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A Note on Modeling High Speed Flow in a Bidisperse Porous Medium

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Abstract A model of a bidisperse porous medium is extended to the case where quadratic drag is significant. The extension may be significant in some geophysical situations as well as when transition to turbulence in this medium is investigated.

Keywords Bidisperse porous medium · Quadratic drag · Forchheimer equation

1 Introduction

A bidisperse porous medium (BDPM) is a medium composed of clusters of large particles that are agglomerations of small particles (Chen et al. 2000). The voids between the clusters are macropores and the voids within the clusters, which are typically much smaller in size, are micropores. Applications of BDPM are found in bidisperse adsorbent or bidisperse capillary wicks in a heat pipe. The bidisperse wick structure significantly increases the area available for liquid film evaporation. For this reason, it has been proposed for use in the evaporator of heat pipes. There are also biological structures, such as bone regeneration scaffolds, that are characterized by bimodal pore distributions.

A BDPM thus may be regarded as a standard porous medium in which the solid phase is replaced by another porous medium. We can then define the *f*-phase (the macropores) and the *p*-phase (the remainder of the structure). An alternative way of looking at the BDPM is to regard it as a porous medium in which fractures or tunnels have been introduced. The *f*-phase can then be viewed as a "fracture phase" and the *p*-phase can be viewed as a "porous phase".

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Extending the Brinkman model for a monodisperse porous medium, Nield and Kuznetsov (2005a) proposed to model the steady-state momentum transfer in a BDPM by the following pair of coupled equations for $\mathbf{v}_f^*$ and $\mathbf{v}_p^*$, where the asterisks denote dimensional variables,

$$\mathbf{G} = \left(\frac{\mu}{K_f}\right)\mathbf{v}_f^* + \zeta(\mathbf{v}_f^* - \mathbf{v}_p^*) - \tilde{\mu}_f \nabla^{*2}\mathbf{v}_f^* \tag{1}$$

$$\mathbf{G} = \left(\frac{\mu}{K_p}\right)\mathbf{v}_p^* + \zeta(\mathbf{v}_p^* - \mathbf{v}_f^*) - \tilde{\mu}_p \nabla^{*2}\mathbf{v}_p^*. \tag{2}$$

Here $\mathbf{G}$ is the negative of the applied pressure gradient, μ is the fluid viscosity, K_f and K_p are the permeabilities of the two phases, and ζ is the coefficient for momentum transfer between the two phases. The quantities $\tilde{\mu}_f$ and $\tilde{\mu}_p$ are the respective effective viscosities. In this pioneering paper, the effect of quadratic (Forchheimer) drag was neglected, and the hydrodynamic interaction between the two phases was modeled by the simplest possible expression.

This model has now been used in a number of papers on forced, natural and mixed convection. Nield and Kuznetsov (2005a) treated forced convection in a parallel-plate channel occupied by a BDPM, using a two-temperature model commonly used to model local thermal non-equilibrium. Nield and Kuznetsov (2004) extended the analysis to the case of a conjugate problem with plane solid slabs bounding the channel. They found that the effect of the finite thermal resistance due to the slabs is to reduce both the heat transfer to the porous medium and the degree of local thermal non-equilibrium. An increase in the value of the Péclet number leads to decrease in the rate of exponential decay in the downstream direction, but does not affect the value of a suitably defined Nusselt number. The case of thermally developing convection in a BDPM was treated by Kuznetsov and Nield (2006). The case of asymmetric heating of a channel was studied by Kuznetsov and Nield (2010). A channel partly filled with a porous medium was treated by Nield and Kuznetsov (2011). Heat transfer in a BDPM has been reviewed by Nield and Kuznetsov (2005b). A three-velocity three-temperature model of a tridisperse porous medium was applied by Kuznetsov and Nield (2011). The hydrodynamic aspect of bidisperse porous media in the context of thermal management has been studied by Narasimhan et al. (2012).

In the context of the Horton–Rogers–Lapwood problem (the onset of convection in a horizontal layer uniformly heated from below), a BDPM was studied by Nield and Kuznetsov (2006, 2007) and Straughan (2009). Their results were extended to a tridisperse porous medium by Kuznetsov and Nield (2011). Convection in a BDPM enclosure was studied by Narasimhan and Reddy (2010, 2011b) and Revnic et al. (2009), while Narasimhan and Reddy (2011a) investigated forced convection in a parallel-plate channel occupied by a heat generating BDPM. Natural convection adjacent to a vertical plate was examined by Nield and Kuznetsov (2008) and Rees et al. (2008). Mixed convection over a horizontal cylinder was studied by Kumari and Pop (2009).

We are currently interested in the hydrodynamic stability of flow in a channel occupied by a BDPM. It is well known that a simple linear instability analysis of shear flow in a channel gives a poor estimate for the value of the critical Reynolds number for the onset of instability, and hence a nonlinear stability analysis, taking into account terms quadratic in the velocity, is required. Thus if a similar analysis for a BDPM is to be made then it is desirable for consistency that a term quadratic in the velocity should be incorporated in the determination of the basic flow. That means that the model for the BDPM needs to be extended to incorporate the effect of quadratic drag.

2 The Extended Model

Hence, we are proposing to extend Eqs. (1) and (2) by incorporating Forchheimer drag terms while retaining the simplicity of the phase interaction term. Thus, we now consider the equations

$$\mathbf{G} = \left(\frac{\mu}{K_f}\right)\mathbf{v}_f^* + \zeta(\mathbf{v}_f^* - \mathbf{v}_p^*) - \tilde{\mu}_f \nabla^{*2}\mathbf{v}_f^* + \frac{c_f \rho}{K_f^{1/2}}|\mathbf{v}_f^*|\mathbf{v}_f^* \tag{3}$$

$$\mathbf{G} = \left(\frac{\mu}{K_p}\right)\mathbf{v}_p^* + \zeta(\mathbf{v}_p^* - \mathbf{v}_f^*) - \tilde{\mu}_p \nabla^{*2}\mathbf{v}_p^* + \frac{c_p \rho}{K_p^{1/2}}|\mathbf{v}_p^*|\mathbf{v}_p^*. \tag{4}$$

Here ρ is the density of the fluid, and the Forchheimer coefficients c_f and c_p are dimensionless quantities that, like the permeabilities, depend on the volume fractions ϕ (volume fraction of macropores) and ε (porosity of the p-phase). Estimates of the Forchheimer coefficients (based on the well-known equations of Ergun and Kozeny applicable to beds of spherical particles) are

$$c_f = \frac{1.75}{\sqrt{150\phi^3}}, \quad c_p = \frac{1.75}{\sqrt{150\varepsilon^3}}. \tag{5a,b}$$

We believe that in general the Forchheimer term is required in each of Eqs. (3) and (4). Although the f-phase is a fluid phase, the momentum Eq. (3) is the result of averaging over a representative elementary volume, and quadratic inertia terms in the Navier–Stokes equation give rise to a quadratic drag. It is true that in many situations the velocity in the p-phase will be small compared with that in the f-phase and in that case the quadratic drag term for the p-phase may be negligible. One can also envision BDPMs of special structure in which one or both of the Forchheimer terms would be negligible. For example, if the f-phase is in the form of a set of parallel capillary tubes then the flow in that phase would be almost unidirectional and so the contribution of inertial terms would be negligible.

3 Parallel-Plate Channel

As an example, we consider unidirectional flow in a parallel-plate channel of width $2H$. We take axes so that the flow is in the x-directions and the boundaries are at $y = -H$ and $y = H$, with

$$\mathbf{v}_f^* = (u_f^*(y^*), 0, 0), \quad \mathbf{v}_p^* = (u_p^*(y^*), 0, 0), \quad \mathbf{G} = (G, 0, 0).$$

For simplicity, we assume that $\tilde{\mu}_f = \tilde{\mu}_p = \mu$. We introduce dimensionless variables by letting

$$y = y^*/H, \quad u_f = u_f^* \mu / G H^2, \quad u_p = u_p^* \mu / G H^2 \tag{6}$$

and defining Darcy numbers Da_f and Da_p, a velocity transfer parameter η, and Forchheimer parameters γ_f and γ_p by

$$\mathrm{Da}_f = \frac{K_f}{H^2}, \quad \mathrm{Da}_p = \frac{K_p}{H^2}, \quad \eta = \frac{\zeta H^2}{\mu}, \quad \gamma_f = \frac{c_f \rho G H^4}{\mu^2 K_f^{1/2}}, \quad \gamma_p = \frac{c_p \rho G H^4}{\mu^2 K_p^{1/2}}. \tag{7}$$

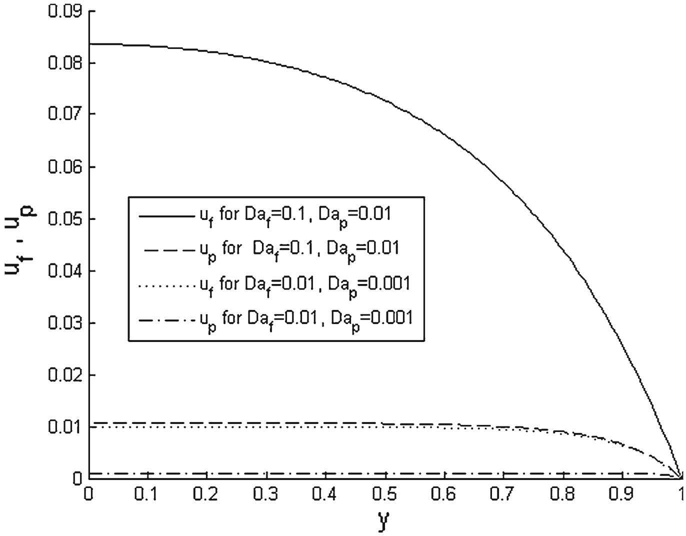

Fig. 1 Velocity profiles for representative values of the parameters, $\eta = 1, \gamma_f = 1, \gamma_p = 1$ and for various Darcy numbers as shown

The momentum equations then take the form

$$1 = \frac{u_f}{\mathrm{Da}_f} + \eta(u_f - u_p) - \frac{d^2 u_f}{dy^2} + \frac{\gamma_f}{\mathrm{Da}_f^{1/2}} u_f^2, \tag{8}$$

$$1 = \frac{u_p}{\mathrm{Da}_p} + \eta(u_p - u_f) - \frac{d^2 u_p}{dy^2} + \frac{\gamma_p}{\mathrm{Da}_p^{1/2}} u_p^2. \tag{9}$$

These equations can now be solved simultaneously subject to the no-slip boundary conditions

$$u_f = u_p = 0 \text{ at } y = -1 \text{ and } y = 1. \tag{10}$$

Once u_f and u_p have been found, the appropriate average velocity is obtained from the expression for the weighted algebraic mean of the two quantities,

$$u_{\mathrm{ave}} = \phi u_f + (1 - \phi) u_p. \tag{11}$$

Velocity profiles for some representative values of the parameters introduced in Eq. (7) are displayed in Fig. 1. The degree of flatness of the profile for the average velocity is of interest, because in general one would expect that the flatter the velocity profile for a shear flow then the greater the stability of that flow. As one would expect, the figure shows that the profiles are reduced in height and become flatter as the Darcy numbers decrease.

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