Remarks on lectures for

The 2nd Seminar on Harmonic Analysis and Applications

IPM, Tehran, January 5-7, 2014

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Main references.

[**1**] M. Ilie and N.S. The spine of a Fourier-Stieltjes algebra. *Proc. London Math. Soc. (3)* 94 (2):273–301, 2007. arXiv: `math/0505591`.

[**2**] M. Ilie and N.S. Corrigendum: The spine of a Fourier-Stieltjes algebra. *Proc. Lond. Math. Soc. (3)* 104: 859-863, 2012. `arXiv:1106.335`.

For arXiv, I recommend `http://front.math.ucdavis.edu`.

[**3**] G. Arsac. Sur l'espace de Banach engendré par les coefficients d'une représentation unitaire. *Publ. Dép. Math. (Lyon)* 13 (1976), no. 2, 1–101. [Contact me `nspronk@uwaterloo.ca` for a link to this document, as well as some unfinished notes of mine and the thesis of my comer student, C. Zwarich.]

[**4**] W. Ruppert. *Compact Semitopological Semigroups: An Intrinsic Theory*, Lecture Notes in Mathematics, 1079, Springer, 1984.

New result. All unexplained notation is as in [1].

Theorem. [N.S., in preparation] *Let $\tau \in \mathcal{T}_{nq}(G)$. Then $B(G_\tau)$ is a quotient of* $\mathrm{B}(G)$*, and there is an ideal* $\mathrm{I}_\tau(G)$ *for which* $\mathrm{B}(G) = \mathrm{B}_\tau(G) \oplus_{\ell^1} \mathrm{I}_\tau(G)$*. In particular,* $\mathrm{B}_\tau(G)$ *is the space of all matrix coefficients of representations which are τ-continuous, and* $\mathrm{I}_\tau(G)$ *is the space of all matrix coefficients of representations which contain no subrepresentation which is τ-continuous.*

The proof makes crucial use of II.4.13 (iv) in [4].

Problems. All unexplained notation is as in [1].

(i) Let G be a locally compact abelian group and $\eta : L \to G$ be continuous injective homomorphism form another locally compact, σ-compact, abelian group. Show that

$$\mathrm{I}^{(\eta,L)}(G) = \{\mu \in \mathrm{M}(G) : \mu(s\,\eta(L)) = 0 \text{ for all } s \text{ in } G\}$$

is an ideal in $\mathrm{M}(G)$. (This was misstated in the problem session.)

(i') Show that under Fourier-Stieltjes transform, $\mathrm{I}^{(\eta,L)}(G)$ corresponds to some ideal $\mathrm{I}_\tau(G)$, as outlined in the theorem above.

(ii) Let $C = \{-1,1\}^{\mathbb{N}}$, which is a compact group with product topology. Given a partition $I \sqcup (\mathbb{N} \setminus I)$ of $\mathbb{N}$, consider the topology τ_I on C given by

$$(C, \tau_I) = \underbrace{\{-1,1\}^I}_{\text{prod. top.}} \times \underbrace{\{-1,1\}^{\mathbb{N}\setminus I}}_{\text{disc. top.}}.$$

Show that $\tau_I = \tau_J$ if and only if $I \triangle J$ (symmetric difference) is finite. Show that there are uncountably many distinct topologies.

(ii') Determine if $\widehat{\mathcal{T}}(C) = \{\tau_I : I \subset \mathbb{N}\}$. [Observe that $\widehat{C} = \{-1,1\}^{\oplus\mathbb{N}}$ is the universal group for separable 2-torsion groups. This may give rise to exotic elements of $\mathcal{T}_{nq}(\widehat{C})$.]