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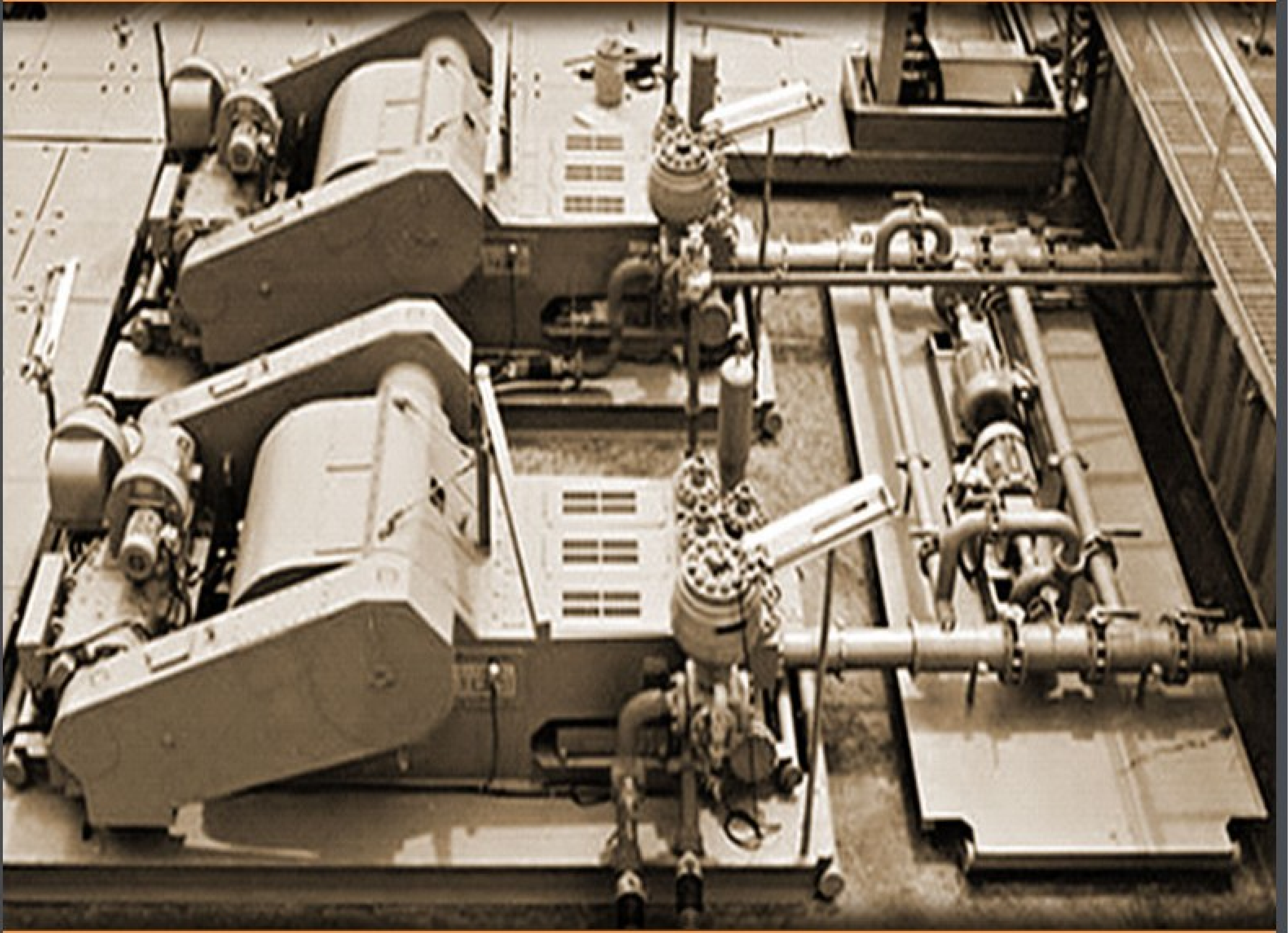
Exercises within Drilling Fluid Engineering

Pål Skalle



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Preface

These exercises have been developed to fit the content of the text book Drilling Fluid Engineering at www.bookboon.dk. The content has the understanding of the physics and mathematics of the processes in focus. Practical applications have also priority, but come after the physics. Most of the exercises have been solved by students in the corresponding course at the Department of Petroleum Engineering and Applied Geophysics at NTNU of Trondheim. If the readers have any comments that could improve the excersises, please contact me at pal.skalle@ntnu.no. Any such comments will be worked into the next year's issue

Pål Skalle
Trondheim, September 2011

1 Fluid Properties

1.1 Filter loss control

a) How can filter loss be optimized in the planning phase?
b) How is filter loss treated if it shows an increasing tendency during drilling.

1.2 Filter loss control

The water phase of a specificdrilling fluid has the identical water activity as the pore waterof the sediments, and will penetrate into the shale in accordance with Darcy's law, although in very small quantities. Is it possible to reduce the flow of water (without changingits salt content)?

Comment: A reduction of water flow into shaleis beneficial because this will reduce unwanted reaction between the drilling fluid's water phase and shale further away from the wall, where the water activity of the pore watermay be different.

1.3 Filter loss control

a) Does particle size in mud has any importance during the drilling process?
b) You have two sand formations of nearly equal pore pressure. Will filtrate invasion be greater in sand with a permeability of 100 mD and 20 % porosity, than in one with 5 mD and 5 % porosity? The final filter cake permeability in both sands is assumed to end up at around 10^{-3}mD.

1.4 Filter loss control

We want to obtain a physical picture of how far the filtrate and the particles penetrate a porous formation and gradually stops due to mud cake formation. A 12 $^1/_4$" hole with open hole length of 4000 ft is being drilled. The bottom 10 % of the borehole length is of porous formation (defining the filter area) with a porosity of 15 %. Assume that this porosity corresponds to the porosity of the filter paper. The filter area A of the filter press is 45 cm^2(r = 3.9 cm.).

A laboratory test of the mud showed an API water loss of 25 ml/30 min. The cumulative loss is proportional to square root of time;$t^{1/2}$. Assume therefore that the accumulative filterloss V_f is expresses as:

$$V_f = A \cdot \sqrt{\frac{k \cdot \Delta p}{\mu}} \cdot \sqrt{t} = A \cdot C \cdot \sqrt{t}$$

We assume the parameters defined by C are constants.

a) To simplify the filter loss estimation imagine that the time of drilling the well is negligible. Construct a plot of filtration loss vs. time. Estimate the filter loss after 24 hours.
b) Calculate the radius of the invaded zone after 24 hours, assuming 100 % displacement of the pore fluid.

1.5 Density control

What is a weighted mud (as opposed to an un-weighted)? Explain how to clean weighted muds.

1.6 Density control

Derive a simple formula of necessary volume increase, ΔV_{add}, involving adding weight material with density ρ_{add} to increase the density from ρ_1 to ρ_2. Use this formula to estimate how much mass of barite (ρ_{add} = 4.3 kg/l) must be added to increase mud density from 1.3 to 1.4 kg/l. Original mud volume was 60 m^3.

1.7 Density control

a) 1 m^3 of mud has a density of 1.5 kg/l. Adjust to 1.72 by adding 100 l mud of density 1.8, 40 kg Bentonite (to adjust rheology) of density 2.3 and x kg Barite of density 4.3 kg/l.
b) Different water based muds defined below are stored in three different mud pits. All 3 pits should be mixed into one tank and water added until the density becomes 1.55. How much volume of water must be added?

V_1 = 10 m^3, ρ_1 = 1.5 kg/l
V_2 = 20 m^3, ρ_2 = 1.6 kg/l
V_3 = 3 m^3, ρ_3 = 1.9 kg/l

1.8 Density control

Sometimes the viscosity increases unintentionally due to accumulation of fines in the mud. These fines are referred to as Low Gravity Solids Content (LGSC). The fines were to fine to be removedby the cleaning equipment. Typical density of LGS is 2.4 kg/l. They are inert, but builds viscosity since particle size is small (< 5 μ). Their unwanted effect can be reduced by diluting the mud with water. Here follows 3 examples:

1. The mud volume is 100 m^3 with a density 1.8 kg/l. The fraction of low specific gravity solids is too high, 5 weight %, and has to be decreased to 3% by water addition. Calculate the mud volume to be discarded and the amounts of fresh water and Barite that should be added. The original volume and density has to be unchanged.
2. A tank of 90 m^3 mud has a density of 1.6 kg/l and should be increased to 1.7 kg/l. Volume fraction of low-gravity solids must first be reduced from 0.055 to 0.030 by water dilution. It is required that you first discard a small part of the mud volume, so that after adding of water the volume of the mud is 90 m^3 before barite is added. How much Barite must be added, and what exactly is the new LGSC?
3. Calculate the volumes of old mud (< 10 m^3) and Barite that has to be mixed in order to fill a 10 m^3 large pit with mud which must balance a pore pressure of 410 bar in a depth of 3000 m. Barite has a density of 4.3 kg/l. The density of the old mud is 1.2 kg/l.

1.9 Rheology control

a) Will YP, PV and μ_{eff} be influenced by Barite addition?
b) Why is lyeNa0H added to the drilling fluid?
c) Define a pseudo plastic, thixotropic and a rheopectic fluid.
d) Why does viscosity of water increase when Bentonite is added?

1.10 Rheology control

a) Explain how dispersed Bentonite is able to hold up to 18 times its own volume of distilled water. Why is it that the water holding effect will be reduced when salt is added to the water?
b) Explain the reason behind the non-Newtonian behavior of Bentonite suspensions.

1.11 Rheology - control

Out on a drilling rig the questions asked are practical of nature: In the upper wellbore section sea water is often used as drilling fluid.If the viscosifying effect of clay drilled through does not produce the proper viscosity, addition of Bentonite to the water has to be considered. Use Figure 1 and assume the quality of the drilled out clay corresponds to Premium Drilling Clay. Assume that ROP is 35 m/hr when using a 26" bit. The pump rate is 3000 l/min. The required mud viscosity must be above 15 cP.

a) Will the formation provides the required viscosity?
b) What is the yield of Wyoming Bentonite?
c) What is the maximum increase of water density (originally 1.0 kg/l) by Bentonite addition, when an effective viscosity of 50 cP is the upper boundary? ($\rho_{Bentonite}$ = 2.4 kg /l).

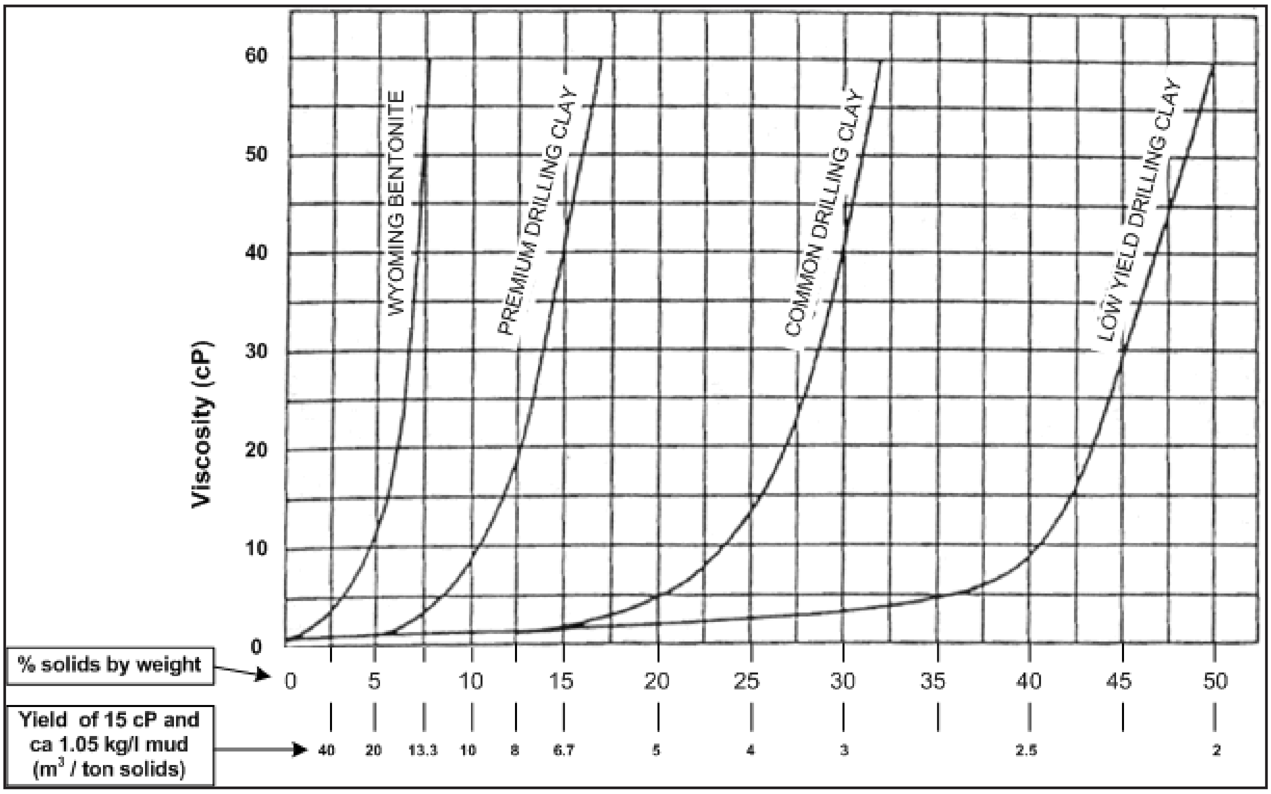


Figure 1-1: The ability of different solids to produce viscosity

1.12 Flocculation

Bentonite and polymers are the dominating viscosity agents. Bentonite is still widely used and we have to understand its behavior properly. Bentonite behaves different from other additives as it tends to swell and flocculate.

a) Why does Bentonite flocculate (weak flocculation)?
b) What will happen if untreated water based mud is used for drilling through the cement in the casing shoe area? Explain what happens to the mud (strong flocculation) and the respective operational consequences. Sketch a flow curve for the mud before and during drilling through the cement.
c) Name 3 factors which enhance flocculation of Bentonite.

1.13 Mud contamination

a) What is a contaminated mud, and how do you restore the drilling fluid parameters?
b) The geologist expected that layers of silty anhydrite would soon be penetrated at a vertical depth of 1600 m. The mud engineer was therefore told to make measurements every 15 min. of the returning mud. Soon afterwards he observed that the viscosity of the mud, a dispersed WBM system, started to rise, and became abnormally thick. Drilling continued and after a few hours of drilling/pumping, the viscosity fell back to a lower level than the original viscosity as in the table shown below.

Time	Shear stress (lb/100 ft²)			ρ	V_f
	600 rpm	30/0	10/0	kg/l	ml/30 min.
0900	42	28	16	1.31	7
0915	41	27	17	1.31	6
0930	68	54	37	1.31	18
0945	31	17	5	1.30	7

Find PV, YP and μ_{eff} (at 100 rpm) for all 4 time points.

Together with the mud engineer you are responsible for the maintenance of the drilling fluid/drilling program. Explain changes in the recorded parameters observed at 0930 and at 0945. Suggest countermeasures against these changes.

c) Assume two clay suspensions are flocculating for two different reasons; 1. Edge-to-face. 2. Calcium attack. How do the two clay suspensions behave rheologically? Make a sketch of the shear stress vs. time; during drilling into the Ca^{++} containing layer at still stand and after resuming circulation.

1.14 Flocculation

a) Find necessary YP to keep a spherical particle suspended in a mud of ρ_{mud} = 1.1 kg/1. The particle has these characteristics:

d_p = 5 mm
ρ_p= 2.3 kg/l

Similarly, find what size particle can be kept suspended when YP = 15 Pa.

b) At 12:00 the ROP became very low and it was decided to change the bit. A 15 min. stop in the operation was made before the tripping-out job was initiated. When first pipe was broken mud spilled out on the drill floor. Could this spill have been prevented? How high up can the string be hoisted before gravity pulls the mud down when the situation isas follows:

τ_{gel} 15 min. = 30 lb/100 ft^2 (14.4 Pa)
Inner diameter of the drill pipe = 4.27" (108.5 mm)

1.15 Fluid additives

Define the different words below and explain their relevance for drilling fluids.

Anhydrite	Lignite / Lignosulfonat
Bentonite	NaOH
Caustic soda	MBT
CEC	PAC
Chalk	Pre hydrated
CMC	PHPA
Colloid	SAPP
Dispergator	Sodium Sulphate
Deflocculators	Starch
HEC	Xanthan
NaOH	Lignosulphonates

1.16 Fluid Additives. Drag reducer

How does the additive called drag reducer reduce turbulent pressure so dramatically?

1.17 Fluid additives

a) How many moles/liter of hydroxyl (OH^-) concentration is required to change the pH of a drilling fluid from 7.5 to 11?
b) How much caustic soda (weight per liter) will be required to increase the pH in question a?
c) Why is potassium hydroxide (KOH) preferred to sodium hydroxide (NaOH) in controlling the pH of mud?

1.18 Fluid additive

How does drilling fluid achieves the following functions:

a) Lift cuttings from the bottom to the surface
b) Releases cuttings at surface
c) Cools and lubricates drill bit and drill stem
d) Prevent blowouts

2 Rheological models

To simplify the evaluation of drilling fluids out in the field, the simplified Bingham field method was developed. In this collection of exercises we distinguish between the field and the standard method of determining rheological model constants.

The field method is referring to the simplified method in conjunction with the Fann viscometer.

The Fann readings are converted to SI units and multiplied with the factor 1.06 (see SPE's Applied Drilling Engineering textbook Appendix A, eqn. A-6b).

2.1 Bingham

Explain why the Bingham field model is so useful for explaining mud behavior.

2.2 Bingham/ Power law

In the laboratory the following data were obtained (θ is the dial reading in the viscometer):

RPM	γ (s-1)	Θ (-)	τ (Pa)
600	1022	100	50.75
300	511	75	38.06
100	170	45	23.84
6	10	10	5.08

a) Find rheological constants for the two rheological models Bingham and Power law. For Bingham model use both field and standard method in accordance with field method and with standard method.
b) Which of the two models, Bingham or Power law, would give the best answer on basis of the given rheology while pumping 1000 l/min through a pipe of 10 cm ID.

2.3 Bingham/Power law

Rheological data are tabulated and presented graphically in Figure 2-1.

RPM	γ	θ	τ
-	s^{-1}	-	Pa
600	1022	140	71.05
300	511	98	49.75
200	340		39.60
100	170		27.39
6			8.14
3			6.61

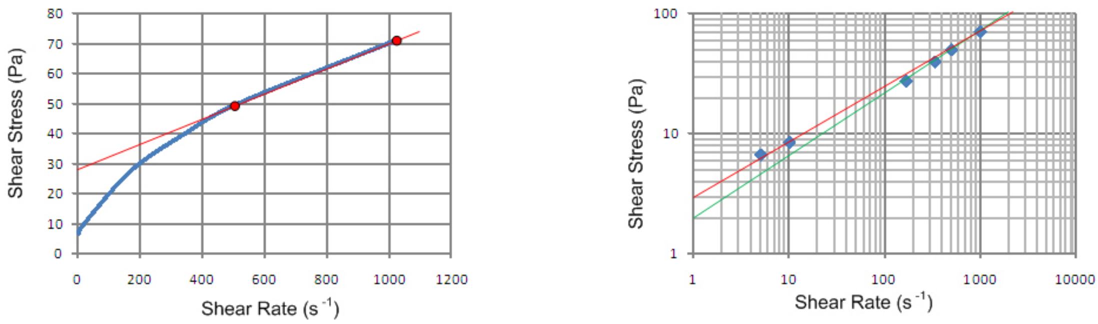


Figure 2-1: Graphical representation of rheological data (flow curve).

a) Select the best 2-data-point-rheology model between Bingham and Power law (at an viscometer RPM of 100 RPM).
b) Observe the log-log plot. It represents a typical clay-dispersed system.
Why do the two data points of the lowest shear rate deviates from the straight line made from the upper four data points?

2.4 Bingham / Power law. Regression

After measuring the rheology of the fluid it is always useful to plot its flow curve. The following data are obtained:

Speed (rpm)	γ (s^{-1})	θ (-)	τ (lbf/100 ft^2)	τ (Pa)
600	1022	60.4	64	30.6
300	511	39.6	42	20.2
200	340	32.1	34	16.3
100	170	24.5	26	12.5
6	10	14.2	15	7.2
3	5	9.4	10	4.8

1. Plot τvs. γfor three rheological modelsNewtonian, Bingham, Power-law according to

   Field procedure (2 data points)
   Standardprocedure (2 data points)
   Regression (6 data points). Use Excel linear regression.

   Find the constants of the three models. Determine which of the model are fitting best to the readings at high shear rates, i.e. for determining pressure loss in nozzles and inside the drill string (300 rpm and higher), and what model fits best to the lower rates of shear in the annulus (100 rpm and lower).

2. An exercise without calculations: Will plug flow occur in this mud system?

2.5 Effective viscosity

a) Rheology data:

600 opm	43 lb/100 ft^2
300 opm	30 lb/100 ft^2
200 opm	23 lb/100 ft^2
100 opm	16 lb/100 ft^2
6 opm	8 lb/100 ft^2
3 opm	7 lb/100 ft^2

A useful exercise is to plot the effective viscosity (Newtonian model) as a function of shear rate. The non-Newtonian and the shear thinning effect will then appear clearly.

The mud is flowing in a 1000 m long pipe with inner diameter of 4 in, flow rate is 6 000 l/min (two pumps) and density = 1.1 kg/l. Use Power Law to estimate effective viscosity at this flow rate.

b) Why is mud often so well suited to the Bingham model?

2.6 All models

The flow rate is 2500 lpm in a 1000 m long pipe with inner diameter of 10 cm. Rheological data points are given below. Mud density is 1.1 kg/l.

Shear rate (s-1)	Shear stress (Pa)
1022	55
511	40
340	35
5	19

a) Find the rheological constants for Bingham and Power law (useonly upper 2 data points) .
b) For Herschel Bulkley model, discuss three different ways of obtaining the constants.
c) Show that the effective viscosity for Bingham fluids is

$$\mu_{eff} = \mu_{pl} + \frac{\tau_0 d}{6v}$$

2.7 All models. Regression

γ	RPM	Θ	τ	τ
H_z	rpm	-	lb/100 ft^2	Pa
1022	600	50	53	25.4
511	300	34	36	17.2
340	200	26	27.6	13.2
170	100	17	18	8.6
102	60	13	13.8	6.6
51	30	9	9.5	4.5
10	6	4	4.2	2.0
5	3	3	3.2	1.5

Use the Fann-viscometer data above to determine model-constants for the first 3 modelslisted below through standard (2 data points) procedure and linear regression procedure (use Excel spread sheet).

Bingham	τ	=	$\tau_o + \mu \cdot \gamma$
Power law	τ	=	$K.\gamma^n$
Herschel Bulkley (H-B)	τ	=	$\tau_o + K\,\gamma^n$

To solve H-B, non-linear regression has to be applied. The procedure is presented in Chapter 10.6 in the Drilling Fluid Engineering text book.

Make a plot of the two models for which linear regression is possible. The lower three models are presented just to give an overview of rheological models.

Collin-Graves (C-G)	τ	=	$(\tau_o + K\,\gamma^n)\,(1-e^{-\beta\gamma})$
Robertson-Stiff (R-S)	τ	=	$K(\gamma_o + \gamma)^n$
Casson	τ	=	$\left[\sqrt{\tau_0} + \sqrt{\mu \cdot \gamma}\right]^2$

2.8 All models

Hz	lb / 100 fl²	Pa
1022	49.3	25
511	33.5	17
340	23.6	12
170	12.8	6.5
10	3.9	3.0
5		2.0

Which of the following models would you select for estimating τ at γ = 170 Hz. Use Field and standard procedure for the Bingham 2-data point model. Use the Iteration approach for the Herschel-Bulkley model. First guess of$\tau_1 = \tau_5$ reading. Start iteration with the 'n' found from Power law.

$$\tau = \tau_o + \mu \cdot \dot{\gamma}$$

$$\tau = K \cdot \dot{\gamma}^n$$

$$\tau = \tau_o + K \cdot \dot{\gamma}^n$$

3 Drilling fluid dynamics

3.1 Velocity profile. Continuity equation

The incompressible steady flow between two parallel plates with breath b in Figure 3.1 is initially uniform; $v = \overline{v} = 8$ cm/s at the inlet area, while downstream the flow develops into the parabolic laminar profile $v(z) = az\,(z_0 - z)$, where a is constant and Z_o the plate distance. If $z_0 = 4$ cm what is the value of v_{max}?

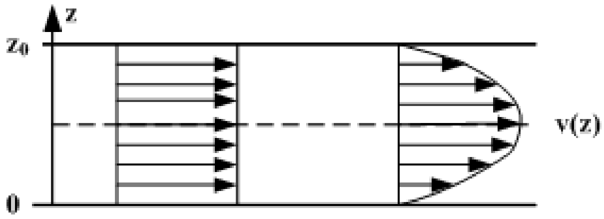


Figure 3-1: Data of exercise 3.1.

3.2 Velocity profile. Momentum flux

a) The fully developed laminar pipe-flow velocity profile $v = v_{max}(1 - r^2 / R^2)$, $v_\theta = 0$, $v_r = 0$: This is an exact solution to the cylindrical Navier-Stoke equation. Neglect gravity and compute the pressure distribution in the pipe; p(r,z), and the shear-stress distribution; τ(r,z), using R, v_{max} and μ as parameters. Why does the maximum shear occur at the wall? Why does the density not appear as a parameter?

b) For flow between parallel plates, compute 1) wall shear stress and 2) the average velocity.From the Text book, Chapter 3, we find that $v(y) = dp/dx \cdot h^2/2\mu \cdot (1 - y^2/h^2$.The general definition of wall shear stress is given by:

$$\tau_w = \tau_{xy\,wall} = \mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$$

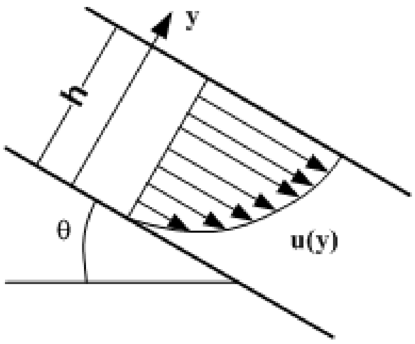


Figure 3-2: Data of exercise 3.2. The flow is in the x-direction. θ = 0 in this exercise.

3.3 Velocity profile

a) Discuss this expression, assumptions etc.

$$\frac{dp}{dx} = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\mu\,\frac{\partial v_x}{\partial r}\right)$$

b) For stationary non rotational and laminar flow of Newtonian fluids in circular horizontal pipes (z = r), show that $v(r) = \frac{dp/dx}{4\mu}\left(R^2 - r^2\right)$

c) Determine average velocity. When is the max velocity largest compared to average velocity (i.e. when is axial dispersion largest)?

d) Find wall shear stress and average pipe velocity when the pressure loss is recorded to be 0.9 bar along a 1000 m long pipe.

3.4 Pressure loss vs. rheology

Use viscometer readings from exercise 2.7.

a) Determine pressure loss pr. 1000 m in a 10 cm ID pipe with ID, flow rate of 1000 l/min. and fluid density of 1000 kg/m^3. Check Reynolds number.
b) Calculate shear rates in the pipe for Bingham, Power-law and Newtonian model. Read shear stress from the flow curve, and determined pressure drop through the universal pressure loss model.

$$\Delta p = \tau_w \cdot \frac{L}{d} \cdot 4$$

3.5 Pressure loss vs. rheology

Mud is pumped at a rate of 800 l/min with these rheological data:

RPM	θ	τ (lb/100 ft^2)
600	66.0	70
300	47.2	50
100	25.5	27
6	9.4	10

a) Find Δp_{pipe} in a 1000 m long pipe of d = 0.109 m for a field Bingham fluid.
b) Which rheological model, Newtonian or field Bingham gives a better answer when the actual Δp_{pipe} was recorded to be 0.7 MPa.

3.6 Pressure loss. Power law

a) Derive laminar pressure loss expression for Power Law fluids from a force balance.
b) Show that:

$$\frac{\rho v \ d}{\mu_{eff}} = \mathrm{Re}_{generalized}$$

when μ_{eff} for a Power-law fluid is applied.

c) Show that the Reynolds number for a Power-law fluid increases as the inner pipe diameter of the annulus decreases, while for a Newtonian it decreases. Apply annular diameter in terms of $d_{hydr} = d_o - d_i$, and let flow rate be constant.

3.7 Pressure loss. Turbulent. Energy equation

Oil of densityr = 900 kg/m^3 and kinematic viscosity n = 0.00001 m^2/s, flows at a rate of 0.2 m^3/s through a 500 m new cast-iron pipe with a diameter of 200 mm and a roughness of 0.26 mm. Determine (a) the head loss and (b) the pressure drop when the pipe slopes down at 10° in the flow direction. See Chapter 3.5 in the Textbook for further information.

3.8 Pressure loss vs. flow rate

Make a graph of pressure loss vs. flow rate in a 1000 m long pipe with inner diameter of 10 cm. Rheological data points are given in Exercise 3.4, and mud density is 1.1 kg/l. select Power law model.

3.9 Pressure loss. Field data

Prior to a pre flush/cementing operation the driller performed pressure tests to verify theoretical pressure estimations. Previously it had been difficult to duplicate the actual pressure readings during drilling with theoretically estimated pressure loss. He was convinced that the errors could be back-tracked to the pressure drop across the mud motor and the bit. These two losses are empirical and are highly uncertain. Now he had the chance to record pressure loss without these two pieces of equipment installed. After lowering the 5” DP (without the bit) down to the casing shoe he circulated for 45 min. to neutralize temperature effects. The casing, a 13 3/8”, 68 lb/ft, was set to 4 500 mMD. The pump was a relatively new Garden-Denver PZ-11-1600 HP triplex mud pump, 6” liner. One pump stroke delivered 15.29 l. At the following pump speeds he reads the average stand pipe pressures (SPP) (the average of 3 tests).

5 spm - 13 (+/-2) bars
20 spm - 20 (+/-2) bars
50 spm - 25 (+/-2.5) bars
100 spm - 50 (+/-3) bars

At the lowest and the highest pump speeds (5 and 100) he made an additional test by rotating the drill string at 100 RPM for a short interval and saw that the average reading changed to 12 and 55 bars respectively at the two pump speeds.

During the pump tests the derrick man took a mud sample and recorded its rheology:

Shear rate (s^{-1})	Shear stress (Pa)
1000	30
500	22
340	18
170	13
10	8
5	6

Mud density was 1.21 kg/l. The recorded pressure losses are presented in Figure 3.3. Evaluate pressure readings a) during pure circulation and b) during simultaneous rotation.

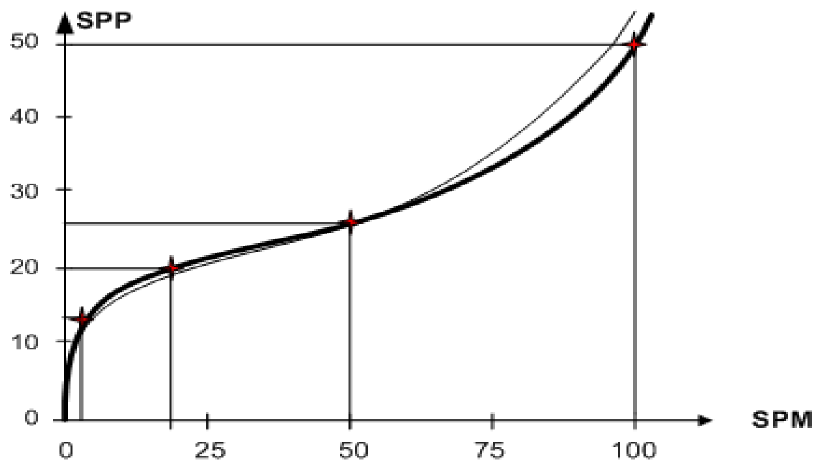


Figure 3-3: Recorded SPP with rotation (thin line) and without.

3.10 Pressure loss. Nozzles.OFU

First a 17.5" hole was drilled, into whicha 13 3/8" * 12.41" casing was set and cemented at MD of 2300 m. Finally a 12 1/4" holewas drilled down to the reservoir at 2500 m.

The drill string consisted of a drill pipe (5*4.276") and 100 m of drill collars (6.25 * 3.1"). The circulating rate was 800 GPM during drilling and the mud density 12.9 PPG. A Fann-VG viscometer gave the following readings:

$\theta_{300} = 50$
$\theta_{600} = 85$

Assume the mud rheology is best described through the Bingham model. OFU-equations are copied from Applied Drilling Engineering SPE-text book Table 4.6:

$$Re = 928\,\rho\bar{v}d_h / \mu \quad \bar{v} = q/(2.448\,d_h^2)$$

$$\Delta p_{lam} / \Delta l = \frac{\mu_{pl}\bar{v}}{1500d^2} + \frac{\tau_y}{225d}$$

$$\Delta p_{turb} / \Delta l = \frac{\rho^{0.75}\mu_{pl}^{0.25}\bar{v}^{1.75}}{1800d^{1.25}}$$

$$\Delta p_{bit} = \frac{8.311 \cdot 10^{-5} \cdot \rho \cdot q^2}{C_d^2 \cdot A_t^2} \,[psi]$$

The units are: ρ in PPG

q in GPM
A in in^2
p in psi
$C_d = 0.95$

a) The pressure drop through the annulus above the BHA wasequal to100 psi, while unknown along the BHA. What is the equivalent mud density at a depth of 2500 m?
b) The bit has 5 nozzles, each with diameter (14/32) inches. Pressure loss through the surface pipes is 200 psi.
c) What pressure is required from the pump when drilling at a depth of 2500 m?
d) Finally, compare the results with bit pressure loss estimated in SI-units:

$$\Delta p_{bit} = 1.11 \cdot \frac{1}{2}\rho\, v_{Av}^2$$

3.11 Swab pressure. Cling factor

The cling factor is used during estimation of surge & swab pressure. How would you stepwise go about to define and estimate the cling factor?

4 Hydraulic program

The hydraulic programexercises distinguish between two different approaches of preparing the hydraulic program:

1. Standard method where each liner represents one pump operating range. The hydraulic program is planned for one well section at a time
2. Extended method where the mud pumpis divided into two operating ranges; the hydraulic program is planned for all sections at the same time

Maximize ROP, which is expressed through:

In this book the following optimization criteria is the default criteria: $ROP = A^{*}\ (q/d_{nozzle})^{a8}$

4.1 Mud pump issues

a) Characterize a mud pump as detailed as possible with respect to

- Effect
- Efficiency

b) Why are mud pumps sometimes arranged in parallel or series?
c) Compare centrifugal with piston pumps
d) Explain hydraulic knocking in pumps

4.2 Parasitic pressure

A tricone bit has 3 nozzles, each 15/32nd inch diameter, ρ_{mud} = 1.3 kg/l, drilling at 2500 mMD. The following pump rates/pressures (which are close to the actual operating flow rates)are recorded;

q_{pump} (lpm)	p_p (bar)
1000	43
2000	160

Determine the value of K_1 and m in the parasitic pressure equation.

4.3 Optimal nozzles? Section wise

The triconebit is equipped with 3 * 26/32" nozzles. The pump runs at optimum flow rate. Check if bit nozzles are optimal at 3000 m. The following data are given:

$p_{p\,1}$ = 230 bar at 2500 l/min pump rate
$p_{p\,2}$ = 120 bar at 1750 l/min pump rate
p_{bit} = $1.11 \times 1/2\ \rho v^2_{nozzle}$
$p_{pumpe,max}$ = 270 bar for piston in use
$q_{pumpe,max}$ = 2700 l/min
$q_{min.ann}$ = 1700 l/min (cuttings accumulation)
$q_{max.ann}$ = 2600 l/min (wellbore erosion)
D = 3000 m
ρ_{mud} = 1400 kg/m³

$$q_{opt} = \left[\frac{2p_{pump.\max}}{K_1 D(m+2)}\right]^{1/m}$$

4.4 Liner selection. Section wise

The pump data of a National 12-P-160 (this number indicates a1600 HP pump)triplex pump operating data are presented in Chapter 6.1.

$D = 2500$ m

$\rho = 1200$ kg/m^3

$K_1 = 1.1 \cdot 10^6$

$m = 1.7$

$q_r = 0.018\ m^3/s\,(d_{bit} = 12\ 1/4")$

$q_{\max}(turb) = 0.04 m^3/s$

1. Derive qopt and determine. What is the optimal liner at this depth?
2. Determine optimal bit pressure if 6 ¾" lines is used
3. At what depth would you change from 6" to 5 ¾" liners?

4.5 Optimal parameters for BHHP. OFU. Section wise

The bit has 3 . 12/32" nozzles, and ρ_{mud} = 10 PPG while drilling in 8 200 ftMD.

The following pump rates/pressures (which are within the operating flow rates) are recorded while the bit is close to bottom:

q_{pump} (GPM)	p_p (psi)
250	800
500	3000

The pump is characterized through:

p_{max} = 3000 psi, $E_{p,max}$ = 1000 Hp, q_r = 240 GPM.

Determine optimal pump parameters and nozzle size when applying BHHP (Bit Hydraulic HP) as the optimization criteria (maximize it). The pump volume efficiency is 0.9.

$$BHHP = \frac{\Delta\, p_{bit} \cdot q}{1714} \quad (Hp)$$

Show first that the optical flow rate is:

$$q_{opt,q/d_{nozzle}} = \left[\frac{2 p_{pump}}{K_1 D (m+2)} \right]^{\frac{1}{m}}$$

Pressure drop in OFU are:

$$\Delta p_{bit} = 8.311 * 10^{-5} * \rho\ (ppg) * [q\ (gpm)]^2 / \{0.952^2 * [\ A_{nozzle}\ (in)]^2\ \}$$

4.6 Hydraulic program. Section wise

Assume the rate of penetration (ROP) is a function of bottom hole cleaning.

$$ROP = A \cdot \left(\frac{q}{d_e} \right)^{a_8}$$

Show that ROP will decrease with depth expressed through pump flow rate, q, and equivalent nozzle diameter, d_e, and the exponent a_8 (< 1) in the equations below:

$$p_{loss} = K_1 D q^m$$

$$p_{bit} = 1.11 \frac{1}{2} \rho \bar{v}^{\,2}$$

The 12 ¼" section starts at the 13 3/8" casing shoe at 1950 m MD, and is planned to reach a depth of 4000 m MD before setting next casing. The following is known:

K_1 = $2 * 10^6$
m = 1.65
$q_{required}$ = 0.025 m^3/s
q_{max} = 0.035 m^3/s

What flow rate and liners would you recommend through this depth interval with the 1600 HP pump in Chapter 6.1?

$$q_{opt} = \left(\frac{2 p_p}{(m+2) K_1 D} \right)^{\frac{1}{m}}$$

4.7 Liner selection. Complete well

Characteristics of a 1600 HP piston pump (see Chapter 6.1) and their hydraulic functions are presented graphically in Figure 4.1.Assume that the optimal pump pressure values are the maximum, given for each liner size in the pump characteristics in Chapter 6.1 (maximum value here is the recommended maximum and is in fact around 85 % of the absolute maximum).In this figure only flow rate is correctly scaled, R_p-values are only quantitative. Hydraulic parameters, the bit program, minimum and maximum flow rate and a hydraulic test is presented below:

A 1660 HP pump is used (see data in Chapter 6.1)
$K_1 = 3 * 10^6$
m = 1.47

$K_{1\ in\ range\ II}$ = $2.20 * 10^6$,m_{II} = 1.6
$K_{1\ in\ range\ I}$ = $1.25 * 10^6$, m_I = 1.5

Bit program:

Bit diameter	Start depth	Permissible annular flow rate (m^3/s)	
in	m	min	max
36	0	0.035	0.050
26	100	0.030	0.045
17 ½	500	0.027	0.035
12 ¼	1500	0.015	0.025
8 ½	3000	0.010	0.020

a) Area I is defined as the pump operating range of the smallest liner. Derive optimal flow rate, $q_{opt\ II}$, in pump area II by maximizing ROP:

$$ROP = K\left(\frac{q}{d_n}\right)^{a_8}$$

$$q_{opt\,II} = \left[\frac{(E_p)_{max}}{K_1 D(m+2)}\right]^{\frac{1}{m+1}}$$

b) Determine at which depth Range II stop (downwards) and at what depth Range I start.

c) Draw also into the graph the optimal hydraulic program (the graph is a principal drawing of ROP vs.q and thus not a quantitative correct drawing)

d) What pump area is optimum at 2000 m.

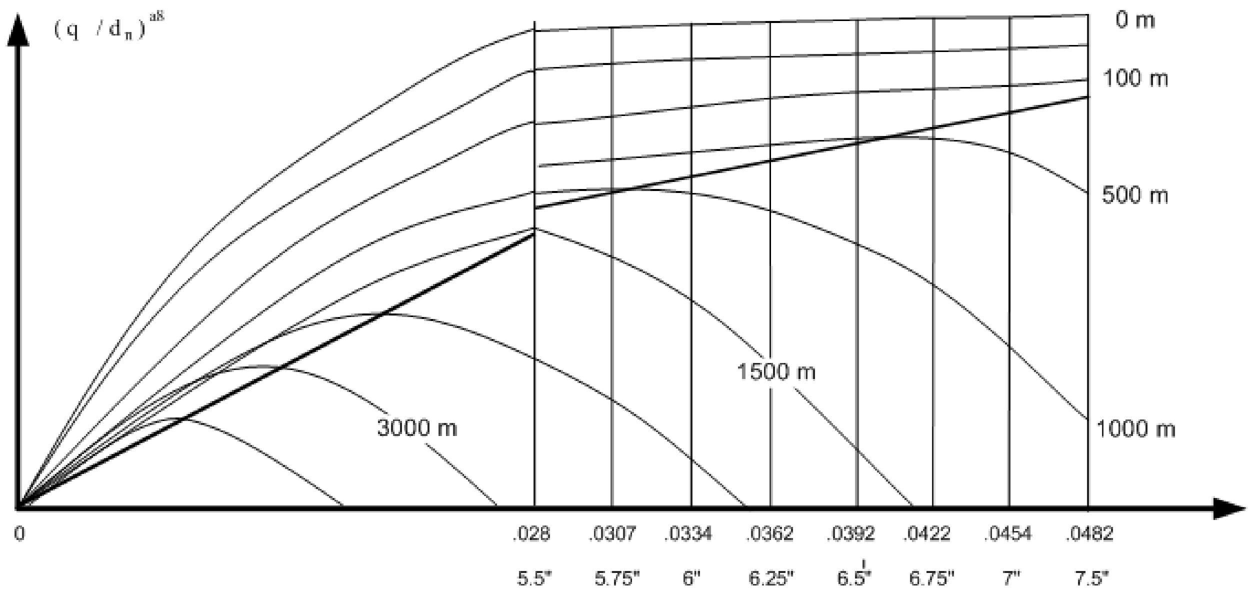


Figure 4-1: Data of exercise 4.7.

If optimal flow rate is not directly achievable, the strategy should be to approach the optimal solution, given as a bold, dotted line in the figure above.Recommend also which liners should be used in each well section.

4.8 Liner selection. Complete well

a) Find at what depths the transition from operating area II to I occur.
A 1660 HP pump is used (see data in Chapter 6.1)

$K_1 = 1.72 * 10^6$
$m = 1.51$

b) Make a program on how to determine when to change from working area II to I of the pump during drilling?

5 Well challenges

5.1 ECD. Cuttings concentration

If the cuttings concentration generated at the bottom during drilling is $c_1 = 0.02$, discuss what could be the concentration at these positions:

a) At the end of the horizontal section (= c_2)
b) At the surface in the return flow line (= c_3)
c) What determines cuttings bed height?
d) Why is cuttings accumulation in wellbore expansions (washouts) a problem during tripping?

5.2 ECD. Solids control

Mention 5 issues related to poor solids control during drilling.

5.3 ECD. Barite

Explain and suggest solution to this problem:

While POOH to change the 12 1/4" bit, the driller experienced no problems. When GIH with the new bit some weight reductions (took weight) were observed in the build-up zone (below the kick-off point KOP at 1500 m). It took around 5 h from the bit left the KOP area of the well till it returned on its trip back into the well. With the bit reached bottom the well was circulated for some time, and the returning mud behaved strangely. The mud weight, which originally was 15 PPG first decreased sharply, then increased as shown in Figure 5.1 before decreasing again.

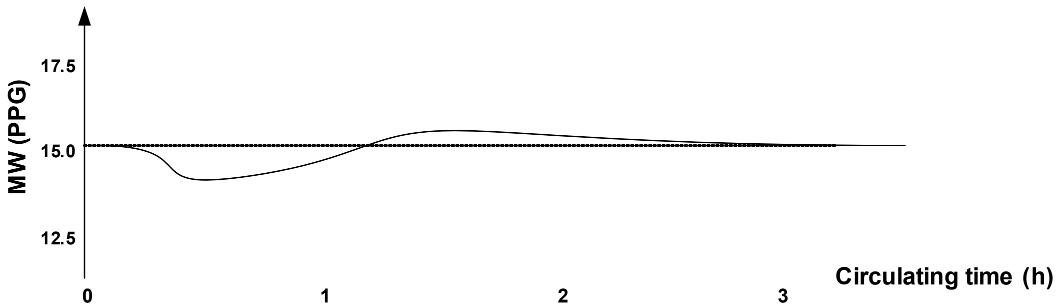


Figure 5-1: Density of returning mud after tripping.

It took 3.5 h of circulation before MW flattened out to a level close to 15 PPG. While performing back reaming the string got stuck at 2000 m(close to the end of the build-up section). The string could be neither rotated nor moved upwards, but could be moved downwards.

5.4 ECD. Flow rate and fluid consistency

A 17 ½” hole wasinitiated from the 20” casing shoe at 1100mTVDand drilled to 2100 mTVD. The bottom hole assembly consistedof 120 m of 9 ½” Drill Collars (DC). 5 ½”DrillPipe (DP) was used.

Capacities:

• 17 ½” open hole capacity:	155.2 l/m
• DC / Open hole capacity:	109.4 l/m
• DP Open hole capacity:	139.2 l/m
• DP / Casing capacity:	161.8 l/m
• 9 ½” DC / capacity:	4.56 l/m
• 5 ½” DP capacity	10.77 l/m

Mud Parameters.

• Mud density:	1.25 kg/l	
• Rheology:	600/300 rpm:	51.7 / 30.6 Pa
	200/100 rpm:	22 / 12 Pa
	6/3 rpm:	3 / 4 Pa
Gel:	10s/10 min:	5/ 13 Pa

a) Prior to drilling, the hole was circulated at 3500 l/min. What is the annular pressure loss, and what is the corresponding ECD?
b) The drilling commenced from 2100 m, and at 2300 m the average drilling rate, ROPwas 50m/hr.Formation bulk density was 2.4 kg/liter. What is the ECD in this situation?
c) At 1800 m the formation fracture pressure is 228 bar. Check if everything is OK.

5.5 ECD. Temperature change

A new mud was being prepared for the 8 ½” section. The 13 3/8” csg shoe was located at 15 000 ft vertical depth. While drilling at 16 500 ft the well started losing mud and it was decided to lower the MW from 15 to 14 PPG. The well had very narrow pressure window, and the equivalent pore pressure gradient at this depth was 13.5 PPG. The mud was mixed to 14 PPG with an effective viscosity of 40 and 30 cP at 600 and 300 RPM respectively, at an average surface mud temperature of 40 °C. After the new mud was circulated, the pump was shut off, and a flow check indicated that the well was dead (no influx of pore fluid). While repairing the power swivel the well started to flow by itself, and soon afterwards the kicking well had to be shut in to avoid a complete unloading.

How would you go about and which additional info will be required to estimate the pressure profile of an initially cold, static fluid column as a function of time.

5.6 Water activity

Define water activity, A_w and how do you determine A_w of water solutions (no clay involved)?

5.7 Water activity

a) Why is water activity a function of water salinity?
b) Explain why high water activity causes clay swelling problems?
c) How do you prevent clay swelling problem?
d) How does water activity of the water phase in OBM influence on wellbore stability?

5.8 Shale stability

For water not to enter and cause swelling of shale, in which the "mechanical" confining pressure has been relieved by drilling through it, the activity of the water in the mud (including the water phase in oil based) and in the shale must be equal.

The activity of the water in the water phase in shale cuttings is measured in the field using an electro hygrometer. The probe of the electro hygrometer is placed in the equilibrium vapor over the sample being tested. The electrical resistance of the probe is sensitive to the amount of water vapor present. Since the test always is conducted at atmospheric pressure, the water vapor pressure is directly proportional to the volume fraction of water in the air/water vapor mixture. The instrument is normally calibrated with saturated solutions of known activity shown in Table 5.1.

Table 5.1: Salt saturated solutions and its vapor's water activity for calibrating electro hygrometer

Salt	Activity
$ZnCl_2$	0.10
$CaCl_2$	0.30
$MgCl_2$	0.33
$Ca(NO_3)_2$	0.51
NaCl	0.75
$(NH_4)_2SO_4$	0.80
Pure water	1.00

Sodium chloride and calcium chloride are the salts generally used to alter the activity of the water in the mud. Calcium chloride is quite soluble, allowing the activity to be varied over a wide range. In addition, it is a relatively inexpensive additive. The resulting water activity for various concentrations of NaCl and $CaCl_2$ are shown in Fig. 5.2.

a) The activity of a sample of shale cuttings drilled with OBM (no foreign fluid invasion) is determined to be 0.69 by an electro hygrometer. Determine the concentration of calcium chloride needed in the water phase of the mud in order to have the activity of the mud equal to the activity of the shale.

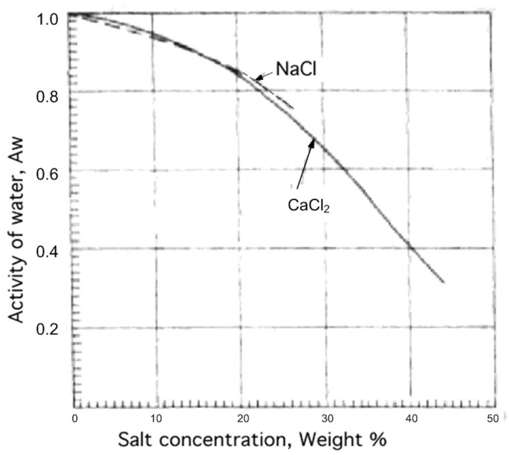


Figure 5-2: Water activity in calcium chloride and sodium chloride at room temperature.

b) A core is taken from a swelling formation. Can you retrieve any useful information from its specific weight, useful with respect to avoid swelling while drilling through it?
c) Explain why wellbores and cuttings stability is so much better when applying OBM instead of WBM. As part of the answer, please explain the principal function of the two different surface active additives that are always added to Oil based mud.

5.9 Clay behavior

a) When drilling into swelling clay, problems like sloughing (soft) shale and stuck pipe may occur. Explain what is MBT and why is the test said to be qualitative? Se lengerframme (kap 2).

b) What significance does the $^{K+}$ concentration have for the shale?

c) How do you convert Bentonite mud into gyp or lime mud?

5.10 Wellbore problem

a) What are the dominating mechanisms or factors leading up tomechanically stuck pipe. Explain the mechanisms of differential sticking in porous and permeable formation.

b) What are the consequences of stuck and how do you suggest combating the problem?

c) What is the most likely stuck pipe mechanism in the salt? In case of presence of halite type salts what kind of mud and why do you want to use it for safe drilling in such salt section.

d) Lost circulation problems: Makes a note on wellbore breathing (ballooning) and Seepage zone.

6 Additional information

6.1 Pump and hydraulic program data

Line size	in	5 ½	5 ¾	6	6 ¼	6 ½	6 ¾	7	7 ½
Discharge pressure	Psi 10^5.Pa	5555 383.0	5085 350.6	4670 322	4305 296.8	3980 274.4	3690 254.4	3430 236.5	3200 220.2
Pump rated at 120 spm	GPM m^3/s	444 0.0280	486 0.0307	529 .0334	574 .0362	621 .0392	669 .0422	720 .0454	772 .0482
HP		1439.0	1441.8	1441.3	1441.7	1442.0	1440.3	1440.8	1441

Efficiency= $1441.7 \cdot 745.7 = 1.0748 \cdot 10^6$ or $322 \cdot 10^5 \cdot 0.0334 = 1.0755 \cdot 10^6$(watt)

$$R_{p-q/d_n} = C \cdot (q/d_n)^{a_8}$$

$$R_{p-BHHP} = C \cdot (BHHP)^{a_8} \quad BHHP = \Delta p_{bit} \cdot q$$

$$P_p = q \cdot p_p$$

$$p_p = \Delta p_{bit} + \Delta p_d$$

$$p_{bit} = 1.11 \frac{1}{2} \rho v^2$$

$$q_{opt_I - q/d_n} = \left(\frac{2 p_p}{(m+2) K_1 D} \right)^{\frac{1}{m}}$$

$$q_{opt_I - q/d_n} = \left[\frac{(E_p)_{max}}{K_1 \cdot D (m+2)} \right]^{\frac{1}{m+1}}$$

6.2 Fluid mechanic

	Newtonian fluid	Bingham model	Powerlaw model
Lam/pipe	$\Delta p_p = \frac{32\,\bar{v}\,\mu\,L}{d^2}$	$\Delta p_p = \frac{32\,\mu_{pl}\cdot L\cdot \bar{v}}{d^2} + \frac{16\,L\tau_o}{3d}$	$\Delta p_p = 4K\left(\frac{8\bar{v}}{d}\cdot\frac{3n+1}{4n}\right)^n \cdot \frac{L}{d}$
Lam/annulus	$\Delta p_a = \frac{48\,\bar{v}\,\mu\,L}{\left(d_o - d_i\right)^2}$	$\Delta p_a = \frac{48\,\mu_{pl}\cdot L\cdot \bar{v}}{\left(d_o - d_i\right)^2} + \frac{6\,L\tau}{d_o - }$	$\Delta p_a = 4K\left(\frac{12\bar{v}}{d_o - d_i}\cdot\frac{2n+1}{3n}\right)^n \cdot \frac{L}{d_o - d_i}$
Turb/pipe/ann	$\Delta p = \frac{0.092\rho_m^{0.8}\,\bar{v}^{1.8}\,\mu^{0.2}\,L}{d_h^{1.2}}$	$\Delta p = \frac{0.073\,\rho_m^{0.8}\cdot\bar{v}^{1.8}\cdot\mu_{pl}^{0.2}\cdot L}{d_h^{1.2}}$	$\Delta p = a\cdot N_{\text{Re}}^{-b}\cdot\frac{4L}{d_h}\cdot\frac{1}{2}\rho\bar{v}^2$ $a = \left(\log n + 3.93\right)/50$ $b = \left(1.75 - \log n\right)/7$
Eff. visc. pipe	$\mu_{eff} = \tau/\gamma$	$\mu_{eff} = \mu_{pl} + \frac{\tau_o d}{6\bar{v}}$	$\mu_{eff} = \left(\frac{8\,\bar{v}}{d}\cdot\frac{3\,n+1}{4\,n}\right)^n \cdot \frac{Kd}{8\,\bar{v}}$
Eff. visc.ann	$\mu_{eff} = \tau/\gamma$	$\mu_{eff} = \mu_{pl} + \frac{\tau_o\left(d_o - d_i\right)}{8\bar{v}}$	$\mu_{eff} = \left(\frac{12\,\bar{v}}{d_h}\cdot\frac{2\,n+1}{3\,n}\right)^n \cdot \frac{Kd_h}{12\,\bar{v}}$
Shear-r. pipe	$\dot{\gamma} = \frac{8\bar{v}}{d}$	$\dot{\gamma} = \frac{8\bar{v}}{d} + \frac{\tau_o}{3\,\mu_{pl}}$	$\dot{\gamma} = \left(\frac{8\bar{v}}{d}\cdot\frac{3n+1}{4n}\right)$
Shear-r. ann.	$\dot{\gamma} = \frac{12\bar{v}}{d_y - d_i}$	$\dot{\gamma} = \frac{12\bar{v}}{d_o - d_i} + \frac{\tau_o}{2\mu_{pl}}$	$\dot{\gamma} = \left(\frac{12\bar{v}}{d_o - d_i}\cdot\frac{2n+1}{3n}\right)$
General $N_{re,pipe}$	$N_{\text{Re}} = \frac{d^n\cdot\bar{v}^{2-n}\cdot\rho}{K_p\cdot\left(8^{n-1}\right)}$	$K_p = K\cdot\left(\frac{3n+1}{4n}\right)^n$	*Fanning* $f_{lam} = 16/N_{re}$
General $N_{re,ann}$	$N_{\text{Re}} = \frac{d^n\cdot\bar{v}^{2-n}\cdot\rho}{K_a\cdot\left(12^{n-1}\right)}$	$K_a = K\cdot\left(\frac{2n+1}{3n}\right)^n$	*Fanning* $f_{lam} = 24/N_{re}$

Continuity equation: $-\frac{\partial \rho}{\partial t} = \nabla \cdot (\rho v)$

Microscopic Cylindrical coordinates $-\frac{\partial \rho}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho w)$

Macroscopic $\frac{d\int v\rho dV}{dt} = -\Delta \rho \bar{v} A = \rho_1 \bar{v}_1 A_1 - \rho_2 \bar{v}_2 A_2$

<u>Momentum equation</u> $\rho \frac{Dv}{Dt} = \rho g - \nabla p - \nabla \cdot \tau$

Microscopic Cylindrical coordinates (only the r-component)

$$\rho\left(\frac{\partial v_z}{\partial t} + v_z \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_z)\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Macroscopic $\frac{d}{dt}\int \rho dV = \rho_1 \bar{v}_1^2 A_1 - \rho_2 \bar{v}_2^2 A_2 + p_1 A_1 - p_2 A_2 - F + Mg$

The steady state, one dimensional pipe flow form is: $p_1 A_1 - p_2 A_2 - F = Mg \sin\theta$ ..

<u>Energy equation</u>

Macroscopic $\left(\frac{p}{\gamma} + \frac{\bar{v}^2}{2g} + z\right)_{in} + h_{pump} = \left(\frac{p}{\gamma} + \frac{\bar{v}^2}{2g} + z\right)_{out} + h_{friction}$

6.3 Conversion factors:

1 Hp = 745.7 W

Fann VG readings = Θ

Shear stress: $\tau_{OFU} = \Theta * 1.06$

Shear stress: 1 lb/100ft^2 (OFU) = 0.4788 Pa (SI)

$\tau_s = \tau_{OFU} \cdot 0.4788)$

Shear rate: $\gamma(s^{-1}) = RPM \cdot 1.703$

1 inch = 0.0254 m

1 bar = 10^5 Pa

1cP = 10^{-3} Pas

P = q*p

Solutions to Exercises in Drilling Fluid Engineering

1. Fluid Properties
2. Rheological models
3. Drilling fluid dynamics
4. Bit hydraulics
5. Wellbore challenges

1 Fluid Properties

1.1 Fluid loss control

a) Filter loss can be optimized in three ways:

1. Make a good filter by including a wise range of particles in the mud (Bentonite, Barite, cuttings are already there. It may be enough)
2. Make a good filter cake by making sure the mud is dispersed
3. Decrease losses through the filter cake by increasing the viscosity of fluid phase (starch, CMC))

b) Increased filter loss is detected through the filter press test. Depending on its rheology and other tests you normally find out which of the 3 qualities in question a. above is the problem.

1.2 Filter loss control

Yes, it can be reduced by creating a filter in the shale. This can be done by adding particles to the water phase where the average particle size distribution is in the range of 1/3 of the average pore throat size distribution. Small sized polymers (low molecular weight) will work.

1.3 Filter loss control

a) Correct particle size distribution leads to an optimal filter (1/7 to 1/3 of pore throuat opening).
b) Filtrate will lead to approximately the same invasion depth (around 3 feet). Particles will theoretically invade both sands approximately 1 ft, assuming the particle size distribution of the added or the existing particles in the mud covers at least the range of 1/7th to 1/3rd of the pore size distribution. That assumption is normally fulfilled through the presence of Bentonite / polymers + Barite + cuttings (disintegrated)

1.4 Filter loss control

a) Standard water loss = filtrate after 30 min

$$V_f = AC\cdot\sqrt{t}$$

API filter press $A_1 = 45\ cm^2 = 45\ .\ 10^{-4}\ m^2$
Flow into porous formation:
Effective formation height (10 %) h = 4000 ft · 0.3048 · 0.10 = 122 m
Wellbore surface area $A_2 = 15\cdot 0.0254\cdot\pi\cdot 122 = 149 m^2$
Assuming that an identical filter is built up on the sandstone surface, the loss during the first 30 minutes can be estimated:

$$\frac{V_{f_2}}{V_{f_1}} = \frac{A_2}{A_1}$$

$$V_{f_{2,0,5}} = V_{f_1} \cdot \frac{A_2}{A_1} = 25 \cdot 10^{-6} \cdot \frac{149}{45 \cdot 10^{-4}} = 0.83\ m^3$$

$$V_{f\ 2.24} = V_{f\ 2,0.5} \cdot \frac{\sqrt{24}}{\sqrt{0,5}} = 0.83 \cdot 6.93 = 5.74\ m^3$$

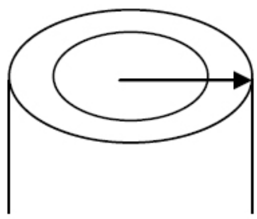
The filter will have the same progress and same constants.

b) Intrusion after 24 h with 100% displacement of 5.74 m³

To find the invasion depth we have to divide by porosity

Formation volume $= V_{form} = \frac{V_f}{0.10} = 57.4\ m^3$

Hole radius: $\frac{18.25}{2} \cdot 0.0254 = 0.13\ m$



Invaded volume becomes:

$$57.4\ m^3 = \left(\pi \cdot x^2 - \pi \cdot 0.13^2\right) \cdot 122 \rightarrow \quad x = 0.53$$

Instruction depth is 0.535 – 0.3 = 0.4 m = 40 cm

1.5 Density control

When weight material has to be added to the mud it is called weighted mud. To clean it normally a 60 + 250 Mesh shakers + a centrifuge would be perfect. Barite is preliminary taken out by the centrifuge and later re-injected.

1.6 Density control

By combining these information we obtain

$$\rho_2 = \frac{\sum m}{\sum V} \qquad m_1 = V_1/\rho_{1,}\ \Delta m_B = \Delta V_B/\rho_B \qquad \Rightarrow \Delta V_B = \frac{\rho_B\left(\rho_2 - \rho_1\right)}{\rho_B - \rho_2} V_1$$

Using it for specified task:

$$\Delta V_B = \frac{60\,000\,(1.4-1.3)}{4.3-1.4} = \underline{\underline{2\,092\ l}} \rightarrow$$

However, Barite is measured by its mass, not volume:

$$\Delta m_B = \Delta V_B \cdot \rho_B = 2.092 \cdot 4.3 \frac{ton}{m^3} = 9\ ton$$

1.7 Density control

In this task we simply take the weighted average of all densities:

a) $\bar{\rho} = \frac{\sum \rho V}{\sum V} = \frac{1500 \cdot 1 + 1800 \cdot 0.1 + 40/(40/2300) + m_B/(m_B/4300)}{1 + 0.1 + 40/2300 + m_B/4300} = 1720 \qquad \rightarrow m_B = 336.6\ kg$

b) $\bar{\rho} = 1.55 = \frac{\rho_1 V_1 + \rho_2 V_2 + \rho_3 V_s + \rho_w \cdot V_w}{V_1 + V_2 + V_3 + V_w}$

$$\frac{1500 \cdot 10 + 1600 \cdot 20 + 1900 \cdot 3 + V_w \cdot 1000}{10 + 20 + 3 + V_w} = 1550 \rightarrow \ V_w = 2.8\ m^3$$

1.8 Density control

1. Remaining volume $V_r = \frac{3}{5} V = 60\ m^3 \longrightarrow$ Remove 40 m³ and add then $V_{add} = V_w + V_b = 40\ m^3$
   Density of added volume must also be 1.8 kg/l

$$\rho_{add} = \frac{m_1 + m_w + m_b}{V_1 + V_w + V_b}$$

$$m_1 = \rho_1 \cdot V_1 = 1800 \cdot 60 = 108\,000\ \text{kg}$$

$$V_w + V_b = 40$$

$$1800 = \frac{108\,000 + m_w + m_b}{100}$$

$m_w + m_b = 72\,000$

$m_w = V_w \cdot 1000$

$m_b = V_b \cdot 4300 = (40 - V_w)\,4300$

$V_w \cdot 1000 + (40 - V_w)\,4300 = 72000$ kg

$V_w = 30.3\ m^3$

$V_b = 40 - 30.3 = 9.7\ m^3$

$m_b = 41710$ kg

Conclusion: Remove 40 m³ mud, add 30.3 m³ of water and 41 710 kg Barite to obtain 100 m³ mud of r = 1800 kg/ m³ and L.S.G.S of 3%.

2. Dump first $V_2 = 90 \cdot (0.055 - 0.03)/0.055 = 40.9\ m^3$
   Remaining volume $V_1 = 49.1$, $\rho_1 = 1600$, $m_1 = 78\ 560$ kg

   $\rho_w = 1000$, $\rho_b = 4300$

   Step 1. Add 40 900 kg of water and find new density ρ_{new} = (78 560 + 40 9009) / 90 = 1327 kg/m³ = ρ_{old} in step 2.

   Step 2: $V_{add} = V_1 \dfrac{\rho_{new} - \rho_{old}}{\rho_{add} - \rho_{new}} = 90 \dfrac{1.7 - 1.327}{4.3 - 1.7} = 11.3\ m^3$

   $V_b = 11.3\ m^3$, $LSGC_2 = 0.03 * (90-11.3)/90 = 0.026 \rightarrow 2.6$ % R

3. Necessary ρ_2: $\rho_2 gh = 410 \times 10^5 \rightarrow \quad \rho_2 = \dfrac{410 \cdot 10^5}{9.81 \cdot 3000} = \underline{1393\ kg/m^3}$

   Fill a tank of 10 m³ with new mud of ρ_2:

   From data: V_1 = volume old mud $\quad V_2 = 10\ m^3$
   $\rho_1 = 1200\ kg/m^3 \quad V_b$ = volume barite $\quad \rho_b = 4300\ kg/m^3$

Increase from ρ_1 to ρ_2:

$$\rho_2 = \frac{m_1 + m_b}{V_1 + V_b} \ , \ V_b = \frac{m_b}{\rho_b} \ , \ \rho_1 = \frac{m_1}{V_1}$$

$$V_1 + V_b = 10 \text{ m}^3 \quad \rightarrow \quad V_1 = (10 - V_b)$$

$$\rho_2 = \frac{(10 - V_b) \cdot \rho_1 + V_b \, \rho_b}{(10 - V_b) + \rho_b} \quad \Rightarrow \quad V_b = 0,6226 \text{ m}^3 \quad \Rightarrow \quad V_l = 9.3774 \text{ m}^3$$

It is shown that 0.6226 m^3 of Bentonite requires 9.3774 m^3 of water to produce 10 m^3 mud of density 1.393 kg/l.

1.9 Rheology control

a) Lots of fine Barite particles leads to increased viscosity. PV and μ_{eff} will increase. YP will be unaffected or decrease, Since the smallest Barite particles will behave as physical dispersants

b) In order to
Suppress Ca^{++} from dissolving.
Keep anionic colloidal particles dispersed.
Suppress corrosion and H_2S and CO_2 attack.

c) Polymers are polymerized monomers, can have organic origin or be synthetic manufactured (copolymers or cross linked). Polymers have high molecular weight and come mostly as charged particles → bind water molecules → increased hydrodynamic volume → influence viscosity and filter behaviour. Some types of polymers can attract or bind charged particles (clay), and therefore influence the swelling process and contribute to selective flocculation (clear water drilling). The purpose of polymers are:
- increase viscosity
- reduce viscosity in turbulent flow (drag reducer)
- control flocculation
- improve both filter cake itself and filter loss through the filter cake

d) The tree expressions are:
- Pseudo plastic (shear thinning) whenever apparent viscosity decreases with increasing shear rate.
- Thixotropic if the fluid displays a change in viscosity over time at a constant shear rate.
- Rheopectic mud gives reduced shear stress when sheared at constant rate over some time, after increasing the shear rate, as demonstrated below:

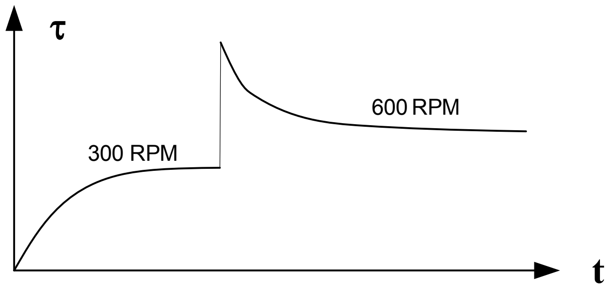


Figure 1-1: Behavior of a rheopectic fluid

e) Clay particles have a static, negative surface charge localized on the edges of the colloidal particles, but with weakly positively spots on the surface of the platelets. This triggers water! When studying the repulsive - attractive forces, it is experienced that at low salt concentration or high colloidal concentration, a very slow flocculation will take place. Edges come closer to the surfaces of other Montmorilonite particles.

1.10 Rheology control

a) Bentonite attracts water molecules (dipoles) in thousands of layers onto its charged surface. Swelled Na-Bentonite sheets separate readily when exposed to shear forces. Salt will neutralize the charged Bentonite surface and in addition bind much of the water (reduce water activity).

At still stand many layers of water molecules are attached to the Bentonite surface, and particles flocculate slowly → higher viscosity. When the dispersion is stirred or pumped, the resultingshear stress will remove attached water layers and break loose the flocks. Dispersed Bentonite with many free water molecules, torn off due to high shear stress, has lower viscosity than at still stand.

1.11 Rheology control

a) Drilling in pure clay with high yield (premium) clay produce following viscosity:

% solids by weight = $f = \frac{q_{clay}}{q_{pump}} = \frac{ROP * A_{bit} * \rho_{clay}}{q_{pump} * \rho_{mud}} = 15\ \%$

$$ROP = 10\ m/t = 0,00028\ m/s$$
$$d_{bit} = 26" = 0,66\ m$$
$$q_{pump} = 3000\ l/\min = 0,05\ m^3/s$$
$$A_{bit} = \pi/\left(4 \cdot d_{bit}{}^2\right) = 0.34\ m^2$$

Drilled clay contributes to a weight increase of 15% and produces a viscosity of 40 %. Yes. Formation will provide the required viscosity.

b) Yield of Bentonite is how many $m^3$of mud of 15 cP the amount of one ton Bentonite is able to produce? From Figure 1-1 one ton of mud pr. ton will produce 15.9 m^3 of 15 cP mud.

c) From Figure 1-1 we see that a viscosity of max. 50 cP, will correspond to 7.5 w % Bentonite.

$$\rho_{bentonite} = 2500\ kg/m^3$$

We take the total mash of: 100 kg mud $\Rightarrow$ 7.5 kg Bentonite + 92.5 kg water

$$Volume = V_{bentonite} + V_{water} = \frac{7.5\ kg}{2500\ kg/m^3} + \frac{92.5\ kg}{1000\ kg/m^3} = 0.094\ m^3$$

$$\rho_{mud} = \frac{m}{v} = 100\ kg/0.094\ m^3 = \underline{\underline{1064\ kg/m^3}}$$

This shows that mud density can be increased to maximum 1064 kg/m³ by the addition of Bentonite.

1.12 Flocculation

a) When two Bentonite flakes are near each other they are electrostatically attracted and will join edge to surface.
b) The cement is by far not hardened and contains a high concentration of lime (CaOH). Lime dissociates and one Ca^{++} ion attack 2 charged clay plates. This binding cannot be broken by mechanical means (shear stress), and the stiff mud has to be dumped when circulated to surface. See Figure 1-3.
c) The drivers behind flocculation:
- High concentration of charged particles (Bentonite or anionic polymer)
- High concentration of Ca^{++}ions (dissolved from salt and carbonates)
- High Temperature (Brownian movements bring colloidal particles in frequent contact)

1.13 Mud contamination

a) Contaminations are salt, chalk, cement etc. all produce Ca^{++}ions which leads to flocculation. Also fines are pollutant. Atthe suface the mud can be treated with thinners. LGSC is treated by running the mud through centrifuges.
b) Rheology monitoring

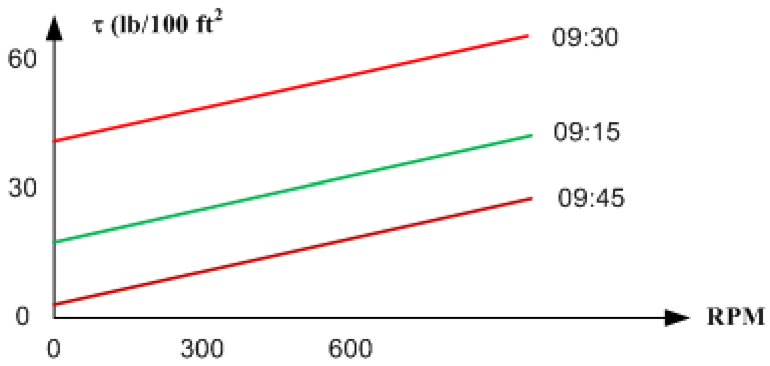


Figure 1-2: The rheology of the mud at 3 time points

To evaluate mentioned changes the Bingham field procedure-model is perfect. The mud is dispersed at first. Salt will dissolve and Ca^{++}contaminatethe mud. From the Figure 1-2 we see that only YP have changed, PV is constant (constant inner friction means solids content is constant). Δ YP is caused by higher attractive forces, caused by drilling into a salty formation. Low concentration of Ca^{++} ions leads to strong cross binding (flocculation). Later, when the Ca^{++} concentration has been high for a while the Ca^{++}ions exchange place with Na^{+}-ions in the Bentonite plates, leading to aggregation of clay platelets. A high concentration of Ca ions will over time lead to lower shear stress than original due to cation exchange. See development in the figure.

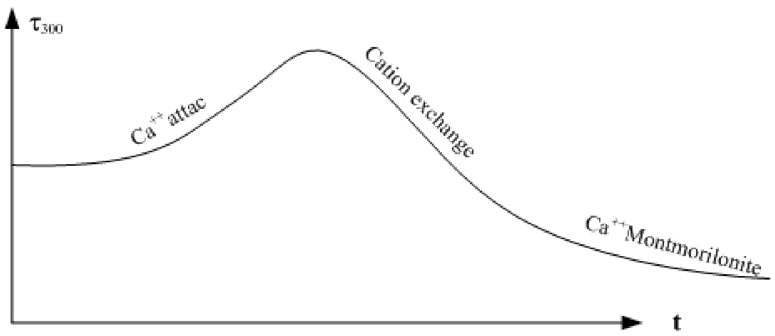


Figure 1-3: Shear stress vs. time after encountering high concentration of contaminants.

Countermeasures:

Add dispersants or use Gyp mud or high pH level in the mud when contamination is expected. This is how to avoid flocculation and aggregation.

c) The edge-to-phase flocculation are weak electrostatic forces and are easily broken when sheared again. The Ca^{++} ion binds two and two clay platelets in a much stronger grip, and is not easily broken. After cation exchange it tends to aggregate.

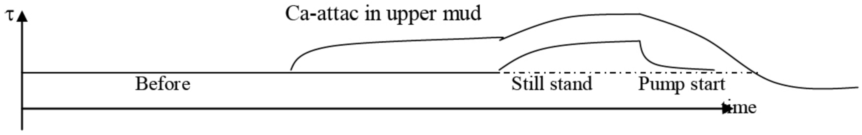


Figure 1-4: Rheological response of normal mud when making a pause (lower curve) and when drilling into contaminants (upper curve).

1.14 Flocculation

a) Consider the force balance between shear and gravity:

$$\frac{1}{4}\pi d_p^2 \cdot \tau_y = \frac{1}{6}\pi d_p^3(\rho_p - \rho_{fl})\, g$$

Solve for yield point

$$\tau_y = \frac{4\, d_p}{6}(\rho_p - \rho_{fl})\, g = \frac{0.005}{6}(2300 - 1100)\; 9.81 = 39.24\; Pa$$

Now solve stokes settling law with respect to particle diameter:

$$d_p = \frac{\tau_y \cdot 6}{(\rho_p - \rho_{fl}) \cdot g} = \frac{15\; Pa \cdot 1.5}{(2300-1100) \cdot 9.81} = 0.0019\; m\; = 1.9\; mm$$

b) Should have broken the gel either by pumping or by inserting a heavy pill in drill string (U-tube the level in the drill string downwards as it will lower the fluid level inside the pipe).

1.15 Fluid additives

Material	Definition	Relevance
Anhydrite	$CaSO_4$ = sedimentary salt	dissolves in water àCa^{++}
Ca SO_4	anhydrite	contaminator, leads to flocculation
Ca $SO_4 \times H_2O$	gypsum	restructure to Ca-Bentonite; shale control
CMC	natural polymer	filter loss/viscosifyer
Chalk	$CaCO_3$ = sedimentary salt	dissolves in water àCa^{++}
Colloid	particles of size < 2 m, clay, silt	build viscosity
CEC	Cation Exchange Capacity	ability of reactive clay to exchange cations
Deflocculator	thinner	reduce/neutralize electrostatic attraction
HEC	natural polymer	viscosifyer, filtrate reducer, defloccul.
Lignite	anionic polymer	Thinner, filtrate reducer
Lignosulfonate	anionic polymer	thinner (dispergator)
Dispergator	thinner	hinder particles to coalesce
MBT	Methylene Blue Test	determinesamount of reactive clay in mud. The test is qualitative because organic material and some other clays present in the mud also will adsorb MBT
NaOH	caustic soda, Sodium hydroxide = lye	adjust pH (dispersant)
PAC	natural polymer	filter reducer, shale control
PHPA	synthetic polymer	shale control, Bentonite extender
Prehydrated	Bentonite hydrated in water	
Salt	Ionic compound, NaCl	dissolves in water àNa^{+}
SAPP	anionic polymer	thinner (dispergator)
Starch	------- " ---------	----- " -----
Xanthan	natural polymer	viscosifyer

1.16 Drag reducer

A drag reducer consists of small amounts of long-chained, neutral polymers. In a turbulent flow regime, the fluid molecules move in a random manner, causing much of the energy applied to them to be wasted as eddy currents (and corresponding high pressure loss). Due to shear stress the polymers are stretched out where the shear is high (at the wall). In outstretched state the diffusion is higher due to lower effective surface area and thus lower resistance to movements. The polymers diffuse more in the direction of the pipe centre since the wall hinders diffusion in the opposite direction. A small concentration of stretched out long polymers along the wall will tend to make turbulent flow laminar or streamlined. Nearest to the pipeline wall is the laminar sub layer. In this zone, the fluid moves in a laminar layer

1.17 Fluid additives

a) The change in OH^- concentration required to increase the pH from a particular value to another is given by:

$\Delta [OH^-] = \Delta [OH^-]_{final} - \Delta [OH^-]_{initial}$

$\Delta [OH^-] = 10^{(11-14)} - 10^{(7.5-14)} = 0.001 - 0.000000316 = 0.00099$ mol/l

b) Mass of NaOH in g/l = concentration * molecular weight (of Caustic Soda)

= 0.00099moles/l * 40 g/moles = 0.0396 g /l

c) K^+ is geometrically suitable and leads to high platelet attraction (low swelling).Recall in your chemistry or periodic table (Group 1) that potassium (atom # 20) is more reactive than sodium (atom # 11). Why? The atomic radius of potassium is greater than that of sodium. Therefore, the single valence electron that exists for all alkali metal is located fartherer from the nukleus for potassium than sodium. This results in less energy required to remove that valence electron from potassium than from sodium, leading to increased reactivity. This trend continues as you move down Group 1 on the periodic table; i.e. Rubidium is more reactive than K.

1.18 Fluid additives

a) The mud must have sufficient viscosity & velocity to exceed the settling velocity of the cuttings
b) The mud must be thixotropic, i.e. it gels or sets when stationary or in low laminar flow, but becomes less viscousduring circulation and when rigorously shaken by the shale shaker
c) A large proportion of the mechanical energy in the form of WOB, rotation and hydraulic energy is dissipated as heat. Mud (or its fluid) has a high heat capacity, is voluminous (typically 100 m^3) and absorbs this heat and allow the drill bit and the rest of the drill string not to be heated
d) The drilling fluid achieves this by providing ahydrostatic pressure at least greater than the formation pressure

2 Rheological models

2.1 Bingham

Increased PV is an indicator of more fines (more mechanical friction). Increased YP is an indicator of more surface interaction between colloidal.

2.2 Bingham / Power law

a) Field $\mu_{pl} = PV = \theta_{600} - \theta_{300} = 100 - 75 = 25$ cP

$YP = \theta_{300} - PV\ 75 - 25 = 50$ lb/100 ft^2 = 23.94

Standard: $\mu_{pl} = \dfrac{50.75 - 38.06}{511} = 0.0248$ Pas = 24.8 cP

$YP = \tau_{511} - 0.0248 \cdot 511 = 25.38$ Pa

$n = 3.32 * \log 100/75 = 0.415$

$K = 38.06/511^{0.415} = 2.86$

b) To answer question b. we need to know at which shear rate to compare the two models. Given data gives us $\overline{v}$ = q/A = 2.12 m/s.
This results in a shear rate (Newtonian model) equal to:

$\gamma = 8\ v/d = 8 \cdot 2.12 \cdot 0.1 = 169.6\ s^{-1}$

We assumed Newtonian fluid just to obtain a quick estimate. And at the shear rate of 170 s^{1} we know the recorded answer; 23.8 Pa. Checking the two models:

$\tau_{Bingh} = 25.38 + 0.0248 \cdot 169.6 = 29.6$ Pa
$\tau_{PL} = 2.86 \cdot 169.6^{0.415} = 24.1$ Pa

Power law is the best model at this specific application.

2.3 Bingham / Power law

a) From the flow curve (τ vs. γ) it is obviously closest to the Power low model, especially for low shear rates. The Bingham model reads about 12 Pa higher than the measured data.

$$n = \frac{\log(71.05 / 49.75)}{\log(600 / 300)} = \frac{\log 1.428}{\log 2} = 0.51$$

$$K = \frac{\tau}{g^n} = \frac{71.05}{1022^{0.51}} = 2.07\ Pas^{-n}$$

$\tau_{100} = 2.07 * 100^{0.51} = 11.2$ Pa.

We see that even the Power law is far off here, around 9 Pa too low.

b) This is a time effect while performing rheological readings in the lab. Starting at 600 rpm and reducing the speed gradually to 6 and 3 rpm low shear rate (laminar of course) and spending time performing the test that colloidal clay particles are given the environment to flocculate (edge to face).

2.4 Bingham / Power law. Regression

1. Newtonian model at 300 rpm:

$$\mu = \frac{\tau}{\dot{\gamma}} = \frac{42 \cdot 0.4788}{300 \cdot 1.703} = 0.039\ Pas$$

Bingham model (Field procedure):

$$\mu_{pl} = \theta_{600} - \theta_{300} = 60.4 - 39.6 = 20.8\ cP = 0.021\ Pas$$
$$\tau_y = \tau_{300} - \mu_{pl} = 39.6 - 20.8 = 18.8 \frac{lb}{100} ft^2 = 18.8 \cdot 0.4788 \cdot 1.06 = 9.6\ Pa$$

Bingham (2 standard procedure data points):

μ_{pl} = (30.4 – 20) / 511 = 0.021 *Pas (same result as Field method)*

$\tau_{300} - \mu_{pl} \cdot \dot{\gamma}$ = 20.2 – 0.021 ž 511 = 9.5 *Pa (1 % lower value)*

Exponent model

$$n = \frac{\log\tau_{600} - \log\tau_{300}}{\log\dot{\gamma}_{600} - \log\dot{\gamma}_{300}} = \frac{\log\frac{\tau_{600}}{\tau_{300}}}{\log\frac{\dot{\gamma}_{600}}{\dot{\gamma}_{300}}} = \frac{\log\frac{64}{42}}{\log\frac{600\cdot 1.703}{300\cdot 1.703}} = \frac{\log\frac{64}{42}}{\log 2} = 0.60$$

$$K = \frac{\tau}{\dot{\gamma}^n} = \frac{64 \cdot 0.4788}{(600 \cdot 1.703)^{0.608}} = 0.454\,Pas^n$$

To plot the curves, t is calculated at 50, 100 and 800 rpm:

Bingham: τ = 9.5 + 0.021 · 50 · 1.703 = 11.3 Pa
τ = 9.5 + 0.021 · 800 · 1.703 = 37.8 Pa

Power law: τ = $0.454 \cdot (50 \cdot 1.703)^{0.61}$ = 6.8 Pa
τ = $0.454\ (100 \cdot 1.703)^{0.61}$ = 10.4 Pa
τ = $0.455\ (800 \cdot 1.703)^{0.61}$ = 36.5 Pa

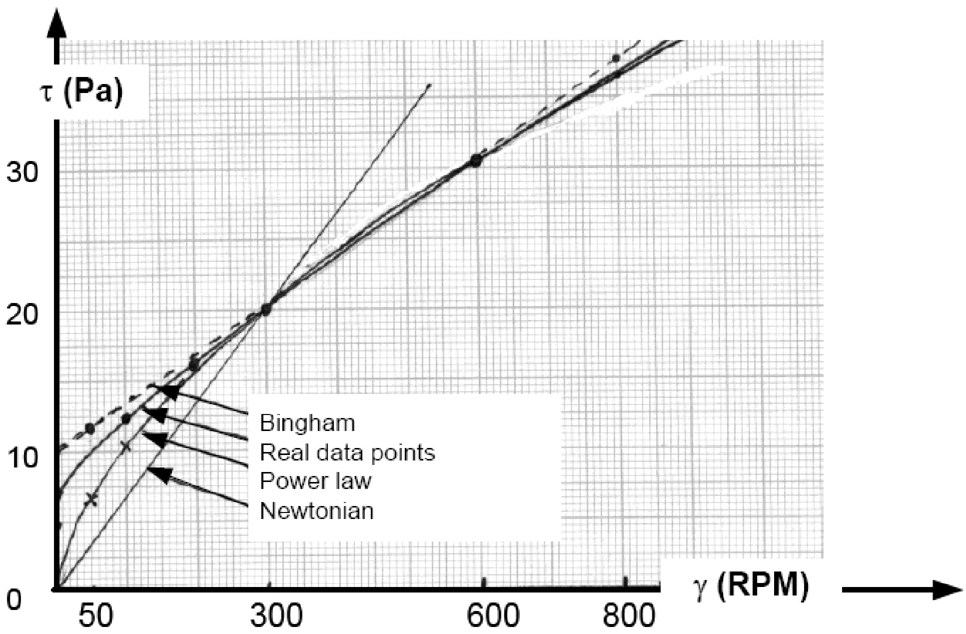


Figure 2-1: Rheogram (flow curve) interpreted through 3 rheological models

Readings on flow curve: 50 rpm = 10.0 Pa

800 rpm = 37.5 Pa

From this we see that the Bingham model suits the mud flow curve best at 50 s^{-1}, since t_{50B-M} is much closer to the real data than t_{50P-L}, while at higher shear rates Power law is better,

2. Plug flow will occur since shear stress is low in the middle of the pipe. The Bingham model predicts an YP = 9.1 Pa. This is a theoretical value, but given time at laminar flow gel strength will develop, in fact approaching the level of the YP. Shear stresses below this level will lead to gel (flocculation) as shown in Figure 2-2

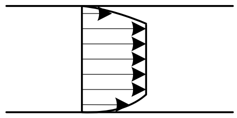


Figure 2-2: Plug flow of a Binghamian fluid.

2.5 Effective viscosity

The effective viscosity (Newtownian model):

For 600 rpm: $\mu = \frac{43 \cdot 0.4788 \cdot 1.06}{600 \cdot 1.703} = 0.0212\,Pas = 21.2\,cP$

For 3 rpm: $\mu = \frac{\tau}{\dot{\gamma}} = \frac{3.2 \cdot 0.4788 \cdot 1.06}{3 \cdot 1.703} = 0.318\,Pas = 318\,cP$

These and more data points result in the curve below:

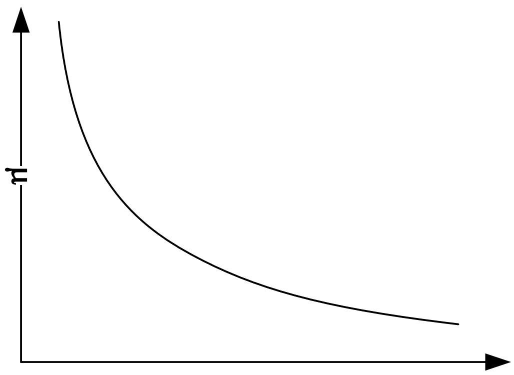


Figure 2-3: Viscoaity for a non-Newtonian fluid.

When determining the effective viscosity we assume the fluid is closest to a Power law fluid:

2 data points-procedure

$$n = 0.52, \quad K = \frac{\tau}{\dot{\gamma}^n} = \frac{43 \cdot 0.4788 \cdot 1.06}{(600 \cdot 1.703)^{0.52}} = \underline{0.59\ Pa\ s^{-n}}$$

$$\bar{v} = q / A = \frac{4 \cdot 0.1}{\pi \cdot 0.102^2} = 12.33\,m/s$$

$$\mu_{eff} = \left(8\frac{\bar{v}}{d} \cdot \frac{3n+1}{4n}\right)^n \cdot \frac{Kd}{8v} 1195^{0.52} \cdot \frac{0.59 \cdot 0.102}{8 \cdot 12.33} = 0.021\,Pas$$

$$\mathrm{Re} = \frac{\rho \bar{v}\, d}{\mu_{eff}} = \frac{1100 \cdot 12.3 \cdot 0.102}{0.021} = \underline{\underline{65\,000}} \rightarrow turbulent$$

b) High YP is caused by slow flocculation/gelling. At long still stand (10 min or more) the gel strength will approach the YP (ok, said before).

2.6 All models

a) $\mu_{pl} = \frac{55-40}{511} = 0.029\ Pas = 29\ cP$

$\tau_y = \tau - \gamma \cdot \mu_p = 55 - 1022 \cdot 0.029 = 25.4\ Pa$

n = 3.32 · log 55/40 = 0.447

k = $\tau / \gamma^n = 55 / 1022^{0.447} = 2.48\ Pas^n$

b) H & B:

1. Field approach: Take τ_3 as τ_y, and n and k from Power law
2. Standard: 3 points, iteration, select the 3 data points closest to actual shear rate
3. Non linear regression: 6 data points

b) Comparing laminar friction expression $\Delta\rho_{Newton} = \Delta\rho_{Bingham}$, solving for $\mu_{Newton} = \mu_{eff}$

2.7 All models. Regression

Standard 2 data points:

Bingham:

$$\mu_{pl} = (25.4 - 17.2)/511 = 0.016\ \ Pas$$

$$\tau_o = \tau_{300} - \mu_{pl} \cdot \dot{\gamma}_{300} = 17.2 - 0.016 \cdot 511 = \underline{9\ \ Pa}$$

Power law:

$$K = \frac{\tau_{600}}{\gamma_{600}^n} = \frac{53 \cdot 0.4788 \cdot 1.06}{1022^{0.56}} = \underline{0.55\ Pas^{Nn}}$$

In the H-B model the constant are found by iteration;

$$0 = \tau_1 - \tau_3 + \gamma_3^n \frac{\tau_2 - \tau_1}{\gamma_2^n - \gamma_1^n} - \gamma_1^n \left(\frac{\tau_2 - \tau_1}{\gamma_2^n - \gamma_1^n} \right) \quad (1)$$

$$K = \frac{\tau_2 - \tau_1}{\gamma_2^n - \gamma_1^n} \quad (2)$$

$$\tau_o = \tau - K \cdot \gamma_1^n \quad (3)$$

Try initially n = 0.56 and K = 0.526. The n-value which gives best result of (1) is taken. When n and K are found use (3) to find τ_o (here = - 0.08).

Alternatively take the τ_0 = 3 rpm-reading = 1.5 Pa

Regression results for Bingham and Power law models are presented in Figure 2-4:

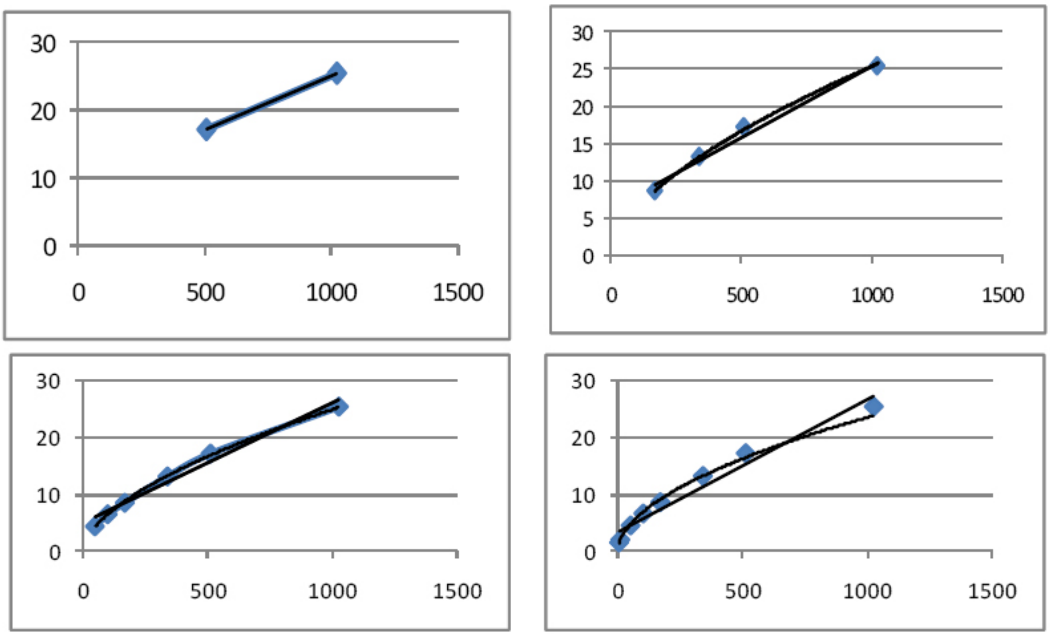


Figure 2-4: Regression result as a function of # of viscometer data points. (4, 6, and 8 data points)

A summary of the results of model fitting:

Data points	Bingham model			Power Law model		
	PV	YP	R^2	n	K	R^2
2	0.0160	9.0	-	0.56	0.52	-
4	0.0192	6.3	0.983	0.61	0.39	0.999
6	0.0212	4.8	0.977	0.58	0.45	0.999
8	0.0232	3.4	0.960	0.53	0.59	0.997

The 4 upper points are the most reliable data points. The two lower ones (3 and 6 RPM) are influenced by the time delayinflicted by the test procedure (see task 2.3 above).

2.8 All models

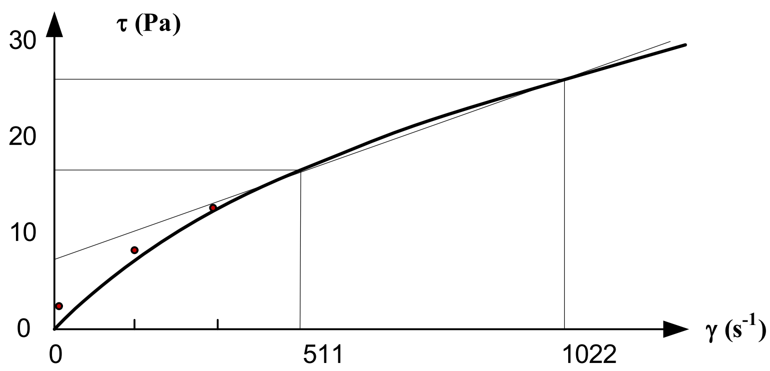


Figure 2-5: The Bingham and the power law model

Bingham Field procedure: from the two upper viscometer data points resulting

μ_{pl}= 49.3 – 33.5 = 15.8 cP

τ_o = 33.5 – 15.8 = 17.7 lb/100ft² (*0.4788ž 1.06 = 9Pa)

Bingham SI:

$$\mu = \frac{25-17}{511} = \underline{0.0157\ Pas}$$

τ_o = 25 – 0.0157 · 1022 = 8.9 Pa

$$\tau_{150} = 8.9 + \cdot 0.0157 \cdot 150 = \underline{11.2\ Pa}$$

Power law:

$$n = 3.32 \cdot \log\ 25/17 = 0.556$$

$$K = \frac{25}{1022^{0.556}} = 0.52$$

$$\tau_{150} = 0.52 \cdot 170^{0.556} = \underline{9.6\ Pa}$$

Herschel Bulkley (iteration):

$\rho_1 = 12$ $\gamma_1 = 300$
$\rho_2 = 17$ $\gamma_2 = 511$
$\rho_3 = 25$ $\gamma_3 = 1022$

n	ε
0.53	0.568
0.44	0.457
0.34	0.014

$$0 = 12 - 25 + 1022^n \frac{17-13}{511^n - 300^n} - 300^n \frac{17-13}{511^n - 300^n}$$

$$\left.\begin{aligned} 25 &= \tau_o + K \cdot 1022^{0.34} \\ 17 &= \tau_o + K \cdot 511^{0.34} \end{aligned}\right\} \qquad \begin{aligned} &25 - K \cdot 10.5 = 17 - K \cdot 8.3 \\ &K = \frac{25-17}{10.5-8.33} = \frac{8}{2.2} = \underline{3.6} \end{aligned}$$

$$\tau_o = 25 - 3.6 \cdot 10.5 = -11.0$$

Summary: $\tau = -11 + 3.6\, \gamma^{0.34}$

$$\tau_{150} = -11 + 3.6 \cdot 150^{0.34} = 8.8\ Pa$$

Conclusion: H.B is closest. Both PL and H-B are relatively far away but as expected since model fit are bases only on the two upper readings, "far away" from 150 $^{s-1}$. Therefore it is, as stated in text book, recommended to apply the three data points closest to the actual shear rate. Please repeat the solving procedure for the lower 3 data points and compare results.

3 Drilling fluid dynamics

3.1 Velocity profile. Continuity equation

Define cross sectional flow area and discretize in the z-direction: by integrating velocity over A.

$$A = z \cdot b \quad \rightarrow \quad dA = b \cdot dz$$

Average velocity:

$$\bar{u}\bar{u}\bar{u}\bar{u}\frac{1}{A}\int \quad (\quad {}_0 \ - \quad) \quad \cdot$$

$$\bar{v} = \frac{ab}{z_0 b}\int\left(z_0 z - z^2\right)dz$$

$$\bar{v} = \frac{a}{z_0}\cdot\left[\frac{z_0}{2}z^2 - \frac{1}{3}z^3\right]$$

$$\bar{v} = \frac{a}{z_0}\left(\frac{z_0^3}{2} - \frac{z_0^3}{3}\right) = az_0^2\left(\frac{1}{2} - \frac{1}{3}\right) = az_0^2 \cdot \frac{1}{6}$$

$v_{\max}$ *when* $v(z)' = 0$

$$\frac{dv}{dz} = \frac{d\left(azz_0 - az^2\right)}{dz} = az_0 - 2az = 0$$

$$2az = az_0 \rightarrow \text{max at } z = \frac{z_0}{2}$$

$$v_{\max} = a\frac{z_0}{2}\left(z_0 - \frac{z_0}{2}\right) = \frac{az_0^2}{4}$$

Find $v_{\max}$ for

$$\bar{v} = 8\ cm/s \ and \ z_0 = 4\ cm:$$

$$\bar{v} = az_0^2 \cdot \frac{1}{6}$$

$$8 = a \cdot 16 \cdot \frac{1}{6}$$

$$a = \frac{8 \cdot 6}{16} = \frac{12}{4} = 3\frac{m}{s}$$

$$v_{\max} = \frac{3 \cdot 4^2}{4} = 12\ cm/s$$

3.2 Velocity profile. Momentum flux

To find pressure distribution etc.: We start with Navier-Stoke in z-direction, cylindrical coordinates

$$v\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} + \frac{\partial^2 v}{\partial z^2}\right] + g_z$$

Under stationary laminar flow the equation reduces to:

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right)\right]$$

$$\frac{\partial p}{\partial z} = \mu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right)$$

To replace latter part of the equation with manageable parameters we go through 3 steps:

Step 1: Define $\partial v / \partial r \ from\ v(r):\ v = v_{max}\left(1 - \frac{r^2}{R^2}\right), v_{\Theta} = v_r = 0$

Step 2: $\frac{\partial v}{\partial r} = v_{max}\left(-\frac{2r}{R^2}\right)$

Step 3: $\frac{\partial}{\partial r} r \cdot v_{max}\left(-\frac{2r}{R^2}\right) = -\frac{v_{max}\cdot 2 \quad 2r}{R^2}$

Pressure distribution or pressure gradient now becomes:

$$\frac{\partial p}{\partial z} = \mu \cdot \frac{1}{r} \cdot \left(-\frac{v_{\max} \cdot 4r}{R^2} \right)$$

Total pressure loss along z:

$$\int_{p(o)}^{p(z)} \partial p = -\int_{o}^{z} \frac{4\mu v_{\max}}{R^2} \partial z$$

$$\Delta p = -\left(0 - \frac{4\mu v_{\max}\ z}{R^2} \right) = 4\mu v_{\max}\ z \ / \ R^2$$

The shear stress is found as: $\tau = -\mu \cdot \frac{\partial v}{\partial r} = \mu\ v_{\max} \left(\frac{2r}{R^2} \right)$.

Maximum of t is found when r = R, i.e. at the wall. Navier Stoke deals with incompressible fluids; density does not change.

$$\tau_{\max} = \mu \cdot v_{\max} \cdot \frac{2}{R}$$

b1.Wall shear stress: The wall shear follows from the definition of a Newtonian fluid:

$$\tau_w = \tau_{xy\,wall} = \mu \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right) \Big|_{y=\pm h} = \mu \frac{\partial}{\partial x} \left[\left(-\frac{dp}{dx} \right) \left(\frac{h^2}{2\mu} \right) \left(1 - \frac{y^2}{h^2} \right) \right] \Big|_{y=\pm h}$$

$$= \pm\, \mu \left(-\frac{dp}{dx}\right) \cdot \frac{h^2}{2\mu} \left(-\frac{2h}{h^2}\right) = \pm \frac{dp}{dx} h = \mp \frac{2 v_{\max}}{h}$$

b2.Average velocity:

It is defined as $\overline{v}$ = q/A, where q = $\int$ v dA over the cross section.

For our particular distribution: A = b·y→ dA = b·dy,

$$\overline{v} = \frac{1}{A} \int v dA = \frac{1}{b(2h)} \int_{-h}^{+h} v_{\max} \left(1 - \frac{y^2}{h^2} \right) b\ dy = \frac{v_{\max}}{2h} \left[h - \frac{h^3}{3h^2} - \left(-h - \frac{-h^3}{3h^2} \right) \right] = \frac{2}{3} v_{\max}$$

In plane Poiseulle flow between parallel plates, the average velocity is two-thirds of the maximum value.

3.3 Flow profile

a) The equation represents the conservation of momentum equation for stationary, incompressible, curl free, laminar pipe flow. Navier Stoke.

b) From momentum equation and stated condition;

$$0 = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}(rv)\right]$$

From boundary condition, rearranging/integrating twice, $v(r)$ is found.

Alternatively, start with the shear stress vs. friction pressure for laminar pipe flow; or, one step later, the general shear stress equation:

$$\tau = \frac{r}{2}\frac{dp}{dx} = -\mu\frac{dv(r)}{dr}$$

$$dv(r) = -\frac{1}{2\mu}\cdot r\frac{dp}{dx}\cdot dr$$

$$v(r) = -\frac{1}{2\mu}\cdot\frac{dp}{dr}\frac{r^2}{2} + C_1$$

$$v(R) = 0,\ C_1 = \frac{1}{4\mu}\cdot R^2\frac{dp}{dx}$$

$$v(r) = \frac{1}{4\mu}\frac{dp}{dx}\left(R^2 - r^2\right)$$

c) Average velocity is found by:

$$\bar{v} = \frac{1}{A}\int v\,dA = \frac{1}{\pi R^2}\frac{dp/dx}{4\mu}\int_o^R\left(R^2 - r^2\right)2\pi\,r\,dr = \frac{R^2}{8}\cdot\frac{dp/dx}{\mu} \qquad \text{(at the wall)}$$

To compare v_{max} and $\bar{v}$ we need fist to express v_{max};

$$v_{\max} = \frac{1}{4\mu}\frac{dp}{dx}R^2 \qquad (for = 0)$$

By comparing the two we find an expression of axial dispersion:

$$\frac{v_{max}}{\bar{v}} = \frac{1\cdot dp/dx\cdot R^2\cdot\mu}{4\mu\cdot R^2/8\cdot dp/dx} = 2$$

From this evaluation we can conclude that axial dispersion is independent of the factors shown above and that it is always positive; the max velocity is twice the average velocity.

d) The wall shear stress is found in two steps:
Step1: dp/dx = 0.09 MPa/1000 m = 90 Pa/m

Step 2: $\tau_w = -\frac{0.05}{2}90 = -2.25\ \text{Pas} \rightarrow$ Pas $\rightarrow \bar{v} = \frac{0.05^2}{8}\cdot\frac{90}{0.0538} = 0.53\,\text{m/s}$

3.4 Pressure loss vs. rheology

a) To find procedure loss we need to check the Reynolds number:

$$\bar{v} = \frac{q}{A} = \frac{\frac{0.75}{60}\frac{m^3}{s}}{\frac{\pi}{4}0.1^2} = 1.6\, m^3/s$$

Viscosity:

Newtonian model (based on $\gamma = 170$: $\mu = \frac{8.6}{170} = 0.051\ Pas$

Bingham: $\mu_{pl} = 0.016\ Pas,\ YP = 8.64\ Pa$

Power law: n = 0.56, K = 0.52 Pa $s^{0.56}$

$$Re = \frac{0.1^{0.56} \cdot 1.6^{1.44} \cdot \rho}{\left(\frac{3 \cdot 0.56 + 1}{4 \cdot 0.56}\right)^{0,56} \cdot 8^{-0.44} \cdot 0.52} = 2177 \quad \Rightarrow$$ laminar, assumed true for all rheologies

Laminar pressure loss equations give these results:

Newton:

$$\Delta p_N = \frac{32\, \mu_{pl}\ L\ \bar{v}}{d^2} = 2.1 \cdot 10^5\ Pa$$

Power law:

$$\Delta p_{PL} = 4\,K\left(\frac{8\,\bar{v}}{d}\cdot\frac{3n+1}{4\,K}\right)^{n}\frac{1}{d}\cdot\Delta L = 4\cdot 0.52\left(\frac{8\cdot 1.6}{0.1}\cdot\frac{2.68}{2.24}\right)^{0.56}\frac{1}{0.1}\cdot 1000 = 3.7\cdot 10^{5}\ Pa$$

Bingham:

$$\Delta p_{BH} = \frac{32\,\mu_{pl}\,L\,\bar{v}}{d^2} + \frac{16\,L\,\tau_o}{3d} = \frac{32\cdot 0.016\cdot 1000\cdot 1.6}{0.1^2} + \frac{16\cdot 1000\cdot 8.64}{3\cdot 0.1} = 5.2\cdot 10^{5}\ Pa$$

We observe a large deviation between the two latter other models. Bingham "aims" high at low shear rates, and will always produce higher results than power law.

b) Shear rate considerations for Bingham model:

$$\tau_w\cdot\frac{4L}{d} = \frac{32\,\mu_{pl}\cdot L_v}{d^2} + \frac{16\,L\,\tau_o}{3d} \quad\Rightarrow\quad \tau_w = \frac{8\,\mu_{pl}\,\bar{v}}{d} + \frac{4\,\tau_o}{3}$$

$$\tau_o + \mu_{pl}\cdot\dot{\gamma} = \frac{8\mu_{pl}\cdot\bar{v}}{d} + \frac{4}{3}\tau_o \quad\Rightarrow\quad \dot{\gamma}_{BH} = \frac{8\bar{v}}{d} + \frac{\tau_o}{3\,\mu_{pl}}$$

Calculated shear rate for all models:

$$\dot{\gamma}_{Newt} = \frac{8\bar{v}}{d} = \frac{8\cdot 1.6}{0.1} = 128\,s^{-1}$$

$$\dot{\gamma}_{PL} = \frac{8\bar{v}}{d}\frac{3n+1}{4n} = 128\cdot\frac{3\cdot 0.56+1}{4\cdot 0.56} = 153\,s^{-1}$$

$$\dot{\gamma}_{BH} = \frac{8\bar{v}}{d} + \frac{\tau_o}{3\,\mu_{pl}} = 128 + 180 = 308\,s^{-1}$$

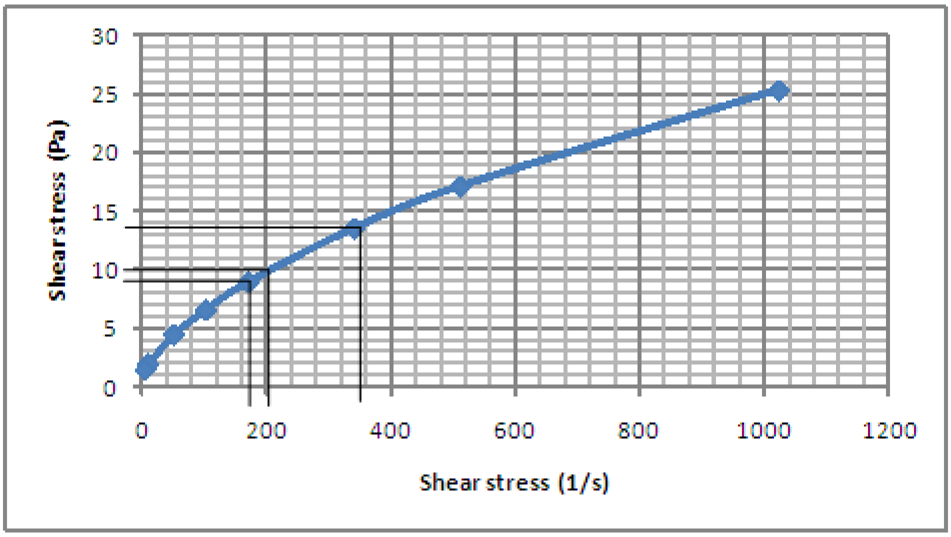


Figure 3-1: Graphical presentation of viscometer data and reading of shear stress for some shear rates.

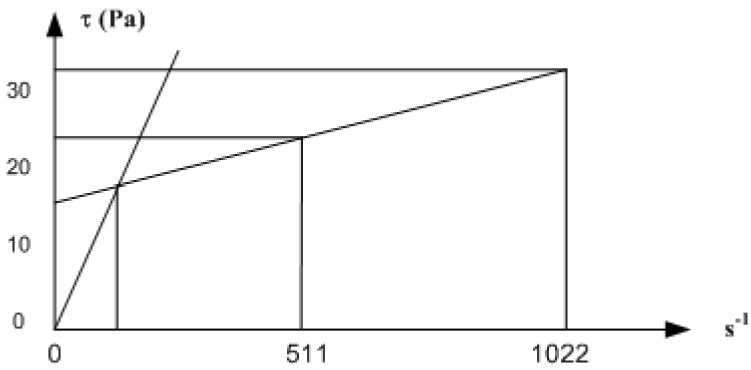
Model	γ (s to the power of -1)	reading stress (Pa)	Universal eqn (Pa)	from ordinary (Pa)
Newton	128	7	2.8	2.1
Power law	153	8	3.2	3.7
Bingham	308	13	5.2	5.2

One conclusion is that the ordinary and universal models give similar results, but different models gave different results.

3.5 Pressure loss vs. Rheology

$\dot{\gamma}$	θ		τ	τ
s^{-1}	-		lb/100 ft²	Pa
1022	66.0		70	33.5
511	47.2		50	23.9
170	25.5		27	12.9
10	9.4		10	4.8

$$\mu_{pl} = 21.2\ cP = 0.02\ Pas$$
$$\tau_0 = 47.2 - 21.2\ 30\ lb/100\ ft^2 = 14.4\ Pa$$



The Newton model can also be based on the average between the two readings!

a) At this low flow rate we assume laminar flow:

$$\bar{v} = q/A = \frac{4}{\pi} \cdot \frac{q}{d^2} = 1.4\ m/s$$

$$\Delta p_{pipe} = \frac{32 \cdot 0.02 \cdot 1.4 \cdot 1000}{0.109^2} + \frac{16 \cdot 1000 \cdot 14.4}{3 \cdot 0.109} = \underline{7.8 \cdot 10^5\ Pa}$$

b) To compare the two we need to compute Newtonian pressure loss at the correct sheer rate

$$\dot{\gamma}_{Newt} = \frac{8\bar{v}}{d} = \frac{8 \cdot 1.4}{0.109} = 102.75\ s^{-1}$$

Read τ at 102.75 s^{-1} to be 10 Pa. From $\Delta p = 4\tau_w \cdot \frac{L}{d}$ we obtain

$\Delta p_N = 4 \cdot 10 \cdot 1\,000 / 0.109 = 3.7 \cdot 10^5$ Pa

Bingham model gives the best answers. This is natural since the mud has a typical Binghamian characteristic at laminar flow.

3.6 Pressure loss. Power law

a) Power law: $\tau = k \cdot \gamma^n$, $\gamma = -\frac{dv}{dr}$

From force balance we derive:

$$\tau \cdot 2\pi r L = (p_1 - p_2)\pi r^2 \quad \rightarrow \quad \tau = \frac{\Delta p \cdot r}{2L}$$

$$K \cdot \gamma^n = \frac{\Delta p \cdot r}{2L} \quad \rightarrow \quad \gamma = \left(\frac{\Delta p}{2LK}\right)^{\frac{1}{n}} \cdot r^{\frac{1}{n}}$$

$$\int_{v(r)}^{0} -\,dv = \int_r^R \left[\frac{\Delta p}{2LK}\right]^{\frac{1}{n}} r^{\frac{1}{n}}\,dr$$

$$v(r) = \left[\frac{\Delta p}{2K}\right]^{\frac{1}{n}} \cdot \frac{n}{n+1}\left[R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right]$$

$$q = \bar{v}\,\pi r^2 \quad \rightarrow \quad dq = v(r)\,\pi\,2r\,dr$$

$$q = \int_0^R \left[\frac{\Delta p}{2LK}\right]^{\frac{1}{n}} \cdot \frac{n}{n+1}\left[R^{\frac{n+1}{n}} - r^{\frac{2n+1}{n}}\right]\pi \cdot 2 \cdot rdr = \pi \cdot 2\left[\frac{\Delta p}{2LK}\right]^{\frac{1}{n}} \frac{n}{n+1}\left[R^{\frac{n+1}{n}} \cdot \frac{R^2}{2} - \frac{n}{3n+1} \cdot R^{\frac{3n+1}{n}}\right]$$

$$= \pi \cdot 2\left[\frac{\Delta p}{2LK}\right]^{\frac{1}{n}} \frac{n}{n+1}\left[\frac{1}{2}R^{\frac{3n+1}{n}} - \frac{n}{3n+1} \cdot R^{\frac{3n+1}{n}}\right] = \pi \cdot 2\left[\frac{\Delta p}{2LK}\right]^{\frac{1}{n}} \frac{n}{n+1}\left[\frac{n+1}{2(3n+1)} - \frac{2n}{2(3n+1)}\right]R^{\frac{3n+1}{n}}$$

$$q = \pi R^2 \cdot \bar{v} \Rightarrow \pi R^2 \cdot \bar{v} = \pi\left[\frac{\Delta p}{2LK}\right]^{\frac{1}{n}} \cdot n \cdot \left[\frac{n}{3n+1} \cdot R^{\frac{3n+1}{n}}\right]$$

Solved for Δp:

$$\Delta p = 4K\left[\frac{8\bar{v}}{d} \cdot \frac{3n+1}{4n}\right] \cdot \frac{L}{d}$$

b) Effective (Newtonian) viscosity for Power law fluids:

$$\mu_{eff,P} = \underline{\underline{=\left(\frac{8\,\bar{v}}{d}\right)^{n-1} \cdot K_p}} \quad \text{where} \quad K_p = K \cdot \left(\frac{3 \cdot n + 1}{4 \cdot n}\right)^n$$

$$\mathrm{Re} = \frac{\rho \cdot \bar{v}\, d}{\mu_{eff}} = \frac{d^{n-1+1}}{v^{n-1-1}} \cdot \frac{\rho}{K_p \cdot 8^{n-1}} = \frac{d^n \cdot \bar{v}^{2-n} \cdot \rho}{K_p \cdot 8^{n-1}} = \mathrm{Re}_{pipe}$$

c) Power law fluids:

$$\mathrm{Re} = \frac{\rho\, v_{Av}^{\;2-n}\, d_h^n}{K_a\, 12^{n-1}}, \quad where\ K_a = K\left(\frac{2n+1}{3n}\right)^n \text{ and } d_h = d_o - d_i$$

When d_i increases, the annular gap becomes smaller but v^2 increases more and the Reynolds number increases for our non-Newtonian drilling fluids.

Newtonian fluids:

$$\bar{v} = q/A, \ where\ A = \frac{\pi\left(d_o^2 - d_i^2\right)}{4} = \frac{\pi\left(d_o - d_i\right)\left(d_o + d_i\right)}{4} = \frac{\pi d_h\left(d_o + d_i\right)}{4}$$

$$\mathrm{Re}_{Newt} = \frac{\rho_1\, v \cdot d_h}{\mu} = \frac{C_1\, 4\, q\, d_h}{C_2 \cdot \pi \cdot d_h\left(d_y + d_i\right)} = \frac{c}{d_o + d_i}$$

Re decreases for Newtonian fluids when d_i decreases!

The physical explanation for this behavior is the forced streamlining of the flow in narrow gaps. Out on the ocean, on the other hand, ocean water turns turbulent at very low shear rates.

3.7 Pressure loss. Turbulent flow. Energy equation

Average velocity:

$$\overline{v} = \frac{q}{\pi R^2} = \frac{0.2\, m^3 \;/\; s}{\pi\left(0.1\, m\right)^2} = 6.4\; m\;/\;s$$

$$\mathrm{Re} = \frac{\rho\, \overline{v}\, d}{\mu} = \frac{\left(6.4\; m\;/\;s\right)^2}{0.00001\, m^2\;/\;s} = 128\,000 \quad --> turbulent$$

From Chapter 3.5 in textbook we find ε = 0.26 mm for cast-iron pipe. Then

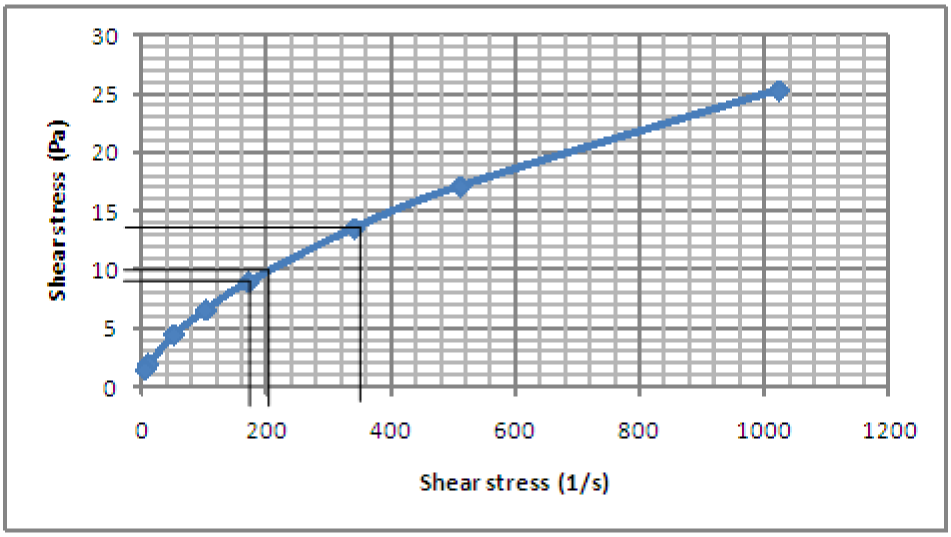
$$\frac{\varepsilon}{d} = \frac{0.26\, mm}{200\, mm} = 0.0013$$

Enter the Moody chart (Chapter 3.5 in Textbook) on the right hand side at ε/d = 0.0013 (need to interpolate), and move to the left to intersect with Re =128,000. Read f = 0.0225.The head loss becomes:

$$h_f = f\,\frac{L}{d}\cdot\frac{\overline{v}^2}{2g} = \left(0.0225\right)\,\frac{500\; m}{0.2\; m}\cdot\frac{\left(6.4\; m\;/\;s\right)^2}{2\cdot(9.81\, m\;/\;s^2)} \;=\; 117\; m$$

3.8 Pressure loss vs. flow rate

The copied Figure 3.1:



From exercise 4 we copy:

n = 0.56, K = 0.52 Pa $s^{0.56}$

Pressure loss equations:

$$\Delta p_{lam} = 4K\left(\frac{8\bar{v}}{d}\cdot\frac{3n+1}{4K}\right)^{n}\frac{1}{d}\cdot\Delta L = 4\cdot 0.52\left(\frac{8\cdot v}{0.1}\cdot\frac{2.68}{2.24}\right)^{0.56}\frac{1}{0.1}\cdot 1000 = 267\,553\cdot v^{0.56}$$

$$\Delta p_{turb} = a\cdot N_{\mathrm{Re}}^{-b}\cdot\frac{4L}{d_h}\cdot\frac{1}{2}\rho\bar{v}^{2} = 1.619\cdot 10^{6}\cdot \mathrm{Re}^{-0.214}\cdot \mathrm{v}^{2}$$

$$a = (\log n + 3.93)/50$$
$$b = (1.75 - \log n)/7 \quad \rightarrow \quad a = 0.0736,\ b = 0.286$$

$$\bar{v} = \frac{q}{A} = \frac{q}{\pi/4\cdot 0.1^{2}} = 127.3q$$

$$Re = \frac{0.1^{0.56}\cdot v^{1.44}\cdot 1100}{\left(\frac{3\cdot 0.56+1}{4\cdot 0.56}\right)^{0.56}\cdot 8^{-0.44}\cdot 0.52} = 1\,206\cdot v^{1.44}$$

Select three data points in the laminar and two in turbulent regime:

q		v	N_{Re}	Δp_{lam}	Δp_{turb}
l/min	m^3/s	m/s	-	MPa	MPa
100	0.00170	0.22	136	0.11	
250	0.00417	0.53	1083	0.18	
500	0.00833	1.06	2170	0.28	0.20
1000	0.017	2.12	4340	0.41	0.66
1500	0.025	3.18	6380		1.34
2000	0.033	4.25	8687		2.19

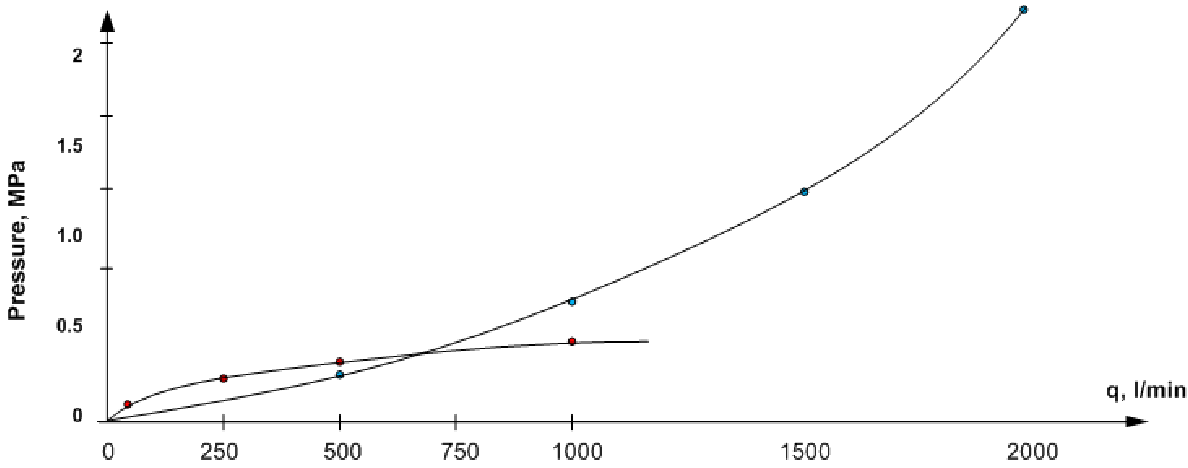


Figure 3-4: Laminar flow is the dominating flow regime at low flow rates.

3.9 Pressure loss. Field data

a) Pressure reading at 5 spm = 13 bars. To check theoretical Δp at 5 spm we go through these steps:

Step 1: Find v and check flow regime: Re = ρvd/ μ_{eff}, both in drill pipe and annulus

Step 2: Find total pressure drop in flow system: $\Delta p = \Delta p_{dp} + \Delta p_{ann}$

Step 3: Make short cuts and assumptions and since this is merely a rough estimate)

Choose simplest possible model for laminar and turbulent flow

Assume surface pipes as 100 m prolongation of drill string

Ignore the effect of tool joints

Step 4: Find all the relevant information

$q = 15.29 * 5 \rightarrow 0.0013\ m^3/s$

$v = q / A = 3.14 / 4 * (4.2 * 0.0254)^2 = 0.015\ m/s$

$\mu_{eff} = \mu_{pl} + yp * d / (6*v) = 1.35\ Pas$

$\mu_{pl} = 60 - 44\ 0\ 16\ cp = 0.016\ Pas$

$YP = 44 - 16 = 28 = 12\ Pa$

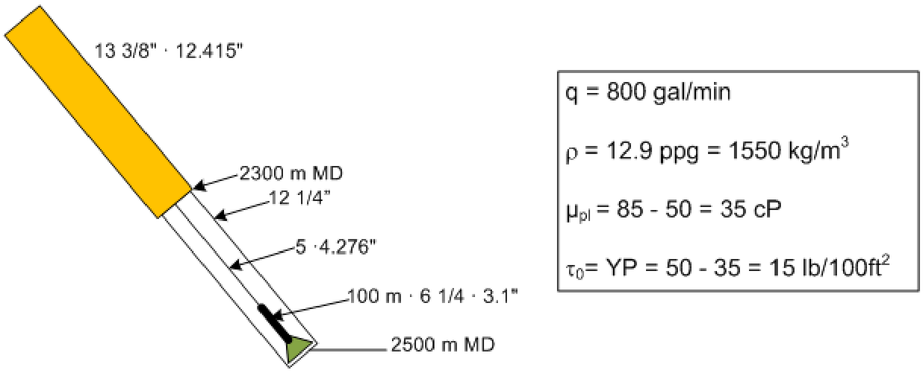
$\Delta p = 32\ v\ \mu_{eff} *l / d^2 = 32 * 0.15 * 1.35 * 4500 / 0.1^2 = 29 * 10^5\ Pa$

The task was to evaluate and compare the estimated pressure with the recorded pump pressure. Many error sources exist; selecting the correct rheology model; fluid is very Binghamian. When does flow turn from laminar to turbulence? In real wells it turns into turbulence earlier than theoretical prediction of transition, due to roughness and uneven flow path. Turbulent pressure loss is highly empirical.

b) During rotation of the pipes, external energy is added to the fluid. If the pipe is exposed to strict, controlled rotation the mud will be stirred (like in a viscometer). The mud is non-Newtonian and shear-thinning (n < 1.0) during laminar flow. Friction loss will therefore be lower (from 13 to 12 bars). If the pipe is exposed to vigorous rotation and possibly vibrations, which often is the case in a real well, then additional turbulence introduces higher pressure loss. At highest pump speed (100 spm) flow is already turbulent and the complex motion of the drill string will increase the level of turbulence.

3.10 Pressure loss. Bit nozzle. OFU

The exercise information are summarized here:



a) Annular pressure drop. Mud velocity in the BHA-annulus when drilling in 2500 m:

$$\bar{v} = \frac{q}{A} = \frac{q\,\dfrac{Gal}{\min}\cdot\dfrac{0.1377\,ft^3/gal}{60\ \ s/\min}}{\dfrac{\pi}{4}d\left(in\cdot\dfrac{1}{12\,in/ft}\right)^2} = 0.40856\frac{q}{d_h^2} = \frac{q}{2.448\,d_h^2} = \frac{800}{2.448\,\left(12.25^2 - 6.25^2\right)} \rightarrow 2.94\ \ ft/s$$

First we need to check flow regime:

Apparent viscosity:

$$\mu_a = \mu_{PL} + \frac{5\,\tau_y \cdot d_b}{\bar{v}} = 35 + \frac{5\cdot 15\cdot(12.25-6.25)}{2.94} = \underline{179}\ cP$$

$$\mathrm{Re} = \frac{757\rho\bar{v}(d_2 - d_1)}{\mu_a} = \frac{757\cdot 12.9\cdot 2.94\,(12.25-6.25)}{\mu_a} = 945 \Rightarrow \textit{not turbulent}$$

$$\frac{\Delta p}{\Delta l_{dc}} = \frac{\mu_{pl}\cdot\bar{v}}{1000\left(d_2 - d_1\right)^2} + \frac{\tau_y}{200\left(d_2 - d_1\right)} = 0.0154\,psi/ft \quad \Rightarrow \quad 0.0154\cdot\frac{100}{0.3048} = 5.05\ psi$$

Eqv.rin 2500 MD:

$$\rho_{ekv} = \rho + \frac{\Delta p_{ann}}{0.052\cdot L} = 12.9 + \frac{5.05+100}{0.052\cdot\dfrac{2500}{0.3048}} = \underline{\underline{13.15\ PPG}}$$

b) Bit pressure drop

$$\Delta p_{bit} = \frac{8.311 \cdot 10^{-5} \cdot 12.9 \cdot 800^2}{0.95^2 \cdot \left(5 \cdot \frac{\pi}{4} \cdot \left(\frac{14}{32}\right)^2\right)^2} = \underline{\underline{1\,346\ psi}}$$

c) Total pressure loss in the system:

DP: $$\overline{v} = \frac{800}{2.448 \cdot 4.276^2} = \underline{17.87\ \ ft/s}$$

DC: $$\overline{v} = \frac{800}{2.448 \cdot 3.1^2} = \underline{34.0\ \ \ ft/s}$$

Turbulent or laminar in largest area?

$$\mathrm{Re} = \frac{928 \rho \overline{v} d}{\mu_{eff}} = \frac{928 \cdot 12.9 \cdot 17.87 \cdot 4.276}{35+} = \underline{26\,136}$$

DP: $$\frac{d_{pf}}{d_L} = \frac{12.9^{0.75} \cdot 17.87^{1.75} \cdot 35^{0.25}}{1800 \cdot 4.276^{1.25}} = 0.2323\ \mathrm{psi/ft}$$

DC: $$\frac{d_{pf}}{d_L} = \frac{12.9^{0.75} \cdot 17.87^{1.75} \cdot 35^{0.25}}{1800 \cdot 4.276^{1.25}} = 0.2323\ \mathrm{psi/ft}$$

Total pressure loss in drill string:

$$\Delta p_{dp} = 0.2323 \cdot 2400 \cdot 3.28 = \underline{\underline{1858\ \ psi}}\ , \qquad \Delta p_{dc} = 300\ psi\ \ (not\ shown)$$

$$\Delta p_{dc} = 0.2323 \cdot 2400 \cdot 3.28 =$$

$$\Delta p_{tot} = 200 + 1858\ \ +\ \ 300 + 1346\ )$$

d) Comparing results from OFU and SI units.

$$\Delta p_{bit} = 1.11 \cdot \frac{1}{2} \cdot 1550 \left[\frac{800 \cdot \frac{3.78533}{60 \cdot 1000}}{5\ \frac{\pi}{4}\left(\frac{14}{32} \cdot 0.0254\right)^2} \right]^2 = 9.318\ \ MPa = \underline{1369\ \ psi}$$

Conclusion: Choice of units does not make any difference for this formula.

3.13 Swab pressure. Clinging factor

1. Make a detailed drawing (see below)
2. Develop velocity profile with the correct boundary condition

$v(r)_{r=R0} = 0$
$v(r)_{r=Rcling} = 0$
$v(r)_{r=R1} = v_p$

3. Mass balance: $q_{up} = v_{pipe} * V_{pipe} + q_{cling}$. An equivalent fluid mass is moving down
4. Cling factor = q_{cling} / q_{down}

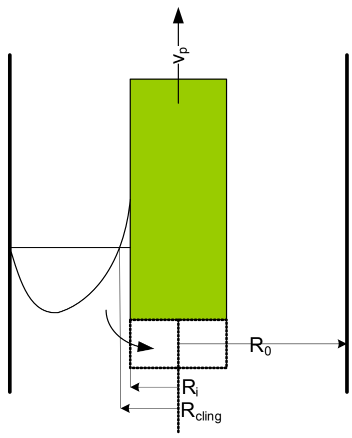


Figure 3-6: Flow streaming down the annulus when pulling a closed end drill string, the pressure loss causing a pressure deficit below the string bottom.

4 Hydraulic program

4.1 Mud pump issues

a) Characteristic diagrams of mud pumps are shown below:

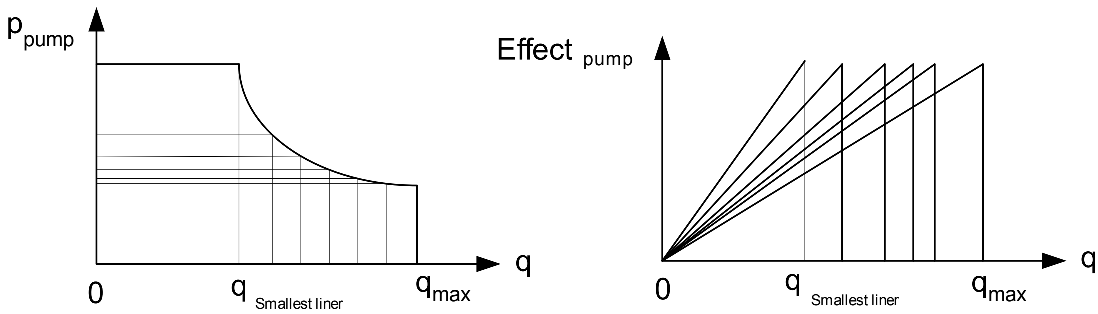


Figure 4-1: Maximum pump pressure for each liner (left) and maximum effect for same (right)

Effect = E = p * q, and is expressed in Horse Power (HP) or in watt (W). As shown in figure the maximum effect is constant for all liners

Volumetric efficiency = 0.96 for new pumps. When it reaches 0.93 the pump is maintained (replace valves or piston / liner)

b) Parallel → higher q, used for upper wellbore sections. Series → higher p (only special application)
c) Centrifugal can deliver high q but limited pressure (max 3 bars)
d) Knocking is due to high acceleration -> vacuum effect -> boiling -> implosion of vapor bubbles

4.2 Parasitic pressure

Find first $\Delta p_d = \Delta p_p - \Delta p_{bit}$

$$p_{bit} = 1.1 \ \frac{1}{2}\rho v^2$$

$\bar{v}$ = q/A_{nozzle}

$A_{nozzles}$= (p/4)de^2= p/4 (d_1^2 + d_2^2 + d_3^2) = p/4 *3 * (15/32)^2*(2.54*10^{-2})^2 = 3.3333 * 10^{-4} m^2

$$\text{or } d_e = \sqrt{d_1^{\ 2} + d_2^{\ 2} + d_3^{\ 2}} = \sqrt{3} \cdot d_1 = \sqrt{3} \cdot \frac{15}{32} 0.0254 = 0.0206\, m$$

q_1 = 0.03333 m 3/s

q_2 = 0.017 m 3/s

v_1 = 100 m/s

v_2 = 51 m/s

Resulting in:

q (lpm)	p_p(bar)	Δp_{bit}	Δp_d
2000	160	72	88
1000	43	18	25

K_1 and m are determined from: $\Delta p_d = K_1 D\, q^m$

$$m = \frac{\ln\left(\Delta p\, d_1/\Delta p d_2\right)}{\ln\left(q_1/q_2\right)} = \frac{\ln\ 88/25}{\ln\ 2000/1000} = 1.82$$

$$K_1 = \Delta p_d \big/ q^m \cdot D = \frac{88\cdot 10^5}{0.0333^{1.82}\cdot 2500} = 1.7 * 10^6$$

4.3 Nozzle selection. Section wise

To find the optimal flow rate we go through all these steps:

$$d_e = \sqrt{3}\cdot d_1 = \sqrt{3}\cdot\frac{15}{32}\cdot 0.0254 = 0.0206$$

$$v_{nozzle,1} = \frac{q}{A} = \frac{2500}{1000\cdot 60\cdot\frac{\pi}{4}0.0206^2} = \frac{0.0417}{0.000333} = 125.2\ m/s$$

$$v_{nozzle,2} = \qquad = 87.6\, m/s$$

$$p_{bit1} = 1.11\cdot\frac{1}{2}\cdot 1400\cdot 125.2 = 121.4\cdot 10^5\, Pa \quad / 59.6\cdot 10^5\, Pa \quad (represents\ pump \quad rate\, 1$$

$$\Delta p_{loss1} = 230 - 121.4 = 108.6\, bar$$
$$\Delta p_{loss2} = 120 - 59.6 = 60.4\, bar$$

$$\Delta p_{loss} = K_1 D\cdot q^m$$

$$m = \frac{\log\Delta p_{loss1}/\Delta p_{loss2}}{\log q_1/q_2} = \frac{\log 108.6/60.4}{\log 2500/1750} = 1.64$$

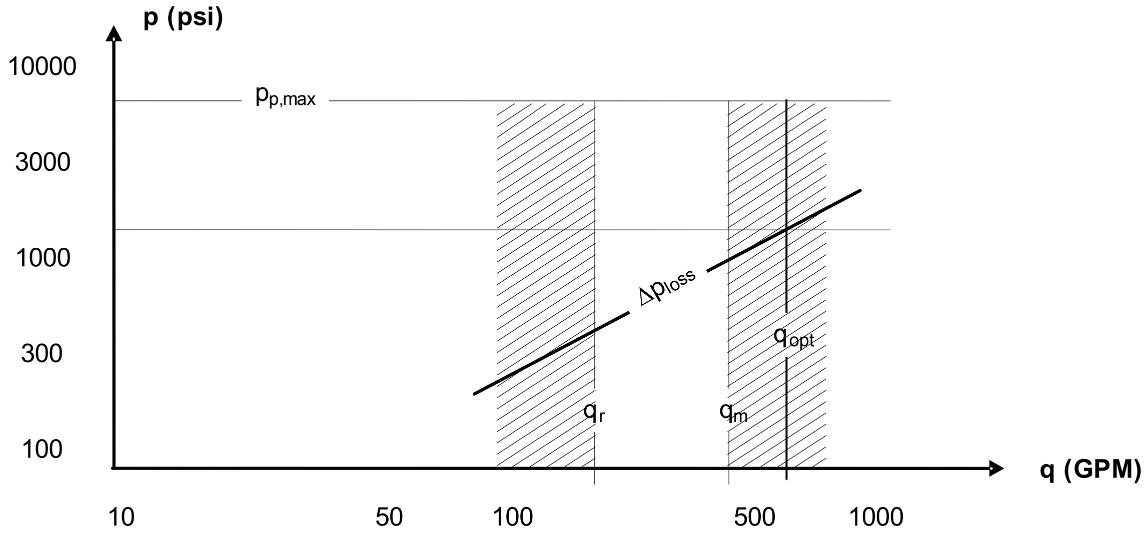
$$K_1 = \frac{108.6\cdot 10^5}{3000\cdot 0.0417^{1.64}} = 0.663\cdot 10^6$$

$$q_{opt} = \left[\frac{2\, p_{pump,\max}}{(m+2)K, D}\right]^{\frac{1}{m}} = \left[\frac{2\cdot 270\cdot 10^5}{3.64\cdot 0.663\cdot 10^6\cdot 3000}\right] = 0.0103^{0.61} = \underline{\underline{0.0504\ m^3/s}}$$

Check then which flow rate ae the boundary conditions (in SI-units to be able to compare):

$$q_{\max,ann} = 2600/60000 = 0.0433\ m^3/s$$
$$q_{\min,ann} = 1700/60\cdot 1000 = 0.0290\ m^3/s$$

Here is the graphical solution also: The optimal liner is the smallest possible since that one delivers the highest pressure. The largest liner must be selected to cover all the flow rate ranges. The max pump pressure of this liner is 220.2 bars (and max flow rate is large enough).Parasitic pressure loss; $\Delta p_{loss} = p_p \cdot 1/(m+2) = 100\, bar$, must be subtracted to see what is left for the bit.



Since q_{opt} is $> q_{max,ann}$, latter must be chosen as the practical optimal solution: $q_{opt} = 0.0433\, m^3/s$

Now we can find the nozzles:

$$\Delta p_{bit} = p_{pump,\max} - K_1 D \cdot q^m = 226.2 \cdot 10^5 - 0.663 \cdot 10^6 \cdot 3000 \cdot 0.0433^{1.64} = \underline{104.6 \cdot 10^5}$$

$$1.11 \cdot \frac{1}{2} \rho \left(\frac{q}{\frac{\pi}{4} d^2} \right)^2 = 104.6 \cdot 10^5$$

$$d = \sqrt{a^2 + b^2} \sqrt[4]{\frac{1.11 \cdot 1400 \cdot 4^2 \cdot 0.0433^2}{2 \cdot \pi^2 \cdot 104.6 \cdot 10^5}} = \underline{0.0218\ m}$$

Nozzle diameter is not optimal at 3000 meters depth but close to. It should have been:

$$d_1 = \cdot \frac{d}{\sqrt{3}} = \frac{0.0218}{\sqrt{3}} = 0.0126 \quad \Rightarrow \quad 3 \cdot \frac{0.0126 \cdot 32}{0.0254} = 15.9$$

i.e.: 12/32", 12/32", 13/32"

4.4 Liner selection. Section wise

1. In order to cover the complete range of possible flow rates we need to select between 6 ¾ and all the smaller ones.The optimum flow rate is:

$$q_{opt} = \left[\frac{2(p_p)_m}{K_1 \cdot D(m+2)} \right]^{\frac{1}{m}} = \left[\frac{2 \cdot 383.0 \cdot 10^5}{1.1 \cdot 10^6 \cdot 2500(1.7+2)} \right]^{\frac{1}{1.7}} = 0.0364\ m^3/s$$

At this depth we select the smallest possible liner to obtain the maximum pump pressure, i,e, is 6 1/2 ".

2. Bit pressure is found from the optimal solution of

$$ROP = A \cdot (d_{nozzle})^{a8}$$

$p_p = \Delta p_{loss} + \Delta p_{bit}$

$$p_p = K_1 D q^m + k_{bit} \cdot \left(\frac{q}{d_e} \right)^2 \rightarrow \frac{q}{d_e} = \left(p_p - k_1 D q^m \right)^{0.5} \cdot \frac{1}{k_{bit}}$$

Enter the expression of d_e (same as d_{bit}) into equation of ROP, differentiate and find maximum when = 0:

$$2 p_p q - K_1 D(m+2) q^{m+1} = 0$$

$$2 p_p - (m+2) p_d = 0$$

$$2 p_p - (m+2)(p_p - p_{bit}) = 0$$

Optimum bit pressure drop is thus:

$$p_{bit} = \frac{m}{m+2} p_p$$

From pump data: $p_{pump} = 254.4 \times 10^5$ Pa

$$p_{bit} = \frac{1.7}{3.7} \cdot 254.4 \cdot 10^5\,Pa = \underline{116.9 \cdot 10^5\,Pa}$$

3. The depth of altering liner is found when q_{opt_1} at liner change = 0.0307 m³/s

$$\left[\frac{2(p_p)_{max}}{K_1 \cdot D(m+2)}\right]^{\frac{1}{m}} = 0.0307$$

$$K_1 \cdot D(m+2) = \frac{2(p_p)_{max}}{(0.0307)^m}$$

$$D = \frac{2(p_p)_{max}}{K_1 \cdot (m+2)(0.0307)^m} \qquad = \frac{2 \cdot 350.6}{1.1 \cdot 10^6 \cdot (3.7)(0.0307)^{1.7}} = 6\,428\ m$$

4.5 Optimal parameters with BHHP. OFU. Section wise

Optimum parameters for this criterion can be expressed as follows:

$$ROP = 1/1714 \cdot C_1 \cdot \Delta p_{bit} \cdot q = C_1 \cdot (p_p - \Delta p_{loss}) \rightarrow \frac{d\ ROP}{dq} = p_p - (m+1)K_1 D\, q^m$$

$$\Delta p_p = (m+1)\,K_1 D\ q^m \qquad \Rightarrow q_{opt} = \left[\frac{p_p}{(m+1) \cdot K_1 D}\right]^{1/m}$$

We observe also that optimal parasitic and bit pressure drop can be derived from the equation above:

$$\Delta p_{loss} = p_p \frac{1}{m+1}, \quad \Delta p_{bit} = p_p \frac{m}{m+1} p_p$$

To estimate q_{opt}, K_1D and m must be determined from: $\Delta p_{loss} = K_1 D q^m$

Input		**Output**	
q(GPM)	p_p(psi)	Δp_{bit}(psi)	Δp_d(psi)
500	3000	2097.0	903.0
250	800	524.3	275.7

$$m = \frac{\ln\left(\Delta p_{loss1}/\Delta p_{loss2}\right)}{\ln\left(q_1/q_2\right)} = \frac{\ln\ 903\ /\ 275.7}{\ln\ 500\ /\ 250} = 1.712$$

$$K_1 D = \Delta p_{loss}/q^m = \frac{903}{500^{1.712}} = 0.02168 = c_1$$

Now q_{opt} can be found:

$$q_{opt} = \left[\frac{\Delta p_p}{(m+1)K_1 D}\right]^{1/m} = \left[\frac{3000}{(1.71+1)\,0.0217}\right]^{1/1.712} = 562\ \text{gpm}\ (\text{mech. eff. not included})$$

Including the volumetric efficiency factor, e_{vol}, we obtain the practical q_{opt}

$$q_{opt} = 562 \cdot 0.9 = 506 \quad GPM$$

Now the bit nozzles can be estimated

$$A_{nozzle,opt} = \sqrt{\frac{8.311 \cdot 10^{-5} \cdot \rho q_{opt}^2}{C_d^{\,2} \cdot \Delta p_{bit}}} = \sqrt{\frac{8.311 \cdot 10^{-5} \cdot 10 \cdot 506^2}{0.95^2 \cdot 2050}} = 0.342\ in^2$$

$$\Delta p_{bit} = 3000 - 950 = 2050$$

$$\Delta p_{loss} = C \cdot q^m = 0.0217 \cdot 506^{1,712} = 950\ psi$$

$$\Delta p_{loss} = p_p \frac{1}{m+1} = 3000 \cdot \frac{1}{2.71} = 1103\ psi$$

Here the e_{vol} is not included and this result cannot be used directly.

$$A = 3 \cdot \frac{\pi}{4}\left(\frac{d_e}{32}\right)^2 \rightarrow d_e = \sqrt{\frac{A \cdot 4 \cdot 32^2}{3 \cdot \pi}} = \sqrt{\frac{0.345 \cdot 4 \cdot 32^2}{3 \cdot \pi}} = 12.24$$

➔12/32", 12/32", 13/32"

4.6 Hydraulic program. Section wise

b) $p_p = \Delta p_{loss} + \Delta p_{bit}$

$$p_p = K_1 D q^m + k_{bit} \cdot \left(\frac{q}{d_e}\right)^2 \rightarrow \frac{q}{d_e} = \left(p_p - k_1 D q^m\right)^{0.5} \cdot \frac{1}{k_{bit}}$$

Substitute into the ROP equation:

$$ROP = k \cdot \left[\left(p_p - k_1 D q^m\right)^{0.5} \cdot \frac{1}{k_{bit}}\right]^{a_8} = k^1 \left(p_p - k_1 D q^m\right)^{a'}$$

This is a decreasing function for increasing depth.

c) Within this narrow flow rate interval only 6 ¼, 6, 5 ¾ and 5 ½" liner s can be used. The strategy now is to obtain a full overview through the graph below. We need the following data points, starting at the shallowest depth where q_{opt} is permissible:

$$q_{opt,6\,1/4"upper} = \left(\frac{2p_{max}}{(m+2)k_1 D}\right)^{1/m} = 0.035$$

$$0.035^{1.65} = \frac{2 \cdot 296.8 \cdot 10^5}{3.65 \cdot 2 \cdot 10^6\ D} \Rightarrow D_{0.035} = \frac{8.11}{0.035^{1.65}} = 2053\ m$$

Depth when changing to 6 " liner:

p_{max} = 322 bars
$q_{opt,\,6"}$= 0.0334 m^3/s

$$0.0334^{1.65} = \frac{2 \cdot 322 \cdot 10^5}{3.65 \cdot 2 \cdot 10^6 D} \Rightarrow D = 2406\ m$$

Depth when to change to 5 ¾ " liner:

p_{max} = 350.6 bars
q_{opt} = 0.0307 m^3/s

$$0.0307 = \left(\frac{2 \cdot 350.6 \cdot 10^5}{3.65 \cdot 2 \cdot 10^6 \cdot D}\right)^{1.65} \quad \Rightarrow \quad D = 3\,011\ m$$

Depth when to change to 5 ½ " liner:

p_{max} = 383 bars
$q_{opt,6"} = 0.0280\ m^3/s$

$$0.028 = \left(\frac{2 \cdot 383 \cdot 10^5}{3.65 \cdot 2 \cdot 10^6 \cdot D}\right)^{1.65} \quad \Rightarrow \quad D = 3\,831\, m$$

Depth when the required flow rate is met

p_{max} = 383 bars

$q_{opt,6"} = 0.025\, m^3/s$

$0.025 = (\frac{2 \cdot 383 \cdot 10^5}{3.65 \cdot 2 \cdot 10^6 \cdot D})^{1.65} \quad \Rightarrow \quad D = 4\,619\, m$

Resultinggraph:

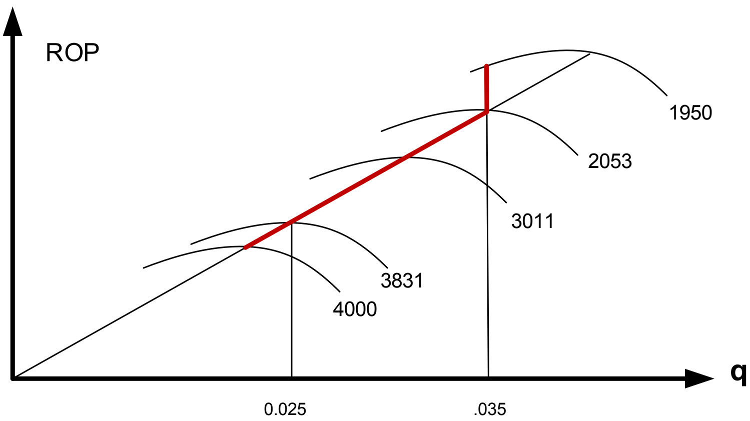


Figure 4-2: ROP vs. q showing at what depths the different liners are changed, from start to end. The data needs to be adjusted to fit this exercise

At shallower depths than "optimal ROP – depths" 15(<2048 m) use highest possible flow rate?

Depth (m)	q (m³/s)	p (bar)	liner
1950 – 2053	0.035	296.8	6 ¼
2053 – 2406	0.0334	296.8	6 ¼
2406 – 3011	0.0307	322	6
3011 – 3831	0.028	350.6	5 ¾
3831 - 4000	0.025	383	5 ½

4.7 Liner selection. Complete well

a) Optimal flow rate in pump range II:

$$ROP = K \cdot \left(\frac{q}{d_{nozzle}}\right)^{a_8} = \frac{K}{K_{bit}}\left(p_p q^2 - K_1 D \cdot q^{m+2}\right)^{\frac{a_8}{4}}$$

$$ROP_I = K^1\left(p_{\max} \cdot q^2 - K_1\ D \cdot q^{m+2}\right)^{a_8/4} \qquad \mathrm{ROP_{II}} = \mathrm{K^1}(\mathrm{E}_p \cdot q - K_1 D q^{m+2})^{\frac{a_8}{4}}$$

$$\frac{\partial ROP_I}{\partial q} \qquad \frac{\partial ROP_{II}}{\partial q} = \frac{K^1\left(E_p - (m+2)\,K_1 D q^{m+1}\right)^{\frac{a_8}{4}}}{(\quad)^{\frac{a_8}{4}1}} = 0$$

$$q_{opt\ II} = \left[\frac{E_p}{K_1 \cdot D(m+2)}\right]^{\frac{1}{m+1}}$$

b) Range II stop, range I start:

The two optimal solutions are solved with respect to D at q = 0.028 m³/s

$$q_{opt,I} = \left(\frac{2p_p}{(m+2)K_1 D}\right)^{1/m}\left(\frac{2 \cdot 383 \cdot 10^5}{3.5 \cdot 1.25 \cdot 10^6 \cdot D}\right)^{1/1.65} = 0.028 \quad \Rightarrow \quad D_I = 3\,737$$

c) The graphical solution:

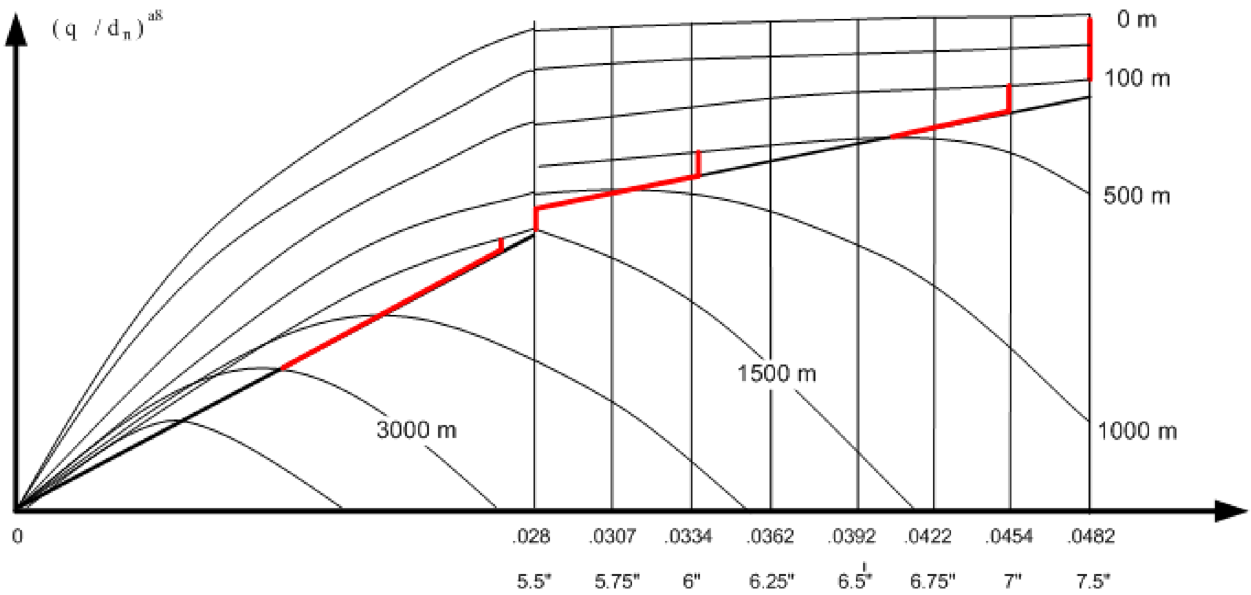


Figure 4-3: Graphical solution.

d) Check optimal pump rate in 2000 m depth for both pump ranges:

$$q_{opt_{II}} = \left[\frac{1.076\cdot10^6}{3\cdot10^6\cdot2000(1.47+2)}\right]^{\frac{1}{1.47+1}} = 0.018\ m^3/s$$

$$q_{opt_I} = \left[\frac{2\cdot383\cdot10^5}{3\cdot10^6\cdot2000\cdot(1.47+2)}\right]^{\frac{1}{1.47}} = 0.022\ m^3/s$$

$q_{opt_{II}}$ is outside its pump area (0.028) while q_{opt_I} is inside.

4.8 Liner selection. Complete well

a) Find first what working area prevails in the boundary zone between pump are II and I, which are valid as indicated in Figure 4.3:

$$q_{opt,II} = 0.028 = \left[\frac{E}{K_1 D(m+2)}\right]^{1/(m+1)}$$

$$D = \frac{E}{K_1\,(m+2)\cdot(0.028)^{2.7}} = \frac{1439\cdot745.7}{1.72\cdot10^6\cdot(1.51+2)\,(0.028\,)^{2.7}} = 1868\ m$$

First in 1 868 m depth the pump area I is entered, coming from II. From pump chart we note that 5 ½" liner has p_{max}= 383 bar. Solving q_{opt} eqn. for D we find where q_{opt} starts.

$$q_{opt,I} = 0.028 = \left[\frac{2p_p}{K_1 D(m+2)}\right]^{1/m} \quad\Rightarrow\quad D = 2809$$

b) In Figure 4-4 we see the division between the two pump areas. A preliminary solution to how to distinguish between them is given in Figure 4.5, its upper part.

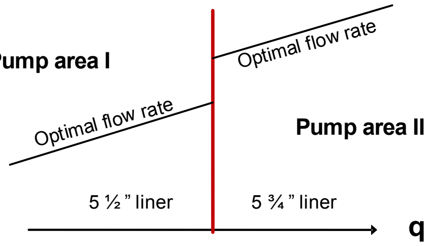


Figure 4-4: Boundary area between pump area II and I

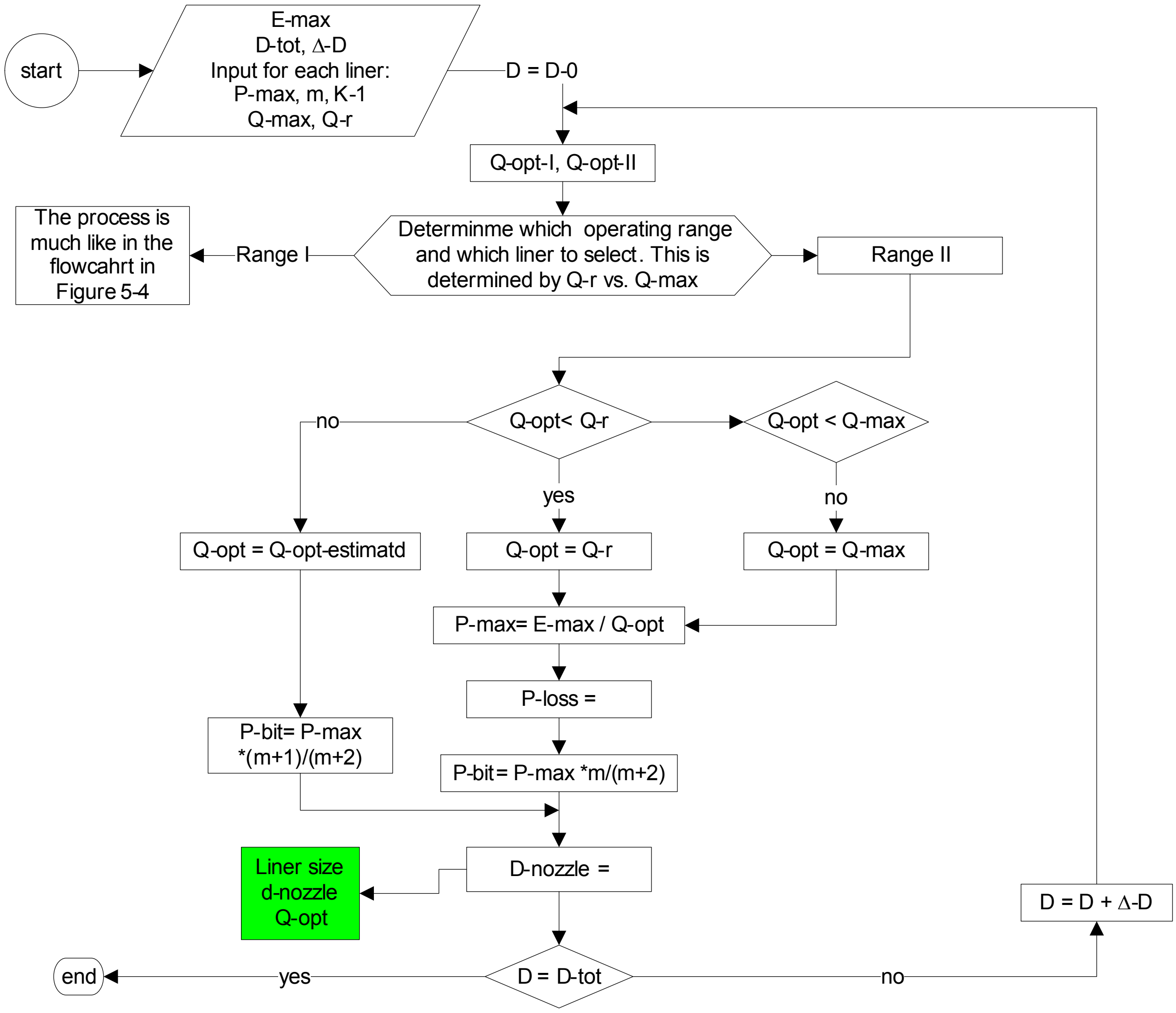


Figure 4-5: Flow chart of estimating extended hydraulic parameters. In the upper part it distinguish between the pump area ranges

5 Wellbore challenges

5.1 ECD. Cuttings concentration

a) At the end of the horizontal section the concentration would probably be close to 0.02. However, some cuttings are always deposited. The bed height demonstrates this. Settling and lifting of cuttings is a function of q, RPM, A_{flow} etc. My guess is that 10 % of the wellbore cross sectional area is in average filled with cuttings. This means 10 % of what is drilled out is left in the well. The initial concentration is accordingly reduced;

$c_2 = c_1 * (1 - 0.1) = 0.18$

b) In the vertical section the concentration will increase due to the slipping of cuttings, expressed through the transport ratio R_t, which is typically 0.75. We then have:

$c_3 = c_2 / R_t = 0.18 / 0.75 = 0.024$

c) The cuttings removal forces are opposed by gravity and cohesive forces. Gravitational forces are given by Stokes law. Removal forces are given by drag and lift forces. The drag force is a function of particle Reynolds number and sphericity. The drag and lift forces can be developed info the critical lift velocity or rolling velocity. At a given cuttings feed rate, a stationary bed height will form as a function of all involved input variables.

d) Here the bed height will be higher due to higher cross sectional area and thus lower velocity. When BHA during tripping is shoveling much cuttings into a narrower wellbore it is easy to imagine that the BHA may become jammed.

5.2 Solids control

Five issues related to poor solids control:

- Building of solids bed →Mechanical stuck pipe
- Building of viscosity / high gel ->high pressure losses
- High cuttings concentration in annulus →heavy mud in annulus →high ECD
- Reduction in ROP due to higher mechanical friction -> loss of WOB
- Loss of filter control ->thicker mud cake ->differential sticking

5.3 ECD. Barite

When ρ_{mud} > ~ 1.5 kg/l, the barite concentration is high. At stillstand, most mud types quickly develop a yield point, and barite is suspended in the gelled mud. During pumping of mud in laminar mode, any existing gel is broken, and barite will not be held in suspension as during stillstand with gelled mud. However, the mud will under these conditions exhibit a high effective viscosity but will, nevertheless, slip slowly downwards due to gravity. Agglomeration may also affect the settling rate. Barite has a specific density of 4.2, and an average particle size of approximately 20 μm. In accordance with Stokes's law of settling, barite particles will settle, but as indicated above, very slowly. However, wellbore inclination will shorten the settling distance to only a few centimetres, and a stratified bed of Barite and cuttings are formed. The stratified layers of solids will, when angle of repose is surpassed, slide downwards. Angle of repose in found mainly in the build up section of the well, and here sliding takes place. After first "landslide", Barite concentration in the mud is lower and accordingly, also the viscosity. Barite settling will now be faster, and the process continues at accelerated speed. The self preserving dynamics may separate out almost all the barite in the build-up section, especially during still stand after tripping out of the hole. The pile of Barite at the bottom of the Build-up section may cause pipe sticking. Worse is the reduction of mud weight. It may disrupt the safety.

The following summary of guidelines to obtain efficient hole cleaning is based on field experiences and on many laboratory investigation:

1. Apply micro barite or equivalent. Such systems have an average particle size of 2 μm
2. Use top drive rigs to allow for pipe rotation and redistribution of settling barite and cuttings while tripping and drilling (rotation of string possible also with down hole motor)
3. Maximize fluid velocity during drilling and reaming by increasing pumping power and/or use large diameter drill string. This improves hydraulic and agitation of cuttings. Use in addition downhole flow enhancers
4. Design mud rheology so that it enhances turbulence in the annulus (if ECD allows). Turbulent flow will bring the cuttings and barite back into the flow stream if point 3 above "failed"
5. Perform frequent wiper trips

5.4 ECD. Flow rate + fluid consistency

a) In order to determine the ECD we need to estimate the pressure loss in the annulus. The mud has some gel, so we assume the Bingham model will suit well.

$$\mu_{pl} = \frac{\tau_{600} - \tau_{300}}{\gamma_{600} - \gamma_{300}} = \frac{51.7 - 30.6}{1022 - 511} = 0.0413\ Pas$$

$$\tau_0 = 51.7\ Pa - 1022 \cdot 0.0413\ Pas = 9.5\ Pa$$

Although the flow rate is rather high, we assume laminar flow in the annulus. However, if the purpose were to find the exact Dp we would need to checkthe Reynolds number to confirm this.

Annular pressure loss:

$$\Delta p_a = 48 \cdot \mu_{pl} \cdot \left[\frac{L_{DC/OH} \cdot v_{DC/OH}}{(d_o - d_i)_{DC/OH}{}^2} + \frac{L_{DP/OH} \cdot v_{DP/OH}}{(d_o - d_i)_{DP/OH}{}^2} + \frac{L_{DP/C} \cdot v_{DP/C}}{(d_o - d_i)_{DP/C}{}^2} \right] +$$

$$6 \cdot \tau_0 \cdot \left[\frac{L_{DC/OH}}{(d_o - d_i)_{DC/OH}} + \frac{L_{DP/OH}}{(d_o - d_i)_{DP/OH}} + \frac{L_{DP/C}}{(d_o - d_i)_{DP/C}} \right] = \underline{\Delta p_a = 385\,179\ Pa = 3.85\ bar}$$

Now the ECD:

$$ECD = \frac{P_{hydr} + \Delta p_{ann}}{g \cdot h} = \frac{p \cdot g \cdot h + \Delta p_{ann}}{g \cdot h}$$

$$ECD = \frac{1250 \cdot 0.0981 \cdot 2100 + 3.85 \cdot 10^5}{0.0981 \cdot 2100} = 1270\ kg/m^3$$

b) What is the ECD during drilling?
Production of mud and cuttings:

q_{mud}= 3 500 /(1000 * 60) = 0.058 m^3/s

$q_{cuttings}$ = ROP * A_{bit}= 3 500/(1000 *60) *3.14/4 * 17.5 *0.0254)^2 = 0.0022 m^3/s

Volumetric concentration of cuttings:

$c_{cuttings}$ = $q_{cuttings}$ /(q_{mud} + $q_{cuttings}$) = 0.0022 / 0.058 = 0.038

We assume slip; R_t = 0.75

$c_{cuttings}$ = 0.038 / 0.75 = 0.051

ECD contribution will be enhanced by the higher density of cuttings:

Density increase = 0.022/0.75 * 2.4 kg/l / (0.058 * 1.25) = 0.097

ECD = 1.25 (1 + 0.097) = 1.37 kg /l

5.5 ECD. Temperature influence

Suggested procedure to determine pressure profile, which is a f(T):

1. Find the stable temperature profile in the well in accordance with Chapter 7 of the text book. Default temperature gradient in sedimentary rocks = (10 + 0.03)°C/m.
2. Look up the thermal coefficients of materials, a, in a handbook;

α_{steel} =
α_{water} =

3. $\rho_i = \rho_o - \rho_o \cdot \alpha_{(T_i - T_{i-1})}$ at each depth interval Δh
4. Estimate $\rho_i = f(D(f(T)))$

5. Find $p_{bottom}\ as \sum \rho_i\ g\, \Delta h$

6. $\rho_{equiv} = \dfrac{p_{bottom}}{g \sum \Delta h}$

5.6 Water activity

One definition of water activity is the vapor pressure of its salt water solution and compares it to the water vapor pressure of distilled water.

$$A_w = \frac{p_{saline\ water}}{p_{destiilled\ water}}$$

Distilled water thus have a water activity of 1.00. All water molecules are freely moving around, not electrostatically bound to salt ions. A_w will diminish at increasing salt content.

Find the A_w of clay through its in situ weight and then by comparing with clays with known A_w. Thus A_w of water phase is determined, and type of salt.

5.7 Water Activity

a) Salt ions attract polar water and make it in-active. Distilled water has a water activity 1.0 whereas for saline water is less than 1.0.

b) Clay swelling:

$$p_{swelling} = -\frac{RT}{V_w} \ln\left(\frac{p_{w.clay}}{p_{w.mud}}\right)$$

V_w = Molar volume of water vapor;

Most shale formations contain water since shale is (water wet). Water sensitive clay materials, such as smectite, illite and mixed – layer clays, will adsorb water and swell. This leads to an elevated localized pressure around the wellbore. The pressure may surpass the material strength; the material will disintegrate and collapse.Water is transported into the shale through different mechanisms (hydraulic pressure, osmosis, diffusion etc)

c) Prevent: Adding K-salt with WBM. Dissolved salt will bind water and hinder it to evaporate; the water vapor pressure will declines and so also the measured A_w.

d) If $A_{w,\ OBM} \geq A_{w,\ pore}$, water will be drawn from the formation, and into the OBM making the water brittle.
If $A_{w,\ OBM} \leq A_{w,\ pore}$, shale will swell.

e) Yes, it can be reduced by creating a filter in the shale. This can be done by adding particles to the water phase where the average particle size distribution $\simeq$ 1/3 of the average pore throat size distribution.

5.8 Shale stability

a) Using Figure 5.2 a calcium chloride concentration of 28.2 % by weight is needed to give an activity of 0.69.
b) Take an ”in-situ” cut of the core, crush it and place it in a desiccator (completely air tight), expose it to different humilities (vapour pressure) through designed brines. When weight of the exposed sample has been converted to typical weight evolution of this type of shale. Through this comparison the water activity in the core can be determined. Since this is the water activity of the shale's pore water, the water activity on the mud should be identical to avoid swelling or shrinking of the shale.
c) Oil does not penetrate clay because the capillary forces are too large;

$$p_c = \frac{2 \cdot \sigma_f \cdot \cos\theta}{r}$$

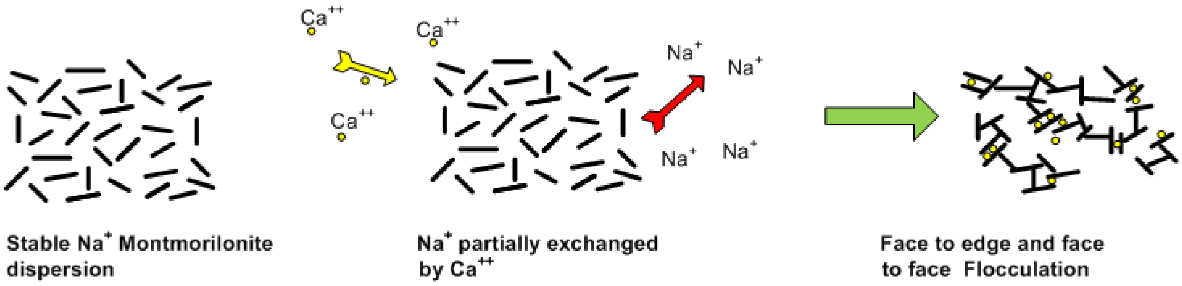
This depends on the wettability (which governs the q). For oil to be the wetting phase against the formation/cuttings/steel a wetting agent must be added to the base oil. If water is emulsified into the oil, the water activity of the water phase must be controlled properly.

Emulsifier is used to reduce the surface tension between oil and water, enabling smaller droplets, enhancing stability. Wetting agents make sure that oil is wetting the shale.

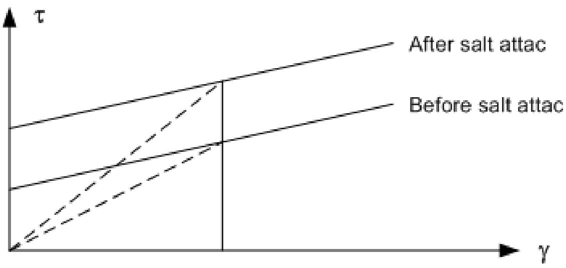
5.9 Clay behavior

a) MBT indicates the amount of reactive clays (Bentonite) present in drilling fluid. Or simply, it is a measure of the Cation Exchange Capacity (CEC) of clay.
This test is qualitative because organic material and some other clays present in the mud also will adsorb methylene blue.

b) The higher concentration, the more K^+ is exchanged with Na^+ shale, the less shale swelling is seen. K+ is geometrically suitable and leads to high platelet attraction.



Ion-exchange is dependent on Ca^{++} concentration. Low concentration → Partly cation exchange, high concentration → ccomplete cation exchange. Partly exchange leads to flocculation tendency.



Yield point (and effective viscosity) increases when low concentration of salt contamination. Effective viscosity indicated by dotted line.

c) The graph below shows both how a Ca attack is changing the dispersed mud, but also how Gyp mud is made. Bentonite is composed mainly of mineral called Montmorilonite. It is an aluminium silicate in which the silicon-oxygen sandwich is a layer of aluminium hydroxide. Some of the aluminium in the lattice structure is, during creation, replaced with ions of a lower charge (electron valence) such as magnesium (from 3 to 2). The *substitution*, without any other change of structure, creates surface negative charges in the lattice.
The negative charge sites created in the clay sheet by magnesium substitution are partly balanced by close association of positively charged ions (cation), normally Na^+. When the negative charge arises in the aluminium layer, the silica layers act as a physical barrier that prevents effective neutralisation of either the cations or the charge sites on the clay surface. The creation of these charged ions and charged clay surfaces creates very strong attractive forces for polar water molecules that readily force themselves in between the unit layers.

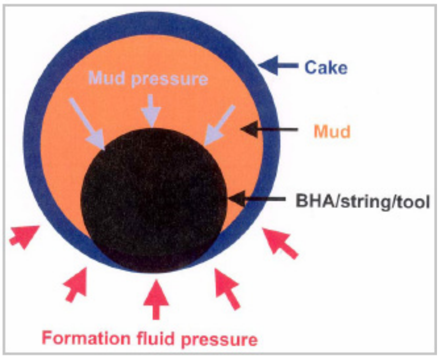
Gypsum is added to prevent shale swelling. When lime or gypsum is added to Bentonite – treated mud, sodium Montmorilonite will convert to calcium Montmorilonite, which first produces flocculation and eventually aggregation of the Montmorilonite. Caustic soda is added for pH control to suppress further dissolution of Ca^{++}. Lignosulphate is added for deflocculating. CMC may be added for fluid control. Then we get gyp mud.

If a lot swellable clays are drilled, apply gypsum mud; Pre-hydrate Na^+-Bentonite Add gypsum to the mud. Gypsum ($Ca\,SO_4$) dissolves and Ca^{++} ions are formed . First reaction causes flocculation. Over time Ca^{++} replaces Na^+ in Bentonite platelets, to make Ca^{++} - Bentonite and they aggregate and viscosity goes down.

But from now on, there is an access of Ca^{++} in the mud (and in filtrate). No new flocculation will occur, and swellable clay does not swell much.

5.10 Wellbore problems

a) Lots of material (cuttings, cavings, sloughing shale) combined with small and large washouts where material tends to accumulate. During tripping, material is scraped and squeezed.



Differential sticking: Once the drill string touches the wellbore wall, the mud pressure, which is higher than the formation fluid pressure, holds the string in place. As mud cake builds around the string, a pressure seal is formed and sticking occurs.

b) The consequences of a stuck pipe are very costly. They include:
   - Lost drilling time spent on freeing the pipe, and on fishing if not ask to jar out string
   - Abandon tools in the hole because fishing was given up.

   Rules of Thumb
   1. Begin working the string immediately. Jar in the opposite direction to the pipe movement prior to becoming stuck.
   2. Work the pipe to the limits.
   3. If getting movement down, concentrate on expanding downward movement – and vice versa.
   4. In cases of hole bridging/packing off, concentrate on downward working. Increase applied working force in gradual increments up to the maximum.

c) Salt tends to ”flow” at high temperature and pressure. Use high mud weigth and a mud that tend to dissolve salt (to keep the wellbore diameter as large as possible).
   WBM can be used with some additives. Halite has little creep tendency.

d) Dominating mechanisms at lost circulation are:

   - low pressure window
   - high ECD (for many reasons)

   Counter measures are:
   - improved hole cleaning
   - improved ECD - control

Wellbore breathing (ballooning) (any two sentence is enough for get full marks)

- The onset of wellbore breathing, often referred to as wellbore ballooning, is typically an indicator of imminent lost circulation.
- Wellbore breathing is associated with fractures that open when annular pressure is applied to the Wellbore and close when the pressure is reduced.
- These fractures fill with drilling fluid when open and subsequently return the fluid is observed as a flow out of the Wellbore when the pumps are off.
- One of the more severe consequences of wellbore breathing is the misinterpretation of the observed flow as a kick when the pumps are shut down.
- Implementing well- control procedures and increasing the mud weight is often enough to propagate the existing fractures leading to severe loss of circulation, a situation that is much more difficult to manage.