Problem set 2

1. Answer True or False.

- A function cannot have more than one global minimum point.
- Gradient of a function at a point gives a local direction of maximum decrease in the function.
- A quadratic form can have first-order terms in the variables.
- Every quadratic form has a symmetric matrix associated with it.
- A positive definite quadratic form must have positive value for any $x \neq 0$.
- A point satisfying KKT conditions for a general optimum design problem can be a local max-point for the cost function.
- In the general optimum design problem formulation, the number of independent equality constraints must be "≤" to the number if design variables.
- A function is convex if and only if its Hessian is positive definite everywhere
- For a convex design problem, the Hessian of the cost function must be positive semidefinite everywhere.

2. Find stationary points for the following functions (use a numerical method or a software package like MATLAB, if needed). Also determine the local minimum, local maximum, and inflection points for the functions (inflection points are those stationary points that are neither minimum nor maximum).

The annual operating cost U for an electrical line system is given by the following expression

$$U = \frac{(21.9 \times 10^7)}{V^2 C} + (3.9 \times 10^6)C + 1000V$$

where V = line voltage in kilovolts and C = line conductance in mhos. Find stationary points for the function, and determine V and C to minimize the operating cost.

3. A minimum weight tubular column design problem is formulated in Section 2.7 using mean radius R and thickness t as design variables. Solve the KKT conditions for the problem imposing an additional constraint R/t > 50 for the following data:
P = 50 kN, l = 5.0m, E = 210 GPa, sa = 250 MPa and r = 7850kg/m3.
Interpret the necessary conditions at the solution point graphically.

4. Consider the problem of designing the "can" formulated in Section 2.2. Check convexity of the problem.