The Pythagorean Theorem

? ESSENTIAL QUESTION

How can you use the Pythagorean Theorem to solve real-world problems?

Real-World Video

The sizes of televisions are usually described by the length of the diagonal of the screen. To find this length of the diagonal of a rectangle, you can use the Pythagorean Theorem.

my.hrw.com

© Houghton Mifflin Harcourt Publishing Company • Image Credits: ©Yuri Arcurs/Shutterstock

GO DIGITAL

my.hrw.com

my.hrw.com

Go digital with your write-in student edition, accessible on any device.

Math On the Spot

Scan with your smart phone to jump directly to the online edition, video tutor, and more.

Animated Math

Interactively explore key concepts to see how math works.

Personal Math Trainer

Get immediate feedback and help as you work through practice sets.

Are YOU Ready?

Complete these exercises to review skills you will need for this module.

Find the Square of a Number

> **EXAMPLE**   Find the square of 2.7.
>
> $$\begin{array}{r} 2.7 \\ \times\, 2.7 \\ \hline 189 \\ 54 \\ \hline 7.29 \end{array}$$
>
> *Multiply the number by itself.*
>
> *So, $2.7^2 = 7.29$.*

Find the square of each number.

1. 5 _____   **2.** 16 _____   **3.** -11 _____   **4.** $\frac{2}{7}$ _____

Order of Operations

> **EXAMPLE** $\sqrt{(5-2)^2 + (8-4)^2}$ *First, operate within parentheses.*
>
> $\sqrt{(3)^2 + (4)^2}$ *Next, simplify exponents.*
>
> $\sqrt{9 + 16}$ *Then add and subtract left to right.*
>
> $\sqrt{25}$ *Finally, take the square root.*
>
> 5

Evaluate each expression.

5. $\sqrt{(6+2)^2 + (3+3)^2}$ _____   **6.** $\sqrt{(9-4)^2 + (5+7)^2}$ _____

7. $\sqrt{(10-6)^2 + (15-12)^2}$ _____   **8.** $\sqrt{(6+9)^2 + (10-2)^2}$ _____

Simplify Numerical Expressions

> **EXAMPLE** $\frac{1}{2}(2.5)^2(4) = \frac{1}{2}(6.25)(4)$ *Simplify the exponent.*
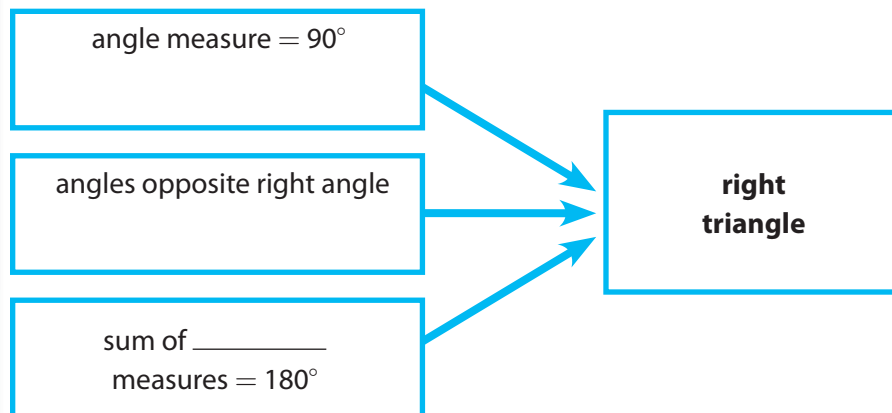>
> $= 12.5$ *Multiply from left to right.*

Simplify each expression.

9. $5(8)(10)$ _____   **10.** $\frac{1}{2}(6)(12)$ _____   **11.** $\frac{1}{3}(3)(12)$ _____

12. $\frac{1}{2}(8)^2(4)$ _____   **13.** $\frac{1}{4}(10)^2(15)$ _____   **14.** $\frac{1}{3}(9)^2(6)$ _____

Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the graphic.

angle measure = 90°	
angles opposite right angle	→ **right triangle**
sum of _____ measures = 180°	

Understand Vocabulary

Match the term on the left to the correct expression on the right.

1. hypotenuse

2. theorem

3. legs

A. An idea that has been demonstrated as true.

B. The two sides that form the right angle of a right triangle.

C. The side opposite the right angle in a right triangle.

Active Reading

Booklet Before beginning the module, create a booklet to help you learn about the Pythagorean Theorem. Write the main idea of each lesson on each page of the booklet. As you study each lesson, write important details that support the main idea, such as vocabulary and formulas. Refer to your finished booklet as you work on assignments and study for tests.

Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

COMMON CORE **8.G.7**

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Key Vocabulary

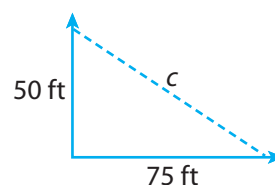**Pythagorean Theorem**
(Teorema de Pitágoras)
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

What It Means to You

You will find a missing length in a right triangle, or use side lengths to see whether a triangle is a right triangle.

UNPACKING EXAMPLE 8.G.7

Mark and Sarah start walking at the same point, but Mark walks 50 feet north while Sarah walks 75 feet east. How far apart are Mark and Sarah when they stop?

$$a^2 + b^2 = c^2$$
$$50^2 + 75^2 = c^2$$
$$2500 + 5625 = c^2$$
$$8125 = c^2$$
$$90.1 \approx c$$

Pythagorean Theorem

Substitute.

Mark and Sarah are approximately 90.1 feet apart.

COMMON CORE **8.G.8**

Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Key Vocabulary

coordinate plane
(plano cartesiano)
A plane formed by the intersection of a horizontal number line called the x-axis and a vertical number line called the y-axis.

What It Means to You

You can use the Pythagorean Theorem to find the distance between two points.

UNPACKING EXAMPLE 8.G.8

Find the distance between points A and B.

$$(AC)^2 + (BC)^2 = (AB)^2$$
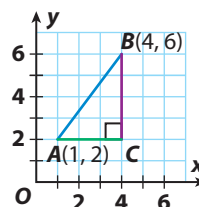$$(4-1)^2 + (6-2)^2 = (AB)^2$$
$$3^2 + 4^2 = (AB)^2$$
$$9 + 16 = (AB)^2$$
$$25 = (AB)^2$$
$$5 = AB$$

The distance is 5 units.

Visit **my.hrw.com** to see all the **Common Core Standards** unpacked.

my.hrw.com

The Pythagorean Theorem

? ESSENTIAL QUESTION

How can you prove the Pythagorean Theorem and use it to solve problems?

**EXPLORE ACTIVITY** COMMON CORE 8.G.6

Proving the Pythagorean Theorem

In a right triangle, the two sides that form the right angle are the **legs**. The side opposite the right angle is the **hypotenuse**.

Leg   Hypotenuse

Leg

The Pythagorean Theorem

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

If a and b are legs and c is the hypotenuse, $a^2 + b^2 = c^2$.
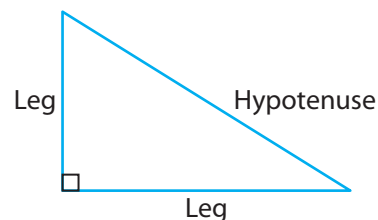
A Draw a right triangle on a piece of paper and cut it out. Make one leg shorter than the other.

B Trace your triangle onto another piece of paper four times, arranging them as shown. For each triangle, label the shorter leg a, the longer leg b, and the hypotenuse c.

C What is the area of the unshaded square?

_____

Label the unshaded square with its area.
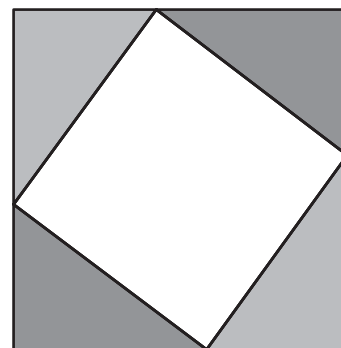
D Trace your original triangle onto a piece of paper four times again, arranging them as shown. Draw a line outlining a larger square that is the same size as the figure you made in **B**.

E What is the area of the unshaded square at the top right of the figure in **D**? at the top left?

_____

Label the unshaded squares with their areas.
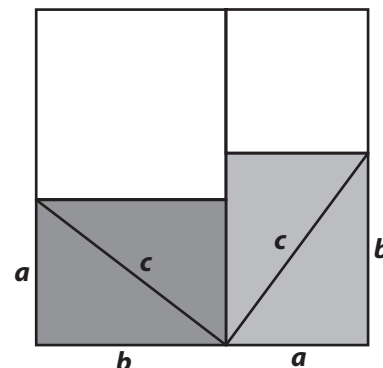
F What is the total area of the unshaded regions in **D**?

_____

a c c b

b a

Reflect

1. Explain whether the figures in **B** and **D** have the same area.

   _____

2. Explain whether the unshaded regions of the figures in **B** and **D** have the same area.

   _____

   _____

   _____

3. **Analyze Relationships** Write an equation relating the area of the unshaded region in step **B** to the unshaded region in **D**.
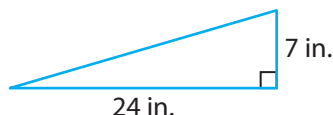
   _____

Using the Pythagorean Theorem

You can use the Pythagorean Theorem to find the length of a side of a right triangle when you know the lengths of the other two sides.

EXAMPLE 1

Find the length of the missing side.

A

7 in.

24 in.

$$a^2 + b^2 = c^2$$
$$24^2 + 7^2 = c^2 \qquad \text{Substitute into the formula.}$$
$$576 + 49 = c^2 \qquad \text{Simplify.}$$
$$625 = c^2 \qquad \text{Add.}$$
$$25 = c \qquad \text{Take the square root of both sides.}$$

The length of the hypotenuse is 25 inches.

> **Math Talk**
>
> **Mathematical Practices**
>
> If you are given the length of the hypotenuse and one leg, does it matter whether you solve for a or b? Explain.

B

15 cm

12 cm
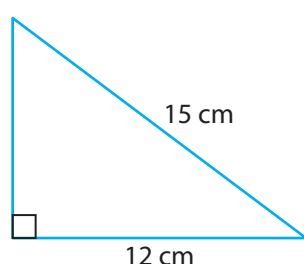
$$a^2 + b^2 = c^2$$
$$a^2 + 12^2 = 15^2 \qquad \text{Substitute into the formula.}$$
$$a^2 + 144 = 225 \qquad \text{Simplify.}$$
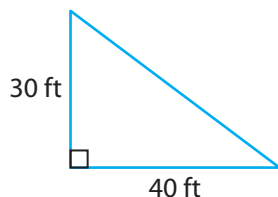$$a^2 = 81 \qquad \text{Use properties of equality to get } a^2 \text{ by itself.}$$
$$a = 9 \qquad \text{Take the square root of both sides.}$$

The length of the leg is 9 centimeters.

Find the length of the missing side.

4.

30 ft

40 ft

_____

5.

41 in.

40 in.

_____
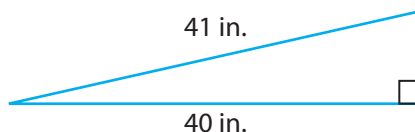
# Pythagorean Theorem in Three Dimensions

You can use the Pythagorean Theorem to solve problems in three dimensions.

EXAMPLE 2 Real World

COMMON CORE **8.G.7**

A box used for shipping narrow copper tubes measures 6 inches by 6 inches by 20 inches. What is the length of the longest tube that will fit in the box, given that the length of the tube must be a whole number of inches?

$h = 6$ in.

r

s

$w = 6$ in.

$l = 20$ in.
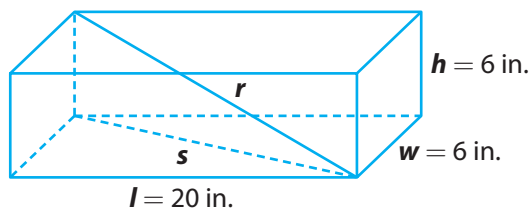
STEP 1 You want to find r, the length from a bottom corner to the opposite top corner. First, find s, the length of the diagonal across the bottom of the box.

$$w^2 + l^2 = s^2$$

$$6^2 + 20^2 = s^2 \qquad \text{Substitute into the formula.}$$

$$36 + 400 = s^2 \qquad \text{Simplify.}$$

$$436 = s^2 \qquad \text{Add.}$$

STEP 2 Use your expression for s to find r.

$$h^2 + s^2 = r^2$$

$$6^2 + 436 = r^2 \qquad \text{Substitute into the formula.}$$

$$472 = r^2 \qquad \text{Add.}$$

$$\sqrt{472} = r \qquad \text{Take the square root of both sides.}$$

$$21.7 \approx r \qquad \text{Use a calculator to round to the nearest tenth.}$$

The length of the longest tube that will fit in the box is 21 inches.

Math Talk
Mathematical Practices

Looking at Step 2, why did the calculations in Step 1 stop before taking the square root of both sides of the final equation?

YOUR TURN

6. Tina ordered a replacement part for her desk. It was shipped in a box that measures 4 in. by 4 in. by 14 in. What is the greatest length in whole inches that the part could have been?
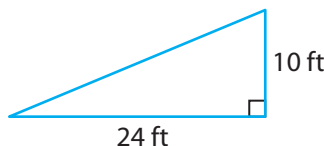
4 in.

r

s

4 in.

14 in.
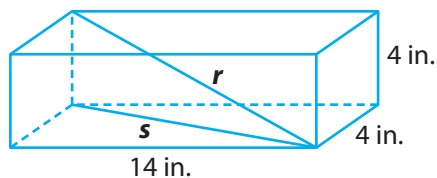
Guided Practice

1. Find the length of the missing side of the triangle. **(Explore Activity 1 and Example 1)**

$a^2 + b^2 = c^2 \rightarrow 24^2 + \boxed{} = c^2 \rightarrow \boxed{} = c^2$

The length of the hypotenuse is $\boxed{}$ feet.

10 ft

24 ft

2. Mr. Woo wants to ship a fishing rod that is 42 inches long to his son. He has a box with the dimensions shown. **(Example 2)**

$h = $ **10 in.**

$w = $ **10 in.**

$l = $ **40 in.**

  a. Find the square of the length of the diagonal across the bottom of the box.

        _____

  b. Find the length from a bottom corner to the opposite top corner to the nearest tenth. Will the fishing rod fit?
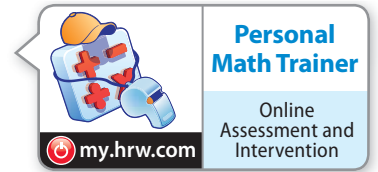
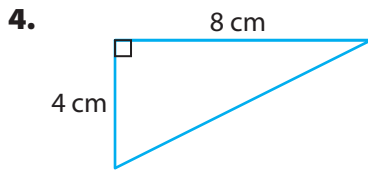   _____

? ESSENTIAL QUESTION CHECK-IN

3. State the Pythagorean Theorem and tell how you can use it to solve problems.
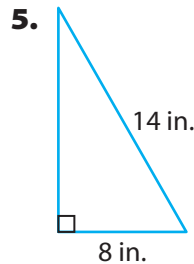
_____

_____

_____

_____

12.1 Independent Practice

COMMON CORE 8.G.6, 8.G.7

**Find the length of the missing side of each triangle. Round your answers to the nearest tenth.**

4.

8 cm

4 cm

_____

5.

14 in.

8 in.

_____

6. The diagonal of a rectangular big-screen TV screen measures 152 cm. The length measures 132 cm. What is the height of the screen?

_____

7. Dylan has a square piece of metal that measures 10 inches on each side. He cuts the metal along the diagonal, forming two right triangles. What is the length of the hypotenuse of each right triangle to the nearest tenth of an inch?

_____

8. **Represent Real-World Problems** A painter has a 24-foot ladder that he is using to paint a house. For safety reasons, the ladder must be placed at least 8 feet from the base of the side of the house. To the nearest tenth of a foot, how high can the ladder safely reach?

_____

9. What is the longest flagpole (in whole feet) that could be shipped in a box that measures 2 ft by 2 ft by 12 ft? _____

r

s

12 ft

2 ft

2 ft
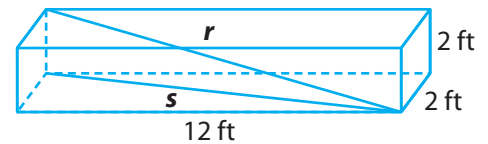
10. **Sports** American football fields measure 100 yards long between the end zones, and are $53\frac{1}{3}$ yards wide. Is the length of the diagonal across this field more or less than 120 yards? Explain.

_____

11. **Justify Reasoning** A tree struck by lightning broke at a point 12 ft above the ground as shown. What was the height of the tree to the nearest tenth of a foot? Explain your reasoning.

12 ft

39 ft

_____

_____

_____
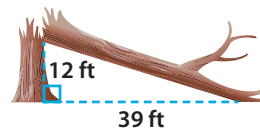
12. Multistep Main Street and Washington Avenue meet at a right angle. A large park begins at this corner. Joe's school lies at the opposite corner of the park. Usually Joe walks 1.2 miles along Main Street and then 0.9 miles up Washington Avenue to get to school. Today he walked in a straight path across the park and returned home along the same path. What is the difference in distance between the two round trips? Explain.

_____

_____

_____

13. Analyze Relationships An isosceles right triangle is a right triangle with congruent legs. If the length of each leg is represented by x, what algebraic expression can be used to represent the length of the hypotenuse? Explain your reasoning.

_____

_____

14. Persevere in Problem Solving A square hamburger is centered on a circular bun. Both the bun and the burger have an area of 16 square inches.
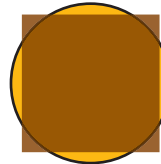
a. How far, to the nearest hundredth of an inch, does each corner of the burger stick out from the bun? Explain.

_____

_____

_____

_____

b. How far does each bun stick out from the center of each side of the burger?

_____

c. Are the distances in part **a** and part **b** equal? If not, which sticks out more, the burger or the bun? Explain.

_____

_____

_____

Converse of the Pythagorean Theorem

? **ESSENTIAL QUESTION**

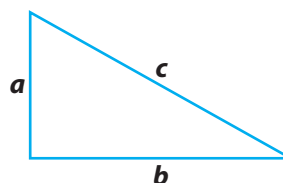How can you test the converse of the Pythagorean Theorem and use it to solve problems?

EXPLORE ACTIVITY **COMMON CORE** 8.G.6

Testing the Converse of the Pythagorean Theorem

The Pythagorean Theorem states that if a triangle is a right triangle, then $a^2 + b^2 = c^2$.

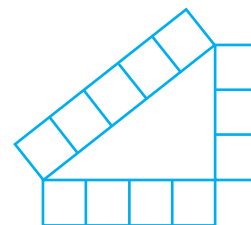The *converse* of the Pythagorean Theorem states that if $a^2 + b^2 = c^2$, then the triangle is a right triangle.

Decide whether the converse of the Pythagorean Theorem is true.

A Verify that the following sets of lengths make the equation $a^2 + b^2 = c^2$ true. Record your results in the table.

a	b	c	Is $a^2 + b^2 = c^2$ true?	Makes a right triangle?
3	4	5		
5	12	13		
7	24	25		
8	15	17		
20	21	29		

B For each set of lengths in the table, cut strips of grid paper with a width of one square and lengths that correspond to the values of a, b, and c.

C For each set of lengths, use the strips of grid paper to try to form a right triangle. An example using the first set of lengths is shown. Record your findings in the table.

Reflect

1. **Draw Conclusions** Based on your observations, explain whether you think the converse of the Pythagorean Theorem is true.

_____

_____

Identifying a Right Triangle

The converse of the Pythagorean Theorem gives you a way to tell if a triangle is a right triangle when you know the side lengths.

EXAMPLE 1

8.G.6

Tell whether each triangle with the given side lengths is a right triangle.

A 9 inches, 40 inches, and 41 inches

Let $a = 9$, $b = 40$, and $c = 41$.

$$a^2 + b^2 = c^2$$

$$9^2 + 40^2 \stackrel{?}{=} 41^2 \qquad \text{Substitute into the formula.}$$

$$81 + 1600 \stackrel{?}{=} 1681 \qquad \text{Simpify.}$$

$$1681 = 1681 \qquad \text{Add.}$$

Since $9^2 + 40^2 = 41^2$, the triangle is a right triangle by the converse of the Pythagorean Theorem.

B 8 meters, 10 meters, and 12 meters

Let $a = 8$, $b = 10$, and $c = 12$.

$$a^2 + b^2 = c^2$$

$$8^2 + 10^2 \stackrel{?}{=} 12^2 \qquad \text{Substitute into the formula.}$$

$$64 + 100 \stackrel{?}{=} 144 \qquad \text{Simpify.}$$

$$164 \neq 144 \qquad \text{Add.}$$

Since $8^2 + 10^2 \neq 12^2$, the triangle is not a right triangle by the converse of the Pythagorean Theorem.

YOUR TURN

Tell whether each triangle with the given side lengths is a right triangle.

2. 14 cm, 23 cm, and 25 cm

3. 16 in., 30 in., and 34 in.

4. 27 ft, 36 ft, 45 ft

5. 11 mm, 18 mm, 21 mm

Using the Converse of the Pythagorean Theorem

You can use the converse of the Pythagorean Theorem to solve real-world problems.

EXAMPLE 2

COMMON CORE 8.G.6

Katya is buying edging for a triangular flower garden she plans to build in her backyard. If the lengths of the three pieces of edging that she purchases are 13 feet, 10 feet, and 7 feet, will the flower garden be in the shape of a right triangle?

Use the converse of the Pythagorean Theorem. Remember to use the longest length for c.

Let $a = 7$, $b = 10$, and $c = 13$.

$$a^2 + b^2 = c^2$$

$$7^2 + 10^2 \stackrel{?}{=} 13^2 \qquad \text{Substitute into the formula.}$$

$$49 + 100 \stackrel{?}{=} 169 \qquad \text{Simpify.}$$

$$149 \neq 169 \qquad \text{Add.}$$

Since $7^2 + 10^2 \neq 13^2$, the garden will not be in the shape of a right triangle.

> **Math Talk**
> **Mathematical Practices**
>
> To what length, to the nearest tenth, can Katya trim the longest piece of edging to form a right triangle?

YOUR TURN

6. A blueprint for a new triangular playground shows that the sides measure 480 ft, 140 ft, and 500 ft. Is the playground in the shape of a right triangle? Explain.

_____

_____

7. A triangular piece of glass has sides that measure 18 in., 19 in., and 25 in. Is the piece of glass in the shape of a right triangle? Explain.

_____

8. A corner of a fenced yard forms a right angle. Can you place a 12 foot long board across the corner to form a right triangle for which the leg lengths are whole numbers? Explain.

_____

_____

© Houghton Mifflin Harcourt Publishing Company

1. Lashandra used grid paper to construct the triangle shown. (Explore Activity)

    a. What are the lengths of the sides of Lashandra's triangle?
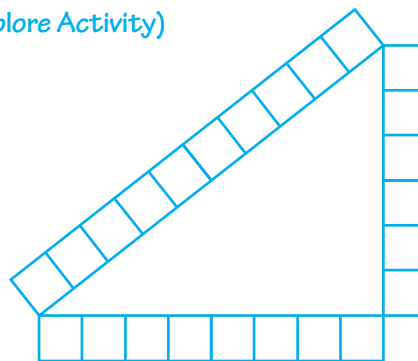
        _____units, _____units, _____units

    b. Use the converse of the Pythagorean Theorem to determine whether the triangle is a right triangle.

$$a^2 + b^2 = c^2$$

$$\boxed{}^2 + \boxed{}^2 \overset{?}{=} \boxed{}^2$$

$$\boxed{} + \boxed{} \overset{?}{=} \boxed{}$$

$$\boxed{} \overset{?}{=} \boxed{}$$

    The triangle that Lashandra constructed $\boxed{\textbf{is / is not}}$ a right triangle.

2. A triangle has side lengths 9 cm, 12 cm, and 16 cm. Tell whether the triangle is a right triangle. (Example 1)

    Let $a =$ _____ , $b =$ _____ , and $c =$ _____ .

$$a^2 + b^2 = c^2$$

$$\boxed{}^2 + \boxed{}^2 \overset{?}{=} \boxed{}^2$$

$$\boxed{} + \boxed{} \overset{?}{=} \boxed{}$$

$$\boxed{} \overset{?}{=} \boxed{}$$

    By the converse of the Pythagorean Theorem, the triangle $\boxed{\textbf{is / is not}}$ a right triangle.

3. The marketing team at a new electronics company is designing a logo that contains a circle and a triangle. On one design, the triangle's side lengths are 2.5 in., 6 in., and 6.5 in. Is the triangle a right triangle? Explain. (Example 2)

_____

? ESSENTIAL QUESTION CHECK-IN

4. How can you use the converse of the Pythagorean Theorem to tell if a triangle is a right triangle?

_____

_____

12.2 Independent Practice

COMMON CORE **8.G.6**
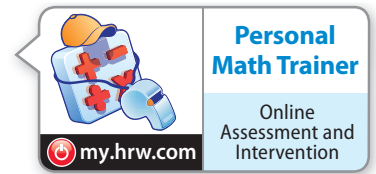
Tell whether each triangle with the given side lengths is a right triangle.

5. 11 cm, 60 cm, 61 cm

_____

6. 5 ft, 12 ft, 15 ft

_____

7. 9 in., 15 in., 17 in.

_____

8. 15 m, 36 m, 39 m

_____

9. 20 mm, 30 mm, 40 mm

_____

10. 20 cm, 48 cm, 52 cm

_____

11. 18.5 ft, 6 ft, 17.5 ft

_____

12. 2 mi, 1.5 mi, 2.5 mi

_____

13. 35 in., 45 in., 55 in.

_____

14. 25 cm, 14 cm, 23 cm

_____

15. The emblem on a college banner consists of the face of a tiger inside a triangle. The lengths of the sides of the triangle are 13 cm, 14 cm, and 15 cm. Is the triangle a right triangle? Explain.

_____

16. Kerry has a large triangular piece of fabric that she wants to attach to the ceiling in her bedroom. The sides of the piece of fabric measure 4.8 ft, 6.4 ft, and 8 ft. Is the fabric in the shape of a right triangle? Explain.

_____

17. A mosaic consists of triangular tiles. The smallest tiles have side lengths 6 cm, 10 cm, and 12 cm. Are these tiles in the shape of right triangles? Explain.

_____

18. **History** In ancient Egypt, surveyors made right angles by stretching a rope with evenly spaced knots as shown. Explain why the rope forms a right angle.

_____

_____

_____
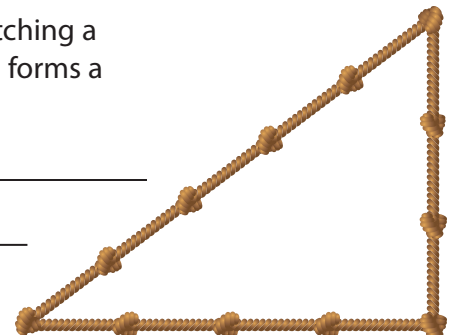
19. Justify Reasoning Yoshi has two identical triangular boards as shown. Can he use these two boards to form a rectangle? Explain.

_____

_____

_____

1 m

0.75 m

1.25 m

1.25 m

0.75 m

1 m
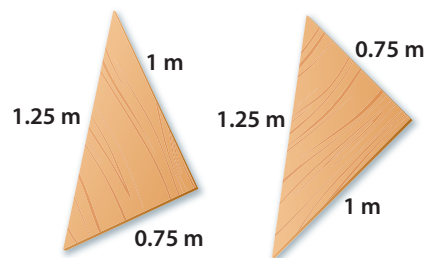
20. Critique Reasoning Shoshanna says that a triangle with side lengths 17 m, 8 m, and 15 m is not a right triangle because $17^2 + 8^2 = 353$, $15^2 = 225$, and $353 \neq 225$. Is she correct? Explain.

_____

_____

H.O.T. **FOCUS ON HIGHER ORDER THINKING**

21. Make a Conjecture Diondre says that he can take any right triangle and make a new right triangle just by doubling the side lengths. Is Diondre's conjecture true? Test his conjecture using three different right triangles.

_____

_____

_____

_____

22. Draw Conclusions A diagonal of a parallelogram measures 37 inches. The sides measure 35 inches and 1 foot. Is the parallelogram a rectangle? Explain your reasoning.

_____

_____

_____

_____

23. Represent Real-World Problems A soccer coach is marking the lines for a soccer field on a large recreation field. The dimensions of the field are to be 90 yards by 48 yards. Describe a procedure she could use to confirm that the sides of the field meet at right angles.

_____

_____

_____

_____

Work Area

LESSON
12.3

COMMON CORE 8.G.8

Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
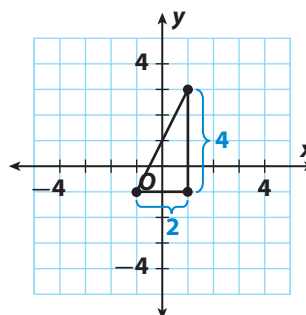
Distance Between Two Points

How can you use the Pythagorean Theorem to find the distance between two points on a coordinate plane?

Pythagorean Theorem in the Coordinate Plane

EXAMPLE 1

COMMON CORE 8.G.8

Math On the Spot
my.hrw.com

The figure shows a right triangle. Approximate the length of the hypotenuse to the nearest tenth using a calculator.

STEP 1 Find the length of each leg.

The length of the vertical leg is 4 units.

The length of the horizontal leg is 2 units.

STEP 2 Let $a = 4$ and $b = 2$. Let c represent the length of the hypotenuse. Use the Pythagorean Theorem to find c.

$$a^2 + b^2 = c^2$$

$$4^2 + 2^2 = c^2 \quad \text{Substitute into the formula.}$$

$$20 = c^2 \quad \text{Add.}$$

$$\sqrt{20} = c \quad \text{Take the square root of both sides.}$$

$$\sqrt{20} \approx 4.5 \quad \text{Use a calculator and round to the nearest tenth.}$$

STEP 3 Check for reasonableness by finding perfect squares close to 20.

$\sqrt{20}$ is between $\sqrt{16}$ and $\sqrt{25}$, or $\sqrt{16} < \sqrt{20} < \sqrt{25}$.

Simplifying gives $4 < \sqrt{20} < 5$.

Since 4.5 is between 4 and 5, the answer is reasonable.

The hypotenuse is about 4.5 units long.

YOUR TURN
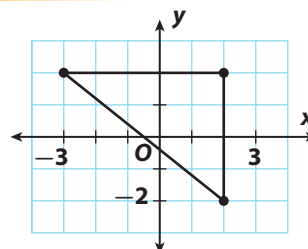
1. Approximate the length of the hypotenuse to the nearest tenth using a calculator.

_____

Personal Math Trainer

Online Assessment and Intervention

my.hrw.com

Finding the Distance Between Any Two Points

The Pythagorean Theorem can be used to find the distance between any two points (x_1, y_1) and (x_2, y_2) in the coordinate plane. The resulting expression is called the Distance Formula.

Distance Formula

In a coordinate plane, the distance d between two points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Use the Pythagorean Theorem to derive the Distance Formula.

A To find the distance between points P and Q, draw segment $\overline{PQ}$ and label its length d. Then draw horizontal segment $\overline{PR}$ and vertical segment $\overline{QR}$. Label the lengths of these segments a and b. Triangle

PQR is a _____ triangle, with hypotenuse _____.

B Since $\overline{PR}$ is a horizontal segment, its length, a, is the difference

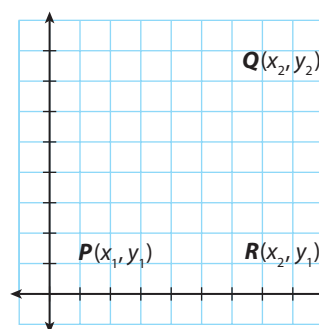between its x-coordinates. Therefore, $a = x_2 -$ _____.

C Since $\overline{QR}$ is a vertical segment, its length, b, is the difference between

its y-coordinates. Therefore, $b = y_2 -$ _____.

D Use the Pythagorean Theorem to find d, the length of segment $\overline{PQ}$. Substitute the expressions from **B** and **C** for a and b.

$d^2 = a^2 + b^2$

$d = \sqrt{a^2 + b^2}$

$d = \sqrt{\left(\boxed{} - \boxed{}\right)^2 + \left(\boxed{} - \boxed{}\right)^2}$

Math Talk

Mathematical Practices

What do $x_2 - x_1$ and $y_2 - y_1$ represent in terms of the Pythagorean Theorem?

Reflect

2. Why are the coordinates of point R the ordered pair (x_2, y_1)?

_____

_____

_____

Finding the Distance Between Two Points

The Pythagorean Theorem can be used to find the distance between two points in a real-world situation. You can do this by using a coordinate grid that overlays a diagram of the real-world situation.
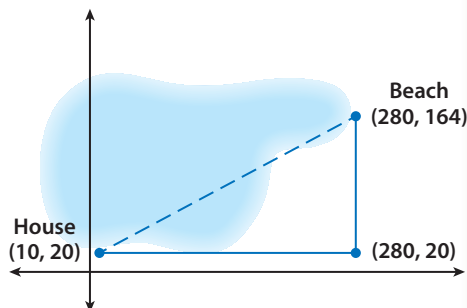
EXAMPLE 2 Real World

COMMON CORE   8.G.8

Francesca wants to find the distance between her house on one side of a lake and the beach on the other side. She marks off a third point forming a right triangle, as shown. The distances in the diagram are measured in meters.

Use the Pythagorean Theorem to find the straight-line distance from Francesca's house to the beach.

Beach
(280, 164)

House
(10, 20)   (280, 20)

STEP 1   Find the length of the horizontal leg.

The length of the horizontal leg is the absolute value of the difference between the x-coordinates of the points (280, 20) and (10, 20).

$$\left|280 - 10\right| = 270$$

The length of the horizontal leg is 270 meters.

STEP 2   Find the length of the vertical leg.

The length of the vertical leg is the absolute value of the difference between the y-coordinates of the points (280, 164) and (280, 20).

$$\left|164 - 20\right| = 144$$

The length of the vertical leg is 144 meters.

STEP 3   Let $a = 270$ and $b = 144$. Let c represent the length of the hypotenuse. Use the Pythagorean Theorem to find c.

$$a^2 + b^2 = c^2$$

$$270^2 + 144^2 = c^2 \qquad \textit{Substitute into the formula.}$$

$$72{,}900 + 20{,}736 = c^2 \qquad \textit{Simplify.}$$

$$93{,}636 = c^2 \qquad \textit{Add.}$$

$$\sqrt{93{,}636} = c \qquad \textit{Take the square root of both sides.}$$

$$306 = c \qquad \textit{Simplify.}$$

The distance from Francesca's house to the beach is 306 meters.

Math Talk
Mathematical Practices

Why is it necessary to take the absolute value of the coordinates when finding the length of a segment?

© Houghton Mifflin Harcourt Publishing Company

Reflect

3. Show how you could use the Distance Formula to find the distance from Francesca's house to the beach.

_____

_____

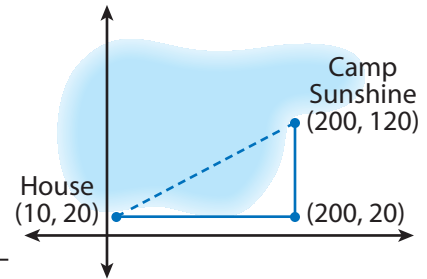_____

YOUR TURN

4. Camp Sunshine is also on the lake. Use the Pythagorean Theorem to find the distance between Francesca's house and Camp Sunshine to the nearest tenth of a meter.

_____

Camp Sunshine (200, 120)

House (10, 20)

(200, 20)

Guided Practice

1. Approximate the length of the hypotenuse of the right triangle to the nearest tenth using a calculator. (Example 1)

_____

2. Find the distance between the points $(3, 7)$ and $(15, 12)$ on the coordinate plane. (Explore Activity) _____

3. A plane leaves an airport and flies due north. Two minutes later, a second plane leaves the same airport flying due east. The flight plan shows the coordinates of the two planes 10 minutes later. The distances in the graph are measured in miles. Use the Pythagorean Theorem to find the distance shown between the two planes.
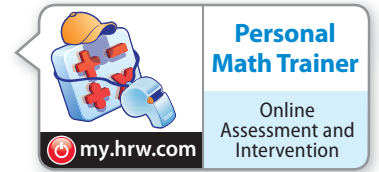
(Example 2) _____

(1, 80)

Airport (1, 1)

(68, 1)
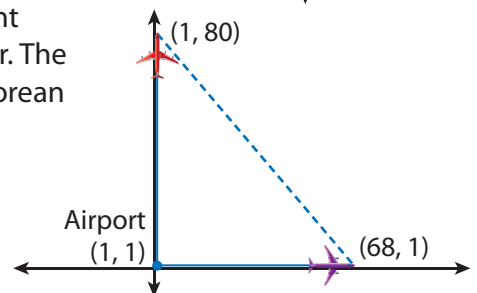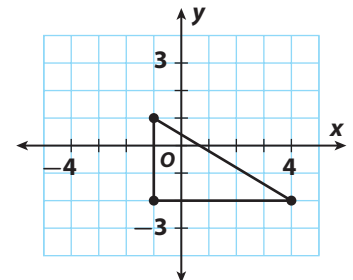
❓ ESSENTIAL QUESTION CHECK-IN

4. Describe two ways to find the distance between two points on a coordinate plane.

_____

_____

_____

12.3 Independent Practice

Personal Math Trainer

Online Assessment and Intervention

my.hrw.com

5. A metal worker traced a triangular piece of sheet metal on a coordinate plane, as shown. The units represent inches. What is the length of the longest side of the metal triangle? Approximate the length to the nearest tenth of an inch using a calculator. Check that your answer is reasonable.

_____
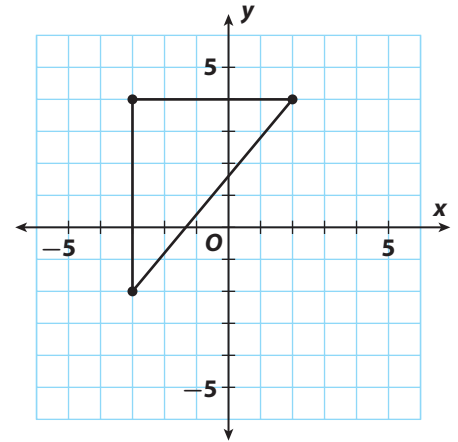
6. When a coordinate grid is superimposed on a map of Harrisburg, the high school is located at (17, 21) and the town park is located at (28, 13). If each unit represents 1 mile, how many miles apart are the high school and the town park? Round your answer to the nearest tenth.

_____

7. The coordinates of the vertices of a rectangle are given by $R(-3, -4)$, $E(-3, 4)$, $C(4, 4)$, and $T(4, -4)$. Plot these points on the coordinate plane at the right and connect them to draw the rectangle. Then connect points E and T to form diagonal $\overline{ET}$.
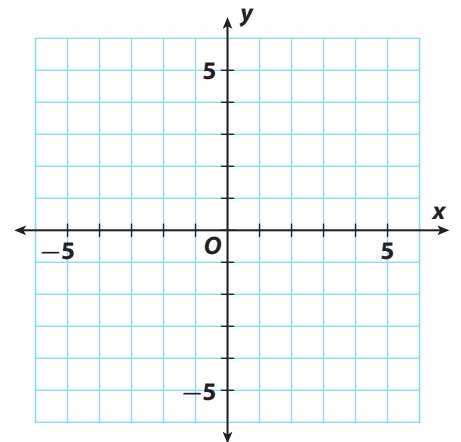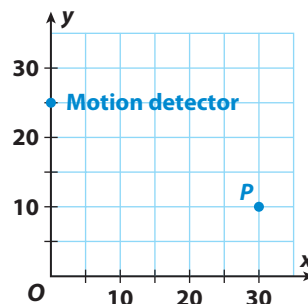
a. Use the Pythagorean Theorem to find the exact length of $\overline{ET}$.

_____

b. How can you use the Distance Formula to find the length of $\overline{ET}$? Show that the Distance Formula gives the same answer.

_____

_____

_____

_____

_____

8. **Multistep** The locations of three ships are represented on a coordinate grid by the following points: $P(-2, 5)$, $Q(-7, -5)$, and $R(2, -3)$. Which ships are farthest apart?

_____

9. Make a Conjecture Find as many points as you can that are 5 units from the origin. Make a conjecture about the shape formed if all the points 5 units from the origin were connected.

_____

_____

_____

10. Justify Reasoning The graph shows the location of a motion detector that has a maximum range of 34 feet. A peacock at point *P* displays its tail feathers. Will the motion detector sense this motion? Explain.

_____

_____

H.O.T. **FOCUS ON HIGHER ORDER THINKING**

11. Persevere in Problem Solving One leg of an isosceles right triangle has endpoints (1, 1) and (6, 1). The other leg passes through the point (6, 2). Draw the triangle on the coordinate plane. Then show how you can use the Distance Formula to find the length of the hypotenuse. Round your answer to the nearest tenth.

_____

_____

_____
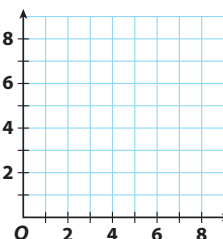
12. Represent Real-World Problems
The figure shows a representation of a football field. The units represent yards. A sports analyst marks the locations of the football from where it was thrown (point *A*) and where it was caught (point *B*). Explain how you can use the Pythagorean Theorem to find the distance the ball was thrown. Then find the distance.
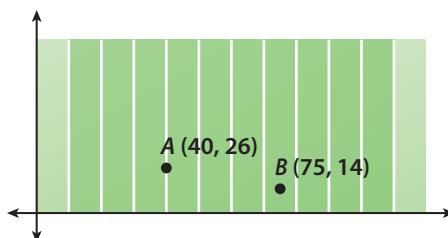
_____

_____

_____

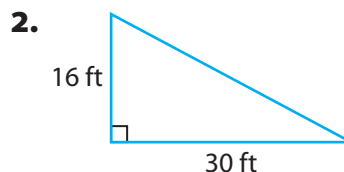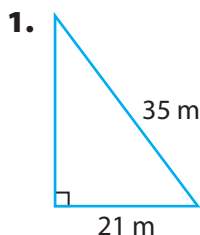_____

Work Area

Ready to Go On?

12.1 The Pythagorean Theorem

Find the length of the missing side.

1.

35 m

21 m

_____

2.

16 ft

30 ft

_____

12.2 Converse of the Pythagorean Theorem

Tell whether each triangle with the given side lengths is a right triangle.

3. 11, 60, 61 _____

4. 9, 37, 40 _____

5. 15, 35, 38 _____

6. 28, 45, 53 _____

7. Keelie has a triangular-shaped card. The lengths of its sides are 4.5 cm, 6 cm, and 7.5 cm. Is the card a right triangle? _____
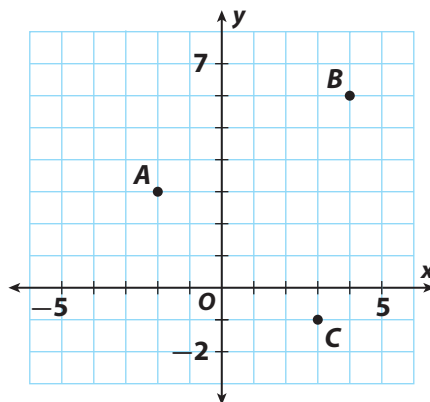
12.3 Distance Between Two Points

Find the distance between the given points. Round to the nearest tenth.

8. *A* and *B* _____

9. *B* and *C* _____

10. *A* and *C* _____

? ESSENTIAL QUESTION

11. How can you use the Pythagorean Theorem to solve real-world problems?

_____

_____

_____

MODULE 12 MIXED REVIEW
Assessment Readiness

COMMON CORE

Personal Math Trainer

Online Assessment and Intervention

my.hrw.com

Selected Response

1. What is the missing length of the side?

89 ft

80 ft

Ⓐ 9 ft   Ⓒ 39 ft
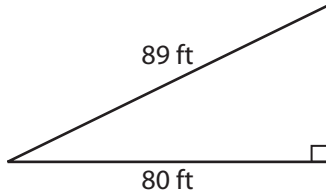
Ⓑ 30 ft   Ⓓ 120 ft

2. Which relation does **not** represent a function?

Ⓐ (0, 8), (3, 8), (1, 6)

Ⓑ (4, 2), (6, 1), (8, 9)

Ⓒ (1, 20), (2, 23), (9, 26)

Ⓓ (0, 3), (2, 3), (2, 0)

3. Two sides of a right triangle have lengths of 72 cm and 97 cm. The third side is **not** the hypotenuse. How long is the third side?

Ⓐ 25 cm   Ⓒ 65 cm

Ⓑ 45 cm   Ⓓ 121 cm

4. To the nearest tenth, what is the distance between point *F* and point *G*?
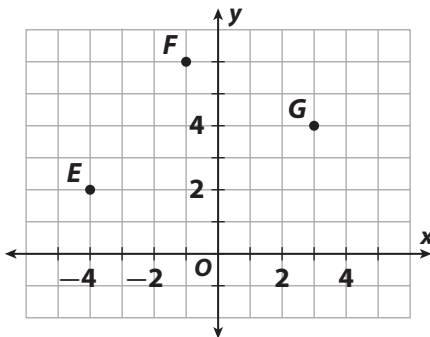
Ⓐ 4.5 units   Ⓒ 7.3 units

Ⓑ 5.0 units   Ⓓ 20 units

5. A flagpole is 53 feet tall. A rope is tied to the top of the flagpole and secured to the ground 28 feet from the base of the flagpole. What is the length of the rope?

Ⓐ 25 feet   Ⓒ 53 feet

Ⓑ 45 feet   Ⓓ 60 feet

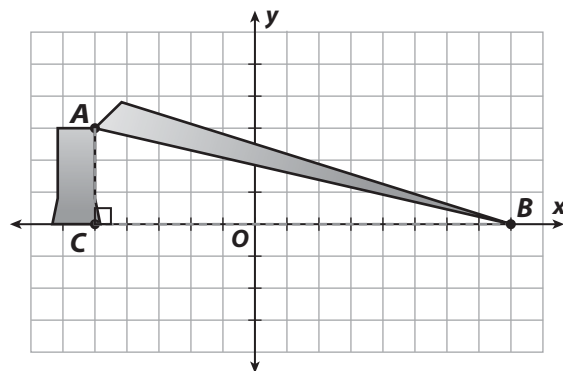**6.** Which set of lengths are **not** the side lengths of a right triangle?

Ⓐ 36, 77, 85   Ⓒ 27, 120, 123

Ⓑ 20, 99, 101   Ⓓ 24, 33, 42

7. A triangle has one right angle. What could the measures of the other two angles be?

Ⓐ 25° and 65°   Ⓒ 55° and 125°

Ⓑ 30° and 15°   Ⓓ 90° and 100°

Mini-Task

8. A fallen tree is shown on the coordinate grid below. Each unit represents 1 meter.

a. What is the distance from *A* to *B*?

_____

b. What was the height of the tree before it fell?

_____