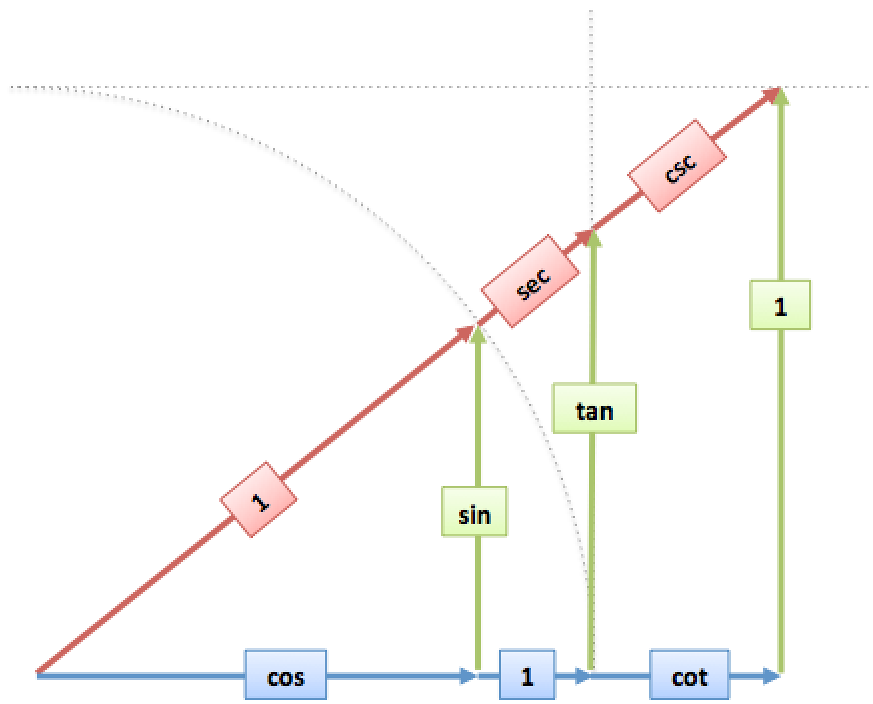
Algebra2/Trig Honors: Trig Unit 2 Packet

In this unit, students will be able to:

- **Learn and apply co-function relationships between trig functions**
- **Learn and apply the sum and difference identities**
- **Learn and apply the double-angle identities**
- **Learn and apply the ½-angle identities**



Name:________________________________

Teacher:______________________________

Pd: _______

Table of Contents

Formulas and Identities

Reciprocal Functions

$\tan x = \dfrac{\sin x}{\cos x}$ $\quad \cot x = \dfrac{\cos x}{\sin x}$ $\quad \sec x = \dfrac{1}{\cos x}$ $\quad \csc x = \dfrac{1}{\sin x}$

$$\cot x = \frac{1}{\tan x}$$

Pythagorean Relationships

$\sin^2 x + \cos^2 x = 1$ $\quad \tan^2 x + 1 = \sec^2 x$ $\quad 1 + \cot^2 x = \csc^2 x$

Functions of the Sum of Two Angles

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

Functions of the Difference of Two Angles

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

Functions of the Double Angle

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

Functions of the Half Angle

$\sin \frac{1}{2} A = \pm\sqrt{\dfrac{1 - \cos A}{2}}$

$\cos \frac{1}{2} A = \pm\sqrt{\dfrac{1 + \cos A}{2}}$

$\tan \frac{1}{2} A = \pm\sqrt{\dfrac{1 - \cos A}{1 + \cos A}}$

Cofunction Identities

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ $\quad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$ $\quad \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$

$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$ $\quad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$ $\quad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$

Negative Angle Identities

$\sin(-\theta) = -\sin\theta$ $\quad \cos(-\theta) = \cos\theta$ $\quad \tan(-\theta) = -\tan\theta$

$\csc(-\theta) = -\csc\theta$ $\quad \sec(-\theta) = \sec\theta$ $\quad \cot(-\theta) = -\cot\theta$

Day 1 – Co-functions

Warm - Up

Directions: Compute the functions below. Try to discover the pattern between the angles.

****ROUND EACH ANSWER TO THREE DECIMAL PLACES****

1. $\cos 40°$	**2.** $\sin 50°$
3. $\csc 20°$	**4.** $\sec 70°$
5. $\cot 65°$	**6.** $\tan 25°$

- **What happened in each row when you plugged the trigonometry functions into your calculator?**
- **What is the relationship between the trigonometry functions in each row?**
- **What do you notice about the angle measures in each row?**
- **What does the prefix '*co-*' remind you of when dealing with angles? (think complementary)**

- **LEARNING GOALS:** What is the relationship between co-functions?
  - How can we use this relationship to find angles?

- ***What are the three sets of "cofunction" identities?***

In mathematics, a function f is **cofunction** of a function g if $f(A) = g(B)$ whenever A and B are complementary angles. This definition typically applies to trigonometric functions.

For example, sine and cosine are cofunctions of each other (hence the "co" in "cosine"):

Sine and **cosine** are cofunctions.	**Tangent** and **cotangent** are cofunctions.	**Secant** and **cosecant** are cofunctions.
$\sin\theta = \cos(90° - \theta)$	$\tan\theta = \cot(90° - \theta)$	$\sec\theta = \csc(90° - \theta)$
$\cos\theta = \sin(90° - \theta)$	$\cot\theta = \tan(90° - \theta)$	$\csc\theta = \sec(90° - \theta)$

- ***How do you know if you are dealing with cofunctions?***

Notice the connection of the letters **C** & **O**:

* sine and **co**sine **co**functions

* tangent and **co**tangent **co**functions

* secant and **co**secant **co**functions

* **co**mplementary

MODEL PROBLEMS

1. Find the smallest positive value of θ if $\sin\theta = \cos 15°$.	**2.** If $\csc(x + 20)° = \sec(2x + 10)°$, find x.
3. If $\cot(4x + 40)° = \sec(2x - 10)°$, find x.	**4.** State the cofunction of $\sin 63°$.

5. If $\csc(24°16') = \sec(\theta)$, find the value of θ, to the *nearest minute*.

`

6. If $\tan\left(\frac{\pi}{3}\right) = \cot(a)$, find the value of a, in radian measure.

7. If $\angle A$ is acute and $\sin A = \frac{4}{5}$, then

(1) $\cos A = \frac{4}{5}$

(2) $\cos A = \frac{1}{5}$

(3) $\cos(90° - A) = \frac{4}{5}$

(4) $\cos(90° - A) = \frac{1}{5}$

Summary/Closure

Sine and **cosine** are cofunctions. $\sin\theta = \cos(90° - \theta)$ $\cos\theta = \sin(90° - \theta)$	**Tangent** and **cotangent** are cofunctions. $\tan\theta = \cot(90° - \theta)$ $\cot\theta = \tan(90° - \theta)$	**Secant** and **cosecant** are cofunctions. $\sec\theta = \csc(90° - \theta)$ $\csc\theta = \sec(90° - \theta)$

Also written:

Sine and **cosine** are cofunctions. $\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$ $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$	**Tangent** and **cotangent** are cofunctions. $\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$ $\cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$	**Secant** and **cosecant** are cofunctions. $\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$ $\csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$

Notice the connection of the letters **C** & **O**:
* sine and **co**sine **co**functions
* tangent and **co**tangent **co**functions
* secant and **co**secant **co**functions
* **co**mplementary

Exit Ticket

If $\sin 6A = \cos 9A$, then $m\angle A$ is equal to

[A] $1\frac{1}{2}$ [B] 36 [C] 6 [D] 54

Day 1 - Homework

1. In right triangle *ABC*, *C* is the right angle and $\sin A = \frac{\sqrt{3}}{2}$. What is the value of $\csc B$?

(1) $\frac{\sqrt{2}}{3}$ (3) $\frac{1}{2}$

(2) 2 (4) $\frac{3\sqrt{3}}{2}$

2. What is the exact value, in *simplest radical form*, of $all\ 6\ trigonometric\ ratios$ if (-5, -2) is on the terminal side of θ in standard position?

3. A circle has a radius of 4 inches. In inches, what is the length of the arc intercepted by a central angle of 2 radians?

(1) 2π

(2) 2

(3) 8π

(4) 8

4. Which angle in radian measure corresponds to 300° ?

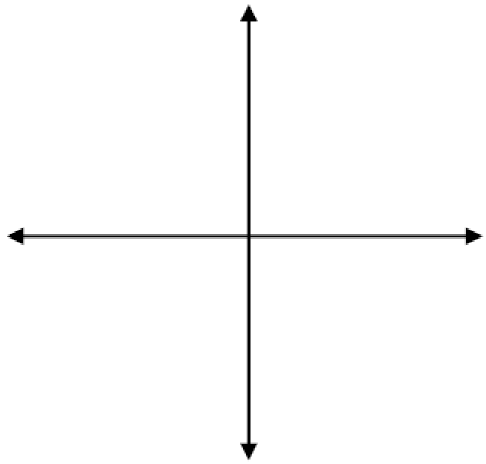
(1) $\frac{5\pi}{3}$ (3) $\frac{11\pi}{8}$

(2) 300π (4) $\frac{\pi}{3}$

5. What is the exact value, *in simplest radical form*, of $\sec 30°$?

6. What is the exact value, *in simplest radical form*, of $\cot\frac{5\pi}{3}$

7. Sketch and label θ and its reference angle in standard position if $\theta = -\frac{4\pi}{3}$. Label the reference angle with α. Find the radian value of the reference angle, α, in terms of π.



8. In which quadrant does θ lie if $\tan\theta < 0$ and $\csc\theta > 0$?

1) I

2) II

3) III

4) IV

9. If $f(x) = 2\cos^2 x + \sin x - 1$, find the value of $f\left(\frac{\pi}{2}\right)$.

10. Which value of *x* satisfies the equation $\sin(3x+5)° = \cos(4x+1)°$?

1) 30

2) 24

3) 12

4) 4

11. If $\angle A$ is acute and $\tan A = \frac{2}{3}$, then

1) $\cot A = \frac{2}{3}$

2) $\cot A = \frac{1}{3}$

3) $\cot(90° - A) = \frac{2}{3}$

4) $\cot(90° - A) = \frac{1}{3}$

12. Find the value of each function, rounded to *three decimal places*.

a. $\csc(-215°)$

b. $\sec\left(\frac{2\pi}{5}\right)$

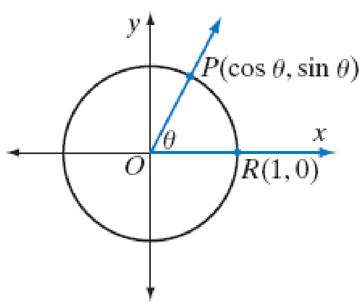
Day 2 - Pythagorean Identities

SWBAT: use Pythagorean Identities to (1) simplify trigonometric expressions
(2) find function values

<u>Warm - Up</u>

If x is a positive acute angle and $\sin x = \cos(x + 20°)$, find the value of x.

Given the unit circle with equation $x^2 + y^2 = 1$, we know x = _____ and y = ______.



Therefore, $(\cos\theta)^2 + (\sin\theta)^2 = 1$

We can write $(\cos\theta)^2$ as $\cos^2\theta$ and $(\sin\theta)^2$ as $\sin^2\theta$.

We can rewrite the above equation as $\mathbf{\cos^2\theta + \sin^2\theta = 1.}$

This equation is called an **identity**. An **identity** is an equation that is true for all values of the variable for which the terms of the variable are defined.

Specifically, the above identity is called a **Pythagorean Identity** since it is based on the Pythagorean Theorem.

Example: Verify that $\cos^2 \frac{\pi}{3} + \sin^2 \frac{\pi}{3} = 1$

Now take the Pythagorean Identity $\mathbf{\cos^2 \theta + \sin^2 \theta = 1}$

Divide it through by $\cos^2 \theta$

Divide it through by $\sin^2 \theta$

SUMMARY

Core Concept

Fundamental Trigonometric Identities

Reciprocal Identities

$\sin \theta = \frac{1}{\csc \theta}$ $\quad \cos \theta = \frac{1}{\sec \theta}$ $\quad \tan \theta = \frac{1}{\cot \theta}$

$\csc \theta = \frac{1}{\sin \theta}$ $\quad \sec \theta = \frac{1}{\cos \theta}$ $\quad \cot \theta = \frac{1}{\tan \theta}$

Tangent and Cotangent Identities

$\tan \theta = \frac{\sin \theta}{\cos \theta}$ $\quad \cot \theta = \frac{\cos \theta}{\sin \theta}$

Pythagorean Identities

$\sin^2 \theta + \cos^2 \theta = 1$ $\quad 1 + \tan^2 \theta = \sec^2 \theta$ $\quad 1 + \cot^2 \theta = \csc^2 \theta$

Cofunction Identities

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$ $\quad \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$ $\quad \tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$

$\csc\left(\frac{\pi}{2} - \theta\right) = \sec \theta$ $\quad \sec\left(\frac{\pi}{2} - \theta\right) = \csc \theta$ $\quad \cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta$

Negative Angle Identities

$\sin(-\theta) = -\sin \theta$ $\quad \cos(-\theta) = \cos \theta$ $\quad \tan(-\theta) = -\tan \theta$

$\csc(-\theta) = -\csc \theta$ $\quad \sec(-\theta) = \sec \theta$ $\quad \cot(-\theta) = -\cot \theta$

Rules of multiplication, division, addition and subtraction can be applied:

Example 2: Simplify by factoring:
$\cos^2 \theta + \cos \theta =$

Example 3: Simplify by factoring:
$1 - \sin^2 \theta =$

Example 4: Simplify $\dfrac{1-\dfrac{1}{\cos\theta}}{\dfrac{\sin\theta}{\cos\theta}}$

Example 5: Simplify (a) $\tan\left(\frac{\pi}{2} - \theta\right)\sin\theta$ and (b) $\sec\theta\tan^2\theta + \sec\theta$.

a. $\tan\left(\frac{\pi}{2} - \theta\right)\sin\theta$

b. $\sec\theta\tan^2\theta + \sec\theta$

Example 6: Express sec θ cot θ as a single function.	Example 7: Write the expression 1 + $\cot^2 \theta$ in terms of sin θ, cos θ, or both.
Example 8: Show that (1 – cos θ)(1 + cos θ) = $\sin^2 \theta$.	

Example 9: a) If cos θ = $\frac{1}{3}$ and θ is in the fourth quadrant, use an identity to find sin θ.

b) Now find:

1) tan θ 2) sec θ 3) csc θ 4) cot θ

Example 10: If $\tan A = \frac{\sqrt{7}}{3}$ and $\sin A < 0$, find $\cos A$.

Summary/Closure

Core Concept

Fundamental Trigonometric Identities

Reciprocal Identities

$\sin\theta = \frac{1}{\csc\theta}$ $\quad \cos\theta = \frac{1}{\sec\theta}$ $\quad \tan\theta = \frac{1}{\cot\theta}$

$\csc\theta = \frac{1}{\sin\theta}$ $\quad \sec\theta = \frac{1}{\cos\theta}$ $\quad \cot\theta = \frac{1}{\tan\theta}$

Tangent and Cotangent Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\quad 1 + \tan^2\theta = \sec^2\theta$ $\quad 1 + \cot^2\theta = \csc^2\theta$

Cofunction Identities

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ $\quad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$ $\quad \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$

$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$ $\quad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$ $\quad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$

Negative Angle Identities

$\sin(-\theta) = -\sin\theta$ $\quad \cos(-\theta) = \cos\theta$ $\quad \tan(-\theta) = -\tan\theta$

$\csc(-\theta) = -\csc\theta$ $\quad \sec(-\theta) = \sec\theta$ $\quad \cot(-\theta) = -\cot\theta$

Exit Ticket

The expression $\frac{\sin^2\theta + \cos^2\theta}{1 - \sin^2\theta}$ is equivalent to

(1) $\cos^2\theta$

(2) $\sin^2\theta$

(3) $\sec^2\theta$

(4) $\csc^2\theta$

Day 2 – HW

#3, 5, 6,7,8,10,11, 12, 14, 16, 21, 22, 23, 25, 26, and 28

In Exercises 3–10, find the values of the other five trigonometric functions of θ. *(See Example 1.)*

3. $\sin\theta = \frac{1}{3},\ 0 < \theta < \frac{\pi}{2}$

4. $\sin\theta = -\frac{7}{10},\ \pi < \theta < \frac{3\pi}{2}$

5. $\tan\theta = -\frac{3}{7},\ \frac{\pi}{2} < \theta < \pi$

6. $\cot\theta = -\frac{2}{5},\ \frac{\pi}{2} < \theta < \pi$

7. $\cos\theta = -\frac{5}{6},\ \pi < \theta < \frac{3\pi}{2}$

8. $\sec\theta = \frac{9}{4},\ \frac{3\pi}{2} < \theta < 2\pi$

9. $\cot\theta = -3,\ \frac{3\pi}{2} < \theta < 2\pi$

10. $\csc\theta = -\frac{5}{3},\ \pi < \theta < \frac{3\pi}{2}$

In Exercises 11–22, simplify the expression. *(See Example 2.)*

11. $\sin x \cot x$

12. $\cos\theta(1 + \tan^2\theta)$

13. $\dfrac{\sin(-\theta)}{\cos(-\theta)}$

14. $\dfrac{\cos^2 x}{\cot^2 x}$

15. $\dfrac{\sin x + \cos x}{1 - \tan(-x)}$

16. $\sin\left(\frac{\pi}{2} - \theta\right)\sec\theta$

17. $\cot(-x)\csc\left(\frac{\pi}{2} - x\right)$

18. $\cos\theta\sec(-\theta)$

19. $\dfrac{\csc^2 x - \cot^2 x}{\sin(-x)\cot x}$

20. $\dfrac{\cos^2 x\tan^2(-x) - 1}{\cos^2 x}$

21. $\dfrac{\cos\left(\frac{\pi}{2} - \theta\right)}{\csc\theta} + \cos^2\theta$

22. $\dfrac{\sec x\sin x + \cos\left(\frac{\pi}{2} - x\right)}{1 + \sec x}$

23. **ERROR ANALYSIS** Describe and correct the error in simplifying the expression.

✗
$$1 - \sin^2\theta = 1 - (1 + \cos^2\theta)$$
$$= 1 - 1 - \cos^2\theta$$
$$= -\cos^2\theta$$

24. **REASONING** Explain how you can use a graphing calculator to determine which of the six trigonometric functions is equal to $\cot x\cos x + \sin x$.

In Exercises 25–34, verify the identity. *(See Examples 3 and 4.)*

25. $\sin x\csc x = 1$

26. $\tan\theta\csc\theta\cos\theta = 1$

27. $\cos\left(\frac{\pi}{2} - x\right)\cot x = \cos x$

28. $\tan x + \tan\left(\frac{\pi}{2} - x\right) = \csc x\sec x$

29. $\dfrac{\cos\left(\frac{\pi}{2} - \theta\right) + 1}{1 - \sin(-\theta)} = 1$

30. $\dfrac{\sin^2(-x)}{\tan^2 x} = \cos^2 x$

31. $\dfrac{1 + \cos x}{\sin x} + \dfrac{\sin x}{1 + \cos x} = 2\csc x$

32. $\dfrac{\sin x}{1 - \cos(-x)} = \csc x + \cot x$

33. $\dfrac{2\sin\theta + \csc(-\theta)}{1 - \cot^2\theta} = \sin\theta$

34. $\dfrac{2\cos\theta - \sec(-\theta)}{1 - \tan^2\theta} = \cos\theta$

Day 3: Proving Identities

SWBAT prove trig identities (day 2)

Do these 27 identities on loose leaf…it will take you several sheets of paper to complete.

EXERCISES

In 1–27, prove that the given statement is an identity for all values of the angle for which the expressions are defined.

1. $\sec\theta - \sin\theta\tan\theta = \cos\theta$
2. $\tan\theta + \cot\theta = \sec\theta\csc\theta$
3. $(\sin A + 1)(\csc A - 1) = \cos A\cot A$
4. $(1 + \csc\theta)(1 - \sin\theta) = \cot\theta\cos\theta$
5. $\dfrac{\tan A + \sin A}{\csc A + \cot A} = \sin A\tan A$
6. $\sin^2 x(1 + \tan^2 x) = \tan^2 x$
7. $\dfrac{1}{\tan x - \cot x} = \dfrac{\sin x\cos x}{2\sin^2 x - 1}$
8. $\dfrac{\cos\theta + \cot\theta}{\cos\theta\cot\theta} = \tan\theta + \sec\theta$
9. $\dfrac{\sin x}{1 + \cos x} + \dfrac{1 + \cos x}{\sin x} = 2\cot x\sec x$
10. $1 + \dfrac{1}{\cos x} = \dfrac{\tan^2 x}{\sec x - 1}$
11. $\dfrac{1 + \tan^2\theta}{1 - \cos^2\theta} = \sec^2\theta\csc^2\theta$
12. $2\cos^2 x - 1 = \dfrac{1 - \tan^2 x}{1 + \tan^2 x}$
13. $\dfrac{\cos x}{\tan x} = \csc x(1 - \sin^2 x)$
14. $\dfrac{\cos\theta\sin\theta + \cos\theta}{\cos^2\theta} = \tan\theta + \sec\theta$
15. $\dfrac{\cos\theta\sin^2\theta}{1 - \cos\theta} = \cos\theta + \cos^2\theta$
16. $\dfrac{\tan\theta - \cot\theta}{\tan\theta + \cot\theta} = 2\sin^2\theta - 1$
17. $\csc x - \sin x = \dfrac{\cot x}{\sec x}$
18. $\dfrac{\tan x\csc^2 x}{1 + \tan^2 x} = \cot x$
19. $\dfrac{\sin x + \tan x}{1 + \sec x} = \sin x$
20. $\dfrac{\sin\theta\tan\theta + \cos\theta}{\cos\theta} = \sec^2\theta$
21. $\dfrac{\sin\theta\cot\theta + \cos^2\theta}{1 + \cos\theta} = \cos\theta$
22. $\dfrac{\cos\theta}{\sin\theta\tan\theta + \cos\theta} = \dfrac{1}{\sec^2\theta}$
23. $2\csc^2\theta = \dfrac{1}{1 + \cos\theta} + \dfrac{1}{1 - \cos\theta}$
24. $\cos\theta(\cos\theta + 1) + \sin^2\theta = \dfrac{\sin\theta + \tan\theta}{\tan\theta}$
25. $\dfrac{\sin x - \cos y}{\sin x + \cos y} = \dfrac{\sec y - \csc x}{\sec y + \csc x}$
26. $\dfrac{1 - \cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1 + \cos\theta}$
27. $(\tan\theta + \sec\theta)^2 = \dfrac{1 + \sin\theta}{1 - \sin\theta}$

Day 4: Sum and Difference Formulas for Sine and Cosine

SWBAT: find trigonometric function values using sum, and difference formulas

Do Now: Which of the following is an identity? Use A = 90^o and B = 60^o to test.

1. cos (A - B) = cos A - cos B

2. cos (A - B) = cos A cos B + sin A sin B

<u>Sum & Difference Identities:</u> The following formulas are used to expand trigonometric functions that have addition & subtraction in brackets.

$\sin(A + B) \neq \sin A + \sin B$, *so we must use these rules whenever we want to expand.*

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Concept 1: Sum and Difference of Angles

Example 1: Expand $\sin(60° - 45°)$

Example 2: Expand $\cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$

Example 3: Find tan (A + B) if tan A = 3 and tan B = $-\frac{1}{2}$

Concept 2: Condensing a sum or Difference

Example 1: Express $\sin 85° \cos 5° + \cos 85° \sin 5°$ as a single trigonometric expression and solve.

For each of the following, express as a single trigonometric expression and solve using the unit circle.

2) $\cos 60^\circ \cos 15^\circ + \sin 60^\circ \sin 15^\circ$

3) $\dfrac{\tan 47° - \tan 17°}{1 + \tan 47° \tan 17°}$

Concept 3: Using special Angles to rewrite a given angle

The sum & difference formulas are useful in determining the exact values of sine & cosine for angles not on the unit circle.

Example 1: Find the exact value of $\sin 15°$

First, think of how you can get 15° by using angles on the unit circle:

$15° = 60° - 45°$

$15° = 45° - 30°$

$15° = 135° - 120°$

$15° = -30° + 45°$

Example 2: Find the exact value of $\cos 105°$

Concept 4: Finding the Sum or Difference with a given Trig Ratio

Example 3:

If A and B are positive acute angles, $\sin A = \frac{5}{13}$,

and $\cos B = \frac{4}{5}$, what is the value of $\sin(A + B)$?

1) $\frac{56}{65}$

2) $\frac{63}{65}$

3) $\frac{33}{65}$

4) $-\frac{16}{65}$

Proofs

1. Prove $\sin(180 + \theta) = -\sin\theta$.

2. Prove $\cos(-\theta) = \cos\theta$.

3. $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

Summary/Closure:

1. If $\sin x = \frac{3}{5}$ and $\cos y = \frac{5}{13}$, and x and y are positive acute angles, find $\cos(x + y)$.

We know that $\sin x = \frac{3}{5}$ and x is a positive acute angle.

$\sin^2 x + \cos^2 x = 1$	Use the Pythagorean identity.
$\left(\frac{3}{5}\right)^2 + \cos^2 x = 1$	Solve for $\cos x$.
$\cos^2 x = 1 - \left(\frac{3}{5}\right)^2$	Combine like terms.
$\cos^2 x = \frac{16}{25}$	Take the square root of both sides.
$\cos x = \pm\frac{4}{5}$	Since x is a positive acute angle, choose the positive value.
$\cos x = \frac{4}{5}$	

Follow the same procedure to obtain $\sin y$.

We know that $\cos y = \frac{5}{13}$, and y is a positive acute angle.

$\sin^2 y + \cos^2 y = 1$	Use the Pythagorean identity.
$\sin^2 y + \left(\frac{5}{13}\right)^2 = 1$	Solve for $\sin y$.
$\sin^2 y = 1 - \left(\frac{5}{13}\right)^2$	Combine like terms.
$\sin^2 y = \frac{144}{169}$	Take the square root of both sides.
$\sin y = \pm\frac{12}{13}$	Since y is a positive acute angle, choose the positive value.
$\sin y = \frac{12}{13}$	

Substitute the above information into the formula.

$$\begin{aligned}\cos(x + y) &= \cos x \cos y - \sin x \sin y \\ &= \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{12}{13}\right) \\ &= \frac{20 - 36}{65} \\ &= -\frac{16}{65}\end{aligned}$$

Exit Ticket

1) The expression $\cos 70° \cos 10° + \sin 70° \sin 10°$ is equivalent to

1) $\cos 60°$
2) $\cos 80°$
3) $\sin 60°$
4) $\sin 80°$

2) $\dfrac{\tan 25° + \tan 15°}{1 - \tan 25° \tan 15°}$ is equivalent to

(1) $\tan 10°$ (2) $\tan 30°$
(3) $\tan 40°$ (4) $\cot 40°$

Day 4 - Homework

1. Find the *exact function value* of $\cos 135^o$ by using $\cos(90^o + 45^o)$.

2. Find the *exact function value* of $\cos 195^o$ by using $\cos(135^o + 60^o)$.

3. If $\cos(A - 30^o) = \frac{1}{2}$, then the measure of $\angle A$ may be

   (1) 30^o (2) 60^o
   (2) (3) 90^o (4) 120^o

4. Find tan (A – B) if tan A = $\frac{3}{4}$ and tan B = -8

5. The value of $(\cos 67^o)(\cos 23^o) - (\sin 67^o)(\sin 23^o)$ is

   (1) 1 (2) $\frac{\sqrt{2}}{2}$ (3) $-\frac{\sqrt{2}}{2}$ (4) 0

6. Find the *exact value* of $\sin 75^o$ by evaluating $\sin(45^o + 30^o)$.

7. If $\angle B$ is acute and $\sin B = \frac{12}{13}$, find the value of $\sin(90^o - B)$.

8. If sin x = $\frac{7}{25}$ and cos y = $\frac{3}{5}$, and x and y are positive acute angles, find tan (x + y).

9. The expression tan $(180° - y)$ is equivalent to

(1) tan y (2) –tan y (3) 0 (4) -1

10. If $\sin x = \frac{4}{5}, \cos y = \frac{4}{5}$, and x and y are the measures of angles in the first quadrant, find the value of $\sin(x + y)$.

11. The expression $\sin 40^o \cos 15^o + \cos 40^o \sin 15^o$ is equivalent to

(1) $\sin 55^o$ (2) $\sin 25^o$ (3) $\cos 55^o$ (4) $\cos 25^o$

12. If $\sin A = \frac{3}{5}$, $\angle A$ is in Quadrant I, $\cos B = -\frac{5}{13}$, and $\angle B$ is in Quadrant II, find $\cos(A + B)$.

13. If $\sin x = -\frac{12}{13}$, x is the measure of an angle in Quadrant III, $\cos y = -\frac{4}{5}$, and y is the measure of an angle in Quadrant II, find $\cos(x + y)$.

14. Find the *exact value* of $\cos 105^o$ by using $\cos(135^o - 30^o)$.

15. If $\sin A = -\frac{12}{13}$, $\angle A$ is in Quadrant III, $\sin B = \frac{4}{5}$, and $\angle B$ is in Quadrant II, find $\cos(A-B)$.

16. The expression $\cos 30^o \cos 12^o + \sin 30^o \sin 12^o$ is equivalent to

(1) $\cos 42^o$ (2) $\cos 18^o$
(2) (3) $\cos 42^o + \sin 42^o$ (4) $\cos^2 42^o + \sin^2 42^o$

17. The expression $\sin(\frac{\pi}{6} - x)$ is equivalent to

(1) $\frac{1}{2} - \sin x$ (2) $\frac{\sqrt{3}}{2} - \sin x$
(3) $\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$ (4) $\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$

18. If $\sin(A - 30^o) = \cos 60^o$, the number of degrees in the measure of $\angle A$ is

(1) 30 (2) 60 (3) 90 (4) 120

19. If x and y are the measures of positive acute angles, $\sin x = \frac{1}{2}$, and $\sin y = \frac{4}{5}$, then $\sin(x+y)$ equals

(1) $\frac{3+4\sqrt{3}}{10}$ (2) $\frac{3-4\sqrt{3}}{10}$ (3) $\frac{\sqrt{3}}{4} + \frac{12}{25}$ (4) $\frac{\sqrt{3}}{4} - \frac{12}{25}$

20. Find tan (A + B) if angle A is in the second quadrant, sin A = 0.6, and tan B = 4.

21. Describe and correct the error in simplifying the expression.

$$\times \quad \sin\left(x + \frac{\pi}{2}\right) = \sin x \sin \frac{\pi}{2} + \cos x \cos \frac{\pi}{2}$$
$$= (1) \sin x + (0) \cos x$$
$$= \sin x$$

22. Verify the identity $\tan(a + b) = \dfrac{\sin(a + b)}{\cos(a + b)}$ by using the angle sum formula for tangent.

23. Verify that the tangent function has a period of π by deriving the identity $\tan(x - \pi) = \tan x$ using the difference formula for tangent.

Day 5: Double Angle Identities

SWBAT: find trigonometric function values using double angle formulas

Warm - Up

- **What are the sine, cosine, and tangent ratios?**

- **If $\cos\theta = -\frac{8}{17}$ and $\sin\theta > 0$, what is the value of $\tan\theta$?**

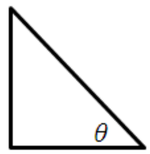
- **Name 3 sets of Pythagorean's triples?**

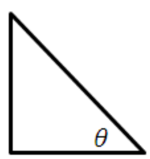
1) ___, ___, ___
2) ___, ___, ___
3) ___, ___, ___

Lesson:

What is a double-angle function? Where can you find the double-angle Identities?

Concept 1: Sine Double-Angle Identity $\sin 2A = 2 \sin A \cos A$

Model Problem
If θ is an acute angle such that $\sin\theta = \frac{5}{13}$, what is the value of $\sin 2\theta$? **Step 1: Create a right triangle**  **Step 2: Find the ratio for ______ =** **Step 3: plug into double-angle Formula**

Model Problem
If $\cos A = -\frac{1}{3}$ and $\angle A$ is in Quadrant III, express, in fractional form for $\sin 2\theta$? **Step 1: Create a right triangle**  **Step 2: Find the ratio for ______ =** **Step 3: plug into double-angle Formula**

Concept 2: Cosine Double-Angle Identity

$\cos 2A = \cos^2 A - \sin^2 A$	$\cos 2A = 1 - 2\sin^2 A$	$\cos 2A = 2\cos^2 A - 1$

Model Problem

If θ is an acute angle such that $\cos\theta = \frac{3}{4}$, what is the value of $\cos 2\theta$?

Step 1: Decide which cosine double angle formula to use

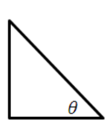
Step 2: plug into "correct" double-angle Formula

Concept 3: Tangent Double-Angle Identity $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

Model Problem

If $\cos A = \frac{4}{5}$ and $\angle A$ is in Quadrant I, find the positive value of $\tan 2A$.

Step 1: Create a right triangle



Step 2: Find the ratio for _______ =

Step 3: plug into double-angle Formula

Proofs

Prove the identity $\cos 2\theta = 2\cos^2\theta - 1$.

$\sin 2A = \tan A(2 - 2\sin^2 A)$

$$\frac{\cos(90-\theta)}{\sin 2\theta} = \frac{\sec\theta}{2}$$

$$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} = \csc x$$

SUMMARY

If $\cos\theta = \frac{7}{25}$, find $\cos 2\theta$.

SOLUTION

Since we are given $\cos\theta = \frac{7}{25}$, we will use the formula that contains cosine.

$\cos 2A = 2\cos^2 A - 1$ — Replace $\cos A$ with the value of $\cos\theta$.

$= 2\left(\frac{7}{25}\right)^2 - 1$ — Simplify.

$= 2\left(\frac{49}{625}\right) - 1$

$= \frac{98 - 625}{625}$

$= -\frac{527}{625}$

Exit Ticket

If θ is an acute angle such that $\sin\theta = \frac{5}{13}$, what is the value of $\sin 2\theta$?

1) $\frac{12}{13}$
2) $\frac{10}{26}$
3) $\frac{60}{169}$
4) $\frac{120}{169}$

<u>Day 5 - Homework</u>

1.Write the identity for sin 2x =

2. Write the identity for the $\cos 2x$ in terms of:

a. $\sin x$ and $\cos x$

b. $\cos x$ only

c. $\sin x$ only

3. Write the identity for tan 2x =

4. If $\cos A = -\frac{24}{25}$ and $\angle A$ is in Quadrant III, express, in fractional form, each value:

a. $\sin A$

b. $\cos 2A$

c. $\sin 2A$

d. tan 2A

5. If $\sin A = -\frac{3}{5}$ and $\angle A$ is in Quadrant III, find:

a. $\sin 2A$

b. $\cos 2A$

c. tan 2A

d. The quadrant in which $\angle 2A$ terminates

6. If $\cos A = \frac{1}{3}$ and $\angle A$ is acute, find

a. $\sin 2A$

b. $\cos 2A$

c. tan 2A

7. If $\cos\theta = \sin\theta$, then $\cos 2\theta$ is equivalent to

(1) 1 (2) 0 (3) $2\cos^2\theta$ (4) $2\sin^2\theta$

8. The expression $(\sin x - \cos x)^2$ is equivalent to

(1) 1 (2) $\sin^2 x - \cos^2 x$ (3) $1 - \cos 2x$ (4) $1 - \sin 2x$

9. If $\sin\theta$ is negative and $\sin 2\theta$ is positive, then $\cos\theta$

(1) Must be positive
(2) Must be negative
(3) Must be 0
(4) May be positive or negative

10. If $\tan \theta = -\frac{\sqrt{7}}{3}$ and θ is a second quadrant angle, find:

a. $\sin 2\theta$ b. $\cos 2\theta$ c. $\tan 2\theta$

11. If $\sec\theta = \frac{\sqrt{13}}{2}$ and is in the fourth quadrant, find $\tan 2\theta$.

12. If $\theta = 225°$, find $\tan 2\theta$.

13. Verify the identity $\cos 2x = \dfrac{1 - \tan^2 x}{1 + \tan^2 x}$.

14. $\dfrac{\sin 2x}{\cos 2x + \sin^2 x} = 2\tan x$

15. $\dfrac{2\sin x \cos x}{\cos^2 x - \sin^2 x} = \tan 2x$

Day 6: Half Angle Identities

SWBAT: find trigonometric function values using half angle formulas

Warm - Up

- **What are the sine, cosine, and tangent ratios?**
- **If θ is located in Quadrant II, such that $\sin\theta = \frac{24}{25}$, what is the value of $\tan\theta$?**
- **If θ is an acute angle, such that $\sin\theta = \frac{5}{13}$, what is the value of $\sin 2\theta$?**
- **If $\cos\theta = \frac{3}{5}$, what is the negative value of $\sin\frac{1}{2}\theta$?**

Lesson:

What is a half-angle function? Where can you find the half-angle Identities?

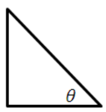
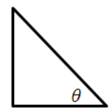
Functions of Half-Angles

Sine Half-Angle $\sin\frac{1}{2}A = \pm\sqrt{\frac{1-\cos A}{2}}$

Cosine Half-Angle $\cos\frac{1}{2}A = \pm\sqrt{\frac{1+\cos A}{2}}$

Tangent Half-Angle $\tan\frac{1}{2}A = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$

Model Problem	**Student Problem**
If $\cos\theta = \frac{1}{8}$, the positive value of $\sin\frac{\theta}{2}$ is **Step 1: plug into half-angle Formula**	**If $\cos\theta = \frac{1}{9}$, what is the negative value of $\sin\frac{1}{2}\theta$?**
If $\cos\theta = \frac{4}{5}$, what is the negative value of $\tan\frac{1}{2}\theta$?	**If $\cos\theta = \frac{5}{13}$, what is the positive value of $\tan\frac{1}{2}\theta$?**

Model Problem	Student Problem
If $\tan A = \frac{24}{7}$ and $\angle A$ is in Quadrant III, find the positive value of $\sin \frac{1}{2}A$. Step 1: Create a right triangle  Step 2: Find the ratio for _______ = Step 3: plug into half-angle Formula	If $\sin A = .6$ and $\angle A$ is in Quadrant I, find the negative value of $\cos \frac{1}{2}A$. **Step 1: Convert .6 to a fraction.** Step 2: Create a right triangle  Step 3: Find the ratio for _______ = Step 4: plug into half-angle Formula

Proof

Show that $\tan \frac{1}{2}A = \pm \frac{\sin A}{1+\cos A}$

SUMMARY

If $\cos \alpha = \frac{3}{5}$ and $\frac{3\pi}{2} < \alpha < 2\pi$, find $\cos \frac{1}{2}\alpha$.

SOLUTION

Since $\frac{3\pi}{2} < \alpha < 2\pi$, $\frac{3\pi}{4} < \frac{1}{2}\alpha < \pi$. Because $\frac{1}{2}\alpha$ lies in Quadrant II, where cosine is negative, we choose the negative value.

$$\cos \frac{1}{2}\alpha = -\sqrt{\frac{1+\cos\alpha}{2}} \quad \text{Given } \cos\alpha = \frac{3}{5}.$$

$$= -\sqrt{\frac{1+\frac{3}{5}}{2}} \quad \text{Simplify the numerator.}$$

$$= -\sqrt{\frac{\frac{8}{5}}{2}} \quad \text{Divide by 2.}$$

$$= -\sqrt{\frac{4}{5}} \quad \text{Simplify the radical.}$$

$$= -\frac{2}{\sqrt{5}} \quad \text{Rationalize the denominator.}$$

$$= -\frac{2\sqrt{5}}{5}$$

Exit Ticket

If x is a positive acute angle and $\cos x = \frac{1}{9}$, what is the value of $\cos \frac{1}{2}x$?

1) $\frac{2}{3}$

2) $\frac{1}{3}$

3) $\frac{2\sqrt{5}}{3}$

4) $\frac{\sqrt{5}}{3}$

Day 6 – Homework

1. If x is a positive acute angle and $\cos x = \frac{1}{9}$, what is the value of $\cos \frac{1}{2}x$?

(1) $\frac{2}{3}$

(2) $\frac{1}{3}$

(3) $\frac{2\sqrt{5}}{3}$

(4) $\frac{\sqrt{5}}{3}$

2. If $\cos \theta = \frac{1}{8}$, the positive value of $\sin \frac{\theta}{2}$ is

(1) $\frac{3}{2}$

(2) $\frac{\sqrt{7}}{4}$

(3) $\frac{9}{16}$

(4) $\frac{3}{4}$

3. If $\sin A = 0.8$ and angle A is a positive acute angle, find the negative value of $\sin \frac{1}{2}A$.

4. If x is an acute angle, and $\cos x = \frac{4}{5}$, then $\cos 2x$ is equal to

(1) $\frac{6}{25}$

(2) $\frac{-1}{25}$

(3) $\frac{2}{25}$

(4) $\frac{7}{25}$

5. If *x* is a positive acute angle and $\sin x = \frac{1}{2}$, what is $\sin 2x$?

(1) $-\frac{1}{2}$

(2) $\frac{1}{2}$

(3) $-\frac{\sqrt{3}}{2}$

(4) $\frac{\sqrt{3}}{2}$

6. If $\cos 72° = \sin x$, find the number of degrees in the measure of acute angle *x*.

7. Find, to the *nearest minute*, the angle whose measure is 1.35 radians.

8. Circle O has a diameter of 12 inches. What is the length of the arc intercepted by a central angle of $150°$, to the *nearest tenth* of an inch?

(1) 4.6 (3) 15.7

(2) 6.3 (4) 31.4

9. What is the measure, in radians, of a central angle formed by cutting a circular pizza into twelve equal wedge shaped pieces?

(1) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$

(2) $\frac{\pi}{5}$ (4) $\frac{\pi}{3}$

10. What is the exact value, in *simplest radical form*, of $\cot\left(-\frac{4\pi}{3}\right)$?

Day 7 – Review of Trig Concepts/Identities

DEGREES AND RADIANS

1. Express -225° in radian measure.

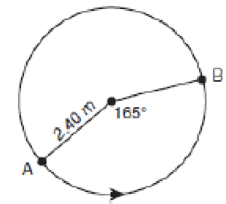
2. Find, to the *nearest minute*, the angle whose measure is 4 radians.

3. What is the radian measure of the smallest angle formed by the hands of a clock at 7:00 p.m.?

ARC LENTH ($s = r\theta$)

4. In a circle whose radius is 10, what is the length of the arc intercepted by a central angle of 4 radians?

5. The accompanying diagram shows the path of a cart traveling on a circular track of radius 2.40 meters. The cart starts at point A and stops at point B, moving in a counterclockwise direction. What is the length of minor arc AB, over which the cart traveled, to the *nearest tenth of a meter*?



RECIPROCAL TRIG FUNCTIONS

6. If the terminal side of angle θ passes through point $(3, -1)$, what is the value of all 6 trigonometric functions, in simplest form?

7. Express the exact value of $\cot 240°$, with a rational denominator.

8. Express the value of $\sec\frac{\pi}{6}$ to four decimal places.

COFUNCTIONS

9. If $\angle A$ is acute and $\csc A = .4$, then

1) $\sec A = .4$
2) $\sec A = .6$
3) $\sec(90° - A) = .4$
4) $\sec(90° - A) = .6$

10. If $\sin(2x+20)° = \cos 40°$, find x.

USING IDENTITIES TO FIND EXACT VALUES

(Pythagorean Identities, Double-Angle, Half-Angle, Sum and Difference Angles)

11. If θ is an acute angle such that $\cos\theta = \frac{12}{13}$, what is the value of $\sin 2\theta$?

12. If x is an acute angle and $\sin x = \frac{8}{17}$, then what is the value of $\cos 2x$?

13. The value of $\cos 16° \cos 164° - \sin 16° \sin 164°$ is

(1) -1
(2) $-\frac{1}{2}$
(3) 0
(4) $\frac{\sqrt{3}}{2}$

14. If $\tan A = \frac{2}{3}$ and $\sin B = \frac{5}{\sqrt{41}}$ and angles *A* and *B* are in Quadrant I, find the value of $\tan(A + B)$.

15. Express, as a single fraction in simplest form, the exact value of $\sin 105°$.

16. If x is the measure of a positive acute angle and $\cos x = \frac{7}{32}$, find the value of $\sin \frac{1}{2} x$.

17. The expression $\sqrt{\frac{1+\cos 80^o}{2}}$ is equivalent to

(1) $\frac{1}{2}\sin 80^o$ (2) $\sin 40^o$ (3) $\frac{1}{2} - \cos 40^o$ (4) $\cos 40^o$

18. If $180° < A < 270°$ and sin A = $-\frac{\sqrt{5}}{3}$, find $\tan \frac{1}{2} A$.

Identities

Verify the identities below.

19. $\dfrac{\cos^2 x + \sin^2 x}{1 + \tan^2 x} = \cos^2 x$	20. $\tan\theta + \cot\theta = \sec\theta\csc\theta$
Simplify the expression below.	
21. $\cot^2 x - \cot^2 x\cos^2 x$	22. $\dfrac{(\sec x + 1)(\sec x - 1)}{\tan x}$
23. $\sin\left(\dfrac{\pi}{2} - x\right)\tan x$	24. $\sin^2 x + \cos^2 x - b^2$