sible, to supply equivalent treatments, in such cases, which are independent of this notion; not necessarily to replace the older theorems, but to afford new theorems which are frequently of more value in practice. Moreover, it is generally difficult to define order of contact in a purely geometric way, especially projectively. Consequently the following theorem is not without interest: *on any non-ruled surface S there exist three one-parameter families of curves, the members of each family of which are union curves of a congruence* Γ' *and at the same time adjoint union curves of the reciprocal congruence* Γ. These are the curves of Segre,[6] and the curves of Darboux may be characterized as composing the three families conjugate to them.

The generalization of surfaces of Voss described above is only one of the many instances in which an important metric property of a configuration is really a particular case of a far more general projective one. I have found a number of others, connected not only with the subject of geodesics but also with apparently unrelated concepts; there appears throughout a unifying feature, however, in the notion of reciprocal congruences.

[1] Green, G. M., these PROCEEDINGS, **3**, 1917, (587–592).

[2] Wilczynski, E. J., *Trans. Amer. Math. Soc., New York,* **9**, 1908, (79–120). Cf. top of p. 83.

[3] Sperry, P., *Amer. J. Math., Baltimore,* **40**, 1918, (213–224).

[4] Green, G. M., *loc. cit.*, pp. 590–591. The congruence referred to is that generated by the lines $y\zeta$.

[5] Green, G. M., *Amer. J. Math., Baltimore,* **38**, 1916, (287–324).

[6] Segre, C., *Ac. dei Lincei, Rend., Rome,* (Ser. 5), **17**$_2$, 1908, (409–411).

[7] Darboux, G., *Bull. Sci. Math., Paris,* (Ser. 2), **4**, 1880, p. 356.

THE RECTANGULAR INTERFEROMETER WITH ACHROMATIC DISPLACEMENT FRINGES IN CONNECTION WITH THE HORIZONTAL PENDULUM

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Communicated, September 16, 1918

1. Introduction.—In the *Reports of the Carnegie Institute of Washington,* 1915, No. 229, Chap. I, part 2, pp. 30 et seq., I adduced a method for the application of the displacement interferometer to the horizontal pendulum with a graphic exhibit of the results obtained during a series of months. The concave mirror design by which the spectrum interference ellipses were made available showed a very satisfactory performance. The attainable accuracy was such that for moderate constants in the installation of the pendulum, an inclination of 3×10^{-4} seconds of arc should have been registered per vanishing interference fringe (ellipse), or about 10^{-3} seconds per 10^{-4} cm. of displacement of the micrometer. The inclination of the line of suspending pivots was here about 1° to the vertical. A smaller angle would have correspondingly increased the sensitiveness.

The apparatus, however, required a space about two meters long between the extreme mirrors, for its installation. This is a disadvantage, since small changes of temperature in the brackets and supports as well as in the pier would interfere with the full realization of the precision of the method. The rectangular interferometer with an auxiliary mirror is thus to be preferred; for here all the necessary parts may easily be placed within a distance of one foot from the wall of the pier carrying the horizontal pendulum. If the achromatic fringes are used, these are straight and intense, so that photographic methods are available.

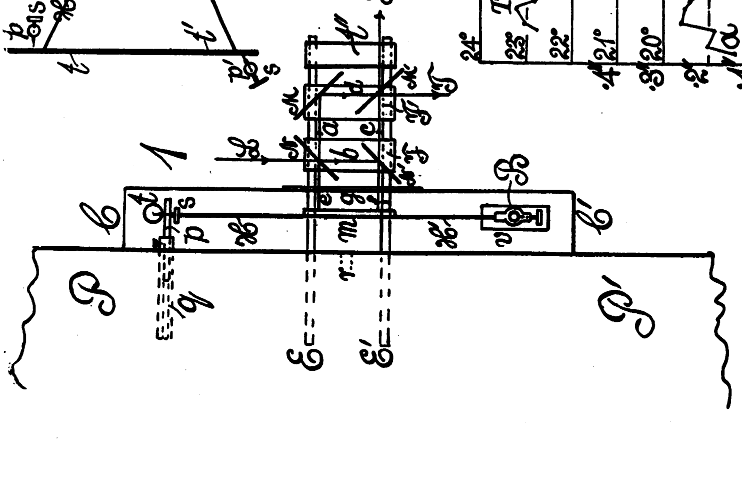
2. *Apparatus.*—The old horizontal pendulum made of thin steel tubing formerly described (l. c.) was again used.

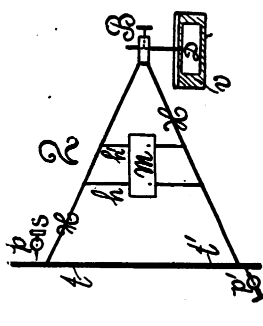
Figure 1 gives a sectional plan of the pendulum installation, and figure 2 a front view of the pendulum HH alone, in a somewhat reduced scale. Its general shape is that of an isosceles triangle and the distance from the line of pivots tt' to apex about 110 cm., while the distance between pivots was 97 cm. These pivots S, S' are fine screws ending in hard steel points, which enter a glasshard steel socket (below) and a steel groove (above). The line tt' of the horizontal pendulum can thus be given any inclination to the vertical, while the rods p, p' which receive the screws S, S' may be moved normally to the wall of the pier PP', inward or outward, and clamped to secure parallelism between the pier PP' and pendulum HH'. The apex B of the pendulum is also provided with a clamp, holding vane D submerged in an oil vat, v, for damping.

The whole pendulum is enclosed by a flat case, CC', of tin plate provided with a plate glass window at g, through which the auxiliary mirror m of the interferometer may be seen. This is attached to one or two vertical tubes hh' of the pendulum, adjustably, so that it can be moved up or down, and rotated slightly about a vertical and a horizontal axis.

The interferometer consists essentially of the 4 plate glass mirrors, M, M', N, N', all but M being half silvers, the collimator (beyond L) and the telescope at T or T', t'' being a telescope support. The collimated white beam L is thus separated into the component rays L, e, m, e, a, d, T, and L, b, f, m, f, c, T, to be observed at either T or T'. M' is on a micrometer slide (not shown) with the screw normal to the face of the mirror. All mirrors should be capable of slight rotation about horizontal and vertical axes and the silvered faces all lie towards m for compensation of glass paths. The rays leaving M' for T must not only be accurately parallel but locally (visible as spots of light) nearly coincident; otherwise the fringes will be weak or invisible.

The telescope T is provided with an ocular micrometer (centimeter divided in tenth millimeters), standardized by aid of the sliding micrometer at M'. Moreover the image of the wide slit of the collimator adapted to the use of


1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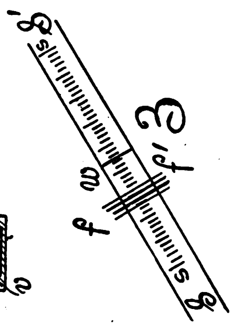
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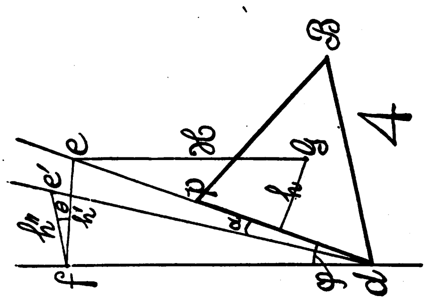
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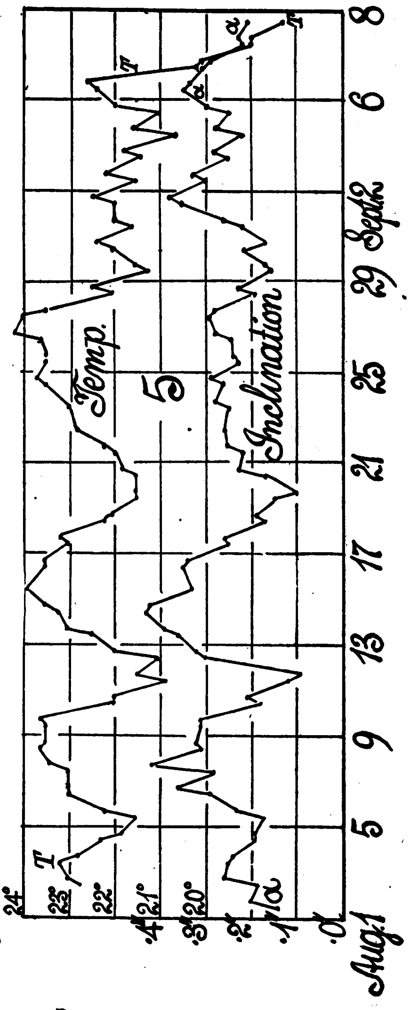
5
Temp.
Inclination
Aug.1
Sept.2
5
9
13
17
21
25
29
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8
24°
23°
22°
21°
20°
.4
.3
.2
.1
.0

the achromatic fringes, should be placed at right angles to them, with the ocular micrometer so placed as to read from end to end of the slit image. A very fine wire beam across the slit gives the fiducial line relative to the ocular micrometer. Figure 3 shows the general arrangement, SS' being the oblique wide slit image, ff the achromatic fringes, w the image of the fiducial wire across the slit, and ss' the ocular scale. Of course the fringes may be made horizontal or vertical; but this requires much adjustment (or else compensators) and is therefore an unnecessary complication of the work. With this fiducial mark at the collimator which is permanently out of reach, if the telescope is accidentally shifted, or it is temporarily removed, it may be replaced without difficulty. It is the telescope, however, which contains the ultimately fiducial scale, and it like the collimator should be held on a standard, t'', suitably attached to the interferometer and the pier. Similarly, the mirrors M and M', N and N' fixed in pairs to slides or carriages F, F' are clamped to two parallel horizontal tubes E, E' (3/8 inch gas pipe smoothed, for instance) anchored in the pier. The highest attainable rigidity in the placement of the mirrors M, M', N, N', and of the telescope is essential. At the outset of the work the viscous yielding of standards and braces is quite apparent.

3. *Equations.*—In figure 4 let pBd denote the horizontal pendulum in the plane of the diagram and dpe the line of pivots prolonged, terminating in e vertically above the center of gravity G. Let the inclination of de to the vertical be φ, a constant of the apparatus, and suppose a perpendicular h' is let fall from e to the vertical df through d. If in consequence of a change in the inclination of the pier the line of pivots passes to de', over a nearly vertical angle α, h' will pass into h'' over a horizontal angle θ. Thus the measurement consists in finding α in terms of the interferometer angle θ. Since these angles are all very small we may write (Δ being a differential symbol) as in the preceding paper.

$$\Delta\alpha = \varphi\Delta\theta \tag{1}$$

In the rectangular interferometer with an auxiliary mirror, if the distance apart of the rays a and c, Fig. 1, be $2R$

$$4R\Delta\theta = 2\Delta N \cos i = n\lambda \tag{2}$$

where $i = 45°$, ΔN the displacement of micrometer to bring the achromatic fringes back to the fiducial line and n the number of fringes which pass that line. Hence

$$\Delta\alpha = \varphi\frac{\Delta N \cos i}{2R} = \varphi\frac{n\lambda}{4R} \tag{3}$$

The smallest angle $\Delta\alpha$ which can thus be measured depends essentially on $2R$ the breadth of the ray parallelogram. There would be no difficulty in making this as long as the line from tt' to B of the horizontal pendulum, i.e. over a meter;

but this would necessitate two mirrors m one at each end. For the present purposes, I preferred to use apparatus which I had at hand in which $2R$ was but 10 cm. and a single mirror could be used at m. Nevertheless if $n = 1$, the limit of angle measurable, if $\varphi = 0.0175$ or one degree is $\Delta\alpha = 5 \times 10^{-8}$ radian per fringe; i.e. about 0.01 second of arc. With an ocular micrometer and well produced achromatic fringes there is no difficulty in estimating 1/10 fringe, so that the limiting angle here is to be a few thousandths of a second, even if $\varphi = 1°$, which may also be reduced. By making $2R = 100$ cm. one should therefore be able to reach 0.0001 second per tenth fringe breadth, if φ is 1°.

Finally on using the ocular micrometer for moderately sized fringes of say one scale part (0.1 millimeter fringes in the ocular) the case is equally promising. A comparison of the two displacements ΔN at M' and Δe of the fringes in the ocular, showed $\Delta e/\Delta N = 265$. Hence

$$\Delta\alpha = \varphi\Delta e \cos i/265 \times 2R$$

If $\Delta e/\Delta N = 10^{-2}$ (one scale part) and $2R = 10$ cm. etc., as above,

$$\Delta\alpha = 2.7 \times 10^{-8}$$

or 0.005″ per scale part of the ocular micrometer. A few tenths of this may be estimated on the scale. The sensitiveness will be ten times greater if $2R$ is a meter.

4. *Observations.*—The interferometer was installed with rather smaller fringes than instanced above and therefore with less sensitiveness, as the inclination of the pier in a heated laboratory would probably run into seconds of arc in the lapse of time. For this reason R was also satisfactory at its small value of $2R = 10$ cm. The angle φ was directly measureable, as the inclination of the line joining the points of the pivots to the plumb line. Under these circumstances $\Delta\alpha = 0.9\ \Delta e$ seconds, roughly, the tenth millimeters of the ocular scale are about 1/100 second of arc in relation to $\Delta\alpha$ and the fringes were of about the same size. There would have been no difficulty in making them much larger and therefore more sensitive as they were clear and strong. The end of the compound pendulum was damped in lubricating oil.

The earlier observations were discarded; but even after January 14 (after which time the apparatus worked smoothly), there are instances of displacement within the apparatus requiring readjustment. These betray themselves in a lack of coincidence of the two wide slit images, Fig. 3, or of the cross wire w (the slit is really superfluous and the collimator lens may be so placed as to widen the illuminated field).

Though observations were made continuously since January, I can here only give the graphs for the days of August and September, just passed. In figure 5, the lower curve shows the apparent change of inclination of the pier in seconds of arc; the upper curve the corresponding temperature (in degrees centigrade) of the basement laboratory room. Observations were usually made at 10 a.m. and 6 p.m. Between August 1 and 21, the resemblance of the two curves is marked. After that the gross resemblance is no longer so striking, but it nevertheless remains in the details and there is a mere lack of equivalence, quantitatively. The question therefore arises where this temperature discrepancy has its seat. It can hardly be in the interferometer, where the parts are of the same metal except in the (virtually) tetrahedral bracket, consisting of the rods, E, E', and the brace downward from t'' to r in the pier. For here one part $t''r$ is of iron and the other from EE' downward to r, of brick. There might thus be differential expansion; but the interferometer would not be sensitive to this motion, of which I convinced myself by bearing down hard at t'' with the hands. No adequate displacement of fringes resulted. Hence it appears probable that what is observed is the warping of the pier, etc., as a result of the inward progress of the successive isotherms through it, beginning at the parts least protected against changes of temperature by the surrounding house walls. At all events this temperature feature is so serious that a few tenths of a degree centigrade can not be overlooked.