**Solution of Assignment #1, CS/191**

Fall, 2014

1. (0 points). Page 12, problem 2, (b) - (d).

(b) What time is it?

sol: This is not a proposition because it is not a declarative sentence.

(c) There are no black flies in Maine.

sol: This is a proposition.

(d) $4 + x = 5$.

sol: This is not a proposition (it contains a variable; the truth value depends on the value assigned to $x$).

2. Page 13, problem 8:

- (0 points) (d), (e), (f).

  (d) $p \wedge q$.

  sol: I bought a lottery ticket this week and I won the million dollar jackpot.

  (e) $p \leftrightarrow q$.

  sol: I bought a lottery ticket this week if and only if I won the million dollar jackpot.

  (f) $\neg p \rightarrow \neg q$.

  sol: If I didn't buy a lottery ticket this week, then I didn't win the million dollar jackpot.

- (0 points) (g), (h).

  (g) $\neg p \wedge \neg q$.

  sol: I didn't buy a lottery ticket this week and I didn't win the million dollar jackpot.

  (h) $\neg p \vee (p \wedge q)$.

  sol: Either I didn't buy a lottery ticket this week, or I bought a lottery ticket this week and I won million dollar jackpot.

3. (0 points), page 13, problem 14, (b) - (d).

$p$: You get an A on the final exam.

$q$: You do every exercise in this book.

$r$: You get an A in this class.

(b): You get an A on the final, you do every exercise in this book, and you get an A in this class.

sol: $p \wedge q \wedge r$.

(c): To get an A in this class, it is necessary for you to get an A on the final.

sol: $r \rightarrow p$.

(d): You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.

sol: $p \wedge \neg q \wedge r$.

Note: the precise meaning of the word "nevertherless" in everyday language is not always clear. In this sentence, it can be replaced by the word "but" without changing its meaning. Thus we have $\wedge r$.

(0 points) (e), (f).

(e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in the class.

sol: $(p \wedge q) \rightarrow r$

(f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

sol: $r \leftrightarrow (q \vee p)$

4. (0 points), page 15, problem 32,

(e) $(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$.

sol:

| $p$ | $q$ | $\neg p$ | $q \rightarrow \neg p$ | $p \leftrightarrow q$ | $(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$ |
|---|---|---|---|---|---|
| $T$ | $T$ | $F$ | $F$ | $T$ | $F$ |
| $T$ | $F$ | $F$ | $T$ | $F$ | $F$ |
| $F$ | $T$ | $T$ | $T$ | $F$ | $F$ |
| $F$ | $F$ | $T$ | $T$ | $T$ | $T$ |

5. (0 points), page 15, problem 36,

(f) $(p \wedge q) \vee \neg r$.

sol:

| $p$ | $q$ | $r$ | $p \wedge q$ | $\neg r$ | $(p \wedge q) \vee \neg r$ |
|---|---|---|---|---|---|
| $T$ | $T$ | $T$ | $T$ | $F$ | $T$ |
| $T$ | $T$ | $F$ | $T$ | $T$ | $T$ |
| $T$ | $F$ | $T$ | $F$ | $F$ | $F$ |
| $T$ | $F$ | $F$ | $F$ | $T$ | $T$ |
| $F$ | $T$ | $T$ | $F$ | $F$ | $F$ |
| $F$ | $T$ | $F$ | $F$ | $T$ | $T$ |
| $F$ | $F$ | $T$ | $F$ | $F$ | $F$ |
| $F$ | $F$ | $F$ | $F$ | $T$ | $T$ |

6. (0 points), page 15, problem 37,

(c) $(p \rightarrow q) \vee (\neg p \rightarrow r)$

sol:

| $p$ | $q$ | $r$ | $p \to q$ | $\neg p$ | $\neg p \to r$ | $(p \to q) \vee (\neg p \to r)$ |
|---|---|---|---|---|---|---|
| $T$ | $T$ | $T$ | $T$ | $F$ | $T$ | $T$ |
| $T$ | $T$ | $F$ | $T$ | $F$ | $T$ | $T$ |
| $T$ | $F$ | $T$ | $F$ | $F$ | $T$ | $T$ |
| $T$ | $F$ | $F$ | $F$ | $F$ | $T$ | $T$ |
| $F$ | $T$ | $T$ | $T$ | $T$ | $T$ | $T$ |
| $F$ | $T$ | $F$ | $T$ | $T$ | $F$ | $T$ |
| $F$ | $F$ | $T$ | $T$ | $T$ | $T$ | $T$ |
| $F$ | $F$ | $F$ | $T$ | $T$ | $F$ | $T$ |

7. (0 points), page 24, problem 36, (a).

sol: **John Did It.** We show the reason as follows: For this problem, we list four cases such that we assume only one statement is true for each case.

**Case 1: Assume Alice's statement "Carlos did it" is true and other statements are false.** From the assumption, Alice's statement "Carlos did it" is true, which implies Carlos did it. Then, this implies John didn't do it and John's statement "I didn't do it." is true. However, we must assume John's statement is false since only one person is telling the truth, and we have already assumed Alice is telling the truth. So, it contradicts our assumption and this case cannot hold.

**Case 2: Assume John's statement "I didn't do it" is true and other statements are false.** From the assumptions, Alice's statement "Carlos did it" is false, which implies Carlos didn't do it. Carlos said "Diana did it." is false. It implies Diana didn't do it. Diana said "Carlos lied when he said that I did it" is false, it implies Carlos didn't lie. It contradicts our assumption that Carlos lied. So this case doesn't hold.

**Case 3: Assume Carlos' statement "Diana did it" is true and other statements are false.** From the assumption, John's statement "I didn't do it" is false, which implies John did it. On the other hand, we also assume Carlos' statement "Diana did it" is true. This implies Diana did it. But only one person did it. So, the assumption cannot hold.

**Case 4: Assume Diana's statement "Carlos lied when he said that I did it" is true and other statements are false**. Alice's statement "Carlos did it" is false. So, Carlos didn't do it. John's statement "I didn't do it" is false. So, John did it. Carlos' statement "Diana did it" is false. So, Diana didn't do it. Diana's statement "Carlos lied when he said that I did it" is true. So, Diana didn't do it. These assertions are consistent, so we can conclude that John did it.

Note: You may be able to reduce the number of possible cases to describe by recognizing that Carlos and Diana's statements are negations of each other, and since at least one of them must be lying, the other must therefore be telling the truth. This reduces the problem to cases 3 and 4.