Types of Signal Systems and their Properties

Academic Resource Center

THE ARC.

Types of Signals

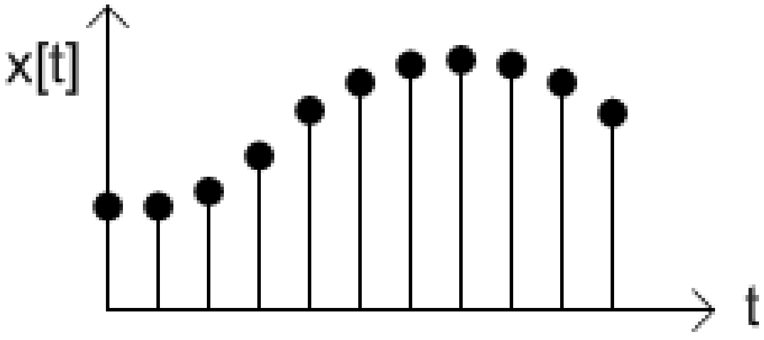
- **Continuous Time Signal**

  If the independent variable (*t*) is continuous, then the corresponding signal is continuous time signal.

Types of Signals

- **Discrete Time Signal**

  If the independent variable (*t*) takes on only discrete values, for example t = ±1, ±2, ±3, ...



- **Periodic Signal**

  If the transformed signal is same as *x*(t+nT), then the signal is periodic.

  $$x(t) = x(t + nT)\ ,\ n = 1, 2, 3, \dots$$

  where T is fun… …iod) of signal *x*(t)

  In discrete-time, the periodic signal is;

  $$x[n + mN] = x[n], \text{ for all integer } n \text{ and } m$$

- **Orthogonal Signal**

  Each comp… …rs.

  Orthogonal signal is denoted as φ(t).

- **Even and Odd Signal**

  One of characteristics of signal is symmetry that may be useful for signal analysis. Even signals are symmetric around vertical axis, and Odd signals are symmetric about origin.

  Even Signal:
  A signal is referred to as an even if it is identical to its time-reversed counterparts; $x(t) = x(-t)$.

  Odd Signal:
  A signal is odd if $x(t) = -x(-t)$.
  An odd signal must be 0 at $t=0$, in other words, odd signal passes the origin.

- Using the definition of even and odd signal, any signal may be decomposed into a sum of its even part, $x_e(t)$, and its odd part, $x_o(t)$, as follows:

$$x(t) = x_e(t) + x_o(t)$$
$$= \frac{1}{2}\{x(t) + x(-t)\} + \frac{1}{2}\{x(t) - x(-t)\}$$

$$\text{where} \quad x_e(t) = \frac{1}{2}\{x(t) + x(-t)\}, \quad x_o(t) = \frac{1}{2}\{x(t) - x(-t)\}$$

- It is an … urier series. In Fourier … of sine and cosine signals. Notice that sine function is odd signal and cosine function is even signal.

Energy of Signals

If we assume for definiteness a unit load (a 1 Ω resistor), then if $f(t)$ is the voltage across the resistor or the current through the resistor, then the energy dissipated is

$$E_f = \int_{-\infty}^{\infty} f^2(t)dt$$

If this quantity is finite, then $f(t)$ is said to be an **energy signal** . Not every signal that we consider analytically is an energy signal: for example, $f(t) = \cos(\omega_o t)$ is not an energy signal.

Substituting in for the $f(t)$ in terms of the inverse F.T.,

$$E_f = \int_{-\infty}^{\infty} f(t)\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega\right]dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\int_{-\infty}^{\infty} f(t)e^{j\omega t}dtd\omega$$

So we can write

$$E_f = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)F(-\omega)d\omega$$

For a real $f(t)$, the F.T. satisfies $F(\omega) = F^*(-\omega)$ (they are conjugates), so that

$$E_f = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2 d\omega$$

This is **Parseval's theorem** for Fourier Transforms. It can be written using inner product notation as

$$\langle f(t), f(t)\rangle = \frac{1}{2\pi}\langle F(\omega), F(\omega)\rangle.$$

It can also be generalized for products of different functions as

$$\langle f(t), g(t)\rangle = \frac{1}{2\pi}\langle F(\omega), G(\omega)\rangle.$$

We can think of an increment of energy lying in an increment of bandwidth:

$$\Delta E_f = \frac{1}{2\pi}|F(\omega)|^2\Delta\omega$$

To get the total energy, add up all of the pieces. The function $|F(\omega)|^2$ is therefore referred to as the **energy spectral density** of the function: it tells where the energy is distributed in frequency.

Power of Signals

Power is a time average of energy (energy per unit time). This is useful when the energy of the signal goes to infinity.

$$P_f = \underset{T \to \infty}{\text{limit}} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \left(|f(t)|\right)^2 \, \mathrm{d}\, t$$

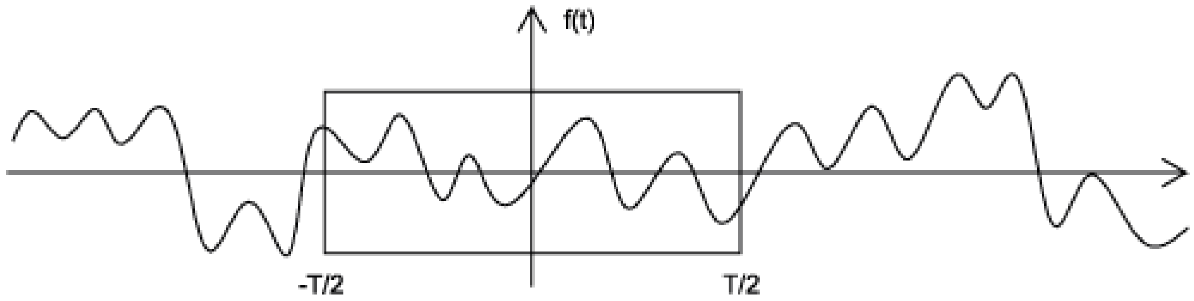


Figure : We compute the energy per a specific unit of time, then allow that time to go to infinity.

1. Compute $\frac{\text{Energy}}{T}$
2. Then look at $\underset{T \to \infty}{\text{limit}} \frac{\text{Energy}}{T}$

P_f is often called the mean-square value of f. $\sqrt{P_f}$ is then called the **root mean squared** (**RMS**) value of f.