length of a diameter of the smaller circle. So the smaller circle has area $π × (6\sqrt{2})^2 = 72π$ in². Therefore, the ratio of the areas of the smaller and larger circles is $72π/144π = $ **1/2**.*

This figure shows a circle inscribed in a rhombus with diagonals of length 6 cm and 8 cm. What is the ratio of the area of the circle to the area of the rhombus? Express your answer as a common fraction in terms of π.
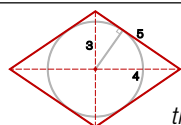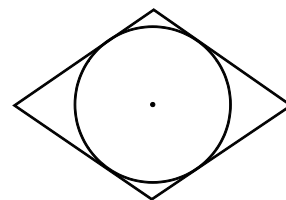
As shown, the diagonals of the rhombus create four 3-4-5 right triangles. Each triangle has area $(1/2) × 3 × 4 = 6$ cm². Thus, the rhombus has area $6 × 4 = 24$ cm². The radius drawn from the center of the circle perpendicular to a side of the rhombus is an altitude of one of these right triangles. We write the following equation for the area of the right triangle: $(1/2) × 5 × r = 6$.
*Solving for r, we see the circle has radius $r = 6 × 2 ÷ 5 = 12/5$ cm. Therefore, the circle has area $π × (12/5)^2 = 144π/25$ cm², and the ratio of the areas of the circle and the rhombus is $(144π/25)/24 = $ **6π/25**.*