CHAPTER II

DIFFRACTION BY A DISK

The problem of diffraction by a disk has a rigorous solution [24-26]; however, it is not suitable for numerical calculations in the quasi-optical region when the dimensions of the disk are large in comparison with the wavelength. The physical optics approach used in such cases sometimes gives erroneous results. In particular, the fringing field calculated in this approach does not satisfy the reciprocity principle.

In this Chapter a refinement of the physical optics approach is carried out. First the diffraction of a plane electromagnetic wave by a disk with normal incidence (§ 7-9) is investigated, and then (§ 10-12) diffraction by a disk with oblique incidence of a plane electromagnetic wave is investigated.

Normal Irradiation

§ 7. The Physical Optics Approach

Let an ideally conducting, infinitely thin disk of radius a (Figure 17) be irradiated by plane wave

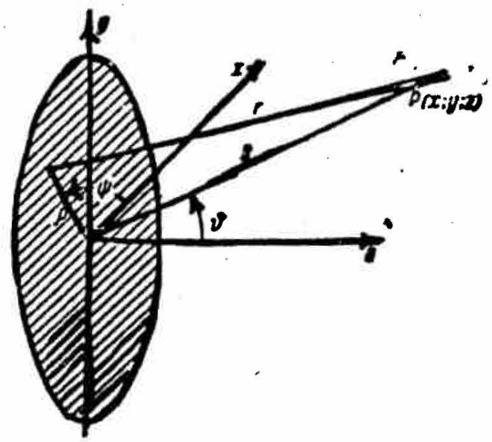


Figure 17. Diffraction by a disk of a plane wave propagated along the z axis.

$$\left.\begin{array}{l} E_y = -H_x = -H_{0x} e^{ikz}, \\ E_x = H_y = 0. \end{array}\right\} \qquad (7.01)$$

The uniform part of the current excited on the disk by wave (7.01) is determined by Equation (3.01) and has the components

$$j_y^0 = -\frac{c}{2\pi} H_{0x}, \quad j_x^0 = j_z^0 = 0. \qquad (7.02)$$

Let us find the field created by this current.

Since the diffraction field in the far zone ($R \gg ka^2$) is of interest to us, the vector potential

$$\mathbf{A}(x, y, z) = \frac{1}{c} \int_0^a \rho d\rho \int_0^{2\pi} \mathbf{j}(\rho, \psi) \frac{e^{ikr}}{r} d\psi \qquad (7.03)$$

may be simplified by using the relationship

$$r = \sqrt{R^2 + \rho^2 - 2\rho R \cos \Omega} \approx R - \rho \cos \Omega, \qquad (7.04)$$

where Ω is the angle between ρ and R, and

$$\cos \Omega = \sin \vartheta \cos (\psi - \varphi). \qquad (7.05)$$

As a result, we obtain the simpler equation

$$\mathbf{A}(x, y, z) = \frac{1}{c} \frac{e^{ikR}}{R} \int_0^a \rho d\rho \int_0^{2\pi} \mathbf{j}(\rho, \psi) e^{-ik\rho \cos \Omega} d\psi. \qquad (7.06)$$

Continuing by using the equations

$$\mathbf{H} = \operatorname{rot} \mathbf{A}, \quad \operatorname{rot} \mathbf{H} = -ik\mathbf{E}, \qquad (7.07)$$

it is easy to show that in the spherical coordinate system the fringing field components with $R \gg ka^2$ equal

$$\left.\begin{aligned} E_\vartheta &= H_\varphi = ikA_\vartheta, \\ E_\varphi &= -H_\vartheta = ikA_\varphi, \\ E_R &= H_R = 0, \end{aligned}\right\} \tag{7.08}$$

where

$$\left.\begin{aligned} A_\varphi &= A_y\cos\varphi - A_x\sin\varphi, \\ A_\vartheta &= (A_x\cos\varphi + A_y\sin\varphi)\cos\vartheta - A_z\sin\vartheta. \end{aligned}\right\} \tag{7.09}$$

Substituting here the values

$$\left.\begin{aligned} A_y &= -H_{0x}\cdot\frac{a}{k\sin\vartheta}J_1(ka\sin\vartheta)\frac{e^{ikR}}{R}, \\ A_x &= A_z = 0, \end{aligned}\right\} \tag{7.10}$$

which result from Equations (7.02) and (7.06), let us find the field radiated by the uniform part of the current in the form

$$\left.\begin{aligned} E_\vartheta &= H_\varphi = -iaH_{0x}\cdot\frac{\sin\varphi}{\sin\vartheta}\cos\vartheta J_1(ka\sin\vartheta)\frac{e^{ikR}}{R}, \\ E_\varphi &= -H_\vartheta = -iaH_{0x}\cdot\frac{\cos\varphi}{\sin\vartheta}J_1(ka\sin\vartheta)\frac{e^{ikR}}{R}. \end{aligned}\right\} \tag{7.11}$$

The function $J_1(ka\sin\vartheta)$ is a first order Bessel function. By using its asymptotic expression

$$J_1(ka\sin\vartheta) = \sqrt{\frac{2}{\pi ka\sin\vartheta}}\cos\left(ka\sin\vartheta - \frac{3\pi}{4}\right), \tag{7.12}$$

which is applicable when $ka\sin\vartheta \gg 1$, one is able to rewrite Equations (7.11) in the following form:

$$\left.\begin{aligned} E_\vartheta &= H_\varphi = -iH_{0x}\sqrt{\frac{a}{2\pi k\sin\vartheta}}\cos\vartheta\frac{\sin\varphi}{\sin\vartheta}\times \\ &\times\left[e^{-i\left(ka\sin\vartheta-\frac{3\pi}{4}\right)} + e^{i\left(ka\sin\vartheta-\frac{3\pi}{4}\right)}\right]\frac{e^{ikR}}{R}, \\ E_\varphi &= -H_\vartheta = -iH_{0x}\sqrt{\frac{a}{2\pi k\sin\vartheta}}\frac{\cos\varphi}{\sin\vartheta}\times \\ &\times\left[e^{-i\left(ka\sin\vartheta-\frac{3\pi}{4}\right)} + e^{i\left(ka\sin\vartheta-\frac{3\pi}{4}\right)}\right]\frac{e^{ikR}}{R}. \end{aligned}\right\} \tag{7.13}$$

The resulting equations show that in the region $R \gg ka^2$, $ka \sin\vartheta \gg 1$ the fringing field may be investigated as the sum of the spherical waves from two "luminous" points on the rim of the disk, the polar angles of which respectively equal $\psi = \phi$ and $\psi = \pi + \phi$. It is not difficult to see that these waves satisfy the Fermi principle. Actually, of all the points on the disk's surface, the point $\rho = a$, $\psi = \phi$ is the closest to the observation point (R, ϑ, φ), and the point $\rho = a$, $\psi = \pi + \phi$ is the furthest from it.

However, Equations (7.13) describe the radiation not only from the two "luminous" points, but they determine the field radiated by the entire "luminous" region which is adjacent to the line connecting the points $\rho = a$, $\psi = \phi$ and $\rho = a$, $\psi = \pi + \phi$.

Let us show that the luminous region actually makes the main contribution to the fringing field. For this purpose, let us calculate the field radiated by the currents which flow inside the sector encompassing the line $\psi = \phi$ (Figure 18). Let us take the angular dimensions of the sector in such a way that its arc, which equals $2a\Phi_0$, would occupy the first Fresnel zone. When this is done, the angle Φ_0 will satisfy the equation

$$a(1 - \cos\Phi_0)\sin\vartheta = \frac{\lambda}{4}. \qquad (7.14)$$

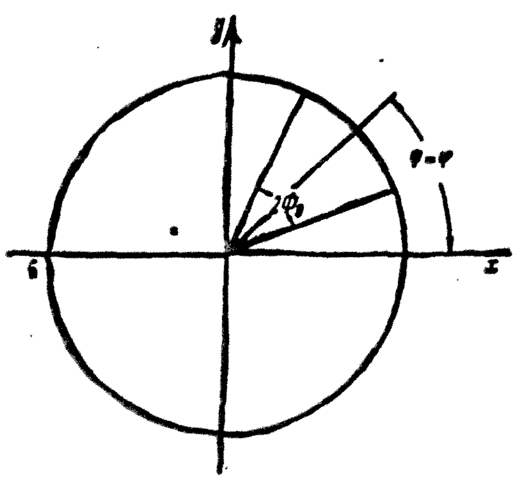


Figure 18. Calculation of the field radiated by the "luminous" region of the disk.

In the case being investigated by us, when the condition $ka \sin\vartheta \gg 1$ is fulfilled, we have from Equation (7.14)

$$\cos\Phi_0 = 1 - \frac{\lambda}{4a\sin\vartheta} \approx 1 - \frac{\Phi_0^2}{2}, \qquad (7.15)$$

hence

$$\Phi_0 = \sqrt{\frac{\pi}{ka\sin\vartheta}}. \qquad (7.16)$$

The vector potential of the currents flowing in the indicated sector is determined by the equation

$$\left.\begin{aligned} A_y &= -\frac{1}{2\pi} H_{0x}\cdot\frac{e^{ikR}}{R}\int_{-\Phi_0}^{\Phi_0} d\psi \int_0^a e^{-ik\rho\sin\theta\cos\psi}\rho\, d\rho, \\ A_x &= A_z = 0. \end{aligned}\right\} \quad (7.17)$$

Taking into account the condition ka sin $\theta \gg 1$, one may show that the field created by the currents of this sector will equal

$$\left.\begin{aligned} E_\theta = H_\varphi \sim H_{0x}\sqrt{\frac{a}{\pi k\sin\theta}}\cos\theta \times \\ \times\frac{\sin\varphi}{\sin\theta}\frac{e^{ikR}}{R}e^{-ika\sin\theta+i\frac{\pi}{5}}+0\left(\frac{1}{\sqrt{ka\sin\theta}}\right), \\ E_\varphi = -H_\theta \sim H_{0x}\sqrt{\frac{a}{\pi k\sin\theta}}\times \\ \times\frac{\cos\varphi}{\sin\theta}\frac{e^{ikR}}{R}e^{-ika\sin\theta+i\frac{\pi}{5}}+0\left(\frac{1}{\sqrt{ka\sin\theta}}\right) \end{aligned}\right\} \quad (7.18)$$

The amplitude of the expressions which have been found is approximately $\sqrt{2}$ times larger than the amplitude of the first terms in Equation (7.13). Moreover, expressions (7.18) and the corresponding terms in Equation (7.13) differ slightly in their phases: the first have the factor $e^{i\frac{\pi}{5}}$, and the latter — the factor $-e^{i\frac{\pi}{4}}$. The result obtained is similar to the well-known thesis in optics that the effect of a wave is equal to the effect of half of the first Fresnel zone (see, for example [27], p. 132).

In the vicinity of the directions $\theta=0$ and $\theta=\pi$, when the azimuthal components lose their meaning, for the purpose of studying the fringing field it is more convenient to use the Cartesian components

$$\left.\begin{aligned} E_x &= (E_\theta\cos\theta + E_R\sin\theta)\cos\varphi - E_\varphi\sin\varphi, \\ E_y &= (E_\theta\cos\theta + E_R\sin\theta)\sin\varphi + E_\varphi\cos\varphi. \end{aligned}\right\} \quad (7.19)$$

Turning to Equations (7.11), we find that when $\theta=0$ and $\theta=\pi$

$$E_x=0, \quad E_y=-iH_{0x}\cdot\frac{ka^2}{2}\cdot\frac{e^{ikR}}{R}. \quad (7.20)$$

Consequently, in the physical optics approach the field scattered in the directions $\vartheta = 0$ and $\vartheta = \pi$ preserves the polarization of the incident wave.

§ 8. The Field From the Uniform Part of the Current

Let us proceed to calculation of the field created by the nonuniform part of the current with normal irradiation of the disk. Since the latter is concentrated mainly in the vicinity of the disk's edge ($\rho = a$), the vector potential corresponding to it will equal, in accordance with Equation (7.06),

$$A = \frac{a}{c}\frac{e^{ikR}}{R}\cdot\int_0^a d\rho\int_0^{2\pi} j^1(\rho, \psi)\, e^{-ik\rho\sin\vartheta\cos(\psi-\varphi)}\, d\psi. \tag{8.01}$$

The inner integral is calculated with $ka\sin\vartheta \gg 1$ based on the stationary phase method (see, for example [21], p. 256), and Equation (8.01) is transformed to the form

$$A(x, y, z) = \frac{a}{c}\sqrt{\frac{2\pi}{ka\sin\vartheta}}\,\frac{e^{ikR}}{R}\,e^{i\frac{\pi}{4}}\times$$
$$\times\left[\int_0^a j^1(\rho, \psi_1)\, e^{-ik\rho\sin\vartheta}\, d\rho - i\int_0^a j^1(\rho, \psi_2)\, e^{ik\rho\sin\vartheta}\, d\rho\right], \tag{8.02}$$

which allows one to interpret the fringing field as the field from a luminous line on the disk. This line is a diameter, the polar angle ψ of the points on which equals

$$\psi_1 = \varphi \text{ and } \psi_2 = \pi + \varphi. \tag{8.03}$$

Assuming the diameter of the disk is sufficiently large in comparison with the wavelength ($ka \gg 1$), one may approximately assume that the nonuniform part of the current near the disk's edge will be the same as on the corresponding half-plane (Figure 19). On the basis of § 4, the field from the nonuniform part of the current flowing on the half-plane $-\infty \leq y_1 \leq a$ may be represented in the form

$$
\left.\begin{aligned}
E_{x_1}(1) = ikA_{x_1}(1) = E_{0x_1}\cdot f^1(1)\frac{e^{i\left(kR+\frac{\pi}{4}\right)}}{\sqrt{2\pi kR}}\, e^{-ika\sin\vartheta},\\
H_{x_1}(1) = -ikA_{y_1}(1)\cos\vartheta = \\
= H_{0x_1}\cdot g^1(1)\,\frac{e^{i\left(kR+\frac{\pi}{4}\right)}}{\sqrt{2\pi kR}}\, e^{-ika\sin\vartheta},
\end{aligned}\right\} \tag{8.04}
$$

and similarly the field from the current flowing on the half-plane $-a<y_1<\infty$, may be represented in the form

$$
\left.\begin{aligned}
E_{x_1}(2) = ikA_{x_1}(2) = E_{0x_1}\cdot f^1(2)\frac{e^{i\left(kR+\frac{\pi}{4}\right)}}{\sqrt{2\pi kR}}\, e^{ika\sin\vartheta},\\
H_{x_1}(2) = -ikA_{y_1}(2)\cos\vartheta = H_{0x_1}\cdot g^1(2)\,\frac{e^{i\left(kR+\frac{\pi}{4}\right)}}{\sqrt{2\pi kR}}\, e^{ika\sin\vartheta}.
\end{aligned}\right\} \tag{8.05}
$$

Here

$$
\left.\begin{aligned}
\mathrm{A}(1) = \frac{1}{c}\sqrt{\frac{2\pi}{kR}}\, e^{i\left(kR+\frac{\pi}{4}\right)}\int_{-\infty}^{a} \mathrm{j}^1(\eta)\, e^{-ik\eta_1\sin\vartheta}\, d\eta,\\
\mathrm{A}(2) = \frac{1}{c}\sqrt{\frac{2\pi}{kR}}\, e^{i\left(kR+\frac{\pi}{4}\right)}\int_{-\infty}^{a} \mathrm{j}^1(\eta)\, e^{ik\eta_1\sin\vartheta}\, d\eta,
\end{aligned}\right\} \tag{8.06}
$$

and the functions f^1 and g^1 are determined for the right half-space $\left(0\leqslant\vartheta\leqslant\frac{\pi}{2}\right)$ by the equations

$$
\left.\begin{aligned}
f^1(1) = f(1)+\frac{1}{\sin\vartheta},\quad f(1) = -\frac{\cos\frac{\vartheta}{2}+\sin\frac{\vartheta}{2}}{\sin\vartheta},\\
f^1(2) = f(2)-\frac{1}{\sin\vartheta},\quad f(2) = \frac{\cos\frac{\vartheta}{2}-\sin\frac{\vartheta}{2}}{\sin\vartheta};
\end{aligned}\right\} \tag{8.07}
$$

$$
\left.\begin{aligned}
g^1(1) = g(1)+\frac{\cos\vartheta}{\sin\vartheta},\quad g(1) = -\frac{\cos\frac{\vartheta}{2}-\sin\frac{\vartheta}{2}}{\sin\vartheta},\\
g^1(2) = g(2)-\frac{\cos\vartheta}{\sin\vartheta},\quad g(2) = \frac{\cos\frac{\vartheta}{2}+\sin\frac{\vartheta}{2}}{\sin\vartheta}.
\end{aligned}\right\} \tag{8.08}
$$

From relationships (8.04) - (8.06), it follows that

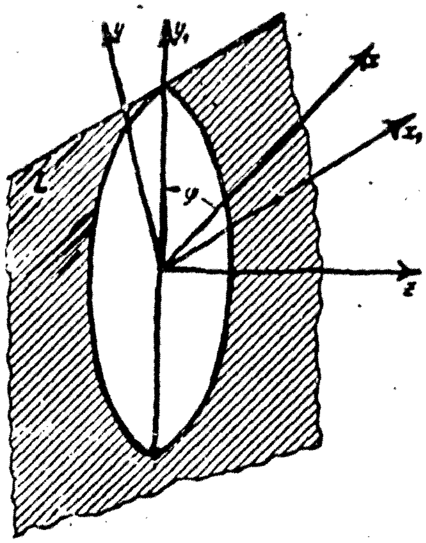


Figure 19. Diffraction by a disk. The half-plane L lies in the plane of the disk. The edge of the half-plane is tangent to the circumference of the disk at the point $y_1 = a$, $x_1 = 0$ (a is the radius of the disk).

$$\left.\begin{aligned}\int_{-\infty}^{a} j^1_{x_1}(\eta)\, e^{-ik\eta\sin\vartheta}\, d\eta &= -\frac{ic}{2\pi k} E_{0x_1}\cdot f^1(1)\, e^{-ika\sin\vartheta},\\ \int_{-\infty}^{a} j^1_{x_1}(\eta)\, e^{ik\eta\sin\vartheta}\, d\eta &= -\frac{ic}{2\pi k} E_{0x_1}\cdot f^1(2)\, e^{ika\sin\vartheta}\end{aligned}\right\} \quad (8.09)$$

and

$$\left.\begin{aligned}\int_{-\infty}^{a} j^1_{y_1}(\eta)\, e^{-ik\eta\sin\vartheta}\, d\eta &= \frac{ic}{2\pi k\cos\vartheta} H_{0x_1}\cdot g^1(1)\, e^{-ika\sin\vartheta},\\ \int_{-\infty}^{a} j^1_{y_1}(\eta)\, e^{ik\eta\sin\vartheta}\, d\eta &= \frac{ic}{2\pi k\cos\vartheta} H_{0x_1}\cdot g^1(2)\, e^{ika\sin\vartheta}.\end{aligned}\right\} \quad (8.10)$$

In accordance with the assumption of equal currents on the disk and on the half-plane, one may consider the following equalities to be valid:

$$\left.\begin{aligned}\int_{0}^{a} \mathbf{j}^1(\rho, \psi_1)\, e^{-ik\rho\sin\vartheta}\, d\rho &= \int_{-\infty}^{a} \mathbf{j}^1(\eta)\, e^{-ik\eta\sin\vartheta}\, d\eta,\\ \int_{0}^{a} \mathbf{j}^1(\rho, \psi_2)\, e^{ik\rho\sin\vartheta}\, d\rho &= \int_{-\infty}^{a} \mathbf{j}^1(\eta)\, e^{ik\eta\sin\vartheta}\, d\eta.\end{aligned}\right\} \quad (8.11)$$

Therefore, the field from the nonuniform part of the current flowing on the disk will equal

$$\left.\begin{aligned}E_\varphi = -H_\vartheta &= \frac{iaE_{0\varphi}}{\sqrt{2\pi ka\sin\vartheta}}\left[f^1(2)\, e^{i\left(ka\sin\vartheta-\frac{3\pi}{4}\right)} - f^1(1)\, e^{-i\left(ka\sin\vartheta-\frac{3\pi}{4}\right)}\right]\frac{e^{ikR}}{R},\\ E_\vartheta = H_\varphi &= \frac{iaH_{0\varphi}}{\sqrt{2\pi ka\sin\vartheta}}\left[g^1(2)\, e^{i\left(ka\sin\vartheta-\frac{3\pi}{4}\right)} - g^1(1)\, e^{-i\left(ka\sin\vartheta-\frac{3\pi}{4}\right)}\right]\frac{e^{ikR}}{R},\end{aligned}\right\} \quad (8.12)$$

where in view of (7.01)

$$E_{0\varphi} = -H_{0x}\cos\varphi, \quad H_{0\varphi} = -H_{0x}\sin\varphi. \quad (8.13)$$

For the direction $\vartheta = 0$, we have according to Equation (8.01)

$$\mathbf{A} = \frac{a}{c} \cdot \frac{e^{ikR}}{R} \cdot \int_0^{2\pi} d\psi \int_0^a \mathbf{j}^1(\rho, \psi)\, d\rho, \tag{8.14}$$

but in accordance with equalities (8.09) - (8.11)

$$\left.\begin{aligned} \int_0^a j^1_{x_1}(\rho, \psi)\, d\rho &= \frac{ic}{4\pi k} H_{0x} \cdot \cos\psi, \\ \int_0^a j^1_{y_1}(\rho, \psi)\, d\rho &= \frac{ic}{4\pi k} H_{0x} \sin\psi. \end{aligned}\right\} \tag{8.15}$$

Consequently,

$$\left.\begin{aligned} A_x &= \frac{a}{c} \frac{e^{ikR}}{R} \cdot \left[\int_0^{2\pi} \cos\psi\, d\psi \int_0^a j^1_{y_1}(\rho, \psi)\, d\rho + \right. \\ &\quad \left. + \int_0^{2\pi} \sin\psi\, d\psi \int_0^a j^1_{x_1}(\rho, \psi)\, d\rho \right] = 0, \\ A_y &= \frac{a}{c} \frac{e^{ikR}}{R} \cdot \left[-\int_0^{2\pi} \cos\psi\, d\psi \int_0^a j^1_{x_1}(\rho, \psi)\, d\rho + \right. \\ &\quad \left. + \int_0^{2\pi} \sin\psi\, d\psi \int_0^a j^1_{y_1}(\rho, \psi)\, d\rho \right] = 0, \end{aligned}\right\} \tag{8.16}$$

that is, in the direction of the main fringe $(\vartheta = 0)$ the field from the nonuniform part of the current equals zero.

By using the Bessel functions J_1 and J_2 for the field from the nonuniform part of the current, one may write the equations

$$\left.\begin{aligned} E_\varphi &= -H_\vartheta = \frac{iaE_{0z}}{2} \{[f^1(2) - f^1(1)] J_1(ka \sin\vartheta) + \\ &\quad + i[f^1(2) + f^1(1)] J_2(ka \sin\vartheta)\} \frac{e^{ikR}}{R}, \\ E_\vartheta &= H_\varphi = \frac{iaH_{0z}}{2} \{[g^1(2) - g^1(1)] J_1(ka \sin\vartheta) + \\ &\quad + i[g^1(2) + g^1(1)] J_2(ka \sin\vartheta)\} \frac{e^{ikR}}{R}, \end{aligned}\right\} \tag{8.17}$$

which with $ka \sin\vartheta \gg 1$ change to Expressions (8.12), which were already found. In the direction $\vartheta = 0$, these equations give a field which equals, in accordance with (8.16), zero, and with intermediate values they are interpolated. Since the transition from Equations (8.12) to Equations (8.17) is not completely unique, in the angular interval $0 < \vartheta \leq \frac{1}{ka}$ Equations (8.17) may give a certain error. This error is not very significant, since in this interval the field from the uniform part of the current is large.

§ 9. The Total Field Being Scattered by a Disk with Normal Irradiation

Turning to Equations (8.07), (8.08) and (8.17), let us represent the field from the nonuniform part of the current in the following form:

$$\left.\begin{aligned} E_\varphi = -H_\vartheta = iaH_{0x}\frac{\cos\varphi}{\sin\vartheta}J_1(ka\sin\vartheta)\frac{e^{ikR}}{R} - \\ -\frac{iaH_{0x}}{2}\{[f(2)-f(1)]J_1(ka\sin\vartheta)+ \\ +i[f(2)+f(1)]J_2(ka\sin\vartheta)\}\cos\varphi\frac{e^{ikR}}{R}, \\ E_\vartheta = H_\varphi = iaH_{0x}\frac{\sin\varphi}{\sin\vartheta}\cos\vartheta J_1(ka\sin\vartheta)\frac{e^{ikR}}{R} - \\ -\frac{iaH_{0x}}{2}\{[g(2)-g(1)]J_1(ka\sin\vartheta)+ \\ +i[g(2)+g(1)]J_2(ka\sin\vartheta)\}\sin\varphi\frac{e^{ikR}}{R}. \end{aligned}\right\} \qquad (9.01)$$

Here the first terms, as is readily apparent, represent the field from the uniform part of the current taken with the opposite sign. As a result, the total field scattered by the disk (that is, the sum of the fields radiated by the uniform and nonuniform parts of the current) will be expressed only in terms of the functions f and g which determine in the rigorous solution the cylindrical wave from the half-plane's edge

$$\left.\begin{aligned} E_\varphi = -H_\vartheta = -\frac{iaH_{0x}}{2}\cos\varphi\{[f(2)- \\ -f(1)]J_1(ka\sin\vartheta)+i[f(2)+ \\ +f(1)]J_2(ka\sin\vartheta)\}\frac{e^{ikR}}{R}, \\ E_\vartheta = H_\varphi = -\frac{iaH_{0x}}{2}\sin\varphi\{[g(2)- \\ -g(1)]J_1(ka\sin\vartheta)+i[g(2)+ \\ +g(1)]J_2(ka\sin\vartheta)\}\frac{e^{ikR}}{R}. \end{aligned}\right\} \qquad (9.02)$$

Substituting here the explicit expressions for the functions f and g, we arrive at the final expressions for the fringing field

$$\left.\begin{aligned} E_\varphi = -H_\vartheta = -\frac{iaH_{0x}}{2}\left[\frac{J_1(ka\sin\vartheta)}{\sin\frac{\vartheta}{2}} - \right. \\ \left. - i\frac{J_2(ka\sin\vartheta)}{\cos\frac{\vartheta}{2}}\right]\frac{e^{ikR}}{R}\cos\varphi, \\ E_\vartheta = H_\varphi = -\frac{iaH_{0x}}{2}\left[\frac{J_1(ka\sin\vartheta)}{\sin\frac{\vartheta}{2}} + \right. \\ \left. + i\frac{J_2(ka\sin\vartheta)}{\cos\frac{\vartheta}{2}}\right]\frac{e^{ikR}}{R}\sin\varphi. \end{aligned}\right\} \tag{9.03}$$

These equations are valid in the right half-space $\left(0\leqslant\vartheta<\frac{\pi}{2}\right)$. In the left half-space $\left(\frac{\pi}{2}<\vartheta\leqslant\pi\right)$, the fringing field is easily found by assuming that its electric field is an even function, and its magnetic field an odd function of the z coordinate:

$$\left.\begin{aligned} E_\varphi(z) = E_\varphi(-z), \\ H_\varphi(z) = -H_\varphi(-z). \end{aligned}\right\} \tag{9.04}$$

Consequently, in the region z < 0 (that is, when $\frac{\pi}{2}<\vartheta\leqslant\pi$)

$$\left.\begin{aligned} E_\varphi = -H_\vartheta = -\frac{iaH_{0x}}{2}\left[\frac{J_1(ka\sin\vartheta)}{\cos\frac{\vartheta}{2}} - \right. \\ \left. - i\frac{J_2(ka\sin\vartheta)}{\sin\frac{\vartheta}{2}}\right]\frac{e^{ikR}}{R}\cos\varphi, \\ E_\vartheta = H_\varphi = \frac{iaH_{0x}}{2}\left[\frac{J_1(ka\sin\vartheta)}{\cos\frac{\vartheta}{2}} + \right. \\ \left. + i\frac{J_2(ka\sin\vartheta)}{\sin\frac{\vartheta}{2}}\right]\frac{e^{ikR}}{R}\sin\varphi. \end{aligned}\right\} \tag{9.05}$$

Assuming that in Equations (9.03) and (9.05) $\vartheta=0$ and $\vartheta=\pi$, respectively, we obtain

$$E_y = -\frac{ika^2}{2}\cdot H_{0x}\cdot\frac{e^{ikR}}{R},\quad E_x=0, \tag{9.06}$$

which is equivalent to the physical optics approach [see Equation (7.20)].

Expressions (9.03) and (9.05) agree with the result obtained by Braunbek [29] for the scalar fringing field in the far zone. It is also interesting to compare these expressions with the precise numerical results obtained by Belkina [34] by the separation of variables method in the spheroidal coordinate system. It turns out that even with ka = 5 a satisfactory agreement is observed between our approximation method and the rigorous theory. In Figures 20 and 21, graphs of the functions $V^{(1)}(\vartheta)$ and $V^{(2)}(\vartheta)$, are presented which allow one to calculate the fringing field on the basis of the equations

$$\left.\begin{aligned} E_{\varphi}=-H_{\vartheta}=\frac{ika^2}{2}E_{0y}\cdot V^{(1)}(\vartheta)\frac{e^{ikR}}{R}\cos\varphi, \\ E_{\vartheta}=H_{\varphi}=\frac{ika^2}{2}E_{0y}\cdot V^{(2)}(\vartheta)\frac{e^{ikR}}{R}\sin\varphi. \end{aligned}\right\} \qquad (9.07)$$

The continuous curve corresponds to the rigorous theory [34]. The dash-dot curve corresponds to the field from the uniform part of the current, and the dashed curve corresponds to the field calculated according to Equation (9.03) and (9.05).

Oblique Irradiation

§ 10. The Physical Optics Approach

Let us investigate the general case when the plane wave

$$\mathbf{E}=\mathbf{E}_0 e^{ik(y\sin\gamma+z\cos\gamma)} \qquad (10.01)$$

falls on the disk at an arbitrary angle to its axis. Let us take the spherical coordinate system in such a way that the normal to the incident wave front, n, would lie in the half-plane $\varphi=\frac{\pi}{2}$ and form an angle γ $\left(0<\gamma<\frac{\pi}{2}\right)$ with the z axis (Figure 22). Adhering to the investigation procedure used in the previous sections, let us first calculate the fringing field in the physical optics approach.

The uniform part of the current excited on the disk by wave (10.01) is determined by Equation (3.01) and has the components

$$j_x^0=\frac{c}{2\pi}H_{0y}\cdot e^{iky\sin\gamma},\quad j_y^0=-\frac{c}{2\pi}H_{0x}e^{iky\sin\gamma},\quad j_z^0=0. \qquad (10.02)$$

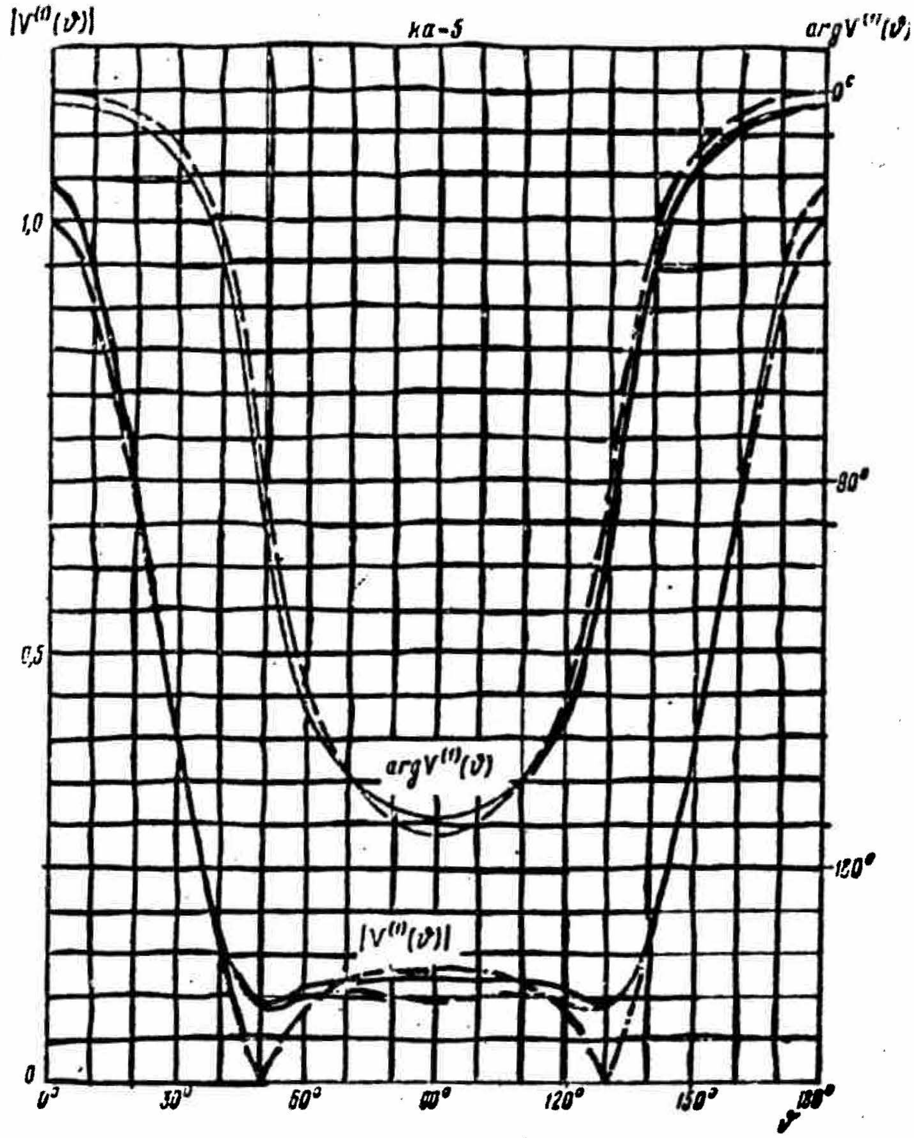


Figure 20. The function $V^{(1)}(\vartheta)$ for a disk with a normal incidence of the wave. The various curves correspond to different approximations.

The field radiated by it is found, as was done in § 7, by integrating (with the condition $R \gg ka^2$). In the case of E-polarization of the incident wave ($\mathbf{E}_0 \perp yoz$), this field equals

$$\left.\begin{aligned} E_\vartheta &= H_\varphi = iaE_{0x}\cdot\cos\gamma\cos\vartheta\cos\varphi\,\frac{J_1(ka\sqrt{\lambda^2+\mu^2})}{\sqrt{\lambda^2+\mu^2}}\,\frac{e^{ikR}}{R},\\ E_\varphi &= -H_\vartheta = -iaE_{0x}\cdot\cos\gamma\sin\varphi\,\frac{J_1(ka\sqrt{\lambda^2+\mu^2})}{\sqrt{\lambda^2+\mu^2}}\,\frac{e^{ikR}}{R}, \end{aligned}\right\} \quad (10.03)$$

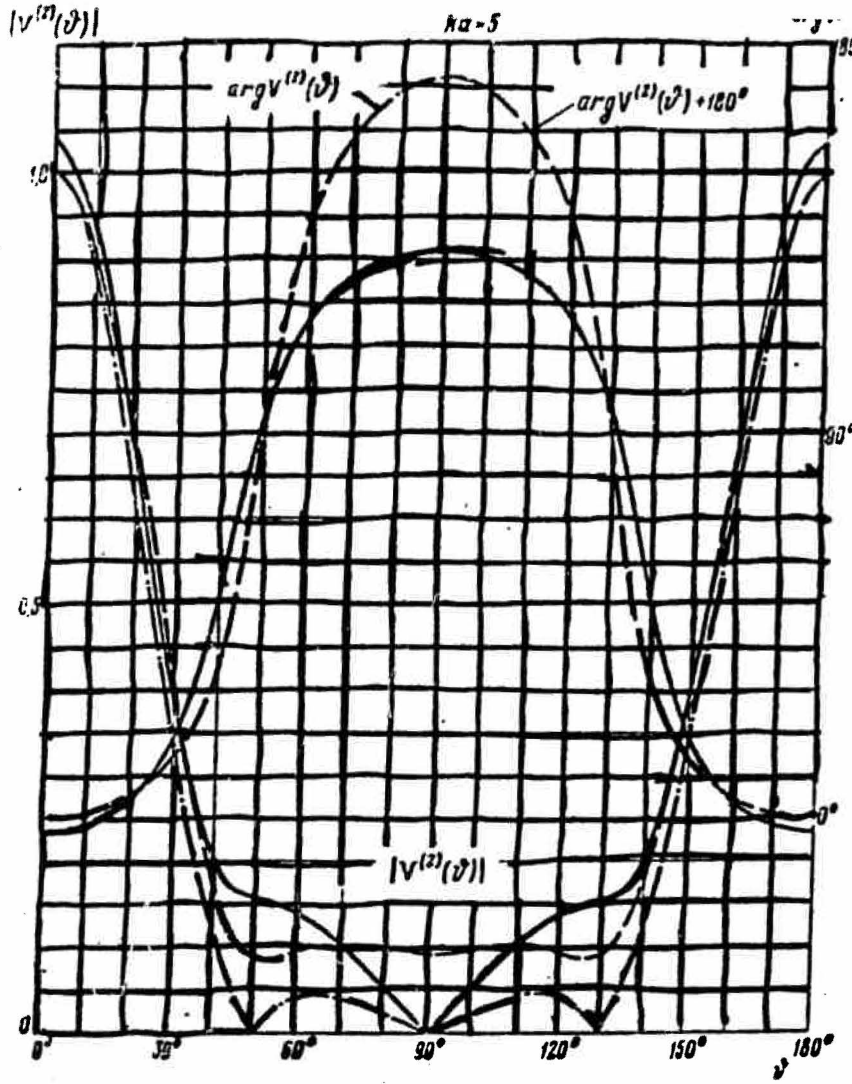


Figure 21. The function $V^{(2)}(\vartheta)$ for a disk with normal incidence of a plane wave. The various curves correspond to different approximations.

and in the case of H-polarization ($\mathbf{H}_0 \perp yoz$)

$$\left.\begin{aligned} E_\vartheta &= H_\varphi = -iaH_{0x}\cos\vartheta\sin\varphi\,\frac{J_1(ka\sqrt{\lambda^2+\mu^2})}{\sqrt{\lambda^2+\mu^2}}\,\frac{e^{ikR}}{R},\\ E_\varphi &= -H_\vartheta = -iaH_{0x}\cos\varphi\,\frac{J_1(ka\sqrt{\lambda^2+\mu^2})}{\sqrt{\lambda^2+\mu^2}}\,\frac{e^{ikR}}{R}. \end{aligned}\right\} \tag{10.04}$$

The quantities λ and μ in Equations (10.03) and (10.04) are determined in the following way:

$$\left.\begin{aligned} \lambda &= \sin\vartheta\cos\varphi,\\ \mu &= \sin\vartheta\sin\varphi - \sin\gamma,\\ &\sqrt{\lambda^2+\mu^2} \geqslant 0. \end{aligned}\right\} \tag{10.05}$$

Assuming $\varphi = -\frac{\pi}{2}$ and $\vartheta = \pi - \gamma \left(\frac{\pi}{2} < \vartheta < \pi\right)$, in the resulting expressions, let us find the field scattered by the disk in the direction toward its source. With E-polarization of the incident wave, it equals

$$\left.\begin{aligned} E_\varphi &= -H_\vartheta = -\frac{iaE_{0x}}{2}\frac{\cos\vartheta}{\sin\vartheta}J_1(2ka\sin\vartheta)\frac{e^{ikR}}{R}, \\ E_\vartheta &= H_\varphi = 0. \end{aligned}\right\} \tag{10.06}$$

and with H-polarization

$$\left.\begin{aligned} E_\vartheta &= H_\varphi = \frac{iaH_{0x}}{2}\frac{\cos\vartheta}{\sin\vartheta}J_1(2ka\sin\vartheta)\frac{e^{ikR}}{R}, \\ E_\varphi &= H_\vartheta = 0. \end{aligned}\right\} \tag{10.07}$$

Using the asymptotic expressions for the Bessel functions, one is able to show that when $R \gg ka^2$ and $ka\sqrt{\lambda^2+\mu^2} \gg 1$ the fringing field is radiated from a luminous region on the disk. In the case when $\sqrt{\lambda^2+\mu^2} \to 0$, the luminous region is increased and in the limit (when $\lambda = \mu = 0$) the entire surface of the disk starts to "shine".

§ 11. The Field Radiated by the Nonuniform Part of the Current

Let us calculate the field in the nonuniform part of the current

$$\mathbf{j}^1(\rho, \psi) = \mathbf{J}(\rho, \psi)\, e^{ik\rho\sin\gamma\sin\psi}. \tag{11.01}$$

Its corresponding vector potential

$$\mathbf{A} = \frac{a}{c}\frac{e^{ikR}}{R}\int_0^a d\rho \int_0^{2\pi} \mathbf{J}(\rho, \psi)\, e^{-ik\rho\sqrt{\lambda^2+\mu^2}\cos(\psi-\delta)}\, d\psi \tag{11.02}$$

by means of the stationary phase method is transformed with $ka\sqrt{\lambda^2+\mu^2} \gg 1$ to the form

$$\mathbf{A} = \frac{a}{c}\sqrt{\frac{2\pi}{ka\sqrt{\lambda^2+\mu^2}}}\, e^{i\frac{\pi}{4}}\left[\int_0^a \mathbf{J}(\rho, \psi_1)\, e^{-ik\rho\sqrt{\lambda^2+\mu^2}}\, d\rho - \right.$$
$$\left. - i\int_0^a \mathbf{J}(\rho, \psi_2)\, e^{ik\rho\sqrt{\lambda^2+\mu^2}}\, d\rho\right]\frac{e^{ikR}}{R}. \tag{11.03}$$

Here

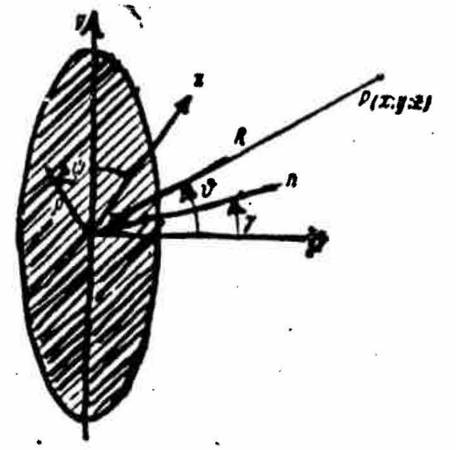


Figure 22. The oblique incidence of a plane wave on a disk. n is the normal to the incident wave front.

$$\psi_1 = \delta, \quad \psi_2 = \pi + \delta \qquad (11.04)$$

are the stationary phase points and the quantity δ is determined by the equalities

$$\sin\delta = \frac{\mu}{\sqrt{\lambda^2+\mu^2}}, \quad \cos\delta = \frac{\lambda}{\sqrt{\lambda^2+\mu^2}}. \qquad (11.05)$$

From Equation (11.03) it follows that with $R \gg ka^2$ and $ka\sqrt{\lambda^2+\mu^2} \gg 1$ the main contribution to the fringing field is given by the luminous region adjacent to the line $\psi = \psi_1$, $\psi = \psi_2$. Thus, the stationary phase points ψ_1, ψ_2 physically correspond to the luminous line on the disk surface.

In order to calculate the vector potential (11.03), it is necessary for us to first express the nonuniform part of the current on the half-plane in terms of its field in the far zone. For this purpose, let us introduce the auxiliary coordinate systems x_1, y_1 and x_2, y_2 (see Figure 23), and let us take the following designations:

α_1, β_1(α_2, β_2) are the angles between the normal to the incident wave front and the coordinate axes x_1, y_1 (x_2, y_2);

$\phi_1^0(\phi_1^0 = -\phi_2^0)$ is the angle between the z axis and the projection of the indicated normal on the plane $x_1 = 0$;

$\phi_1(\phi_1 = -\phi_2)$ is the angle between the z axis and the direction from the coordinate origin to the point $p(y_1, z)$ which lies in the plane $x_1 = 0$ and is the projection of the observation point $P(x, y, z)$;

r_1 is the distance from the origin to the point $p(y_1, z)$.

The quantities introduced here are determined by the equations:

$$\left.\begin{aligned}
&\cos\alpha_1 = -\sin\gamma\cos\psi_1, \quad \cos\beta_1 = \sin\gamma\sin\psi_1, \\
&\alpha_2 = \pi - \alpha_1, \qquad \beta_2 = \pi - \beta_1, \\
&\sin\varphi_1^0 = \frac{\cos\beta_1}{\sin\alpha_1}, \quad \cos\varphi_1^0 = \frac{\cos\gamma}{\sin\alpha_1}, \\
&\sin\varphi_1 = \frac{\sin\vartheta\cos(\psi_1-\varphi)}{\sqrt{1-\sin^2\vartheta\sin^2(\psi_1-\varphi)}}, \\
&\cos\varphi_1 = \frac{\cos\vartheta}{\sqrt{1-\sin^2\vartheta\sin^2(\psi_1-\varphi)}}, \\
&\varphi_2 = -\varphi_1, \quad \varphi_2^0 = -\varphi_1^0, \\
&r_1 = R\sqrt{1-\sin^2\vartheta\sin^2(\psi_1-\varphi)}.
\end{aligned}\right\} \tag{11.06}$$

Furthermore, let us write the expressions for the field from the nonuniform part of the current excited by wave (10.01) on an ideally conducting half-plane $-\infty \leqslant y_1 \leqslant a$. In accordance with § 5, they have the form

$$\begin{aligned}
E_{x_1} &= e^{ikx_1\cos\alpha_1} E_{0x_1} f^1(\varphi_1, \varphi_1^0) \frac{e^{i\left(k_1 r_1 + \frac{\pi}{4}\right)}}{\sqrt{2\pi k_1 r_1}} e^{ik_1 a(\sin\varphi_1^0 - \sin\varphi_1)}, \\
H_{x_1} &= e^{ikx_1\cos\alpha_1} H_{0x_1} g^1(\varphi_1, \varphi_1^0) \frac{e^{i\left(k_1 r_1 + \frac{\pi}{4}\right)}}{\sqrt{2\pi k_1 r_1}} e^{ik_1 a(\sin\varphi_1^0 - \sin\varphi_1)},
\end{aligned} \tag{11.07}$$

where

$$\left.\begin{aligned}
&k_1 = k\sin\alpha_1, \\
&k_1(\sin\varphi_1 - \sin\varphi_1^0) = k\sqrt{\lambda^2+\mu^2},
\end{aligned}\right\} \tag{11.08}$$

$$\left.\begin{aligned}
f^1(\varphi_1, \varphi_1^0) &= \frac{\sin\frac{\varphi_1-\varphi_1^0}{2} + \cos\frac{\varphi_1+\varphi_1^0}{2}}{\sin\varphi_1^0 - \sin\varphi_1} - \frac{\cos\varphi_1^0}{\sin\varphi_1^0 - \sin\varphi_1}, \\
g^1(\varphi_1, \varphi_1^0) &= \frac{-\sin\frac{\varphi_1-\varphi_1^0}{2} + \cos\frac{\varphi_1+\varphi_1^0}{2}}{\sin\varphi_1^0 - \sin\varphi_1} - \frac{\cos\varphi_1}{\sin\varphi_1^0 - \sin\varphi_1},
\end{aligned}\right\} \tag{11.09}$$

$$\left(-\frac{\pi}{2} < \varphi_1 < \frac{3\pi}{2}\right).$$

On the other hand, this field may be expressed in terms of the vector potential

$$\mathbf{A} = \frac{1}{c}\int_{-\infty}^{a} \mathbf{J}(\eta)\, e^{ik\eta\cos\beta_1} d\eta \int_{-\infty}^{\infty} e^{ik\xi\cos\alpha_1} \frac{e^{ik\sqrt{z^2+(y_1-\eta)^2+(x_1-\xi)^2}}}{\sqrt{z^2+(y_1-\eta)^2+(x_1-\xi)^2}} d\xi. \tag{11.10}$$

By means of equation

$$H_0^{(1)}(qD) = \frac{1}{i\pi}\int_{-\infty}^{\infty} \frac{e^{ip\sqrt{D^2+\zeta^2}}}{\sqrt{D^2+\zeta^2}} e^{-it\zeta} d\zeta, \tag{11.11}$$
$$q = \sqrt{p^2 - t^2}, \quad \operatorname{Im} q > 0, \quad D > 0,$$

which follows from Equation (3.10), if one substitutes z = t, w = ζ, d = -ip, k = -iD, in it we find that

$$A = \frac{i\pi}{c} e^{ikx_1\cos\alpha_1} \int_{-\infty}^{a} J(\eta) H_0^{(1)}[k\sqrt{z^2+(y_1-\eta)^2}]\, e^{ik\eta\cos\beta_1} d\eta. \tag{11.12}$$

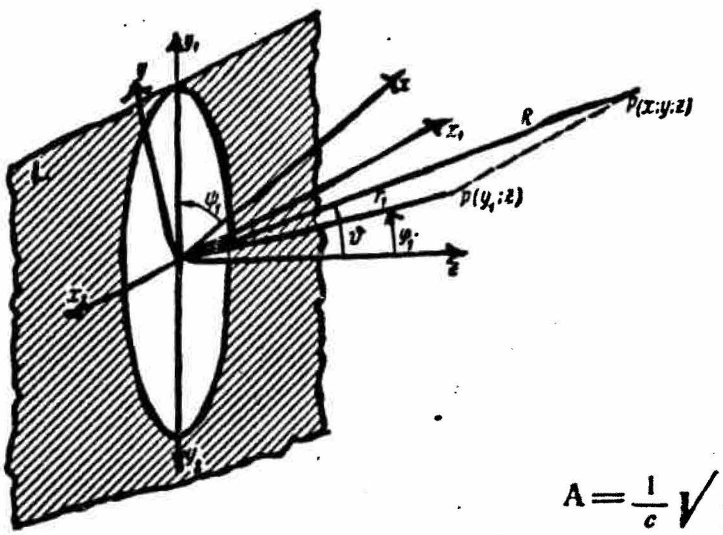


Figure 23. Diffraction by a disk with oblique incidence of a plane wave. The half-plane L lies in the plane of the disk. Its edge is tangent to the circumference of the disk at the point $x_1 = 0$, $y_1 = a$ (a is the radius of the disk).

Taking the fact into account that the nonuniform part of the current is concentrated mainly in the vicinity of the half-plane edge and using the asymptotic representation of the Hankel function, we obtain

$$A = \frac{1}{c}\sqrt{\frac{2\pi}{k_1 r_1}}\, e^{ik(x_1\cos\alpha_1 + r_1\sin\alpha_1) + i\frac{\pi}{4}} \int_{-\infty}^{a} J(\eta)\, e^{ik\eta\Phi_1} d\eta, \tag{11.13}$$

where
$$\Phi_1 = \cos\beta_1 - \sin\alpha_1\sin\varphi_1 =$$
$$= \sin\gamma\sin\psi_1 - \sin\vartheta\cos(\psi_1 - \varphi)$$
$$\sqrt{\frac{1-\sin^2\gamma\cos^2\psi_1}{1-\sin^2\vartheta\sin^2(\psi_1-\varphi)}}. \tag{11.14}$$

In the case when $\psi_1 = \delta$ [see Equations (11.04) and (11.05)] the function Φ_1 takes the value

$$\Phi_1 = -\sqrt{\lambda^2 + \mu^2}. \tag{11.15}$$

Starting from expression (11.13), it is not difficult to show that the fringing field in the far zone is described by the following equations:

$$E_{x_2} = -ik \sin\alpha_1 \cos\alpha_1 \sin\varphi_1 A_{y_1} + ik \sin^2\alpha_1 A_{x_1}, \quad H_{x_2} = -ik \sin\alpha_1 \cos\varphi_1 A_{y_1}, \tag{11.16}$$

where

$$A_{y_1} = \theta_1 I_{y_1}(\psi_1), \quad A_{x_1} = \theta_1 I_{x_1}(\psi_1), \quad \theta_1 = \frac{1}{c}\sqrt{\frac{2\pi}{k_1 r_1}}\, e^{ik(x_1\cos\alpha_1 + r_1\sin\alpha_1) + i\frac{\pi}{4}}, \tag{11.17}$$

$$I_{y_1}(\psi_1) = \int_{-\infty}^{a} J_{y_1}(\eta)\, e^{-ik\eta\sqrt{\lambda^2+\mu^2}}\, d\eta, \quad I_{x_1}(\psi_1) = \int_{-\infty}^{a} J_{x_1}(\eta)\, e^{-ik\eta\sqrt{\lambda^2+\mu^2}}\, d\eta. \tag{11.18}$$

Then by equating expressions (11.07) and (11.16), we find the desired connection between the nonuniform part of the current on the half-plane $-\infty \leqslant y_1 \leqslant a$ and its field in the far zone

$$\begin{aligned} I_{y_1}(\psi_1) &= -\frac{c}{ik2\pi}\frac{H_{0x_1}}{\sin\alpha_1\cos\varphi_1} g^1(\varphi_1, \varphi_1^0)\, e^{-ika\sqrt{\lambda^2+\mu^2}}, \\ I_{x_1}(\psi_1) &= \frac{c}{ik2\pi}\frac{1}{\sin^2\alpha_1}[E_{0x_1} f^1(\varphi_1, \varphi_1^0) - \\ &\quad -\cos\alpha_1 \operatorname{tg}\varphi_1 H_{0x_1} g^1(\varphi_1, \varphi_1^0)]\, e^{-ika\sqrt{\lambda^2+\mu^2}}. \end{aligned} \tag{11.19}$$

One may show in a completely similar way that the nonuniform part of the current excited by wave (10.01) on the half-plane $-\infty \leqslant y_2 \leqslant a$ creates, in the far zone, the fringing field

$$E_{x_2} = -ik \sin\alpha_1 \cos\alpha_1 \sin\varphi_1 A_{y_2} + ik \sin^2\alpha_1 A_{x_2}, \quad H_{x_2} = -ik \sin\alpha_1 \cos\varphi_1 A_{y_2}, \tag{11.20}$$

where

$$A_{y_2} = \theta_2 I_{y_2}(\psi_2), \quad A_{x_2} = \theta_2 I_{x_2}(\psi_2), \quad \theta_2 = \frac{1}{c}\sqrt{\frac{2\pi}{k_1 r_1}}\, e^{ik(r_1\sin\alpha_1 - x_1\cos\alpha_1) + i\frac{\pi}{4}}, \tag{11.21}$$

$$\left.\begin{aligned} I_{y_2}(\psi_2) &= \int\limits_{-\infty}^{a} J_{y_2}(\eta)\, e^{ik\eta\sqrt{\lambda^2+\mu^2}}\, d\eta, \\ I_{x_2}(\psi_2) &= \int\limits_{-\infty}^{a} J_{x_2}(\eta)\, e^{ik\eta\sqrt{\lambda^2+\mu^2}}\, d\eta. \end{aligned}\right\} \tag{11.22}$$

On the other hand, in accordance with § 5 this field equals

$$\left.\begin{aligned} E_{x_2} &= e^{-ikx_2\cos\alpha_1} E_{0x_2} f^1(\varphi_2, \varphi_2^0)\, \frac{e^{i\left(k_1 r_1 + \frac{\pi}{4}\right)}}{\sqrt{2\pi k_1 r_1}}\, e^{ik_1 a(\sin\varphi_2^0 - \sin\varphi_2)}, \\ H_{x_2} &= e^{-ikx_2\cos\alpha_1} H_{0x_2} g^1(\varphi_2, \varphi_2^0)\, \frac{e^{i\left(k_1 r_1 + \frac{\pi}{4}\right)}}{\sqrt{2\pi k_1 r_1}}\, e^{ik_1 a(\sin\varphi_2^0 - \sin\varphi_2)}. \end{aligned}\right\} \tag{11.23}$$

Here

$$k_1(\sin\varphi_2 - \sin\varphi_2^0) = -\sqrt{\lambda^2+\mu^2}, \tag{11.24}$$

and the functions $f^1(\varphi_2, \varphi_2^0)$ and $g^1(\varphi_2, \varphi_2^0)$ are determined by the equations:

$$\left.\begin{aligned} f^1(\varphi_2, \varphi_2^0) &= \frac{\sin\frac{\varphi_1 - \varphi_1^0}{2} - \cos\frac{\varphi_1 + \varphi_1^0}{2}}{\sin\varphi_1^0 - \sin\varphi_1} + \frac{\cos\varphi_1^0}{\sin\varphi_1^0 - \sin\varphi_1}, \\ g^1(\varphi_2, \varphi_2^0) &= -\frac{\sin\frac{\varphi_1 - \varphi_1^0}{2} + \cos\frac{\varphi_1 + \varphi_1^0}{2}}{\sin\varphi_1^0 - \sin\varphi_1} + \frac{\cos\varphi_1}{\sin\varphi_1^0 - \sin\varphi_1} \end{aligned}\right\} \tag{11.25}$$

$$\left(-\frac{3\pi}{2} < \varphi_1 < \frac{\pi}{2}\right).$$

Equating the quantities (11.20) and (11.23), we find

$$\left.\begin{aligned} I_{y_2}(\psi_2) &= -\frac{c}{ik2\pi}\, \frac{e^{ika\sqrt{\lambda^2+\mu^2}}}{\sin\alpha_1\cos\varphi_1}\, H_{0x_2} g^1(\varphi_2, \varphi_2^0), \\ I_{x_2}(\psi_2) &= \frac{c}{ik2\pi}\, \frac{e^{ika\sqrt{\lambda^2+\mu^2}}}{\sin^2\alpha_1}\, [E_{0x_2} f^1(\varphi_2, \varphi_2^0) - \\ &\quad - \cos\alpha_1\, \mathrm{tg}\,\varphi_1 H_{0x_2} g^1(\varphi_2, \varphi_2^0)]. \end{aligned}\right\} \tag{11.26}$$

In this way we established the relationship between the nonuniform part of the current on the half-plane and its field in the far zone. Now let us return to a calculation of the field from the nonuniform part of the current flowing on the disk.

Since the disk is assumed to be large in comparison with the wavelength, the nonuniform part of the current in the vicinity of its edge may be approximately considered the same as on a corresponding half-plane. Consequently, the integrals in Equation (11.03) will approximately equal the corresponding integrals from the current on the half-plane:

$$\left.\begin{aligned}
&\int_0^a J_{x_1}(\rho,\ \psi_1)\, e^{-ik\rho\sqrt{\lambda^2+\mu^2}}d\rho = I_{x_1}(\psi_1),\\
&\int_0^a J_{y_1}(\rho,\ \psi_1)\, e^{-ik\rho\sqrt{\lambda^2+\mu^2}}d\rho = I_{y_1}(\psi_1),\\
&\int_0^a J_{x_2}(\rho,\ \psi_2)\, e^{ik\rho\sqrt{\lambda^2+\mu^2}}d\rho = I_{x_2}(\psi_2),\\
&\int_0^a J_{y_2}(\rho,\ \psi_2)\, e^{ik\rho\sqrt{\lambda^2+\mu^2}}d\rho = I_{y_2}(\psi_2).
\end{aligned}\right\} \qquad (11.27)$$

As a result, the vector components of (11.03) may be represented in the following form:

$$\left.\begin{aligned}
A_{y_1} &= \frac{1}{c}\sqrt{\frac{2\pi a}{k\sqrt{\lambda^2+\mu^2}}}\frac{e^{ikR}}{R}e^{i\frac{\pi}{4}}[I_{y_1}(\psi_1)+iI_{y_2}(\psi_2)],\\
A_{x_1} &= \frac{1}{c}\sqrt{\frac{2\pi a}{k\sqrt{\lambda^2+\mu^2}}}\frac{e^{ikR}}{R}e^{i\frac{\pi}{4}}[I_{x_1}(\psi_1)+iI_{x_2}(\psi_2)].
\end{aligned}\right\} \qquad (11.28)$$

Then substituting these values into the equations

$$\left.\begin{aligned}
E_\varphi &= ikA_\varphi = ik[A_{y_1}\sin(\psi_1-\varphi) - A_{x_1}\cos(\psi_1-\varphi)],\\
E_\vartheta &= ikA_\vartheta = ik[A_{y_1}\cos(\psi_1-\varphi) + A_{x_1}\sin(\psi_1-\varphi)]\cos\vartheta,
\end{aligned}\right\} \qquad (11.29)$$

we find the field from the nonuniform part of the current flowing on the disk

$$\begin{aligned}
E_\varphi = -H_\vartheta = \frac{ae^{i\frac{\pi}{4}}}{\sqrt{2\pi ka\sqrt{\lambda^2+\mu^2}}}\frac{e^{ikR}}{R}\Bigg\{ & -H_{ox_1}\Bigg[\frac{\sin(\psi_1-\varphi)}{\sin\alpha_1\cos\varphi_1} - \\
& - \frac{\cos(\psi_1-\varphi)}{\sin^2\alpha_1}\cos\alpha_1\,\mathrm{tg}\,\varphi_1\Bigg]\times \\
& \times[g^1(\varphi_1,\ \varphi_1^0)e^{-ika\sqrt{\lambda^2+\mu^2}} - ig^1(\varphi_2,\ \varphi_2^0)e^{ika\sqrt{\lambda^2+\mu^2}}] - \\
& - E_{ox_1}\frac{\cos(\psi_1-\varphi)}{\sin^2\alpha_1}[f^1(\varphi_1,\ \varphi_1^0)e^{-ika\sqrt{\lambda^2+\mu^2}} - \\
& - if^1(\varphi_2,\ \varphi_2^0)e^{ika\sqrt{\lambda^2+\mu^2}}]\Bigg\},
\end{aligned} \qquad (11.30)$$

$$E_\vartheta = H_\varphi = \frac{a e^{i\frac{\pi}{4}} \cos\vartheta}{\sqrt{2\pi ka}\sqrt{\lambda^2+\mu^2}} \frac{e^{ikR}}{R} \Big\{ -H_{0x_1} \Big[\frac{\cos(\psi_1-\varphi)}{\sin\alpha_1\cos\varphi_1} +$$
$$+ \frac{\sin(\psi_1-\varphi)}{\sin^2\alpha_1} \cos\alpha_1 \operatorname{tg}\varphi_1 \Big] [g^1(\varphi_1, \varphi_1^0) e^{-ika\sqrt{\lambda^2+\mu^2}} -$$
$$- ig^1(\varphi_2, \varphi_2^0) e^{ika\sqrt{\lambda^2+\mu^2}}] + E_{0x_1} \frac{\sin(\psi_1-\varphi)}{\sin^2\alpha_1} \times \tag{11.31}$$
$$\times [f^1(\varphi_1, \varphi_1^0) e^{-ika\sqrt{\lambda^2+\mu^2}} - if^1(\varphi_2, \varphi_2^0) e^{ika\sqrt{\lambda^2+\mu^2}}] \Big\}.$$

The resulting expressions are valid when $ka\sqrt{\lambda^2+\mu^2} > 1$. They may be slightly simplified to

$$E_\varphi = -H_\vartheta = \frac{a e^{i\frac{\pi}{4}}}{\sqrt{2\pi ka\sqrt{\lambda^2+\mu^2}}} \frac{e^{ikR}}{R} \times$$
$$\times \Big\{ -H_{0x_1} \cos\vartheta \frac{\sin(\psi_1-\varphi)}{\sin^2\alpha_1} [g^1(\varphi_1, \varphi_1^0) e^{-ika\sqrt{\lambda^2+\mu^2}} -$$
$$- ig^1(\varphi_2, \varphi_2^0) e^{ika\sqrt{\lambda^2+\mu^2}}] - E_{0x_1} \frac{\cos(\psi_1-\varphi)}{\sin^2\alpha_1} \times$$
$$\times [f^1(\varphi_1, \varphi_1^0) e^{-ika\sqrt{\lambda^2+\mu^2}} - if^1(\varphi_2, \varphi_2^0) e^{ika\sqrt{\lambda^2+\mu^2}}] \Big\}, \tag{11.32}$$
$$E_\vartheta = H_\varphi = \frac{a e^{i\frac{\pi}{4}}}{\sqrt{2\pi ka\sqrt{\lambda^2+\mu^2}}} \frac{e^{ikR}}{R} \times$$
$$\times \Big\{ -H_{0x_1} \frac{\cos(\psi_1-\varphi)}{\sin^2\alpha_1} [g^1(\varphi_1, \varphi_1^0) e^{-ika\sqrt{\lambda^2+\mu^2}} -$$
$$- ig^1(\varphi_2, \varphi_2^0) e^{ika\sqrt{\lambda^2+\mu^2}}] + E_{0x_1} \cos\vartheta \frac{\sin(\psi_1-\varphi)}{\sin^2\alpha_1} \times$$
$$\times [f^1(\varphi_1, \varphi_1^0) e^{-ika\sqrt{\lambda^2+\mu^2}} - if^1(\varphi_2, \varphi_2^0) e^{ika\sqrt{\lambda^2+\mu^2}}] \Big\},$$

if we use the identities

$$\left.\begin{aligned} &\frac{\sin(\psi_1-\varphi)}{\sin\alpha_1\cos\varphi_1} - \frac{\cos(\psi_1-\varphi)}{\sin^2\alpha_1}\cos\alpha_1 \operatorname{tg}\varphi_1 \equiv \frac{\cos\vartheta\sin(\psi_1-\varphi)}{\sin^2\alpha_1}, \\ &\frac{\cos(\psi_1-\varphi)}{\sin\alpha_1\cos\varphi_1} + \frac{\sin(\psi_1-\varphi)}{\sin^2\alpha_1}\cos\alpha_1 \operatorname{tg}\varphi_1 \equiv \frac{\cos(\psi_1-\varphi)}{\sin^2\alpha_1\cos\vartheta}. \end{aligned}\right\} \tag{11.33}$$

The operations carried out above may be briefly summarized in the following way. The field from the nonuniform part of the current on the disk

$$\mathbf{A} = \frac{a}{c} \frac{e^{ikR}}{R} \int_0^{2\pi} d\psi \int_0^a \mathbf{J}(\rho, \psi) e^{ik\rho\Phi} d\rho$$

is found (without direct calculation of the current) in terms of the known field of an auxiliary half-plane

$$A = \frac{1}{c}\sqrt{\frac{2\pi}{k_1 r_1}}\, e^{ik(x_1 \cos\alpha_1 + r_1 \sin\alpha_1) + i\frac{\pi}{4}} \int_{-\infty}^{\infty} J(\eta)\, e^{ik\eta_1 \Phi_1} d\eta_1$$

by a replacement of

$$\int_0^a J(\rho, \psi)\, e^{ik\rho\Phi} d\rho \text{ by } \int_{-\infty}^{a} J(\eta)\, e^{ik\eta_1\Phi_1} d\eta$$

in those cases when $\Phi = \Phi_1$. The functions Φ and Φ_1 are determined by the equations

$$\left.\begin{aligned} \Phi &= \sin\gamma \sin\psi - \sin\vartheta \cos(\psi - \varphi), \\ \Phi_1 &= \sin\gamma \sin\psi - \sin\vartheta \cos(\psi - \varphi)\sqrt{\frac{1 - \sin^2\gamma \cos^2\psi}{1 - \sin^2\vartheta \sin^2(\psi - \varphi)}}. \end{aligned}\right\} \qquad (11.34)$$

Solution (11.32) was determined exactly in this way with $ka\sqrt{\lambda^2 + \mu^2} \gg 1$, when, for auxiliary half-plane whose edge touches the rim of the disk at the points $\psi = \delta$, $\psi = \pi + \delta$, the phase Φ_1 was equal to Φ.

A solution to the problem using this method also is possible in the case

$$\vartheta = \gamma,\ \varphi = \frac{\pi}{2}, \qquad (11.35)$$

when $\Phi_1 = \Phi = 0$. The direction $\vartheta = \gamma$, $\varphi = \frac{\pi}{2}$ corresponds to the principal maximum of the scattering diagram, and therefore is of special interest. Substituting the relationships

$$\left.\begin{aligned} I_{y_1} &= \frac{c}{ik4\pi} \frac{H_{0x_1}}{\sin\alpha_1} \frac{\sin\varphi_1^0 - 1}{\cos^2\varphi_1^0}, \\ I_{x_1} &= \frac{c}{ik4\pi} \frac{1}{\sin^2\alpha_1} (E_{0x_1} + \cos\alpha_1 \operatorname{tg}\varphi_1^0 H_{0x_1}) \frac{\sin\varphi_1^0 - 1}{\cos\varphi_1^0} \end{aligned}\right\} \qquad (11.36)$$

which follow in this case from (11.19) into the equations

$$\left.\begin{aligned} A_x &= \frac{a}{c} \frac{e^{ikR}}{R} \int_0^{2\pi} (I_{y_1} \cos\psi + I_{x_1} \sin\psi)\, d\psi, \\ A_y &= \frac{a}{c} \frac{e^{ikR}}{R} \int_0^{2\pi} (I_{y_1} \sin\psi - I_{x_1} \cos\psi)\, d\psi, \end{aligned}\right\} \qquad (11.37)$$

we find the field radiated by the nonuniform part of the current in the direction of the principal maximum. With the E-polarization of the incident wave ($E_0 \perp yoz$), it equals

$$\left.\begin{aligned} E_x = H_\vartheta = E_{0x}\frac{a}{\pi \sin^2\gamma}\left(2K\cos\gamma - \frac{1+\cos^2\gamma}{\cos\gamma}E\right)\frac{e^{ikR}}{R}, \\ E_\vartheta = H_x = 0, \end{aligned}\right\} \quad (11.38)$$

and with the H-polarization ($H_0 \perp yoz$) of the incident wave, it equals

$$\left.\begin{aligned} E_\vartheta = -H_x = -E_{0\vartheta}\frac{a}{\pi \sin^2\gamma}\times \\ \times\left(2K\cos\gamma - \frac{1+\cos^2\gamma}{\cos\gamma}E\right)\frac{e^{ikR}}{R}, \\ E_x = H_\vartheta = 0, \end{aligned}\right\} \quad (11.39)$$

where

$$K = \int_0^{\frac{\pi}{2}} \frac{d\psi}{\sqrt{1-\sin^2\gamma\sin^2\psi}}, \quad E = \int_0^{\frac{\pi}{2}} \sqrt{1-\sin^2\gamma\sin^2\psi}\, d\psi \quad (11.40)$$

which are complete elliptic integrals. From the resulting expressions, it follows that with the rotation of the incident wave polarization by 90° the phase of the field from the nonuniform part of the current is changed by 180°, as it was in the case of a half-plane. If $\gamma \to 0$, then the difference between the polarizations disappears, and in the limit (when $\gamma = 0$) we arrive at the previous result (8.16).

Numerical calculations were carried out using Equations (11.38) and (11.39). They showed that for values of γ not exceeding 55° the field from the uniform part of the current is at a minimum ka times larger than the field from the nonuniform part of the current.

§ 12. The Scattering Characteristics with an Arbitrary Irradiation

The total field scattered by the disk equals the sum of the fields radiated by the uniform and nonuniform parts of the current. However, now, in contrast to the case of normal irradiation, in region $ka\sqrt{\lambda^2+\mu^2} \gg 1$ the total field no longer is expressed only in terms of the functions f and g. Therefore, we will first investigate in more detail the fringing field in the incident plane $\left(x=0,\ \varphi=\pm\frac{\pi}{2}\right)$ where the expressions (11.32) take the form

$$\left.\begin{aligned}
&E_{\varphi}=-H_{\vartheta}=\frac{ae^{i\frac{\pi}{4}}}{\sqrt{2\pi ka\mu}}E_{0x}[-f^{1}(1)\,e^{-ika\mu}+if^{1}(2)\,e^{ika\mu}]\frac{e^{ikR}}{R}\\
&\qquad\text{with } \varphi=\frac{\pi}{2},\quad \vartheta>\gamma,\\
&E_{\varphi}=-H_{\vartheta}=\frac{ae^{i\frac{\pi}{4}}}{\sqrt{2\pi ka|\mu|}}E_{0x}[-f^{1}(2)\,e^{ika\mu}+if^{1}(1)\,e^{-ika\mu}]\frac{e^{ikR}}{R}\\
&\qquad\text{with } \varphi=\frac{\pi}{2},\quad \vartheta<\gamma,\\
&E_{\varphi}=-H_{\vartheta}=\frac{ae^{i\frac{\pi}{4}}}{\sqrt{2\pi ka|\mu|}}E_{0x}[f^{1}(2)\,e^{ika\mu}-if^{1}(1)\,e^{-ika\mu}]\frac{e^{ikR}}{R}\\
&\qquad\text{with } \varphi=-\frac{\pi}{2};
\end{aligned}\right\}\qquad(12.01)$$

$$\left.\begin{aligned}
&E_{\vartheta}=H_{\varphi}=\frac{ae^{i\frac{\pi}{4}}H_{0x}}{\sqrt{2\pi ka\mu}}[-g^{1}(1)\,e^{-ika\mu}+ig^{1}(2)\,e^{ika\mu}]\frac{e^{ikR}}{R}\\
&\qquad\text{with } \varphi=\frac{\pi}{2},\quad \vartheta>\gamma,\\
&E_{\vartheta}=H_{\varphi}=\frac{ae^{i\frac{\pi}{4}}H_{0x}}{\sqrt{2\pi ka|\mu|}}[-g^{1}(2)\,e^{ika\mu}+ig^{1}(1)\,e^{-ika\mu}]\frac{e^{ikR}}{R}\\
&\qquad\text{with } \varphi=\frac{\pi}{2},\quad \vartheta<\gamma,\\
&E_{\vartheta}=H_{\varphi}=\frac{ae^{i\frac{\pi}{4}}H_{0x}}{\sqrt{2\pi ka|\mu|}}[g^{1}(2)\,e^{ika\mu}-ig^{1}(1)\,e^{-ika\mu}]\frac{e^{ikR}}{R}\\
&\qquad\text{with } \varphi=-\frac{\pi}{2}.
\end{aligned}\right\}\qquad(12.02)$$

The functions $f^1(1)$ and $g^1(1)$ correspond to the field of the auxiliary half-plane $-\infty\leqslant y\leqslant a$, and the functions $f^1(2)$ and $g^1(2)$ correspond to the field of the half-plane $-a\leqslant y\leqslant\infty$. In accordance with Equations (11.09) and (11.25), they are determined by the expressions

$$\begin{aligned}
&f^{1}(1)=f(1)-\frac{\cos\gamma}{\sin\gamma-\sin\vartheta},\quad f(1)=\frac{\sin\frac{\vartheta-\gamma}{2}+\cos\frac{\vartheta+\gamma}{2}}{\sin\gamma-\sin\vartheta},\\
&g^{1}(1)=g(1)-\frac{\cos\vartheta}{\sin\gamma-\sin\vartheta},\quad g(1)=\frac{-\sin\frac{\vartheta-\gamma}{2}+\cos\frac{\vartheta+\gamma}{2}}{\sin\gamma-\sin\vartheta},
\end{aligned}\qquad(12.03)$$

(Equation continued on next page.)

$$
\left.\begin{aligned}
f^1(2) &= f(2) + \frac{\cos\gamma}{\sin\gamma - \sin\vartheta}, \quad f(2) = \frac{\sin\frac{\vartheta-\gamma}{2} - \cos\frac{\vartheta+\gamma}{2}}{\sin\gamma - \sin\vartheta},\\
g^1(2) &= g(2) + \frac{\cos\vartheta}{\sin\gamma - \sin\vartheta}, \quad g(2) = -\frac{\sin\frac{\vartheta-\gamma}{2} + \cos\frac{\vartheta+\gamma}{2}}{\sin\gamma - \sin\vartheta},
\end{aligned}\right\} \qquad (12.03)
$$

if $\varphi = \frac{\pi}{2}$ and $\vartheta < \frac{\pi}{2}$; and

$$
\left.\begin{aligned}
f^1(1) &= f(1) - \frac{\cos\gamma}{\sin\gamma + \sin\vartheta}, \quad f(1) = -\frac{\sin\frac{\vartheta+\gamma}{2} - \cos\frac{\vartheta-\gamma}{2}}{\sin\gamma + \sin\vartheta},\\
g^1(1) &= g(1) - \frac{\cos\vartheta}{\sin\gamma + \sin\vartheta}, \quad g(1) = \frac{\sin\frac{\vartheta+\gamma}{2} + \cos\frac{\vartheta-\gamma}{2}}{\sin\gamma + \sin\vartheta},\\
f^1(2) &= f(2) + \frac{\cos\gamma}{\sin\gamma + \sin\vartheta}, \quad f(2) = -\frac{\sin\frac{\vartheta+\gamma}{2} + \cos\frac{\vartheta-\gamma}{2}}{\sin\gamma + \sin\vartheta},\\
g^1(2) &= g(2) + \frac{\cos\vartheta}{\sin\gamma + \sin\vartheta}, \quad g(2) = \frac{\sin\frac{\vartheta+\gamma}{2} - \cos\frac{\vartheta-\gamma}{2}}{\sin\gamma + \sin\vartheta},
\end{aligned}\right\} \qquad (12.04)
$$

if $\varphi = -\frac{\pi}{2}$ and $\vartheta < \frac{\pi}{2}$.

It was mentioned above that when ka >> 1 in the direction $\vartheta=\gamma$ $(0^\circ \leqslant \gamma \leqslant 55^\circ)$, $\varphi = \frac{\pi}{2}$ the field from the nonuniform part of the current is negligibly small in comparison with the field from the uniform part. Therefore, for the field from the nonuniform part of the current one may write, with the help of Bessel functions, the following interpolation formulas: with $\varphi = \frac{\pi}{2}$

$$
\left.\begin{aligned}
E_\varphi = -H_\vartheta = \frac{iaE_{0x}}{2}\{[f^1(1) - f^1(2)]\,J_1(\zeta) -\\
- i\,[f^1(1) + f^1(2)]\,J_2(\zeta)\}\frac{e^{ikR}}{R},\\
E_\vartheta = H_\varphi = \frac{iaH_{0x}}{2}\{[g^1(1) - g^1(2)]\,J_1(\zeta) -\\
- i[g^1(1) + g^1(2)]\,J_2(\zeta)\}\frac{e^{ikR}}{R},
\end{aligned}\right\} \qquad (12.05)
$$

and with $\varphi = -\frac{\pi}{2}$

$$
\left.\begin{aligned}
E_\varphi = -H_\vartheta = \frac{iaE_{0x}}{2}\{[f^1(1) - f^1(2)]\,J_1(\xi) +\\
+ i\,[f^1(1) + f^1(2)]\,J_2(\xi)\}\frac{e^{ikR}}{R},
\end{aligned}\right\} \qquad (12.06)
$$

(Equation continued on next page).

$$E_\vartheta = H_\varphi = \frac{iaH_{0x}}{2}\{[g^1(1) - g^1(2)]J_1(\xi) + + i[g^1(1) + g^1(2)]J_2(\xi)\}\frac{e^{ikR}}{R}, \qquad (12.06)$$

where

$$\left.\begin{aligned} \zeta &= ka(\sin\vartheta - \sin\gamma), \\ \xi &= ka(\sin\vartheta + \sin\gamma). \end{aligned}\right\} \qquad (12.07)$$

These expressions are valid in the region $0 < \vartheta < \frac{\pi}{2}$; when $|\zeta| \gg 1$ and $|\xi| \gg 1$ they change to Equations (12.01) and (12.02), and in the direction $\vartheta = \gamma,\ \varphi = \frac{\pi}{2}$ they give a field equal to zero.

Using specific expressions for the functions f^1 and g^1, it is not difficult to establish that the total field scattered by the disk in view of Equations (10.03) and (10.04) may be represented in the following form:

with $\varphi = \frac{\pi}{2}$

$$\left.\begin{aligned} E_\varphi = -H_\vartheta &= \frac{iaE_{0x}}{2}\{[f(1) - f(2)]J_1(\zeta) - \\ &- i[f(1) + f(2)]J_2(\zeta)\}\frac{e^{ikR}}{R}, \\ E_\vartheta = H_\varphi &= \frac{iaH_{0x}}{2}\{[g(1) - g(2)]J_1(\zeta) - \\ &- i[g(1) + g(2)]J_2(\zeta)\}\frac{e^{ikR}}{R}, \end{aligned}\right\} \qquad (12.08)$$

and with $\varphi = -\frac{\pi}{2}$

$$\left.\begin{aligned} E_\varphi = -H_\vartheta &= \frac{iaE_{0x}}{2}\{[f(1) - f(2)]J_1(\xi) + \\ &+ i[f(1) + f(2)]J_2(\xi)\}\frac{e^{ikR}}{R}, \\ E_\vartheta = H_\varphi &= \frac{iaH_{0x}}{2}\{[g(1) - g(2)]J_1(\xi) + \\ &+ i[g(1) + g(2)]J_2(\xi)\}\frac{e^{ikR}}{R}. \end{aligned}\right\} \qquad (12.09)$$

It is convenient to write these expressions as follows

$$\left.\begin{aligned} E_\varphi = -H_\vartheta &= \frac{iaE_{0x}}{2}\overline{\Sigma}(\vartheta, \gamma)\frac{e^{ikR}}{R}, \\ E_\vartheta = H_\varphi &= \frac{iaH_{0x}}{2}\Sigma(\vartheta, \gamma)\frac{e^{ikR}}{R}, \end{aligned}\right\} \qquad (12.10)$$

where the functions $\bar{\Sigma}$ and Σ are determined in the region $0 \leqslant \vartheta \leqslant \frac{\pi}{2}$ by the equations:

$$\left.\begin{array}{l}\bar{\Sigma}(\vartheta,\ \gamma)\\ \Sigma(\vartheta,\ \gamma)\end{array}\right\} = -\frac{J_1(\xi)}{\sin\frac{\vartheta-\gamma}{2}} \pm i\frac{J_2(\xi)}{\cos\frac{\vartheta+\gamma}{2}} \text{ with } \varphi = \frac{\pi}{2},$$

$$\left.\begin{array}{l}\bar{\Sigma}(\vartheta,\ \gamma)\\ \Sigma(\vartheta,\ \gamma)\end{array}\right\} = \frac{J_1(\xi)}{\sin\frac{\vartheta+\gamma}{2}} \mp i\frac{J_2(\xi)}{\cos\frac{\vartheta-\gamma}{2}} \text{ with } \varphi = -\frac{\pi}{2}, \qquad (12.11)$$

and in the region $\frac{\pi}{2} \leqslant \vartheta \leqslant \pi$

$$\left.\begin{array}{l}\bar{\Sigma}(\vartheta,\ \gamma)\\ \Sigma(\vartheta,\ \gamma)\end{array}\right\} = \mp\frac{J_1(\xi)}{\cos\frac{\vartheta+\gamma}{2}} + i\frac{J_2(\xi)}{\sin\frac{\vartheta-\gamma}{2}} \text{ with } \varphi = \frac{\pi}{2},$$

$$\left.\begin{array}{l}\bar{\Sigma}(\vartheta,\ \gamma)\\ \Sigma(\vartheta,\ \gamma)\end{array}\right\} = \pm\frac{J_1(\xi)}{\cos\frac{\vartheta-\gamma}{2}} - i\frac{J_2(\xi)}{\sin\frac{\vartheta+\gamma}{2}} \text{ with } \varphi = -\frac{\pi}{2}. \qquad (12.12)$$

Here assuming $\gamma = 0$, we obtain the previous relationships (9.03) and (9.05).

In the directions $\vartheta = \gamma$ and $\vartheta = \pi - \gamma$ (with $\varphi = \frac{\pi}{2}$), where the scattering diagram has a principal maximum, it follows from Equations (12.11) and (12.12) that

$$\bar{\Sigma}(\gamma) = \Sigma(\gamma) = -ka\cos\gamma \qquad (12.13)$$

and

$$\bar{\Sigma}(\pi-\gamma) = -\Sigma(\pi-\gamma) = -ka\cos\gamma. \qquad (12.14)$$

In the direction toward the source $\left(\vartheta = \pi - \gamma,\ \varphi = -\frac{\pi}{2}\right)$, the functions $\bar{\Sigma}(\vartheta)$ and $\Sigma(\vartheta)$ take the values

$$\left.\begin{array}{l}\bar{\Sigma}(\vartheta)\\ \Sigma(\vartheta)\end{array}\right\} = \pm\frac{1}{\sin\vartheta}J_1(\xi) - iJ_2(\xi). \qquad (12.15)$$

Here considering $\vartheta = \pi$, we obtain

$$\bar{\Sigma}(\pi) = -\Sigma(\pi) = ka. \qquad (12.16)$$

which corresponds to the physical optics approach [Equation (7.20)].

The functions $\bar{\Sigma}$ and Σ allow one to calculate

$$\sigma_E = \pi a^2 |\bar{\Sigma}|^2, \quad \sigma_H = \pi a^2 |\Sigma|^2 \tag{12.17}$$

which are the effective scattering surfaces with the E- and H-polarizations of the incident wave. Let us recall that, by definition, the effective scattering surface is a quantity equal to

$$\sigma = 4\pi R^2 \frac{|\mathbf{S}|}{|\mathbf{S}_0|}, \tag{12.18}$$

where

$$\mathbf{S} = \frac{c}{8\pi} \mathrm{Re}\,[\mathbf{E}\mathbf{H}^*] \tag{12.19}$$

which is the energy flux density averaged over one oscillation cycle (the Poynting vector) in the scattered wave, and S_0 is a similar quantity for the incident wave.

In this way, we obtained the expressions for the fringing field which approximately take into account the nonuniform part of the current. In the incident plane $\left(\varphi = \pm\frac{\pi}{2}\right)$, they have a form which is rather simple and convenient for calculations. It is also interesting that in this case they satisfy the reciprocity principle as distinct from expressions (10.03) and (10.04) which correspond to the uniform part of the current. It is not difficult to prove this by verifying that Equations (12.11) are not changed with the simultaneous replacement of γ by ϑ and of ϑ by γ, and Equations (12.12) are not changed with the replacement of ϑ by $\pi - \gamma$ and of γ by $\pi - \vartheta$ [in the case $\varphi = -\frac{\pi}{2}$].

However, Equations (12.11) and (12.12) lead to a discontinuity of the magnetic field tangential component H_ϕ on the plane z = 0 in which the disk lies. As in the case of diffraction by a strip, the reason for this is that we did not consider the interaction of the edges. It is also necessary to take into account this interaction in the case of glancing incidence of the plane wave $\left(\gamma = \frac{\pi}{2}\right)$, when the fringing field components E_ϑ and H_ϕ must be equal to zero.

Let us point out once again in conclusion to this section that expressions (12.11) and (12.12) near directions $\vartheta = \gamma$, $\vartheta = \pi - \gamma$ (with $\varphi = \frac{\pi}{2}$) have an interpolation character, but in return they allow one to represent the fringing field in the incident plane x = 0 in a convenient (uniform) form which frequently is of greatest importance (compare § 24).

CHAPTER III

DIFFRACTION BY A FINITE LENGTH CYLINDER

The distinctive feature of this problem is that, in addition to the nonuniform part of the current on the cylinder's surface which is caused by the discontinuity, there also exists a nonuniform part of the current arising as a consequence of the smooth curve of the surface. This part of the current has the character of waves travelling over the cylindrical surface along geodesic lines [36] — that is, along spirals on the cylinder. These waves, which as they move strike the edge of the cylinder, undergo diffraction and excite secondary surface currents. In turn, the nonuniform part of the current resulting from the discontinuity undergoes diffraction while being propagated over the cylindrical surface. It is clear that specific consideration of all these effects is a very complicated problem.

However, if all the linear dimensions of the cylinder are sufficiently large in comparison with the wavelength, these effects may be neglected when calculating the fringing field in many cases which are of practical interest. In particular, they may be neglected when calculating the field scattered in the direction toward the source [5, 37]. In this case it is sufficient to consider only the nonuniform part of the current which is caused by the discontinuity of the surface, and we will do this in this Chapter. The equations obtained in this way are generalized to the case when the observation direction does not coincide with the direction of the source.

§ 13. The Physical Optics Approach

Let us investigate the diffraction of plane electromagnetic wave

$$\mathbf{E} = \mathbf{E}_0 e^{ik(y\sin\gamma + z\cos\gamma)} \qquad (13.01)$$

on a finite, ideally conducting cylinder of radius a and length l. Let us position the spherical coordinate system in such a way that its origin is at the center of the cylinder, and the normal n to the incident wave front lies in the half-plane $\varphi = \frac{\pi}{2}$ and forms an angle γ $\left(0 < \gamma < \frac{\pi}{2}\right)$ with the z axis (Figure 24).

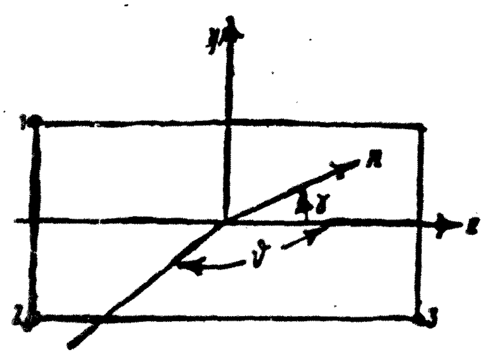


Figure 24. Diffraction of a plane wave by a finite cylinder. **n** is the normal to the incident wave front.

An incident wave having an arbitrary linear polarizaton always may be represented as the sum of two waves with mutually perpendicular polarizations. Therefore, for a complete solution of the problem, it is sufficient to investigate two particular cases of incident wave polarization:

(1) E-polarization, when the incident wave electric vector is perpendicular to the plane ($\mathbf{E}_0 \perp yoz$) and

(2) H-polarization, when $\mathbf{H}_0 \perp yoz$.

The uniform part of the current excited on the cylindrical surface by wave (13.01) has, with the E-polarization, the components

$$\left.\begin{aligned} j_x^0 &= -\frac{c}{2\pi} E_{0x} \cdot \sin\gamma \sin\psi \, e^{ik\Phi}, \\ j_y^0 &= \frac{c}{2\pi} E_{0x} \sin\gamma \cos\psi \, e^{ik\Phi}, \\ j_z^0 &= \frac{c}{2\pi} E_{0x} \cdot \cos\gamma \cos\psi \, e^{ik\Phi}, \end{aligned}\right\} \tag{13.02}$$

and with the H-polarization it has the components

$$\left.\begin{aligned} j_x^0 &= j_y^0 = 0, \\ j_z^0 &= -\frac{c}{2\pi} H_{0x} \sin\psi \, e^{ik\Phi}, \end{aligned}\right\} \tag{13.03}$$

where

$$\Phi = a \sin\gamma \sin\psi + \zeta \cos\gamma. \tag{13.04}$$

Let us calculate the field created by these currents in the region $\varphi = -\frac{\pi}{2}$.

The vector potential of the fringing field is determined by the equations

$$A = \frac{a}{c} \int_0^{2\pi} d\psi \int_{-\frac{l}{2}}^{\frac{l}{2}} j^0(\zeta, \psi) \frac{e^{ikr}}{r} d\zeta \text{ with } \gamma = 0 \tag{13.05}$$

and

$$A = \frac{a}{c} \int_{-\pi}^{0} d\psi \int_{-\frac{l}{2}}^{\frac{l}{2}} j^0(\zeta, \psi) \frac{e^{ikr}}{r} d\zeta \text{ with } \gamma > 0, \tag{13.06}$$

where

$$r = \sqrt{\xi^2 + (y-\eta)^2 + (z-\zeta)^2} . \tag{13.07}$$

Since the field in the far zone ($R \gg ka^2, R \gg kl^2$) is of interest to us, these expressions may be simplified by using the relationship

$$r \approx R + a \sin\vartheta \sin\psi - \zeta \cos\vartheta. \tag{13.08}$$

As a result, we obtain a simpler equation

$$\mathrm{A}=\frac{a}{c}\frac{e^{ikR}}{R}\int e^{ika\sin\vartheta\sin\psi}d\psi\int_{-\frac{l}{2}}^{\frac{l}{2}}\mathrm{j}^{0}(\zeta,\ \psi)\,e^{-ik\zeta\cos\vartheta}d\zeta. \qquad (13.09)$$

Since the current components are described by the functions $f(\psi)\,e^{ik\zeta}$, then the problem of finding the field reduces essentially to a calculation of integrals of the type

$$\int_{-\frac{l}{2}}^{\frac{l}{2}}e^{ik\zeta(\cos\gamma-\cos\vartheta)}d\zeta\int_{-\pi}^{0}f(\psi)\,e^{ip\sin\psi}d\psi=$$
$$=\frac{1}{ik(\cos\gamma-\cos\vartheta)}\left[e^{i\frac{kl}{2}(\cos\gamma-\cos\vartheta)}-\right. \qquad (13.10)$$
$$\left.-e^{-i\frac{kl}{2}(\cos\gamma-\cos\vartheta)}\right]\int_{-\pi}^{0}f(\psi)\,e^{ip\sin\psi}\,d\psi.$$

The integral

$$\int_{-\pi}^{0}f(\psi)\,e^{ip\sin\psi}d\psi,\ \ p=ka(\sin\gamma+\sin\vartheta) \qquad (13.11)$$

when p >> 1 is easily calculated by the stationary phase method. The stationary phase point is determined from the condition $\frac{d}{d\psi}\sin\psi=0$ and equals

$$\psi_0=-\frac{\pi}{2}. \qquad (13.12)$$

Then assuming $\psi=-\frac{\pi}{2}+\delta$, we find

$$\int_{-\pi}^{0}f(\psi)\,e^{ip\sin\psi}d\psi\approx f(\psi_0)\,e^{-ip}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}e^{ip\frac{\delta^2}{2}}d\delta\approx \qquad (13.13)$$
$$\approx\sqrt{\frac{2}{p}}f(\psi_0)\,e^{-ip}\int_{-\infty}^{\infty}e^{it^2}dt=\sqrt{\frac{2\pi}{p}}f(\psi_0)\,e^{-ip+i\frac{\pi}{4}}.$$

As a result, we obtain the following expressions for the vector potential:

with E-polarization

$$A_x = \frac{a}{2\pi} E_{0x} \sin\gamma \frac{e^{ikR}}{R} \cdot I, \quad A_y = A_z = 0 \tag{13.14}$$

and with H-polarization

$$A_z = \frac{a}{2\pi} H_{0x} \cdot \frac{e^{ikR}}{R} \cdot I, \quad A_x = A_y = 0, \tag{13.15}$$

where

$$I = \frac{e^{i\frac{kl}{2}(\cos\gamma - \cos\vartheta)} - e^{-i\frac{kl}{2}(\cos\gamma - \cos\vartheta)}}{ik(\cos\gamma - \cos\vartheta)} \times \\ \times \sqrt{\frac{2\pi}{ka(\sin\gamma + \sin\vartheta)}}\, e^{-ika(\sin\gamma + \sin\vartheta) + i\frac{\pi}{4}}. \tag{13.16}$$

The fringing field in the region $\varphi = -\frac{\pi}{2}$ is determined by the relationships

$$\left. \begin{aligned} E_\varphi &= -H_\vartheta = ikA_x, \\ E_\vartheta &= H_\varphi = -ikA_z \sin\vartheta. \end{aligned} \right\} \tag{13.17}$$

Therefore, with the E-polarization it equals

$$\left. \begin{aligned} E_\varphi &= -H_\vartheta = \frac{ika}{2\pi} E_{0x} \sin\gamma \frac{e^{ikR}}{R} \cdot I, \\ E_\vartheta &= H_\varphi = 0, \end{aligned} \right\} \tag{13.18}$$

and with H-polarization

$$\left. \begin{aligned} E_\vartheta &= H_\varphi = -\frac{ika}{2\pi} H_{0x} \cdot \sin\vartheta \frac{e^{ikR}}{R} \cdot I, \\ E_\varphi &= H_\vartheta = 0. \end{aligned} \right\} \tag{13.19}$$

The resulting equations show that the field scattered by the cylindrical surface is created mainly by a luminous band adjacent to the cylinder's generatrix with $\psi = \psi_0 = -\frac{\pi}{2}$. The radiation from this band may be represented [see Equation (13.16)] in the form of spherical waves diverging from its ends (points 2 and 3 in Figure 24).

Now let us write Expressions (13.18) and (13.19) in a form which is most convenient for calculating the effective scattering area

$$E_\varphi = -H_\vartheta = \frac{ia}{2} E_{0x} \cdot \frac{e^{ikR}}{R} \cdot \overline{\sum\nolimits_{ц}}^0 (\vartheta, \gamma),$$
$$E_\vartheta = H_\varphi = \frac{ia}{2} H_{0x} \cdot \frac{e^{ikR}}{R} \cdot \sum\nolimits_{ц}^0 (\vartheta, \gamma). \quad (13.20)$$

Here

$$\overline{\sum\nolimits_{ц}}^0 = G \sin\gamma, \quad \sum\nolimits_{ц}^0 = -G \sin\vartheta, \quad (13.21)$$

and

$$G = 2\sqrt{\frac{2}{\pi ka(\sin\gamma + \sin\vartheta)}} \times$$
$$\times \frac{\sin\left[\frac{kl}{2}(\cos\gamma - \cos\vartheta)\right]}{\cos\gamma - \cos\vartheta} e^{-ika(\sin\gamma + \sin\vartheta) + i\frac{\pi}{4}}. \quad (13.22)$$

The index "0" on $\overline{\sum}_{ц}^0$ and $\sum_{ц}^0$ means that the field was calculated in the physical optics approach (based on the uniform part of the current), and the index "c" shows that this fringing field is created by a cylindrical surface. The effective scattering area, in accordance with (12.17), is determined for a cylindrical surface by the relationships

$$\sigma_{ц,E}^0 = \pi a^2 \left|\overline{\sum\nolimits_{ц}}^0\right|^2 = \pi a^2 \sin^2\gamma\, |G|^2,$$
$$\sigma_{ц,H}^0 = \pi a^2 \left|\sum\nolimits_{ц}^0\right|^2 = \pi a^2 \sin^2\vartheta\, |G|^2. \quad (13.23)$$

In the direction of the mirror-reflected ray $(\vartheta = \gamma)$, we have

$$\sigma_{ц,E}^0 = \sigma_{ц,H}^0 = kal^2 \sin\vartheta = \frac{2\pi a}{\lambda} l^2 \sin\vartheta. \quad (13.24)$$

In the direction toward the source $(\vartheta = \pi - \gamma)$, the functions $\overline{\sum}_c^0$ and $\sum_c^0$ equal

$$\overline{\sum\nolimits_{ц}}^0 = -\sum\nolimits_{ц}^0 = \sqrt{\frac{\sin\vartheta}{\pi ka}} \cdot \frac{\sin(kl\cos\vartheta)}{\cos\vartheta} \cdot e^{-2ika\sin\vartheta + i\frac{\pi}{4}}. \quad (13.25)$$

These expressions are valid if ka sin $\vartheta \gg 1$. It is not difficult to see, by means of equations (13.02) - (13.05), that the fringing field equals zero if $\gamma = 0$ and $\vartheta = \pi$. Thus in the case of radar (that is, in the direction toward the source) we find an expression for the fringing field in the region ka sin $\vartheta \gg 1$ and in the direction $\vartheta = \pi$. Naturally the desire arises to write interpolation equations — that is, equations which would provide a continuous transition from the region ka sin $\vartheta \gg 1$ to the direction $\vartheta = \pi$. Now let us note that the field scattered by a cylinder is comprised of the fields scattered by the lateral (cylindrical) surface and the base (end) of the cylinder. In the physical optics approach, the field scattered by the end of the cylinder is equivalent to the field scattered by a disk. But the field scattered by a disk is described by Bessel functions. Therefore, it is also advisable to express the field scattered by the cylindrical surface in terms of Bessel functions. As s result, the desired interpolation equations for the field scattered by the cylindrical surface may be represented in the form

$$\overline{\sum}_{\text{ц}}^{\bullet} = -\sum_{\text{ц}}^{\bullet} = -\frac{\sin\vartheta}{2\cos\vartheta}\left(e^{ikl\cos\vartheta} - e^{-ikl\cos\vartheta}\right)\left[J_1(\zeta) - iJ_2(\zeta)\right], \qquad (13.26)$$

$$\zeta = 2ka\sin\vartheta.$$

From this it follows that $\overline{\sum}_{\text{ц}}^{0} = \sum_{\text{ц}}^{0} = 0$ in the direction $\vartheta = \pi$, and with the conditions ka sin $\vartheta \gg 1$ we obtain Equations (13.25).

The field being scattered by the cylinder's end (by the disk), in accordance with equalities (10.06) and (10.07), is described in the physical optics approach by the equations

$$\left.\begin{array}{l} \overline{\sum}_{\text{д}}^{\bullet} \\ \sum_{\text{д}}^{\bullet} \end{array}\right\} = \mp\frac{\cos\vartheta}{\sin\vartheta} J_1(\zeta)\, e^{ikl\cos\vartheta}. \qquad (13.27)$$

Consequently, the field scattered by the entire surface of the cylinder will be determined in the plane $\varphi = -\frac{\pi}{2}$ by the equations:

$$\left.\begin{array}{l} E_{\varphi} = -H_{\vartheta} = \frac{ia}{2} E_{0x}\cdot\frac{e^{ikR}}{R}\overline{\sum}^{\bullet}(\vartheta), \\ E_{\vartheta} = H_{\varphi} = \frac{ia}{2} H_{0x}\cdot\frac{e^{ikR}}{R}\sum^{\bullet}(\vartheta), \end{array}\right| \qquad (13.28)$$

where

$$\left.\begin{array}{l}\overline{\Sigma}^{\bullet}(\vartheta)\\ \Sigma^{\bullet}(\vartheta)\end{array}\right\}=\mp\frac{\sin\vartheta}{2\cos\vartheta}\left(e^{ikl\cos\vartheta}-e^{-ikl\cos\vartheta}\right)\left[J_1(\zeta)-iJ_2(\zeta)\right]\mp \mp\frac{\cos\vartheta}{\sin\vartheta}J_1(\zeta)\,e^{ikl\cos\vartheta}\quad(\zeta=2ka\sin\vartheta). \tag{13.29}$$

These equations allow one to determine in the physical optics approach the effective scattering area of a finite cylinder.

§ 14. The Field Created by the Nonuniform Part of the Current

Let us find the field from the nonuniform part of the current caused by the surface's discontinuity. Figuratively speaking, the field scattered by the cylinder is created by the "luminous" regions on its end and lateral surface. Mathematically this field is described by the sum of spherical waves from the "luminous" points 1, 2 and 3 (see Figure 24). Obviously the field from the nonuniform part of the current also will have the form of spherical waves diverging from these same points.

In the case when the length and diameter of the cylinder are sufficiently large in comparison with the wavelength, one may approximately consider that the nonuniform part of the current near the discontinuity is the same as that on a corresponding wedge. The field radiated by this part of the current in principle may be found in the same way as in the case of the disk. However, such a method is rather complicated. We will find the desired field by a simpler and more graphic method, starting from a physical analysis of the solution obtained for the disk.

For this purpose, let us investigate the structure of waves (12.01) and (12.02) which are radiated by the disk. These equations include the factor

$$\frac{ia}{\sqrt{2\pi ka(\sin\gamma+\sin\vartheta)}}\,\frac{e^{ikR}}{R}=i\,\frac{e^{ikR}}{\sqrt{2\pi kR}}\sqrt{\frac{a}{R}}\,\frac{1}{\sqrt{\sin\gamma+\sin\vartheta}}. \tag{14.01}$$

Here $\sqrt{\frac{a}{R}}$ is the unfolding coefficient of the wave. It shows how the field is formed with increasing distance from the disk: the diffracted wave which is cylindrical near the disk unfolds into a spherical wave as the distance from it increases. The coefficient $(\sin\gamma+\sin\vartheta)^{-1/2}$ is proportional to the width of the luminous region on the disk or, in other words, to the width of the first Fresnel zone. Thus, in Equations (12.01) and (12.12) the functions f^1 and g^1 depend only on the body's geometry — more precisely, on the character of the discontinuity.

Therefore, it is entirely natural to assume that the similar waves which are being scattered by a cylinder have the same structure and differ only in the functions f^1 and g^1 which correspond in this case to a rectangular wedge. Consequently, in the direction toward the source, the field from a nonuniform part of the current flowing on the cylinder may be represented when $ka\sin\vartheta \gg 1$ in the following way:

$$\left.\begin{aligned} E_\varphi = -H_\vartheta = \frac{ia}{\sqrt{2\pi\zeta}} E_{0x}\Big\{ f^1(1)\, e^{i\left(\zeta-\frac{3\pi}{4}\right)+ikl\cos\vartheta} - \\ -[f^1(2)\, e^{ikl\cos\vartheta} + f^1(3)\, e^{-ikl\cos\vartheta}]\, e^{-i\left(\zeta-\frac{3\pi}{4}\right)}\Big\}\frac{e^{ikR}}{R}, \\ E_\vartheta = H_\varphi = \frac{ia}{\sqrt{2\pi\zeta}} H_{0x}\cdot\Big\{ g^1(1)\, e^{i\left(\zeta-\frac{3\pi}{4}\right)+ikl\cos\vartheta} - \\ -[g^1(2)\, e^{ikl\cos\vartheta} + g^1(3)\, e^{-ikl\cos\vartheta}]\, e^{-i\left(\zeta-\frac{3\pi}{4}\right)}\Big\}\frac{e^{ikR}}{R}. \end{aligned}\right| \quad (14.02)$$

In accordance with § 4, the functions f^1 and g^1 are determined by the equations

$$\left.\begin{aligned} \begin{matrix} f^1(1) \\ g^1(1) \end{matrix}\Big\} = \frac{\sin\frac{\pi}{n}}{n}\left(\frac{1}{\cos\frac{\pi}{n}-1} \mp \right. \\ \left. \mp \frac{1}{\cos\frac{\pi}{n}-\cos\frac{\pi-2\vartheta}{n}}\right) \pm \frac{\cos\vartheta}{2\sin\vartheta}, \\ \begin{matrix} f^1(2) \\ g^1(2) \end{matrix}\Big\} = \frac{\sin\frac{\pi}{n}}{n}\left(\frac{1}{\cos\frac{\pi}{n}-1} \mp \frac{1}{\cos\frac{\pi}{n}-\cos\frac{2\vartheta}{n}}\right) \mp \end{aligned}\right| \quad (14.03)$$

(Equation continued on next page.)

$$\mp\frac{\cos\vartheta}{2\sin\vartheta}\mp\frac{\sin\vartheta}{2\cos\vartheta},$$

$$\left.\begin{matrix} f^1(3) \\ g^1(3) \end{matrix}\right\} = \frac{\sin\frac{\pi}{n}}{n}\left(\frac{1}{\cos\frac{\pi}{n}-1}\mp\right.$$

$$\left.\mp\frac{1}{\cos\frac{\pi}{n}-\cos\frac{\pi+2\vartheta}{n}}\right)\pm\frac{\sin\vartheta}{2\cos\vartheta}, \qquad (14.03)$$

where

$$n=\frac{3}{2}. \qquad (14.04)$$

In Chapter IV, we will show [see Equation (17.25)] that in the direction $\vartheta=\pi-\gamma=\pi$ one may neglect the field from the nonuniform part of the current flowing on the cylinder in comparison with the field from the uniform part, if $ka\gg 1$. Therefore, for the field from the nonuniform part of the current, one may write with the help of Bessel functions the following interpolation equations:

$$\left.\begin{aligned} E_\varphi=-H_\vartheta=\frac{ia}{2}E_{0x}\cdot\frac{e^{ikR}}{R}\overline{\sum}^1(\vartheta), \\ E_\vartheta=H_\varphi=\frac{ia}{2}H_{0x}\cdot\frac{e^{ikR}}{R}\sum{}^1(\vartheta). \end{aligned}\right\} \qquad (14.05)$$

Here

$$\begin{aligned} \overline{\sum}^1(\vartheta)=[\overline{M}^1J_1(\zeta)+i\overline{N}^1J_2(\zeta)]\,e^{ikl\cos\vartheta}- \\ -f^1(3)[J_1(\zeta)-iJ_2(\zeta)]\,e^{-ikl\cos\vartheta}, \\ \sum{}^1(\vartheta)=[M^1J_1(\zeta)+iN^1J_2(\zeta)]\,e^{ikl\cos\vartheta}- \\ -g^1(3)[J_1(\zeta)-iJ_2(\zeta)]\,e^{-ikl\cos\vartheta}, \end{aligned} \qquad (14.06)$$

and the functions $\overline{M^1}$, $\overline{N^1}$ and M^1, N^1 respectively equal

$$\left.\begin{matrix}\overline{M}^1 \\ \overline{N}^1\end{matrix}\right\}=f^1(1)\mp f^1(2), \quad \left.\begin{matrix}M^1 \\ N^1\end{matrix}\right\}=g^1(1)\mp g^1(2), \qquad (14.07)$$

or

$$\left.\begin{matrix}\overline{M}^1\\ M^1\end{matrix}\right\} = \frac{\sin\frac{\pi}{n}}{n}\left(\mp\frac{1}{\cos\frac{\pi}{n}-\cos\frac{\pi-2\vartheta}{n}}\pm\right.$$
$$\left.\pm\frac{1}{\cos\frac{\pi}{n}-\cos\frac{2\vartheta}{n}}\right)\pm\frac{\cos\vartheta}{\sin\vartheta}\pm\frac{\sin\vartheta}{2\cos\vartheta},$$
$$\left.\begin{matrix}\overline{N}^1\\ N^1\end{matrix}\right\} = \frac{\sin\frac{\pi}{n}}{n}\left(\frac{2}{\cos\frac{\pi}{n}-1}\mp\right.$$
$$\mp\frac{1}{\cos\frac{\pi}{n}-\cos\frac{\pi-2\vartheta}{n}}\mp$$
$$\left.\mp\frac{1}{\cos\frac{\pi}{n}-\cos\frac{2\vartheta}{n}}\right)\mp\frac{\sin\vartheta}{2\cos\vartheta}. \qquad (14.08)$$

The resulting Equations (14.05) change when $ka \sin\vartheta \gg 1$ into Equations (14.02), and in the direction $\vartheta=\pi$ they give a value equal to zero for the field.

§ 15. The Total Fringing Field

Summing Expressions (13.28) and (14.05), it is not difficult to see that the total field scattered by a cylinder will equal

$$\left.\begin{aligned}E_{\varphi} &= -H_{\vartheta} = \frac{ia}{2}E_{0x}\cdot\frac{e^{ikR}}{R}\cdot\overline{\sum}(\vartheta),\\ E_{\vartheta} &= H_{\varphi} = \frac{ia}{2}H_{0x}\cdot\frac{e^{ikR}}{R}\cdot\sum(\vartheta).\end{aligned}\right\} \qquad (15.01)$$

where

$$\left.\begin{aligned}\overline{\sum}(\vartheta) &= [\overline{M}J_1(\zeta)+i\overline{N}J_2(\zeta)]\,e^{ikl\cos\vartheta}-\\ &\quad -f(3)\,[J_1(\zeta)-iJ_2(\zeta)]\,e^{-ikl\cos\vartheta},\\ \sum(\vartheta) &= [MJ_1(\zeta)+iNJ_2(\zeta)]\,e^{ikl\cos\vartheta}-\\ &\quad -g(3)\,[J_1(\zeta)-iJ_2(\zeta)]\,e^{-ikl\cos\vartheta},\\ \zeta &= 2ka\sin\vartheta\end{aligned}\right\} \qquad (15.02)$$

and the functions $\overline{M}$, $\overline{N}$ and M, N are expressed only in terms of the functions f and g which correspond to the asymptotic solution for a rectangular wedge

$$\left.\begin{matrix}\overline{M}\\ \overline{N}\end{matrix}\right\}=f(1)\mp f(2),\quad \left.\begin{matrix}M\\ N\end{matrix}\right\}=g(1)\mp g(2), \tag{15.03}$$

or

$$\begin{aligned}
\left.\begin{matrix}\overline{M}\\ M\end{matrix}\right\}&=\frac{\sin\frac{\pi}{n}}{n}\left(\mp\frac{1}{\cos\frac{\pi}{n}-\cos\frac{\pi-2\vartheta}{n}}\pm\frac{1}{\cos\frac{\pi}{n}-\cos\frac{2\vartheta}{n}}\right),\\
\left.\begin{matrix}\overline{N}\\ N\end{matrix}\right\}&=\frac{\sin\frac{\pi}{n}}{n}\left(\frac{2}{\cos\frac{\pi}{n}-1}\mp\frac{1}{\cos\frac{\pi}{n}-\cos\frac{\pi-2\vartheta}{n}}\mp\frac{1}{\cos\frac{\pi}{n}-\cos\frac{2\vartheta}{n}}\right).
\end{aligned} \tag{15.04}$$

The functions f(3) and g(3) in turn are determined by the equation

$$\left.\begin{matrix}f(3)\\ g(3)\end{matrix}\right\}=\frac{\sin\frac{\pi}{n}}{n}\left(\frac{1}{\cos\frac{\pi}{n}-1}\mp\frac{1}{\cos\frac{\pi}{n}-\cos\frac{\pi+2\vartheta}{n}}\right). \tag{15.05}$$

Thus only the functions f and g are included in the final expressions for the scattering characteristic of a plane wave by a cylinder.

In the direction $\vartheta=\pi$, the functions $\overline{\Sigma}(\vartheta)$ and $\Sigma(\vartheta)$ take, as in the case of a disk, the values

$$\overline{\Sigma}(\pi)=-\Sigma(\pi)=kae^{-ikl}, \tag{15.06}$$

and with $\vartheta=\frac{\pi}{2}$ they respectively equal

$$\begin{aligned}
\overline{\sum}\left(\frac{\pi}{2}\right)&=-\left(\frac{\frac{2}{n}\sin\frac{\pi}{n}}{\cos\frac{\pi}{n}-1}+\frac{1}{n}\operatorname{ctg}\frac{\pi}{n}+ikl\right)[J_1(\zeta)-iJ_2(\zeta)],\\
\sum\left(\frac{\pi}{2}\right)&=i\left(\frac{\frac{4}{n}\sin\frac{\pi}{n}}{\cos\frac{\pi}{n}-1}-\frac{1}{n}\operatorname{ctg}\frac{\pi}{\pi}-ikl\right)J_2(\zeta)+\\
&\quad+\left(\frac{1}{n}\operatorname{ctg}\frac{\pi}{n}+ikl\right)J_1(\zeta),
\end{aligned} \tag{15.07}$$

where $\zeta = 2ka$. The terms in this equation which contain the factor kl refer to the field from the uniform part of the current, and the remaining terms refer to the field from the nonuniform part of the current.

In accordance with (12.17), the effective scattering area of the cylinder is determined with the E-polarization of the incident wave by the function

$$\sigma_E = \pi a^2 |\overline{\Sigma}(\vartheta)|^2, \tag{15.08}$$

and with the H-polarization of the incident wave by the function

$$\sigma_H = \pi a^2 |\Sigma(\vartheta)|^2. \tag{15.09}$$

Let us note that Expressions (15.02) for the scattering field may be obtained directly on the basis of an analogy with the Equations (12.06), omitting the calculation of the fields from the uniform and nonuniform parts of the current. In the same way, one may obtain the expressions

$$\begin{aligned}\overline{\Sigma}(\vartheta, \vartheta_0) &= [\overline{M}J_1(\xi) + i\overline{N}J_2(\xi)]\, e^{i\frac{kl}{2}(\cos\vartheta + \cos\vartheta_0)} - \\ &- f(3)[J_1(\xi) - iJ_2(\xi)]\, e^{-i\frac{kl}{2}(\cos\vartheta + \cos\vartheta_0)}, \\ \Sigma(\vartheta, \vartheta_0) &= [MJ_1(\xi) + iNJ_2(\xi)]\, e^{i\frac{kl}{2}(\cos\vartheta + \cos\vartheta_0)} - \\ &- g(3)[J_1(\xi) - iJ_2(\xi)]\, e^{-i\frac{kl}{2}(\cos\vartheta + \cos\vartheta_0)},\end{aligned} \tag{15.10}$$

which are suitable for calculating the fringing field in the region $\varphi = -\frac{\pi}{2}$, $\frac{\pi}{2} < \vartheta;\ \vartheta_0 < \pi$ $(\vartheta_0 = \pi - \gamma)$. The quantities here equal

$$\xi = ka(\sin\vartheta + \sin\vartheta_0) \tag{15.11}$$

$$\begin{aligned}&\left.\begin{matrix}\overline{M}\\ M\end{matrix}\right\} = \frac{\sin\frac{\pi}{n}}{n} \times \\ &\times \left(\mp \frac{1}{\cos\frac{\pi}{n} - \cos\frac{\pi - \vartheta - \vartheta_0}{n}} \pm \frac{1}{\cos\frac{\pi}{n} - \cos\frac{\vartheta + \vartheta_0}{n}} \right), \\ &\left.\begin{matrix}\overline{N}\\ N\end{matrix}\right\} = \frac{\sin\frac{\pi}{n}}{n} \left(\frac{2}{\cos\frac{\pi}{n} - \cos\frac{\vartheta - \vartheta_0}{n}} \mp \right. \\ &\left. \mp \frac{1}{\cos\frac{\pi}{n} - \cos\frac{\pi - \vartheta - \vartheta_0}{n}} \mp \frac{1}{\cos\frac{\pi}{n} - \cos\frac{\vartheta + \vartheta_0}{n}} \right),\end{aligned} \tag{15.12}$$

$$\left.\begin{matrix} f(3) \\ g(3) \end{matrix}\right\} = \frac{\sin\frac{\pi}{n}}{n}\left(\frac{1}{\cos\frac{\pi}{n}-\cos\frac{\vartheta-\vartheta_0}{n}} \mp \frac{1}{\cos\frac{\pi}{n}-\cos\frac{\pi+\vartheta+\vartheta_0}{n}}\right). \qquad (15.13)$$

Expressions (15.10) satisfy the reciprocity principle — that is, they do not change their values if one interchanges ϑ and ϑ_0. When $\vartheta=\vartheta_0$, they change into the previous Expressions (15.02).

Equations (15.02) and (15.10) describe the radiation from the currents flowing only on part of the cylinder's surface: on the one end (when $z = \frac{l}{2}$) and on half of the lateral surface ($-\pi \leq \psi \leq 0$). Moreover, these expressions do not take into account the nonuniform part of the current caused by the curvature of the cylindrical surface. Therefore, they must be refined with values of ϑ and ϑ_0 which are close to $\frac{\pi}{2}$ and π. However, in the case $\vartheta=\vartheta_0$ — that is, in the direction towards the source — these corrections may be neglected if the parameters ka and kl are sufficiently large. Numerical calculations performed by us on the basis of Equations (15.02) show that this evidently may be done already when ka = π and kl = 10 π. The graphs of the functions $\frac{\sigma_E}{\pi a^2}=|\bar{\Sigma}(\vartheta)|^2$ and $\frac{\sigma_H}{\pi a^2}=|\Sigma(\vartheta)|^2$ constructed for this case in Figures 25 and 26 agree with the experimental curve(1) (the dashed line): the position of the maxima and minima basically agree, and the number of diffraction fringes is the same. For the purpose of illustrating the effect of the ends, we constructed a graph of the effective scattering area for those same values of ka and kl, taking into account only the uniform part of the current on the cylindrical surface (Figure 27). A comparison of Figures 25, 26 and 27 shows that the effect of the ends begins to appear when ϑ = 120°.

Footnote (1) appears on page 89.

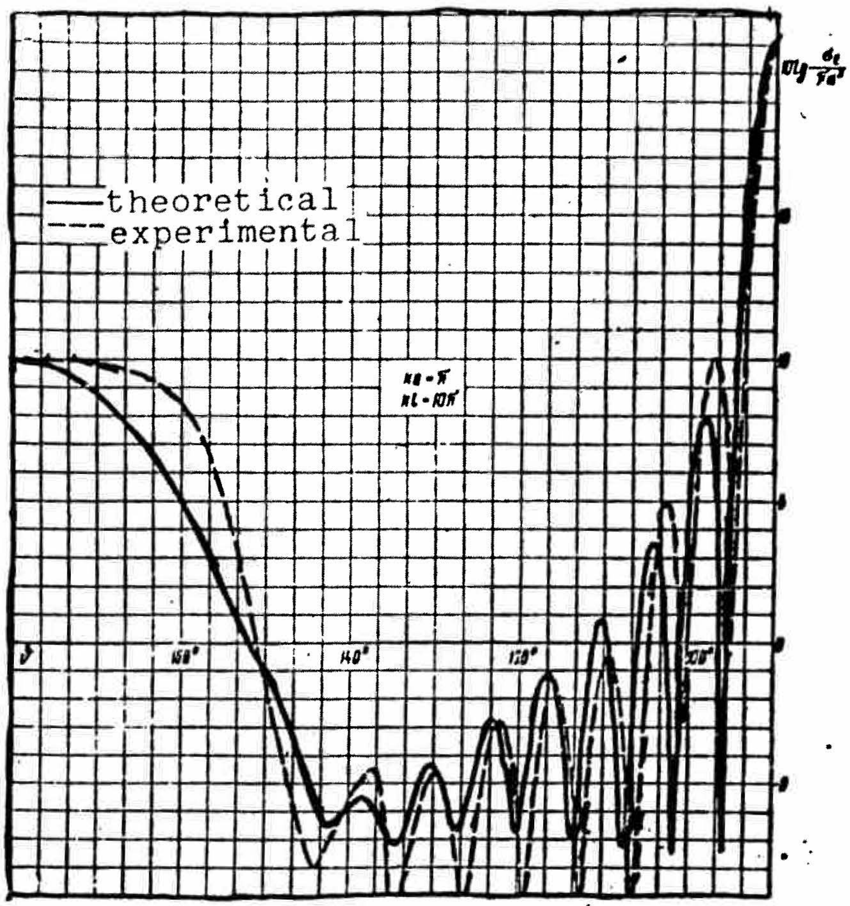


Figure 25. Diagram of the effective scattering surface for a finite cylinder. The case of E-polarization.

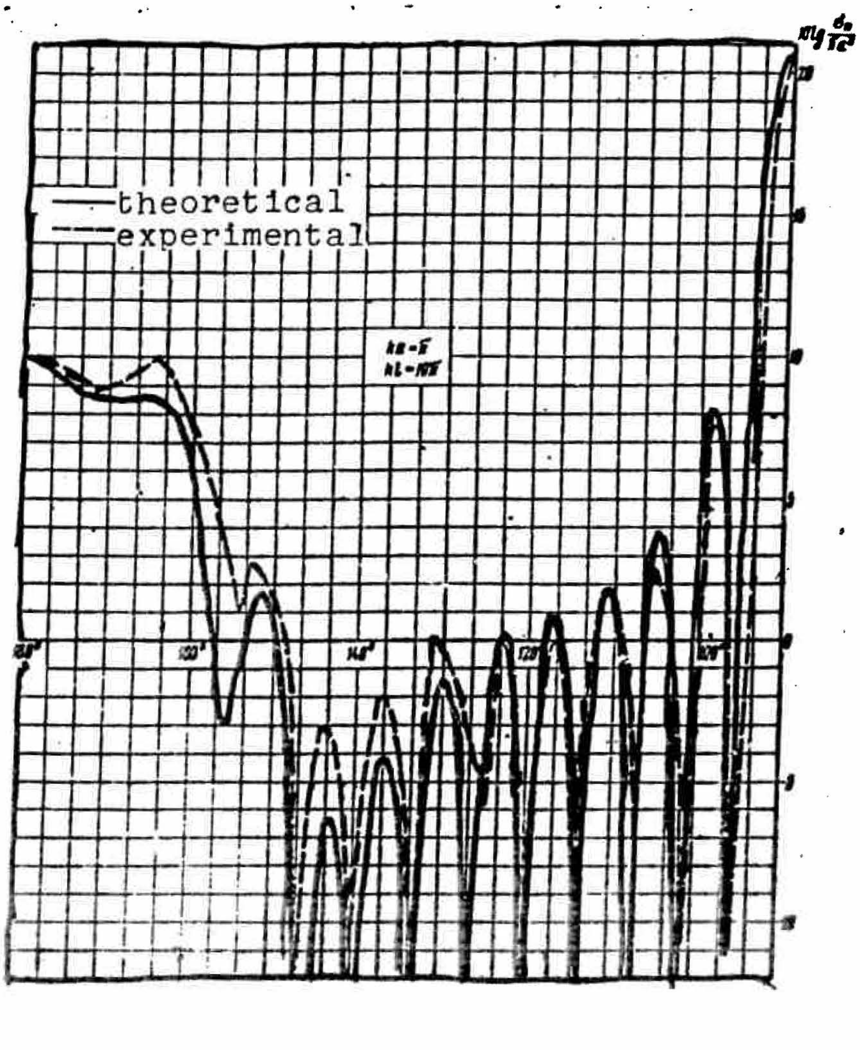


Figure 26. Diagram of the effective scattering surface for a finite cylinder. The case of H-polarization

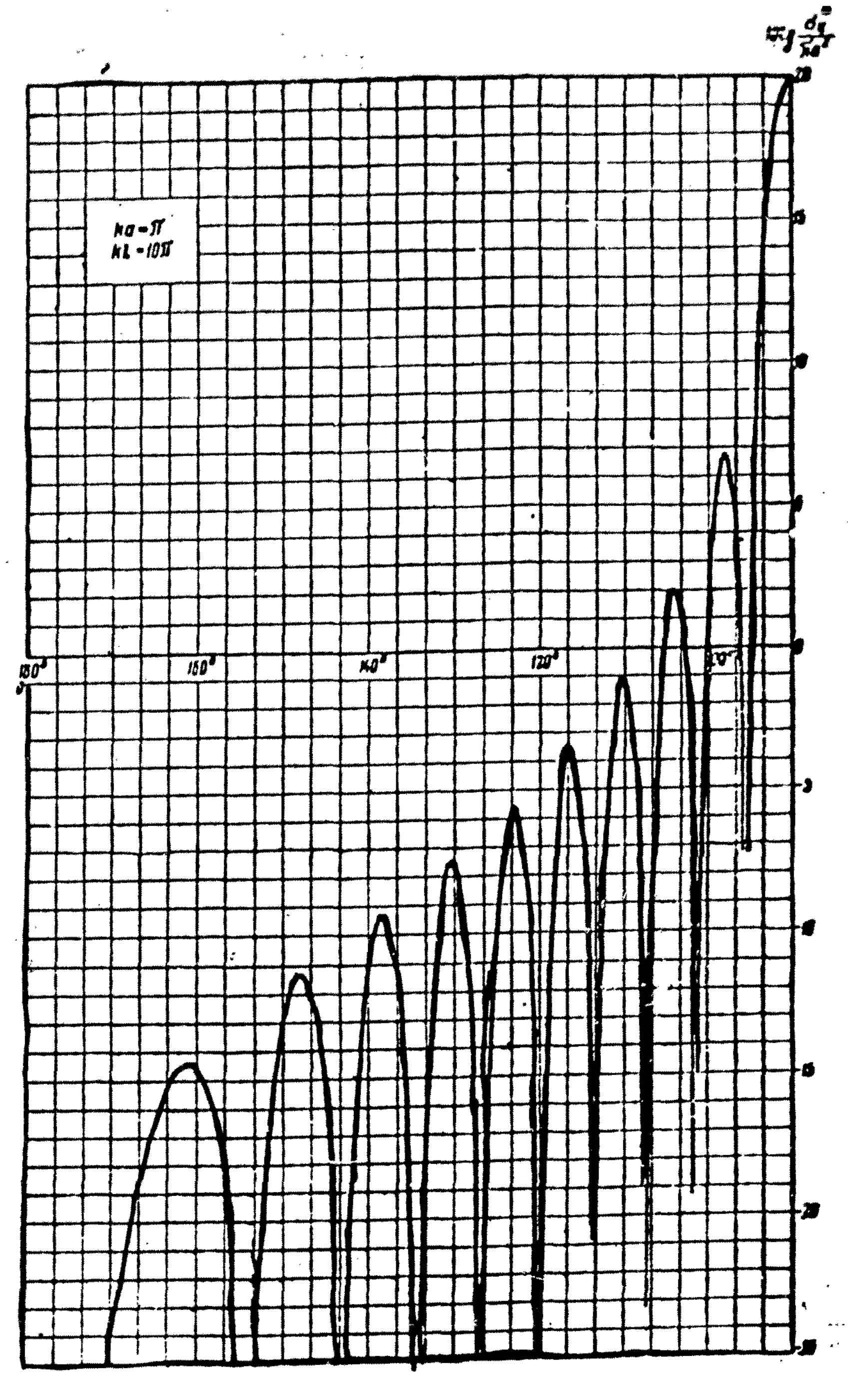


Figure 27. The effective scattering area of the lateral surface of a cylinder in the physical optics approach [see (13.26)].

FOOTNOTES

1. on page 86. The experimental curves shown in Figures 25 and 26 and also those in Figures 31, 32, 65 and 71 were obtained by Ye. N. Mayzels and L. S. Chugunova.

CHAPTER IV

DIFFRACTION OF A PLANE WAVE INCIDENT ALONG THE SYMMETRY AXIS OF FINITE BODIES OF ROTATION

In this Chapter we will refine the physical optics approach for certain other bodies of rotation, whose surfaces have circular discontinuities. We will limit ourselves to the case when a plane electromagnetic wave falls on the bodies along their symmetry axis.

As before, we will assume that the linear dimensions of the bodies are large in comparison with the wavelength. In this case the currents in the vicinity of a circular discontinuity of any convex surface of rotation may be approximately considered to be the same as that on a corresponding conical body. Consequently, it is sufficient to study the field from the nonuniform part of the current which is caused by the circular discontinuity of the surface, using such a body as an example.

§ 16. The Field Created by the Nonuniform Part of the Current

Let a plane electromagnetic wave fall on a conical body in the positive direction of the z axis (Figure 28). From the relationships

$$\left.\begin{aligned} \mathbf{E} &= -\frac{1}{ik}(\text{grad div } \mathbf{A} + k^2\mathbf{A}), \\ \mathbf{H} &= \text{rot } \mathbf{A} \end{aligned}\right\} \tag{16.01}$$

we find the following expressions for the fringing field in the wave zone:

$$\left.\begin{aligned} E_x &= H_y = ikA_x, \\ E_y &= -H_x = ikA_y \end{aligned}\right\} \text{ with } \vartheta = 0 \tag{16.02}$$

and

$$\left.\begin{aligned} E_x &= -H_y = ikA_x, \\ E_y &= H_x = ikA_y \end{aligned}\right\} \text{with } \vartheta = \pi. \tag{16.03}$$

The vector potential is determined by the equation

$$\begin{aligned} \mathbf{A} = \frac{1}{c}\frac{e^{ikr}}{r}\int_0^{2\pi}\Big[\int_0^{l_1} \mathbf{j}_1(\zeta)\, e^{\pm ik\zeta\cos\omega}(a - \zeta\sin\omega)\, d\zeta + \\ + \int_0^{l_2} \mathbf{j}_2(\zeta)\, e^{\mp ik\zeta\cos\Omega}(a - \zeta\sin\Omega)\, d\zeta\Big]\, d\psi. \end{aligned} \tag{16.04}$$

Here r is the distance from the discontinuity to the observation point, $j_1(\zeta)$ is the surface current density flowing on the irradiated side of the body, and $j_2(\zeta)$ is the current density on the shadowed side. The upper sign in the exponents refers to the case $\vartheta = \pi$, and the lower sign refers to the case $\vartheta = 0$. Since the nonuniform part of the current is concentrated mainly in the vicinity of the discontinuity, the vector potential corresponding to it may be represented in the form

$$\begin{aligned} \mathbf{A} = \frac{a}{c}\frac{e^{ikr}}{r}\int_0^{2\pi}\Big(\int_0^{\infty} \mathbf{j}_1^1(\zeta)\, e^{\pm ik\zeta\cos\omega} d\zeta + \\ + \int_0^{\infty} \mathbf{j}_2^1(\zeta)\, e^{\mp ik\zeta\cos\Omega} d\zeta\Big)\, d\psi. \end{aligned} \tag{16.05}$$

Obviously the nonuniform part of the current near the discontinuity of a conical surface may be considered to be approximately the same as on a corresponding wedge (Figure 29). In the local cylindrical coordinate system r_1, ϕ_1, z_1, the field from the nonuniform part

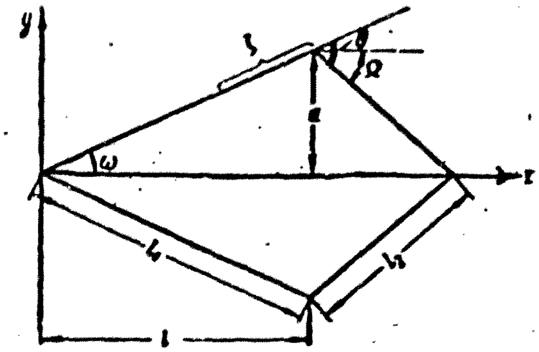


Figure 28. Diffraction of a plane wave by a conical body. The plane wave is propagated along the z axis.

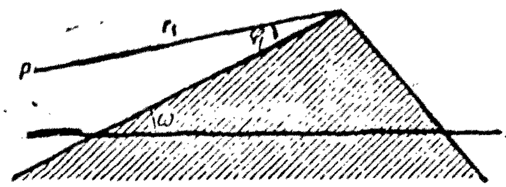


Figure 29. The dihedral angle corresponding to the discontinuity of a conical surface.

of the current flowing on such a wedge is determined in the far zone ($kr_1 >> 1$) by the following equations:

$$\left.\begin{aligned} E_{z_1}(\psi) &= -H_{\varphi_1}(\psi) = ikA_{z_1}(\psi), \\ H_{z_1}(\psi) &= E_{\varphi_1}(\psi) = ikA_{\varphi_1}(\psi), \end{aligned}\right\} \qquad (16.06)$$

where

$$A = \frac{1}{c}\sqrt{\frac{2\pi}{k_1 r_1}}\left[\int_0^\infty j_1^1(\zeta)\, e^{\pm ik\zeta\cos\omega} d\zeta + \int_0^\infty j_2^1(\zeta)\, e^{\mp ik\zeta\cos\Omega} d\zeta\right] e^{i\left(kr_1 + \frac{\pi}{4}\right)}. \qquad (16.07)$$

Here the upper sign in the exponents refers to the case $\phi_1 = \pi + \omega$, and the lower sign — to the case $\phi_1 = \omega$. On the other hand, in § 4 it was shown that this field equals

$$\left.\begin{aligned} E_{z_1}(\psi) &= E_{0z_1}(\psi) f^1 \frac{e^{i\left(kr_1 + \frac{\pi}{4}\right)}}{\sqrt{2\pi kr_1}}, \\ H_{z_1}(\psi) &= H_{0z_1}(\psi) g^1 \frac{e^{i\left(kr_1 + \frac{\pi}{4}\right)}}{\sqrt{2\pi kr_1}}, \end{aligned}\right\} \qquad (16.08)$$

where $E_{0z_1}(\psi)$, $H_{0z_1}(\psi)$ are the values of the incident wave amplitude at the wedge edge, and f^1 and g^1 are angular functions characterizing the scattering diagram.

Let us introduce the designation

$$J=\int_0^\infty j_1^1(\zeta)\,e^{\pm ik\zeta\cos\vartheta}d\zeta+\int_0^\infty j_2^1(\zeta)e^{\mp ik\zeta\cos\vartheta}d\zeta. \tag{16.09}$$

Equating Expressions (16.06) and (16.08), we find

$$J_{z_1}=\frac{cE_{0z_1}(\psi)}{ik2\pi}f^1,\quad J_{\varphi_1}=\frac{cH_{0z_1}(\psi)}{ik2\pi}g^1. \tag{16.10}$$

The components J_{z_1} and J_{ϕ_1} are mutually perpendicular, and when $\vartheta=0$ and $\vartheta=\pi$ they are parallel to the plane x0y (Figure 30). The different orientation of the unit vector e_{ϕ_1} when $\vartheta=0$ and $\vartheta=\pi$ is connected with the fact that the angle ϕ_1 is measured from the irradiated face of the wedge. In the original x, y, z coordinate system the vector **J** has the components

$$\left.\begin{aligned}J_x&=J_{z_1}\sin\psi-J_{\varphi_1}\cos\psi,\\ J_y&=-J_{z_1}\cos\psi-J_{\varphi_1}\sin\psi\end{aligned}\right\}\text{with }\vartheta=0 \tag{16.11}$$

and

$$\left.\begin{aligned}J_x&=J_{z_1}\sin\psi+J_{\varphi_1}\cos\psi,\\ J_y&=-J_{z_1}\cos\psi+J_{\varphi_1}\sin\psi\end{aligned}\right\}\text{with }\vartheta=\pi. \tag{16.12}$$

Substituting Expressions (16.10) here, we obtain

$$\left.\begin{aligned}J_x&=\frac{c}{ik2\pi}[f^1E_{0z_1}(\psi)\sin\psi-g^1H_{0z_1}(\psi)\cos\psi],\\ J_y&=\frac{-c}{ik2\pi}[f^1E_{0z_1}(\psi)\cos\psi+g^1H_{0z_1}(\psi)\sin\psi]\end{aligned}\right\}\text{with }\vartheta=0 \tag{16.13}$$

and

$$\left.\begin{aligned}J_x&=\frac{c}{ik2\pi}[f^1E_{0z_1}(\psi)\sin\psi+g^1H_{0z_1}(\psi)\cos\psi],\\ J_y&=-\frac{c}{ik2\pi}[f^1E_{0z_1}(\psi)\cos\psi-g^1H_{0z_1}(\psi)\sin\psi]\end{aligned}\right\}\text{with }\vartheta=\pi. \tag{16.14}$$

Now identifying the current near the conical surface discontinuity with the current on the wedge, we find the components of vector potential (16.05)

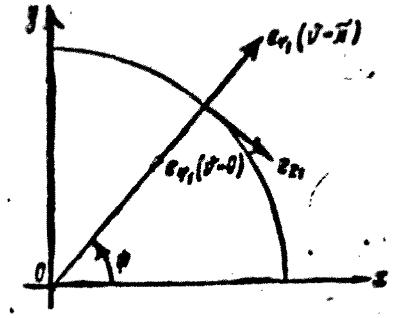


Figure 30. The relative orientation of the unit vectors e_{ϕ_1} and e_{z_1} in the cases $\vartheta=0$ and $\vartheta=\pi$.

$$\left.\begin{aligned} A_x &= \frac{a}{ik2\pi}\frac{e^{ikr}}{r}\int_0^{2\pi}[f^1E_{0z_1}(\psi)\sin\psi - \\ &\quad - g^1H_{0z_1}(\psi)\cos\psi]\,d\psi, \\ A_y &= -\frac{a}{ik2\pi}\frac{e^{ikr}}{r}\int_0^{2\pi}[f^1E_{0z_1}(\psi)\cos\psi + \\ &\quad + g^1H_{0z_1}(\psi)\sin\psi]\,d\psi \end{aligned}\right\} \text{ with } \vartheta=0 \tag{16.15}$$

and

$$\left.\begin{aligned} A_x &= \frac{a}{ik2\pi}\frac{e^{ikr}}{r}\int_0^{2\pi}[f^1E_{0z_1}(\psi)\sin\psi + \\ &\quad + g^1H_{0z_1}(\psi)\cos\psi]\,d\psi, \\ A_y &= -\frac{a}{ik2\pi}\frac{e^{ikr}}{r}\int_0^{2\pi}[f^1E_{0z_1}(\psi)\cos\psi - \\ &\quad - g^1H_{0z_1}(\psi)\sin\psi]\,d\psi. \end{aligned}\right\} \text{ with } \vartheta=\pi. \tag{16.16}$$

Furthermore, let the plane wave be polarized in such a way that $E_0||0x$. Then

$$E_{0z_1}(\psi)=E_{0x}\sin\psi, \quad H_{0z_1}(\psi)=-E_{0x}\cos\psi. \tag{16.17}$$

Considering these relationships and substituting Expressions (16.15) and (16.16) into Equations (16.02) and (16.03), we find the field from the nonuniform part of the current which is caused by the circular discontinuity of the conical surface

$$\left.\begin{aligned} E_x &= H_y = \frac{aE_{0x}}{2}(f^1+g^1)\frac{e^{ikr}}{r} \\ E_y &= H_x = 0 \end{aligned}\right\} \text{ with } \vartheta=0 \tag{16.18}$$

and

$$\left.\begin{aligned} E_x &= -H_y = \frac{aE_{0x}}{2}(f^1-g^1)\frac{e^{ikr}}{r} \\ E_y &= H_x = 0 \end{aligned}\right\} \text{ with } \vartheta=\pi. \tag{16.19}$$

Equation (16.18) is applicable for the values $0\leqslant\omega\leqslant\frac{\pi}{2}$, $\omega<\theta\leqslant\pi$, and Equation (16.19) for the values $0\leqslant\omega\leqslant\frac{\pi}{2}$, $0<\theta\leqslant\pi$. In the case of a disk$\left(\omega=\frac{\pi}{2},\ \theta=\pi\right)$, the field from the nonuniform part of the current equals zero on the z axis, since $f^1 = -g^1 = -1/2$ when $\vartheta=0$, and $f^1 = g^1 = -1/2$ when $\vartheta=\pi$ [compare (8.16)].

Using the resulting relationships in the following sections,we will calculate the effective scattering area (in the direction $\vartheta=\pi$) for specific bodies. We shall assume that they are irradiated by the plane wave

$$E_x = H_y = E_{0x} e^{ikz}, \tag{16.20}$$

and their linear dimensions are large in comparison with the wavelength.

§ 17. A Cone

Let a cone (Figure 28) be irradiated by plane electromagnetic wave (16.20). The uniform part of the current which is excited on the cone's surface has the components

$$\left.\begin{aligned} j_x^0 &= \frac{c}{2\pi} E_{0x} \sin\omega\, e^{ikz}, \\ j_y^0 &= 0, \\ j_z^0 &= \frac{c}{2\pi} E_{0x} \cos\omega \cos\psi\, e^{ikz} \end{aligned}\right\} \tag{17.01}$$

and creates in the direction $\vartheta=\pi$ (with $R \gg ka^2$, $R \gg kl^2$) the field

$$\left.\begin{aligned} E_x = -H_y = -E_{0x}\frac{i}{4k}\operatorname{tg}^2\omega \frac{e^{ikR}}{R} + \\ + E_{0x}\left(\frac{i}{4k}\operatorname{tg}^2\omega + \frac{a}{2}\operatorname{tg}\omega\right)\frac{e^{ikR}}{R} e^{2ikl}, \\ E_y = H_x = 0. \end{aligned}\right\} \tag{17.02}$$

Here the first term describes the spherical wave diverging from the vertex of the cone, and the remaining terms describe the spherical wave from its base.

The field caused by the discontinuity of the surface at the cone base is a spherical wave, and is determined in accordance with

(16.19) by the expression

$$E_x = -H_y = -\frac{a}{2} E_{0x}\left(\operatorname{tg}\omega + \frac{\frac{2}{n}\sin\frac{\pi}{n}}{\cos\frac{\pi}{n} - \cos\frac{2\omega}{n}}\right)\frac{e^{ikR}}{R}\, e^{2ikl}, \quad E_y = H_x = 0, \tag{17.03}$$

where

$$n = 1 + \frac{\omega + \Omega}{\pi}. \tag{17.04}$$

An asymptotic calculation of the rigorous diffraction series for a semi-infinite cone [38-40] shows that in the direction $\theta = \pi$ one may neglect the effect of the nonuniform part of the current caused by the conical point. Therefore, summing (17.02) and (17.03), we obtain the following expression for the fringing field:

$$E_x = -H_y = -E_{0x}\left[\frac{i}{2}\operatorname{tg}^2\omega\,(1 - e^{2ikl}) + ka\frac{\frac{2}{n}\sin\frac{\pi}{n}}{\cos\frac{\pi}{n} - \cos\frac{2\omega}{n}} e^{2ikl}\right]\frac{e^{ikR}}{2kR}, \quad E_y = H_x = 0. \tag{17.05}$$

Let us point out the following important feature of the resulti[ng] equation. In the problems which were investigated in the previous chapters, the edge waves of the fringing field were expressed only in terms of the functions f and g. But now in the equation for the spherical wave from the cone's base, in addition to the term which depends on f and g [the last term in the bracket of Equation (17.05) there is an additional term [term $-i/_2\operatorname{tg}^2\omega e^{2ikl}$ in Equation (17.05)] whi[ch] does not depend on these functions and is determined by the [non]uniform part of the current Therefore, it is impossible to represe[nt] the resulting spherical wave from the cone's base only in terms of the functions f and g which characterize the total edge wave diagra[m] from the corresponding wedge edge. This important fact was not considered in [41, 44], as a consequence of which their authors did not succeed in obtaining correct results for a cone with an arbitrary aperture angle $\omega (0 \leq \omega \leq \pi/2)$.

The effective scattering area in accordance with (12.18) is determined by the equation

$$\sigma = \pi a |\Sigma|^2, \tag{17.06}$$

where the function Σ is connected with the fringing field by the relationship

$$E_x = -H_y = -\frac{a}{2} E_{0x} \frac{e^{ikR}}{R} \Sigma \tag{17.07}$$

and equals

$$\Sigma = \frac{1}{ka} \operatorname{tg}^2 \omega \sin kl \, e^{ikl} + \frac{\frac{2}{n} \sin \frac{\pi}{n}}{\cos \frac{\pi}{n} - \cos \frac{2\omega}{n}} e^{2ikl}. \tag{17.08}$$

The analogous function in the physical optics approach may be written in accordance with (17.02) in the form

$$\Sigma^0 = \frac{1}{ka} \operatorname{tg}^2 \omega \sin kl \, e^{ikl} - \operatorname{tg} \omega \, e^{2ikl}. \tag{17.09}$$

With the deforming of the top part of the cone into a disk $\left(\omega \rightarrow \frac{\pi}{2}, l \rightarrow 0\right)$, Equations (17.08) and (17.09) are transformed, respectively, to the form

$$\left.\begin{aligned} \Sigma &= -ika - \frac{1}{n} \operatorname{ctg} \frac{\pi}{n}, \\ \Sigma^0 &= -ika. \end{aligned}\right\} \tag{17.10}$$

Furthermore, it follows from (17.08) and (17.09) that for large values of the parameter $ka (ka \gg \operatorname{tg}^2 \omega)$ the functions Σ and Σ^0 may be represented in the form

$$\Sigma = \frac{\frac{2}{n} \sin \frac{\pi}{n}}{\cos \frac{\pi}{n} - \cos \frac{2\omega}{n}} e^{2ikl}, \tag{17.11}$$

$$\Sigma^0 = \mathrm{tg}\,\omega\, e^{2ikl}. \qquad (17.12)$$

Thus even in the case of short waves ($ka \gg \mathrm{tg}^2\omega$, but $R \gg kl^2$), our Expression (17.08) does not change into the physical optics equation, but substantially differs from it because

$$\sigma = \pi a^2 \left| \frac{\frac{2}{n}\sin\frac{\pi}{n}}{\cos\frac{\pi}{n} - \cos\frac{2\omega}{n}} \right|^2 \qquad (17.13)$$

and

$$\sigma^0 = \pi a^2\, \mathrm{tg}^2\,\omega. \qquad (17.14)$$

With this

$$\sigma = \sigma^0 \left| \frac{\frac{2}{n}\sin\frac{\pi}{n}}{\left(\cos\frac{\pi}{n} - \cos\frac{2\omega}{n}\right)\mathrm{tg}\,\omega} \right|^2, \qquad (17.15)$$

that is, for sufficiently short waves (or for sufficiently large dimensions of the cone) the function σ is proportional to σ^0. The coefficient of proportionality here does not depend on the cone dimensions, but is determined only by its shape.

This result is graphically illustrated by the curves giving the effective scattering area of a cone ($\omega = 10°25'$, $k = \pi$, $\Omega = 90°$) as a function of its length (Figure 31). Whereas our equation (the continuous line) is in satisfactory agreement with the results of measurements (the small crosses)[(1)], the physical optics approach (the dashed line) gives values which are smaller than the experimental values by 13-15 dB. For sharply pointed cones, the nonuniform part of the current has an especially large values. In Figure 32, a curve is constructed for the effective surface of a cone ($ka = 2.75\,\pi$, $\Omega = 90°$)

Footnote (1) appears on page 113.

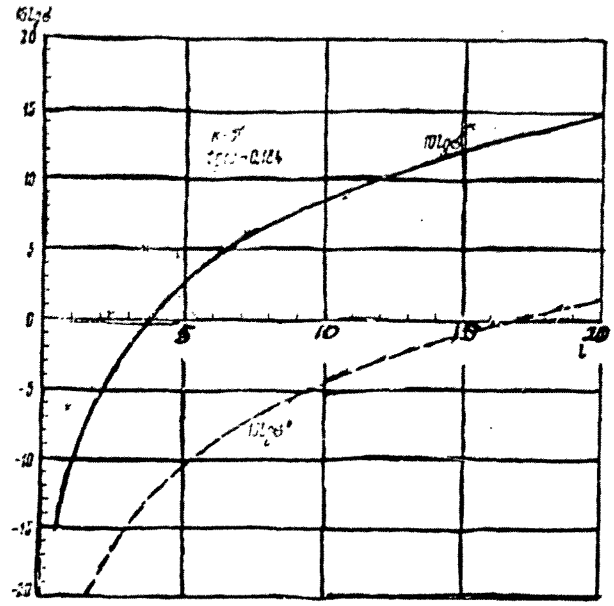


Figure 31. The effective scattering area of a finite cone as a function of its length. The function σ (the continuous line) was calculated on the basis of equation (17.06) which considers the nonuniform part of the current in the vicinity of the circular discontinuity. The function σ^0 (the dashed line) corresponds to the physical optics approach.

with its deformation into a disk ($\omega \to 90°$). The discrepancy between our curve and the physical optics approach here reaches almost 30 dB when $\omega = 2°$.

Expression (17.08) obtained by us also allows one, in contrast to the physical optics approach (17.09), to evaluate the role of the shape of the shadowed part of the body and shows that the reflected signal will be larger, the closer this shape is to a funnel-shaped form ($\Omega \approx \pi - \omega$). Thus, for example, in the case $\omega = 10°$, $kl = 10\pi$ ($k = \pi$) the signal reflected by the cone may exceed by 15 dB the value corresponding to physical optics (see Figure 33) if $\Omega \approx 170°$.

Let us note that our Expression (17.13) is equivalent to the expression presented in the above-mentioned papers [41, 44]. However, the latter expression is applicable only for sharply pointed cones, whereas we have, in addition to (17.13), Equation (17.08) which is suitable for cones with any aperture angle ω $\left(0 \leqslant \omega \leqslant \frac{\pi}{2}\right)$.

The calculation method discussed may be generalized in the case of asymmetric irradiation of the cone. However, with asymmetric irradiation, generally speaking, it is necessary to take into account the nonuniform part of the current caused by the point of the cone.

In concluding this section, let us calculate the effective scattering area for a body which is formed by rotation around the

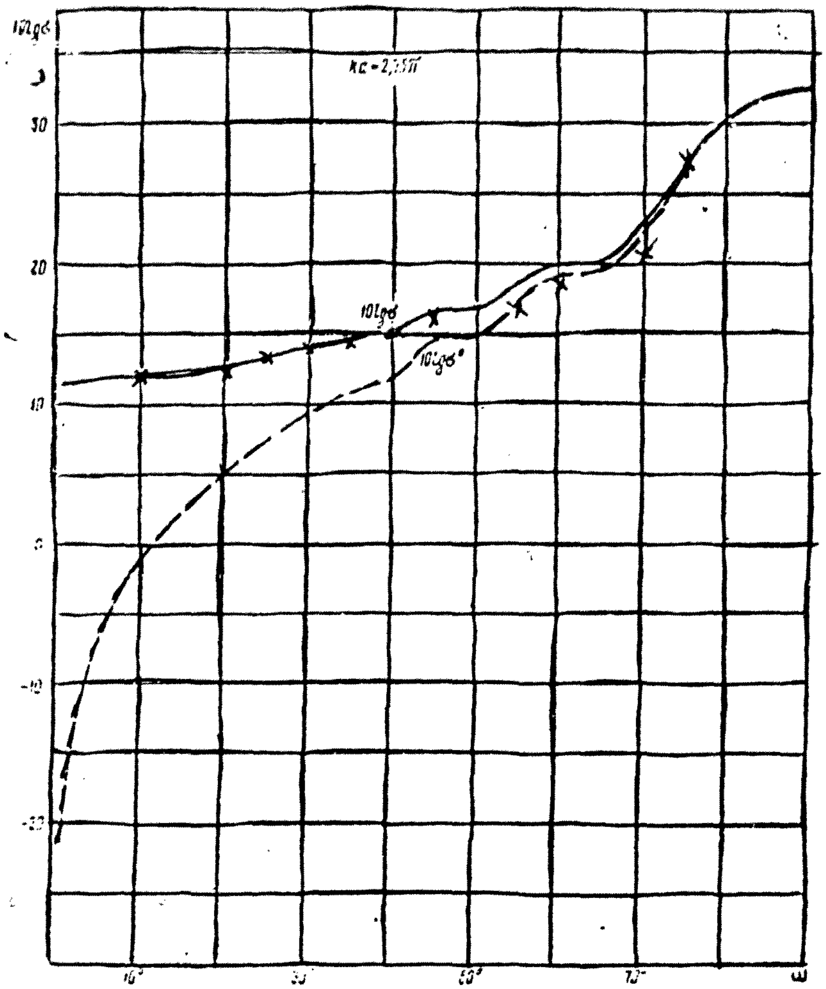
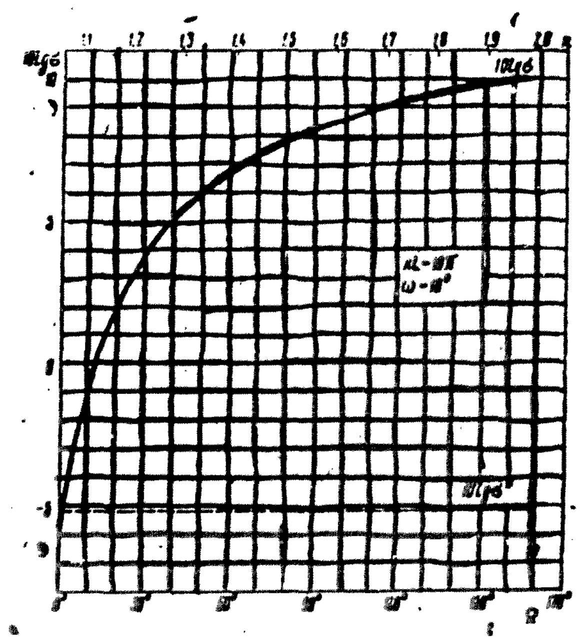


Figure 32. The effective scattering area of a finite cone as a function of the vertex angle.



ɡure 33. The effective scattering area of a finite cone as a ınction of the shape of the ıaded part.

z axis of the plane figure shown in Figure 34. Integrating the uniform part of the current, it is not difficult to show that the field scattered in the direction $\vartheta=\pi$ by the lateral surface of the truncated cone (Figure 35) is determined by the equation

$$E_x = -H_y = E_{0x}\cdot\left[-\left(\frac{i}{4k}\operatorname{tg}^2\omega_1+\frac{a_1}{2}\operatorname{tg}\omega_1\right)e^{2ikl_1}+\right.$$

$$\left.+\left(\frac{i}{4k}\operatorname{tg}^2\omega_1+\frac{a_2}{2}\operatorname{tg}\omega_1\right)e^{2ik(l_1+l_2)}\right]\frac{e^{ikR}}{R}. \quad (17.16)$$

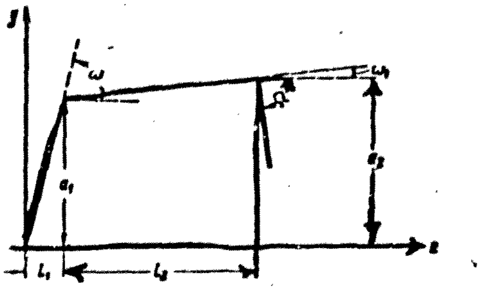


Figure 34. The generatrix of the surface of rotation.

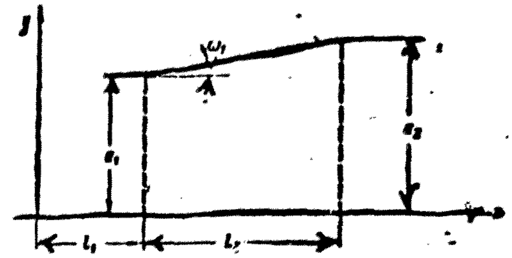


Figure 35. The generatrix of of a truncated conical surface.

Summing this expressing with (17.02), where the quantities l and a must be replaced by l_1 and a_1, we find the field from the uniform part of the current flowing on the entire illuminated side of the body

$$E_x = -H_y = -\frac{a_1 E_{0x}}{2}\left\{\frac{1}{ka_1}\operatorname{tg}^2\omega \sin kl_1 e^{ikl_1} - \operatorname{tg}\omega\, e^{2ikl_1} + \left[\frac{1}{ka_1}\operatorname{tg}^2\omega_1 \sin kl_2\, e^{ikl_2} + \left(1 - \frac{a_2}{a_1} e^{2ikl_2}\right)\operatorname{tg}\omega_1\right] e^{2ikl_1}\right\}\frac{e^{ikR}}{R}. \quad (17.17)$$

The field radiated by the nonuniform part of the current is determined in accordance with § 16 by the equation

$$E_x = -H_y = -\frac{a_1 E_{0x}}{2}\left[\left(\frac{\frac{2}{n_1}\sin\frac{\pi}{n_1}}{\cos\frac{\pi}{n_1} - \cos\frac{2\omega}{n_1}} + \operatorname{tg}\omega - \operatorname{tg}\omega_1\right) e^{2ikl_1} + \frac{a_2}{a_1}\left(\frac{\frac{2}{n_2}\sin\frac{\pi}{n_2}}{\cos\frac{\pi}{n_2} - \cos\frac{2\omega_1}{n_2}} + \operatorname{tg}\omega_1\right) e^{2ik(l_1+l_2)}\right]\frac{e^{ikR}}{R}, \quad (17.18)$$

where

$$n_1 = 1 + \frac{\omega - \omega_1}{\pi}, \quad n_2 = 1 + \frac{\omega_1 + \Omega}{\pi}. \quad (17.19)$$

Now summing (17.17) and (17.18), we obtain a refined expression for the field scattered in the direction $\vartheta = \pi$

$$E_x = -H_y = -\frac{a_1 E_{0x}}{2}\left(\frac{1}{ka_1}\operatorname{tg}^2\omega \sin kl_1 e^{ikl_1} + \right. \quad (17.20)$$

(Equation continued on next page.)

$$+\frac{\frac{2}{n_1}\sin\frac{\pi}{n_1}}{\cos\frac{\pi}{n_1}-\cos\frac{2\omega}{n_1}}e^{2ikl_1}+\frac{1}{ka_1}\mathrm{tg}^2\omega_1\sin kl_2 e^{ikl_2+2ikl_1}+$$

$$+\frac{a_2}{a_1}\frac{\frac{2}{n_2}\sin\frac{\pi}{n_2}}{\cos\frac{\pi}{n_2}-\cos\frac{2\omega_1}{n_2}}e^{2ik(l_1+l_2)}\Bigg)\frac{e^{ikR}}{R}. \quad (17.20)$$

Consequently, the effective scattering area will equal

$$\sigma=\pi a_1^2\left|\frac{1}{ka_1}\mathrm{tg}^2\omega\sin kl_1 e^{ikl_1}+\frac{\frac{2}{n_1}\sin\frac{\pi}{n_1}}{\cos\frac{\pi}{n_1}-\cos\frac{2\omega}{n_1}}e^{2ikl_1}+\right.$$

$$\left.+\frac{1}{ka_1}\mathrm{tg}^2\omega_1\sin kl_2 e^{ikl_2+2ikl_1}+\frac{a_2}{a_1}\frac{\frac{2}{n_2}\sin\frac{\pi}{n_2}}{\cos\frac{\pi}{n_2}-\cos\frac{2\omega_1}{n_2}}e^{2ik(l_1+l_2)}\right|^2. \quad (17.21)$$

In the physical optics approach, the analogous quantity equals

$$\sigma^0=\pi a_1^2\left|\frac{1}{ka_1}\mathrm{tg}^2\omega\sin kl_1 e^{ikl_1}-\mathrm{tg}\,\omega e^{2ikl_1}+\right.$$

$$\left.+\left[\frac{1}{ka_1}\mathrm{tg}^2\omega_1\sin kl_2 e^{ikl_2}+\left(1-\frac{a_2}{a_1}e^{2ikl_2}\right)\mathrm{tg}\,\omega_1\right]e^{2ikl_1}\right|^2. \quad (17.22)$$

When the top part of the cone is deformed into a disk $\left(\omega\to\frac{\pi}{2},\ l_1\to 0\right)$, Equations (17.21) and (17.22) take the form

$$\sigma=\pi a_1^2\left|-ika_1-\frac{1}{n_1}\mathrm{ctg}\frac{\pi}{n_1}+\frac{1}{ka_1}\mathrm{tg}^2\omega_1\sin kl_2 e^{ikl_2}+\right.$$

$$\left.+\frac{a_2}{a_1}\frac{\frac{2}{n_2}\sin\frac{\pi}{n_2}}{\cos\frac{\pi}{n_2}-\cos\frac{2\omega_1}{n_2}}e^{2ikl_2}\right|^2, \quad (17.23)$$

$$\sigma^0=\pi a_1^2\left|-ika_1+\frac{1}{ka_1}\mathrm{tg}^2\omega_1\sin kl_2 e^{ikl_2}+\right.$$

$$\left.+\left(1-\frac{a_2}{a_1}e^{2ikl_2}\right)\mathrm{tg}\,\omega_1\right|^2. \quad (17.24)$$

In these expressions assuming $\omega_1 = 0$, we find the effective scattering area for a finite cylinder

$$\sigma = \pi a_1^2 \left| -ika_1 - \frac{1}{n_1}\,\mathrm{ctg}\,\frac{\pi}{n_1} + \frac{\frac{2}{n_2}\sin\frac{\pi}{n_2}}{\cos\frac{\pi}{n_2} - 1}\, e^{2ikl_0} \right|^2, \tag{17.25}$$

$$\sigma^0 = \pi a_1^2 (ka_1)^2, \tag{17.26}$$

in connection with which

$$n_1 = \frac{3}{2},\ n_2 = 1 + \frac{\Omega}{\pi}. \tag{17.27}$$

Equation (17.25) is more precise than Equation (15.06) which was derived in § 15, where the value of the field in the direction $\vartheta = \pi$ was taken in the physical optics approach.

§ 18. A Paraboloid of Rotation

Let us calculate the effective scattering area of a paraboloid of rotation $r^2 = 2pz$ (Figure 36) which is irradiated by plane wave (16.20). The uniform part of the current excited on the paraboloid's surface has the components

$$\left.\begin{aligned} j_x^0 &= \frac{c}{2\pi} E_{0x} \sin\alpha e^{ikz}, \\ j_y^0 &= 0, \\ j_z^0 &= \frac{c}{2\pi} E_{0x} \cdot \cos\alpha \cos\psi e^{ikz}. \end{aligned}\right\} \tag{18.01}$$

Integrating this current, it is not difficult to show that in the direction $\vartheta = \pi$ it radiates the field

$$\left.\begin{aligned} E_x &= -H_y = -E_{0x} \cdot \frac{a}{2} (1 - e^{2ikl})\,\mathrm{tg}\,\omega \frac{e^{ikR}}{R}, \\ E_y &= H_x = 0. \end{aligned}\right\} \tag{18.02}$$

Here a is the radius of the base of the paraboloid; $l = \frac{a^2}{2p} = \frac{a}{2}\,\mathrm{ctg}\,\omega$ is its length; α is the angle between the z axis and the tangent to the generatrix of the paraboloid ($r^2 = 2pz$). At the point $z = l$, the angle α takes the value $\alpha = \omega \left(\mathrm{tg}\,\omega = \frac{p}{a}\right)$.

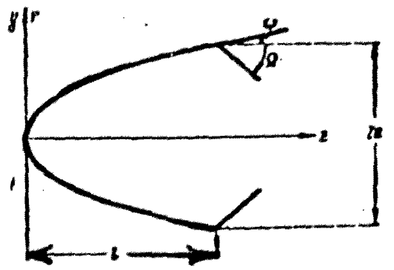


Figure 36. Diffraction of a plane wave by a paraboloid of rotation.

The field from the nonuniform part of the current caused by the discontinuity of the paraboloid's surface is determined by Equation (17.03). The field from the nonuniform part of the current which is caused by the smooth curve of the paraboloid's surface equals zero in the case of symmetric radiation [45]. Therefore, summing (18.02) and (17.03) we find the expression for the resulting fringing field.

$$\left.\begin{aligned} &E_x = -H_y = -\frac{aE_{0x}}{2}\left(\operatorname{tg}\omega + \frac{\frac{2}{n}\sin\frac{\pi}{n}}{\cos\frac{\pi}{n} - \cos\frac{2\omega}{n}} e^{2ikl}\right)\frac{e^{ikR}}{R}, \\ &E_y = H_x = 0 \\ &\left(n = 1 + \frac{\omega + \Omega}{\pi}\right). \end{aligned}\right\} \quad (18.03)$$

Consequently, the effective scattering area of the paraboloid will be determined by the relationship

$$\sigma = \pi a^2 \left| \operatorname{tg}\omega + \frac{\frac{2}{n}\sin\frac{\pi}{n}}{\cos\frac{\pi}{n} - \cos\frac{2\omega}{n}} e^{2ikl} \right|^2, \quad (18.04)$$

which, when the paraboloid is deformed into a disk $\left(\omega \to \frac{\pi}{2},\ l \to 0,\ \Omega = \text{const}\right)$, is transformed to the form

$$\sigma = \pi a^2 \left| ika + \frac{1}{n}\operatorname{ctg}\frac{\pi}{n} \right|^2. \quad (18.05)$$

Comparing Expression (18.04) with the equation

$$\sigma^0 = \pi a^2 \operatorname{tg}^2\omega \left| 1 - e^{2ikl} \right|^2, \quad (18.06)$$

which physical optics gives for the effective scattering area, we see that they differ significantly from one another. First of all, the oscillating character of the function σ^0 draws our attention:

the reflected signal equals zero if a whole number of half-waves $\left(l=\frac{\lambda}{2}n,\ n=1, 2, 3 \ldots\right)$ is fitted into the length of the paraboloid, and it takes a maximum value if a half-integral number of half-waves $\left(l=\frac{\lambda}{2}\left(n+\frac{1}{2}\right),\ n=1, 2, 3 \ldots\right)$ is contained in this length.

A calculation performed by us on the basis of Equation (18.04) for paraboloids with the parameters $\Omega = 90°$, $tg\omega = 0.1$ $(k = \pi)$ shows (Figure 37) that, although the oscillating character of the effective scattering area is preserved, the amplitude of the oscillations is only 2 dB, and the maximum values of the function σ exceed the corresponding values in the physical optics approach by almost 13 dB. A still stronger divergence between the results of our theory and physical optics is detected when the paraboloid is deformed into a disk (Figure 38, $ka = 3\pi$, $k = \pi$, $\Omega = 90°$, $\omega \rightarrow 90°$).

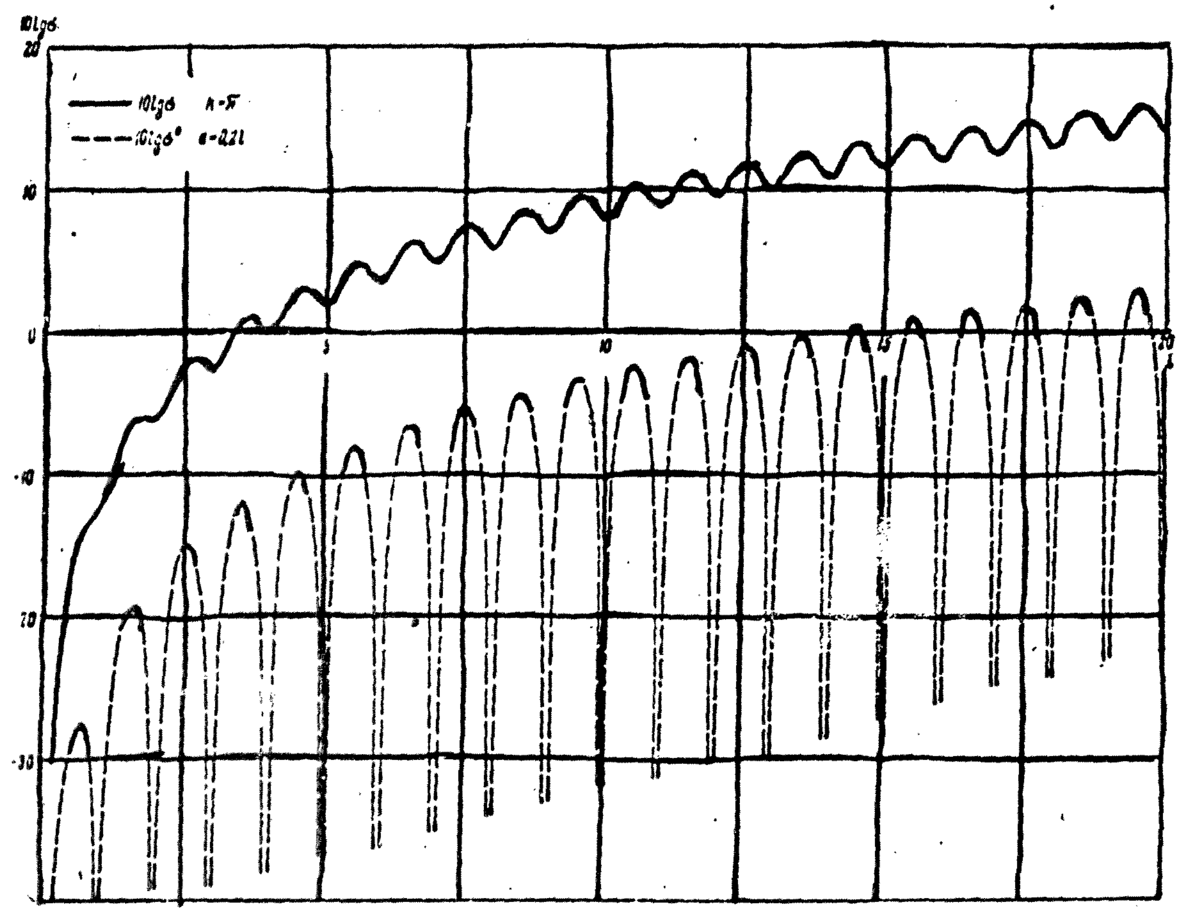


Figure 37. The effective scattering area of a finite paraboloid as a function of its length with a constant value of the angle ω ($tg\omega = 0.1$). The diameter of the base varies.

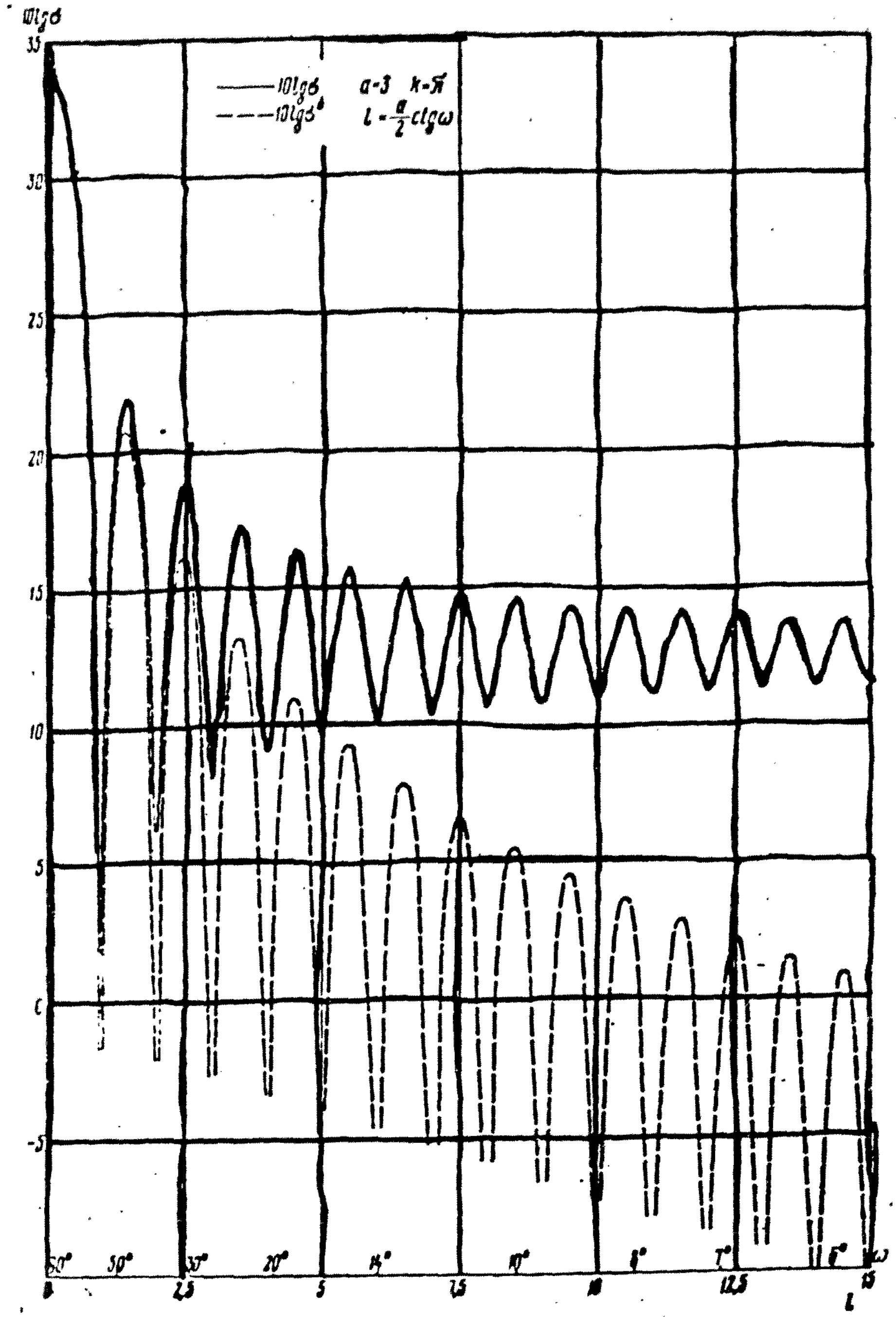


Figure 38. The effective scattering area of a finite paraboloid as a function of its length with a constant radius of the base.

As in the case of a cone, the shape of the shadowed part turns out to have a substantial influence on the reflected signal. For example, for a paraboloid with the parameters $ka = 2\pi$, $kl = 10\pi$, $tg\omega = 0.1$ $(k = \pi)$, the reflected signal increases by 44 dB with an increase of $\Omega(\omega < \Omega < \pi - \omega)$ (Figure 39).

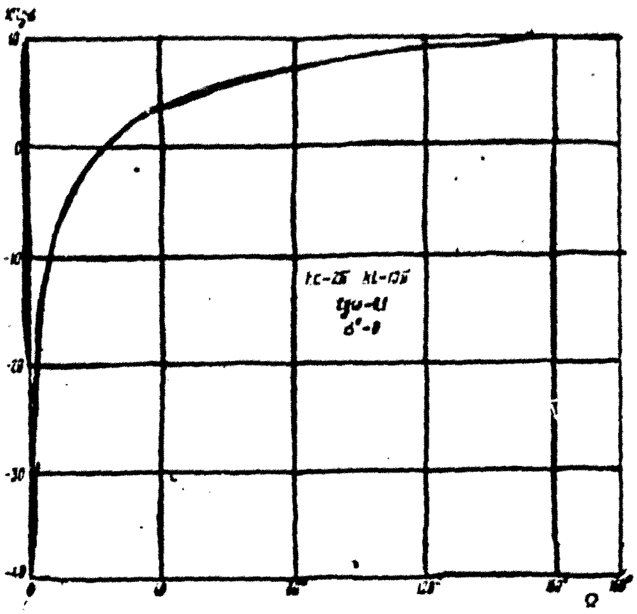


Figure 39. The effective scattering area of a finite paraboloid as a function of the shape of the shaded part.

In concluding this section, let us dwell for a moment on the question of calculating the effective scattering area for bodies of rotation of a complex shape, whose elements are the lateral surfaces of truncated paraboloids. The field from the nonuniform part of the current arising in the vicinity of circular discontinuities may be determined without difficulty from Equation (17.03). The field from the uniform part of the current is found by quadratures. Thus, the field being created in the direction $\vartheta = \pi$ by the uniform part of the current which flows on the lateral surface of the truncated paraboloid $r^2 = 2pz$($p = a_1 tg\omega_1 = a_2 tg\omega_2$; see Figure 40) is determined by the equation

$$\left.\begin{aligned} E_x &= -H_y = -\frac{1}{2}E_{0x}(a_1 \operatorname{tg}\omega_1 - a_2 \operatorname{tg}\omega_2 e^{2ikl_2})e^{2ikl_1}\frac{e^{ikR}}{R},\\ E_y &= H_x = 0. \end{aligned}\right\} \qquad (18.07)$$

Here

$$l_2 = \frac{1}{2}(a_2 \operatorname{ctg}\omega_2 - a_1 \operatorname{ctg}\omega_1) \qquad (18.08)$$

is the height of the truncated paraboloid (the distance between its bases). Let us note that Equation (18.07) is a simple algebraic corollary of Expression (18.02): it is the difference of the fields scattered, respectively, by the paraboloid of height $l_1 + l_2 = \frac{a_2^2}{2p}$ and by the paraboloid of height $l_1 = \frac{a_1^2}{2p}$.

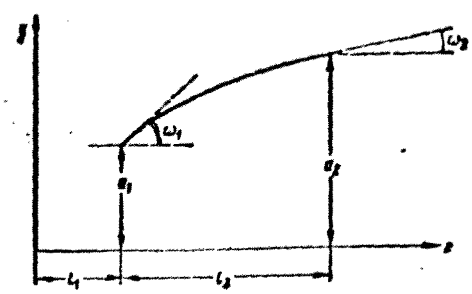


Figure 40. The generatrix of the lateral surface of a truncated paraboloid of rotation.

§ 19. A Spherical Surface

Incident wave (16.20) excites a surface current on the surface of an ideally conducting sphere (a radius of ρ and a center on the z axis at the point $z = \rho$). The uniform part of this current has the components

$$\left.\begin{aligned} j_x^0 &= -\frac{c}{2\pi} E_{0x} \cos\theta e^{ikz}, \\ j_y^0 &= 0, \\ j_z^0 &= \frac{c}{2\pi} E_{0x} \sin\theta \cos\varphi e^{ikz}. \end{aligned}\right\} \quad (19.01)$$

The currents flowing on a spherical ring cut from the sphere's surface by the planes $z = l_1$ and $z = l_1 + l_2$ (Figure 41) create, in the direction $\vartheta = \pi$, the field

$$\left.\begin{aligned} E_x = -H_y = E_{0x}\Big[-\left(\frac{a_1}{2}\operatorname{tg}\omega_1 - \frac{i}{4k}\right) e^{2ikl_1} + \\ + \left(\frac{a_2}{2}\operatorname{tg}\omega_2 - \frac{i}{4k}\right) e^{2ik(l_1+l_2)}\Big]\frac{e^{ikR}}{R}, \\ E_y = H_x = 0, \end{aligned}\right\} \quad (19.02)$$

where

$$\left.\begin{aligned} l_1 &= \rho(1 - \sin\omega_1), \\ l_2 &= \rho(\sin\omega_1 - \sin\omega_2); \end{aligned}\right\} \quad (19.03)$$

$$\rho = \frac{a_1}{\cos\omega_1} = \frac{a_2}{\cos\omega_2}. \quad (19.04)$$

Here a_1 is the radius of the first cross section; a_2 is the radius of the second cross section; $\omega_1(\omega_2)$ is the angle between the z axis and the tangent to the meridian at the point $z = l_1 (z_2 = l_1 + l_2)$.

Furthermore, assuming in Equation (19.02) $\omega_1 = \frac{\pi}{2}$ $(\omega_2 = \text{const})$, we obtain in the physical optics approach an expression for the field scattered by the spherical segment (Figure 42)

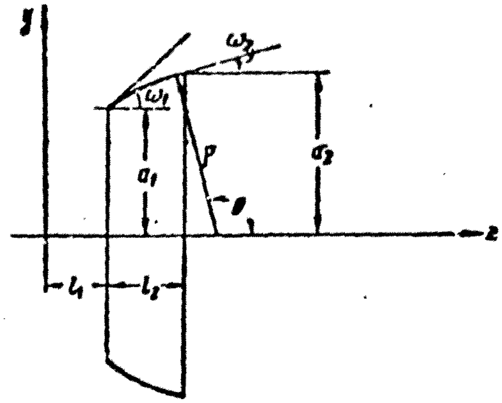


Figure 41. A ring cut from the surface of its sphere by the planes z = l_1 and z = $l_1 + l_2$.

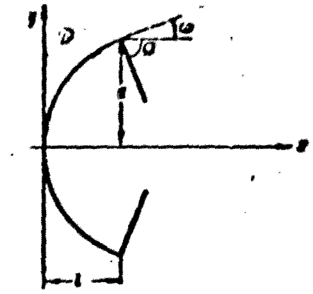


Figure 42. A spherical segment with a conically shaped base.

$$E_x = -H_y = E_{0x}\left[-\frac{a}{2\cos\omega} + \frac{i}{4k} + \left(\frac{a}{2}\operatorname{tg}\omega - \frac{i}{4k}\right)e^{2ikl}\right]\frac{e^{ikR}}{R}. \qquad (19.05)$$

Here we used the new designations

$$\left.\begin{array}{l} a = a_2, \quad \omega = \omega_2, \\ l = l_2 = \rho(1 - \sin\omega). \end{array}\right\} \qquad (19.06)$$

Equations (19.02) and (19.05) are simplified if $ka_1 \gg 1$ and $ka_2 \gg 1$. Thus, the field from the spherical ring will equal

$$E_x = -H_y = -E_{0x}\left(\frac{a_1}{2}\operatorname{tg}\omega_1 - \frac{a_2}{2}\operatorname{tg}\omega_2\, e^{2ikl_2}\right)e^{2ikl_1}\frac{e^{ikR}}{R}, \qquad (19.07)$$

and the field from the spherical segment will equal

$$E_x = -H_y = -\frac{aE_{0x}}{2}\left(\sec\omega - \operatorname{tg}\omega\, e^{2ikl}\right)\frac{e^{ikR}}{R}. \qquad (19.08)$$

If here one assumes ω = 0, then equation

$$E_x = -H_y = -\frac{aE_{0x}}{2}\frac{e^{ikR}}{R} \qquad (19.09)$$

gives us the field scattered by a hemisphere. The value of the effective scattering area corresponding to it will equal, in accordance with (17.06),

$$\sigma^* = \pi a^2. \qquad (19.10)$$

Now let us find the field scattered by the spherical segment considering the discontinuity of the surface; one may neglect the perturbation of the current as a consequence of the smooth curve of the surface if ka >> 1 [74]. The nonuniform part of the current which is caused by the discontinuity creates in the direction $\vartheta=\pi$ the field (17.03). Summing the latter with the field (19.08), we find the desired field

$$E_x = -H_y = -\frac{aE_{0x}}{2}\left(\frac{1}{\cos\omega} + \frac{\frac{2}{n}\sin\frac{\pi}{n}}{\cos\frac{\pi}{n} - \cos\frac{2\omega}{n}} e^{2ikl}\right)\frac{e^{ikR}}{R}. \qquad (19.11)$$

Consequently, the effective scattering area of a spherical segment will equal

$$\sigma = \pi a^2\left|\frac{1}{\cos\omega} + \frac{\frac{2}{n}\sin\frac{\pi}{n}}{\cos\frac{\pi}{n} - \cos\frac{2\omega}{n}} e^{2ikl}\right|^2, \qquad (19.12)$$

$$n = 1 + \frac{\omega + \Omega}{\pi}.$$

In the physical optics approach, a similar quantity is determined by field (19.08) and equals

$$\sigma^0 = \pi a^2\left|\frac{1}{\cos\omega} - \mathrm{tg}\,\omega\, e^{2ikl}\right|^2. \qquad (19.13)$$

With the deforming of the spherical surface into a disk $\left(\omega\to\frac{\pi}{2},\ l\to 0,\ \Omega=\mathrm{const}\right)$, Equations (19.12) and (19.13) are transformed, respectively, to the form

$$\left.\begin{aligned}\sigma &= \pi a^2\left|ika + \frac{1}{n}\,\mathrm{ctg}\,\frac{\pi}{n}\right|^2,\\ \sigma^0 &= \pi a^2 (ka)^2.\end{aligned}\right\} \qquad (19.14)$$

It follows from Equations (19.12) and (19.13) that the effective scattering area of a spherical segment is an oscillating function of its length. The oscillation period equals $\frac{\lambda}{2}$. Numerical calculations performed on the basis of these equations showed (Figure 43) that, with small angles of the discontinuity ($\Omega = 15°$), one may still neglect the field from the nonuniform part of the current. In Figur 44, graphs are constructed for the effective scattering area of a

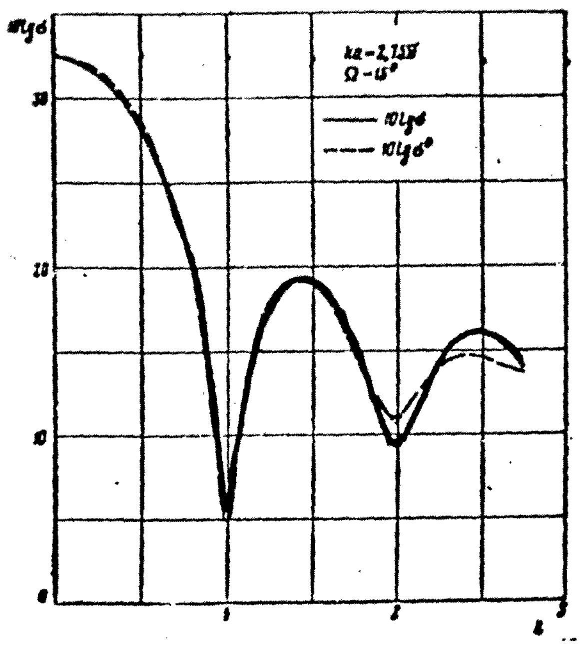


Figure 43. The effective scattering area of a spherical segment as a function of its length with a constant radius of the base. The function σ (the continuous line) is calculated on the basis of Equation (19.12) which considers the nonuniform part of the current near the discontinuity. The function σ^0 (the dashed line) is calculated from Equation (19.13), and corresponds to the physical optics approach.

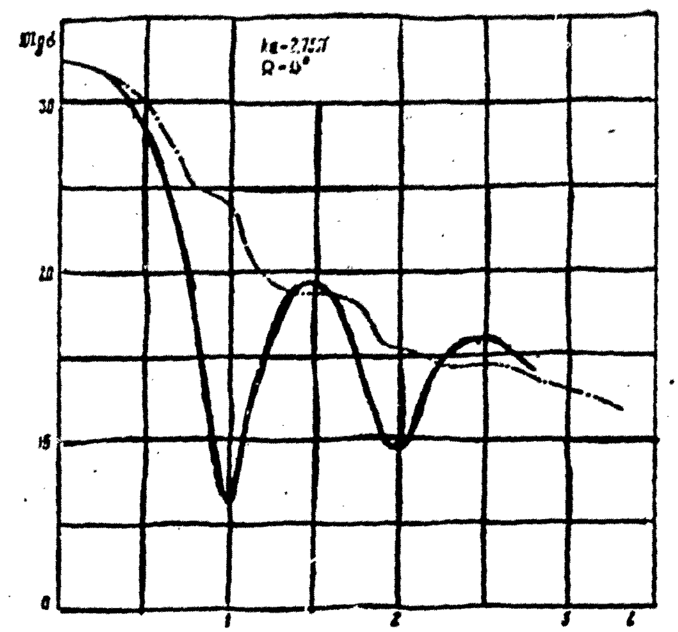


Figure 44. A comparison of the effective scattering area of a spherical segment (continuous line) and a finite cone (dashed line) which have the same bases.

spherical segment and a finite cone (the dashed curve) which have the same diameter and base shape.

* * * * * *

The results obtained in this Chapter show that the reflected signal depends substantially on the shape of the shaded part of the body, and increases with an increase of the concavity. However, since the nonuniform part of the current is concentrated mainly near the discontinuity, that part of the shaded surface which is several wavelengths away from the discontinuity evidently will not have a noticeable effect on the reflected signal and may be an arbitrary shape.

It is interesting that our expressions, which agree satisfactorily with experiments, even with large (in comparison with the wavelengths) dimensions of the bodies, do not change into the physical optics equations, but differ from them substantially. At the same time, physical optics, contrary to the widely held opinion concerning its reliability in such cases, leads to a significant discrepancy with experiments.

The method used in this Chapter allows one to calculate the effective scattering area associated with the symmetric irradiation of any convex body of rotation, the surface of which has circular discontinuities. It may also be generalized to the case of asymmetric irradiation. However, when doing this it is necessary to take into account the nonuniform part of the current caused by the point and the smooth curve of the surface.

FOOTNOTES

1. on page 98. See footnote on page 86.