Precalculus 115, section 7. 3 Trig Double Angle Formulae

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For this section, we introduce two identities, which you'll need to memorize.

$$\sin(2x) = 2\sin x \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

Your text also has two additional versions for cos $2x$ along with a formula for tan $2x$, but my recommendation is that you not memorize these. Instead, it's fairly simple to derive the cosine formulae, and to find sine and cosine values, then use the definition of tangent.

The proofs of the double-angle formulae come directly from the sum of angles formulae.

$$\sin(s+t) = \sin s \cos t + \cos s \sin t$$

$$\begin{aligned} \sin 2x &= \sin(x+x) \\ &= \sin x \cos x + \cos x \sin x \\ &= 2\sin x \cos x \end{aligned}$$

The proof of the double-angle formula is similar. I'll leave it to you to do for yourself, and instead will focus on the two alternate versions.

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$\cos(2x) = \left(1 - \sin^2 x\right) - \sin^2 x = 1 - 2\sin^2 x$$

$$\cos(2x) = \cos^2 x - \left(1 - \cos^2 x\right) = 2\cos^2 x - 1$$

Example A: Given $\tan x = \frac{1}{4}$ and x in Quadrant III, find sin $2x$, cos $2x$, and tan $2x$.

Example B: Rewrite $\cos^4 x$ in terms of the first power of cosine.

You won't need to memorize either the reduction of powers formulae or the half-angle formulas for this class. They won't be needed again until Math 141. At that time, you can derive the reduction of powers formulae from the alternate versions of the cos 2*x* formula, then derive the half-angle formula by taking the square root of both sides of the reduction of powers formulae and substituting $x = \frac{u}{2}$.

Example C (text #76): Prove the identity $\frac{2\tan x}{1+\tan^2 x} = \sin 2x$.

Example D (text #104): A rectangle is to be inscribed in a semicircle of radius 5 cm. a) Show that the area of the rectangle is given by the function $A(\theta) = 25\sin 2\theta$. b) Find the largest possible area for such and inscribed rectangle. c) Find the dimensions of the inscribed rectangle with the largest possible area.