Negative and fractional powers

Introduction

Sometimes it is useful to use negative and fractional powers. These are explained on this leaflet.

1. Negative powers

Sometimes you will meet a number raised to a negative power. This is interpreted as follows:

$$a^{-m} = \frac{1}{a^m}$$

This can be rearranged into the alternative form:

$$a^m = \frac{1}{a^{-m}}$$

Example

$$3^{-2} = \frac{1}{3^2}, \qquad \frac{1}{5^{-2}} = 5^2, \qquad x^{-1} = \frac{1}{x^1} = \frac{1}{x}, \qquad x^{-2} = \frac{1}{x^2}, \qquad 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$$

Exercises

1. Write the following using only positive powers:

a) $\frac{1}{x^{-6}}$, b) x^{-12}, c) t^{-3}, d) $\frac{1}{4^{-3}}$, e) 5^{-17}.

2. Without using a calculator evaluate a) 2^{-3}, b) 3^{-2}, c) $\frac{1}{4^{-2}}$, d) $\frac{1}{2^{-5}}$, e) $\frac{1}{4^{-3}}$.

Answers

1. a) x^6, b) $\frac{1}{x^{12}}$, c) $\frac{1}{t^3}$, d) 4^3, e) $\frac{1}{5^{17}}$.

2. a) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$, b) $\frac{1}{9}$, c) 16, d) 32, e) 64.

2. Fractional powers

To understand fractional powers you first need to have an understanding of roots, and in particular square roots and cube roots. If necessary you should consult leaflet *1.2 Powers and Roots*.

When a number is raised to a fractional power this is interpreted as follows:

$$a^{1/n} = \sqrt[n]{a}$$

So,

$$\begin{array}{ll} a^{1/2} & \text{is a square root of } a \\ a^{1/3} & \text{is the cube root of } a \\ a^{1/4} & \text{is a fourth root of } a \end{array}$$

Example

$$3^{1/2} = \sqrt[2]{3}, \qquad 27^{1/3} = \sqrt[3]{27} \;\text{ or }\; 3, \qquad 32^{1/5} = \sqrt[5]{32} = 2,$$
$$64^{1/3} = \sqrt[3]{64} = 4, \qquad 81^{1/4} = \sqrt[4]{81} = 3$$

Fractional powers are useful when we need to calculate roots using a scientific calculator. For example to find $\sqrt[7]{38}$ we rewrite this as $38^{1/7}$ which can be evaluated using a scientific calculator. You may need to check your calculator manual to find the precise way of doing this, probably with the buttons x^y or $x^{1/y}$.

Check that you are using your calculator correctly by confirming that

$$38^{1/7} = 1.6814 \quad (4\,\text{dp})$$

More generally $a^{m/n}$ means $\sqrt[n]{a^m}$, or equivalently $(\sqrt[n]{a})^m$.

$$a^{m/n} = \sqrt[n]{a^m} \text{ or equivalently } \left(\sqrt[n]{a}\right)^m$$

Example

$$8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4, \qquad \text{and} \qquad 32^{3/5} = (\sqrt[5]{32})^3 = 2^3 = 8$$

Exercises

1. Use a calculator to find a) $\sqrt[5]{96}$, b) $\sqrt[4]{32}$.
2. Without using a calculator, evaluate a) $4^{3/2}$, b) $27^{2/3}$.
3. Use the third law of indices to show that

$$a^{m/n} = \sqrt[n]{a^m}$$

and equivalently

$$a^{m/n} = \left(\sqrt[n]{a}\right)^m$$

Answers

1. a) 2.4915, b) 2.3784. 2. a) $4^{3/2} = 8$, b) $27^{2/3} = 9$.