

Math 150 Lecture Notes

Double-Angle, Half-Angle, and Product-Sum Formulas

Double-Angle Formulas

$$\sin 2x = 2 \sin x \cos x$$

$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - 2 \sin^2 x \\ &= 2 \cos^2 x - 1\end{aligned}$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Half-Angle Formulas

$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\tan \frac{x}{2} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$$

Determination of + or – sign depends on
quadrant of $\frac{x}{2}$

Lowering Powers

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

Product-to-Sum Formulas

$$\sin u \cos v = \frac{1}{2} [\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2} [\sin(u + v) - \sin(u - v)]$$

$$\cos u \cos v = \frac{1}{2} [\cos(u + v) + \cos(u - v)]$$

$$\sin u \sin v = \frac{1}{2} [\cos(u - v) - \cos(u + v)]$$

Sum-to-Product Formulas

$$\sin x + \sin y = 2 \sin \frac{x + y}{2} \cos \frac{x - y}{2}$$

$$\sin x - \sin y = 2 \cos \frac{x + y}{2} \sin \frac{x - y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x + y}{2} \cos \frac{x - y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x + y}{2} \sin \frac{x - y}{2}$$

Example 1: Find $\sin 2x$, $\cos 2x$, and $\tan 2x$ given that $\tan x = -\frac{4}{3}$ and x is in quadrant IV.

Example 2: Use the formulas for lowering powers to rewrite the expression in terms of the first power of cosine:
 $\cos^4 x \sin^2 x$

Example 3: Use an appropriate half-angle formula to find the exact value of the expression.

$$\tan \frac{5\pi}{12}$$

Example 4: Simplify the expression by using a double-angle formula or a half-angle formula.
 $\cos^2 5\alpha - \sin^2 5\alpha$