ANALYTICAL DYNAMICS

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Dedicated in memory of my dear mother, Ester Baruh, ע"ה

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PREFACE

APPROACH

Two occurrences in the second part of the 20th century have radically changed the nature of the field of dynamics. The first is the increased need to model and analyze complex, multibodied and often elastic-bodied structures, such as satellites, robot manipulators, and vehicles. The second is the proliferation of the digital computer, which has led to the development of numerical techniques to derive the describing equations of a system, integrate the equations of motion, and obtain the response. This new computational capability has encouraged scientists and engineers to model and numerically analyze complex dynamical systems which in the past either could not be analyzed, or were analyzed using gross simplifications.

The prospect of using computational techniques to model a dynamical system has also led dynamicists to reconsider existing methods of obtaining equations of motion. When evaluated in terms of systematic application, ease of implementation by computers, and computational effort, some of the traditional approaches lose part of their appeal. For example, to obtain Lagrange's equations, one is traditionally taught first to generate a scalar function called the Lagrangian and then to perform a series of differentiations. This approach is computationally inefficient. Moreover, certain terms in the differentiation of the kinetic energy cancel each other, resulting in wasted manipulations.

As a result of the reevaluation of the methods used in dynamics, new approaches have been proposed and certain older approaches that were not commonly used in the past have been brought back into the limelight. What has followed in the literature is a series of papers and books containing claims by proponents of certain methods, each extolling the virtues of one approach over the other without a fair and balanced analysis. This, at least in the opinion of this author, has not led to a healthy environment and fruitful exchange of ideas. It is now possible for basic graduate level courses in dynamics to be taught at different schools with entirely different subject material.

These developments of recent years have inspired me to compile my lecture notes into a textbook. Realizing that much of the research done lately in the field of dynamics has been reported very subjectively, I have tried to present in this book a fair and balanced description of dynamics problems and formulations, from the classical methods to the newer techniques used in today's multibody environments. I have emphasized the need to know both the classical methods as well as the newer techniques and have shown that these approaches are really complementary. Having the knowledge and experience to look at a problem in a number of ways not only facilitates the solution but also provides a better perspective. For example, the book discusses Euler parameters, which lead to fewer singularities in the solution and which lend themselves to more efficient computer implementation.

The focus of this book is primarily the kinematics and derivation of the describing equations of dynamics. We also consider the qualitative analysis of the response. We discuss a special case of quantitative analysis, namely the response of motion linearized about equilibrium.

We discuss means to analyze the kinematics and to describe the equations of motion. We study force and moment balances, as well as analytical methods. In most dynamics problems, the resulting equations of motion are nonlinear and lengthy, so that closed-form solutions are generally not available. We discuss analytical solutions, motion integrals and basic stability concepts. We make use of integrals of the motion, which are derived quantities that give qualitative information about the system without having to solve for the exact solution.

For linearized systems, we discuss the closed-form response. We outline concepts from vibration theory and eigenvector expansions. This also is done for continuous systems, in the last chapter of the book. We discuss the importance of numerical solutions.

CONTENTS

The book is organized into eleven chapters and three appendixes. The first eight chapters are intended for an introductory level graduate or advanced undergraduate course. The later chapters of the book can be used as part of a second, more advanced graduate level course. The book follows a classical approach, in which one first deals with particle mechanics and then extends the concepts into rigid bodies. The Lagrange's equations are initially discussed for a system of particles and plane motion of rigid bodies.

We follow in this book this school of thought for a number of reasons. First, graduate students come from a variety of backgrounds. Many times, students have not considered dynamics since the sophomore dynamics, or the freshman physics course. Also, this organization presents a more natural flow of the concepts used in dynamics. Nevertheless, the book is suitable also for instructors who prefer to teach three-dimensional rigid body dynamics before introducing analytical methods. Following is a description of the chapters:

In Chapter 1 we study fundamental concepts of dynamics and see their applications to particle mechanics problems. We discuss Newton's laws and energy and momentum principles. We look at integrals of motion and basic ideas from stability theory. The chapter outlines the response of linearized systems, which forms an introduction to vibration theory. This chapter should be covered in detail if the course is an undergraduate one. Less time should be spent on it for a graduate course or if the students taking the course are familiar with the basic ideas.

Chapter 2 discusses relative motion. Coordinate frames, rotation sequences, angular velocities, and angular accelerations are introduced. The significance of taking time derivatives in different coordinate systems is emphasized. We derive the relative motion equations and consider motion with respect to the rotating earth.

Chapter 3 is a chapter on systems of particles and plane kinetics of rigid bodies. It is primarily included for pedagogical considerations. I recommend its use for an undergraduate course; for a graduate level course, it should serve as independent reading. A number of sections in this chapter are devoted to an introduction to celestial mechanics problems, namely the two-body problem. The sections on plane kinetics of rigid bodies basically review the sophomore level material. This review is included here mainly because the approaches in the next chapter are described in terms of particles and plane motion of rigid bodies.

The subject of classical analytical mechanics is discussed in Chapters 4 and 5. Chapter 4 introduces the basic concepts, covering generalized coordinates, constraints, and degrees of freedom. We derive the principle of virtual work, D'Alembert's principle, and Hamilton's principle and then we develop Lagrange's equations. Analytical mechanics makes use of the calculus of variations, a subject covered separately in Appendix B. While Chapters 4 and 5 are written so that one does not absolutely need to learn the calculus of variations as a separate subject, it has been the experience of this author that some initial exposure of students to the calculus of variations is very helpful.

Chapter 5 revisits the concept of equilibrium and outlines the distinction between natural and nonnatural systems. We derive the linearized equations about equilibrium. The response of linearized systems is analyzed, which in essence is vibration theory for multidegree of freedom systems. Generalized momenta and motion integrals are considered.

In Chapter 6, we discuss the internal properties of a rigid body. In a departure from traditional approaches, we discuss moments of inertia independent of the kinetic energy and angular momentum.

Chapter 7 is devoted to a detailed analysis of the kinematics of a rigid body, where we learn of methods of quantifying the angular velocity vector. We present a discussion of Euler angles and Euler parameters. We then discuss constraints acting on the motion and quantify these constraints and the resulting kinematic relations.

Chapter 8 explores basic ideas associated with the kinetics of rigid bodies. We first begin with the application of force and moment balances. We express the equations of motion in terms of both the Euler angles and angular velocities. We discuss the relative merits of deriving the equations of motion in terms of generalized coordinates as well as in terms of angular velocity components. We analyze impulse-momentum and work-energy principles. We discuss the physical interpretation of Lagrange's equations and integrals of the motion associated with the Lagrangian.

Chapter 9 introduces more advanced concepts in the analysis of rigid body motion. We analyze the modified Euler's equations and then consider the moment equations about an arbitrary point. We discuss quasi-velocities, also known as generalized speeds, and their applications. We demonstrate that such coordinates are desirable when dealing with nonholonomic systems. We demonstrate the equivalence of the Gibbs-Appell and Kane's equations and discuss momentum balances in terms of the generalized speeds.

Many of the analytical methods described in this chapter could have been introduced in Chapters 4 or 5. However, the power of these methods, which are equally applicable to both particles and rigid bodies, is better appreciated when we consider applications to complex rigid body problems.

Chapter 10 covers the qualitative analysis of rigid body motion, and in particular gyroscopic effects. The chapter initially goes into a qualitative study of torque-free motion and the differences in the response between axisymmetric and arbitrary bodies. We then discuss interesting classical applications of gyroscopic motion, such as a spinning top, a rolling disk, and gyroscopes.

Chapter 11 investigates the subject of dynamics of lightly flexible bodies. Recent problems in dynamics have demonstrated the importance of including the elasticity of a body in the describing equations. The emphasis is the analysis of bodies that undergo combined rigid and elastic motion, typical examples being robot manipulators and spacecraft with appendages. We derive the classical boundary value problem and examine a shortcoming in the traditional formulation. The combined large-angle rigid and elastic motions are modeled in terms of the superposition of a primary motion, as the motion of a moving reference frame, and a secondary motion, the motion of the body as observed from the moving reference frame.

Appendix A is a historical survey of dynamics and a synopsis of the work of the many people who contributed to this field.

Appendix B presents an introduction to the calculus of variations. It is recommended that at least part of this appendix be studied before Chapter 4. However, Chapter 4 is written such that a brief introduction to virtual displacements should be sufficient to understand basic concepts from analytical mechanics.

Appendix C gives the mass moments of inertia of common shapes.

PEDAGOGICAL TOOLS

The book contains several examples and homework problems. It has been my experience that students understand a subject best when they see many examples. I encourage anyone teaching dynamics, either at the undergraduate or the graduate level, to use as many examples as possible.

Another pedagogical tool emphasized in the book is computational techniques. While we do not go into details of numerical integration, we discuss the numerical integration of equations of motion. Many of the examples and homework problems in the book can be assigned as computer projects. I encourage every student to keep pace with new advances in scientific software, such as symbolic manipulators, because, as discussed earlier, the availability of computational tools has changed the nature of dynamics.

SUPPLEMENT

The book is supplemented by an Instructor's Solutions Manual which includes detailed solutions to all of the problems in the book.

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Writing a textbook is a lengthy and difficult task and I would like to acknowledge the support of many people. First, I am richly indebted to all the teachers I have had in my life, from my elementary school teacher in Istanbul, Mrs. Cahide Şen, to my advisor in graduate school, Dr. Leonard Meirovitch, who has always been a source of advice and inspiration. I am very grateful to my employer, Rutgers University, and especially to the leadership of the Mechanical and Aerospace Engineering Department, the current chair, Professor Abdelfattah Zebib, and the previous chair, Professor Samuel Temkin, for giving their faculty the time and resources to conduct scholarly work. I thank Verica Radisavljevic emphatically, who read the entire manuscript, helped with figures and made very valuable suggestions. Professor Leon Bahar gave valuable suggestions on variational mechanics. Many graduate students who used earlier versions of the manuscript as their course notes helped correct errors and improve the text. Many thanks are due to McGraw-Hill, and especially to Senior Editor Debra Riegert, who gave me the opportunity to publish with them. Victoria St. Ambrogio was an outstanding copy editor and the competent staff of McGraw-Hill and Publication Services facilitated the production of the book.

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I dedicate this book to the memory of my mother, who passed away a year before the publication of this book. Ester Baruh was a symbol of honesty, integrity, decency, kindness, compassion, and common sense. She was unfailingly devoted to her family and to her community. May her memory be a blessing.

While both the author and the publisher have made every effort to produce an error-free book, there may nevertheless be errors. I would be most appreciative if a reader who spots an error or has a comment about the book would bring it to my attention. My current e-mail addresses are baruh@jove.rutgers.edu or baruh@rci.rutgers.edu. To inform the reader of any discovered errors I am establishing a page on my professional Web site (www-srac.rutgers.edu/~baruh). Just go to my Web site and you will find the link. If my web address at any time has changed, try www.rutgers.edu and find me from there.