Math Analysis

Warm Up

Homework Review

Double and Half Angles

Practice

Warm Up Back

Watch out $\cot^{-1}(-4)$

☆ Not only add π to $\tan^{-1}$

but $\tan^{-1}\left(-\frac{1}{4}\right)$ not $\frac{1}{\tan^{-1}(4)}$

$\frac{\pi}{12} = 15^\circ$

$\frac{\pi}{4} = \frac{3\pi}{12}$ $\quad \frac{\pi}{6} = \frac{2\pi}{12}$

$\frac{\pi}{3} = \frac{4\pi}{12}$

Sum and Differences

p. 455 - 457

#7 F $\cancel{\tan 75 = \tan 30^\circ + \tan 45^\circ}$

$\tan 75 = \tan(30+45)$

#11 $\cos\frac{7\pi}{12} = \cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$

$\cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$\cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right) = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2}-\sqrt{6}}{4}$

15 $\tan 15^\circ = \tan(45-30)$

$\frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$

$\frac{1-\frac{\sqrt{3}}{3}}{1+\frac{\sqrt{3}}{3}} = \frac{3-\sqrt{3}}{3+\sqrt{3}}\left(\frac{3-\sqrt{3}}{3-\sqrt{3}}\right)$

$\frac{9-6\sqrt{3}+3}{9-3} = \frac{12-6\sqrt{3}}{6}$

$\tan 15^\circ = 2-\sqrt{3}$

17 $\sin\frac{17\pi}{12} = \sin\left(\frac{5\pi}{4}+\frac{\pi}{6}\right)$

$\sin\left(\frac{5\pi}{4}+\frac{\pi}{6}\right) = \sin\frac{5\pi}{4}\cos\frac{\pi}{6} + \cos\frac{5\pi}{4}\sin\frac{\pi}{6}$

$= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{-\sqrt{6}-\sqrt{2}}{4}$

19. $\sec\left(-\frac{\pi}{12}\right)$

$\frac{1}{\cos\left(-\frac{\pi}{12}\right)}$; $\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$

$\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\cos\left(\frac{\pi}{12}\right) = \frac{\sqrt{2}+\sqrt{6}}{4}$

$\sec\left(\frac{\pi}{12}\right) = \frac{4}{\sqrt{2}+\sqrt{6}}\left(\frac{\sqrt{2}-\sqrt{6}}{\sqrt{2}-\sqrt{6}}\right) = \frac{\cancel{4}(\sqrt{2}-\sqrt{6})}{\cancel{2-6}_{-4}}$

$\sec\left(-\frac{\pi}{12}\right) = \sqrt{6}-\sqrt{2}$

23. $\cos 90 = 0$

21. $\sin(20+10) = \sin 30^\circ = \frac{1}{2}$

25. $\tan(20+25) = \tan 45^\circ = 1$

27. $\sin\left(\frac{\pi}{12} - \frac{7\pi}{12}\right)$

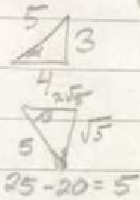
$\sin\left(\frac{-6\pi}{12}\right)$

$\sin\left(\frac{-\pi}{2}\right) = -1$

51. $\frac{\sin(\alpha+\beta)}{\sin\alpha\cos\beta} = 1 + \cot\alpha\tan\beta$

$\frac{\sin\alpha\cos\beta}{\sin\alpha\cos\beta} - \left(\frac{\cos\alpha}{\sin\alpha}\right)\left(\frac{\sin\beta}{\cos\beta}\right)$

$1 - \cot\alpha\tan\beta$



$25 - 20 = 5$

31. a) $\sin(\alpha+\beta)$

$\sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\left(\frac{3}{5}\right)\left(\frac{2\sqrt{5}}{5}\right) - \frac{4}{5}\left(\frac{\sqrt{5}}{5}\right)$

$\frac{6\sqrt{5} - 4\sqrt{5}}{25} = \frac{2\sqrt{5}}{25}$

67. 

$\sin\left(\sin^{-1}\frac{3}{5}\right)\cos\left(\cos^{-1}\left(-\frac{4}{5}\right)\right) +$

$\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right)$

$\cos\left(\sin^{-1}\frac{3}{5}\right)\sin\left(\cos^{-1}\left(-\frac{4}{5}\right)\right)$

$\left(\frac{4}{5}\right)\left(\frac{-3}{5}\right)$

$\frac{-12}{5} + \frac{-12}{5} = \frac{-24}{25}$

37. a) $\cos\theta = \frac{-2\sqrt{2}}{3}$

b) $\sin\left(\theta + \frac{\pi}{6}\right) = \sin\cos + \cos\sin$

$\left(\frac{1}{3}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{-2\sqrt{2}}{3}\right)\left(\frac{1}{2}\right)$

$\frac{\sqrt{3}}{6} - \frac{2\sqrt{2}}{6} = \frac{\sqrt{3} - 2\sqrt{2}}{6}$

77. $\cos\left(\cos^{-1}u + \sin^{-1}v\right)$

$\cos(\cos^{-1}u)\cos(\sin^{-1}v) -$

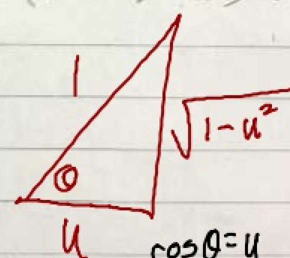
$\sin(\cos^{-1}u)\sin(\sin^{-1}v)$

$u\left(\sqrt{1-v^2}\right) - \left(\sqrt{1-u^2}\right)(v)$

c) $\cos\left(\theta - \frac{\pi}{3}\right) = \cos\cos + \sin\sin$

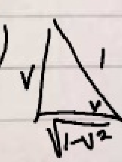
$\left(\frac{-2\sqrt{2}}{3}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{3}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\frac{-2\sqrt{2} + \sqrt{3}}{6}$



$\cos\theta = u$

$\cos^{-1}(u)$



d) $\tan\left(\theta + \frac{\pi}{4}\right) = \frac{\tan\theta + \tan\frac{\pi}{4}}{1 - \tan\theta\tan\frac{\pi}{4}}$

$\frac{\frac{1}{-2\sqrt{2}} + 1}{1 - \frac{1}{-2\sqrt{2}}} = \frac{\frac{1-2\sqrt{2}}{-2\sqrt{2}}}{\frac{-2\sqrt{2}-1}{-2\sqrt{2}}} \frac{(-2\sqrt{2}+1)}{(-2\sqrt{2}+1)}$

$\frac{-2\sqrt{2} + 8 + 1 - 2\sqrt{2}}{8 - 1} = \frac{9 - 4\sqrt{2}}{7}$

39. $\sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta$

$\sin\frac{\pi}{2}\cos\theta - \cos\frac{\pi}{2}\sin\theta$

$(1)(\cos\theta) - (0)(\sin\theta)$

$\cos\theta = \cos\theta$

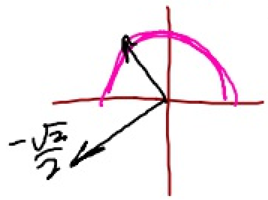
$\sin\theta = \frac{1}{3}$ $\quad \sin(\pi + \theta)$

$-\sin\theta = -\frac{1}{3}$ $\quad \cancel{\sin\pi\cos\theta} + \cos\pi\sin\theta$

$0 \quad (-1)\sin\theta$

1. $\cos^{-1}\left(\cos\frac{5\pi}{4}\right) = \frac{3\pi}{4}$

$\frac{\sqrt{2}}{2}$



2. $\frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$

$$\frac{1-2\sin\theta+\sin^2\theta}{1-\sin^2\theta}$$

$$\frac{1-2\sin\theta+\sin^2\theta}{\cos^2\theta}$$

$$\frac{1}{\cos^2\theta} - 2\frac{\sin\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}$$

$$\sec^2\theta - 2\sec\theta\tan\theta + \tan^2\theta$$

$$(\sec\theta-\tan\theta)^2$$

$$\boxed{\begin{array}{c} x^2-2xy+y^2 \\ (x-y)^2 \end{array}}$$

Name:______________________
Date:____________ Pd:_______

Use sum or difference formulas to find the exact value of:

1. $\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{4\pi}{12}+\frac{3\pi}{12}\right) = \sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$

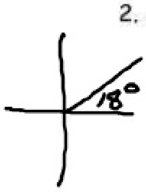
$\sin\frac{\pi}{3}\cos\frac{\pi}{4}+\cos\frac{\pi}{3}\sin\frac{\pi}{4}$

$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

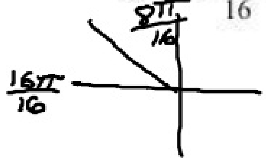
In which quadrant would the following angles lie?

2. $\theta = 18^\circ$



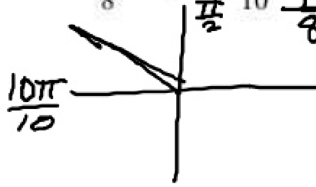
I

3. $\alpha = \frac{13\pi}{16}$



II

4. $\frac{5\pi}{8} < \beta < \frac{9\pi}{10}$ $\frac{4\pi}{8}$



II

5. 15° is half of what?

$15 = \frac{1}{2}(?)$ 30°

$2(15) = ?$

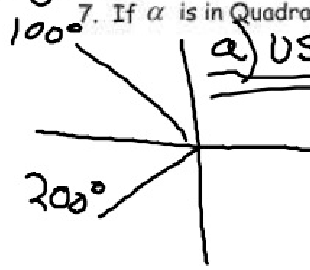
$30 = ?$

6. $\frac{5\pi}{8}$ is half of what?

$\frac{10\pi}{8} = \frac{5\pi}{4}$

7. If α is in Quadrant III then where is $\frac{\alpha}{2}$ located? Show work.

a) use a value



b) calculate

$180 < a < 270$

$\frac{\pi}{2} < \frac{a}{2} < \frac{3\pi}{2}\left(\frac{1}{2}\right)$

$\frac{\pi}{2} < \frac{a}{2} < \frac{3\pi}{4}$

Math Analysis Notes **Date _____**

6.5 – Double and Half-angle Formulas

Double Angle Formulas

Use the angle sum formulas to develop new formulas for double angles:

$\sin(2\alpha) = \sin(\alpha + \alpha) =$ $\sin\alpha\cos\alpha + \cos\alpha\sin\alpha$

$\boxed{2\sin\alpha\cos\alpha}$

$\cos(2\alpha) = \cos(\alpha + \alpha) =$ $\cos\alpha\cos\alpha - \sin\alpha\sin\alpha$

$\boxed{\cos^2\alpha - \sin^2\alpha}$

$1 - \sin^2\alpha - \sin^2\alpha$

$\boxed{1 - 2\sin^2\alpha}$

$\cos^2\alpha - (1 - \cos^2\alpha)$

$\boxed{2\cos^2\alpha - 1}$

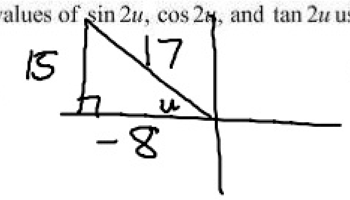
$\tan(2\alpha) = \tan(\alpha + \alpha)$

$\tan(2\alpha) = \tan(\alpha + \alpha) = \dfrac{\tan\alpha + \tan\alpha}{1 - \tan\alpha\tan\alpha} = \boxed{\dfrac{2\tan\alpha}{1 - \tan^2\alpha}}$

Example 1.] Find the **EXACT** values of $\sin 2u$, $\cos 2u$, and $\tan 2u$ using the double-angle formulas.

$\tan u = \dfrac{-15}{8}$, $\dfrac{\pi}{2} < u < \pi$.

Q2



$c^2 = 15^2 + 8^2$

$= 225 + 64$

$c^2 = 289$

$c =$

sin 2u

$2\sin u\cos u$

$2\left(\dfrac{15}{17}\right)\left(\dfrac{-8}{17}\right)$

$\boxed{\dfrac{-240}{289}}$

cos 2u

$\cos^2 u - \sin^2 u$

$\left(\dfrac{-8}{17}\right)^2 - \left(\dfrac{15}{17}\right)^2$

$\dfrac{64 - 225}{289}$

$\boxed{\dfrac{-161}{289}}$

tan 2u

$\dfrac{2\tan u}{1 - \tan^2 u}$

$\dfrac{2\left(\frac{-15}{8}\right)}{1 - \left(\frac{-15}{8}\right)^2}$

$\dfrac{\frac{-30}{8}}{\frac{64 - 225}{64}} = \dfrac{-30}{\cancel{8}} \cdot \dfrac{\overset{8}{\cancel{64}}}{-161}$

$\boxed{\dfrac{240}{161}}$

check

$\dfrac{-240}{\cancel{289}}$

$\dfrac{-161}{\cancel{289}}$

$\dfrac{240}{161}$

Example 2.] Find the value of $\sin 60°$, using the double- angle formulas.
Compare that answer with the unit circle value you have memorized.

$\sin(60°) = \sin(2 \cdot \underset{u}{30})$

$2\sin(30)\cos(30)$

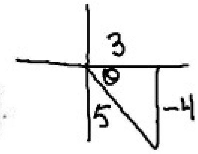
$\cancel{2}\left(\dfrac{1}{\cancel{2}}\right)\left(\dfrac{\sqrt{3}}{2}\right)$

$\boxed{\dfrac{\sqrt{3}}{2}}$

Example 3.] Find the exact value of $\cos\left(2\tan^{-1}\left(-\frac{4}{3}\right)\right)$.

θ

$\cos(2\theta)$



$\cos 2\theta$

$1-2\sin^2\theta$

$1-2\left(-\frac{4}{5}\right)^2$

$1-\frac{2(16)}{25}$

$\frac{25-32}{25}$

$\boxed{\frac{-7}{25}}$

Example 4.] Verify the identity algebraically. Use a TI-83 to confirm the identity graphically.

$y_1=\left(\frac{1}{\sin 2\theta}\right)$

$y_2=\frac{\left(\frac{1}{\sin\theta}\right)}{2\cos\theta}$

$$\csc 2\theta = \frac{\csc\theta}{2\cos\theta}$$

$\frac{1}{\sin 2\theta}$

$\frac{1}{2\sin\theta\cos\theta}$

$\frac{1}{2\cos\theta}\cdot\frac{1}{\sin\theta}$

$\boxed{\frac{\csc\theta}{2\cos\theta}=\frac{\csc\theta}{2\cos\theta}}$

Half-Angle Formulas

$\boxed{\sin\frac{u}{2}} = \pm\sqrt{\frac{1-\cos u}{2}}$ $\qquad$ $\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$

(** The **signs** of $\sin\frac{u}{2}$ and $\cos\frac{u}{2}$ depend on the quadrant in which $\frac{u}{2}$ lies.)

$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$

Example 5.] Use the half-angle formulas to find $\cos 15°$.

$\cos(15°) = \cos\left(\frac{30}{2}\right)$

$\frac{u}{2}$ $\qquad$ $u = 30$

$+\sqrt{\frac{1+\cos 30}{2}}$

$\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$

$\sqrt{\frac{2+\sqrt{3}}{2}\cdot\frac{1}{2}}$

$\sqrt{\frac{2+\sqrt{3}}{4}}$

$\frac{\sqrt{2+\sqrt{3}}}{\sqrt{4}}$

$\frac{\sqrt{2+\sqrt{3}}}{2}$

Example 6.] Use the half-angle formula to find the exact value of sin75°.
Compare this value to the answer from our notes in section 6.4.

$\sin 75^\circ$
$\sin(30+45)$
$\dfrac{\sqrt{2}+\sqrt{6}}{4}$

$\sin(75) = \sin\left(\frac{150}{2}\right)$ $\quad \frac{u}{2}$

$+\sqrt{\dfrac{1-\cos 150}{2}}$

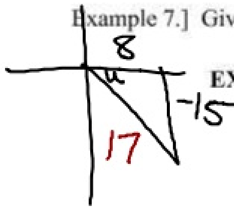
$\sqrt{\dfrac{1-\left(\frac{-\sqrt{3}}{2}\right)}{2}}$

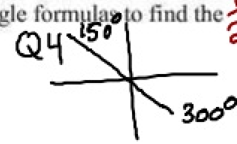
$\sqrt{\dfrac{\frac{2+\sqrt{3}}{2}}{2}}$

$\sqrt{\dfrac{2+\sqrt{3}}{4}}$

$\dfrac{\sqrt{2+\sqrt{3}}}{2}$

Example 7.] Given that $\tan u = \dfrac{-15}{8}$, $\dfrac{3\pi}{2} < u < 2\pi$, use the half-angle formulas to find the EXACT values of $\sin\dfrac{u}{2}$, $\cos\dfrac{u}{2}$, and $\tan\dfrac{u}{2}$.





$\frac{1}{2}\left(\frac{3\pi}{2} < u < 2\pi\right)$

$\frac{3\pi}{4} < \frac{u}{2} < \pi$

$\sin\left(\frac{u}{2}\right)$

$+\sqrt{\dfrac{1-\cos u}{2}}$

$\sqrt{\dfrac{1-\frac{8}{17}}{2}}$

$\sqrt{\dfrac{\frac{17-8}{17}}{2}}$

$\sqrt{\frac{9}{17}\cdot\frac{1}{2}}$

$\dfrac{\sqrt{9}}{\sqrt{34}}$

$\dfrac{3}{\sqrt{34}} = \boxed{\dfrac{3\sqrt{34}}{34}}$

$\cos\left(\frac{u}{2}\right)$

$-\sqrt{\dfrac{1+\cos u}{2}}$

$-\sqrt{\dfrac{1+\frac{8}{17}}{2}}$

$-\sqrt{\dfrac{\frac{17+8}{17}}{2}}$

$-\sqrt{\dfrac{25}{34}}$

$\dfrac{-5}{\sqrt{34}}$

$\boxed{\dfrac{-5\sqrt{34}}{34}}$

$\tan\left(\frac{u}{2}\right)$

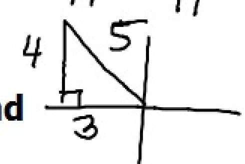
$\dfrac{1-\cos u}{\sin u}$

$\dfrac{1-\cos u}{\sin u}$

$\dfrac{1-\frac{8}{17}}{\frac{-15}{17}}$ $\boxed{\dfrac{-3}{5}}$

$\dfrac{\frac{17-8}{17}}{\frac{-15}{17}} = \dfrac{\frac{9}{17}}{\frac{-15}{17}} = \dfrac{-9}{15}$

QUIZ NEXT CLASS: Blk 3 sin (pi/2 -0)
Review Sum and Diff formulas (memorize)
Review Trig Identities (Practice) & Know
Review Trig Inverse (know the unit circle and right triangle trig)



Homework: pp. 455 – 457: 7, 11, 17, 19, 21, 25, 30(use angle sum formulas, 3x = 2x +1x), 35, 41, 49, 57, 59, 61, 79

Worksheet

1. $\cos\left(\frac{11\pi}{12}\right)$

   $\cos\left(\frac{1}{2}\cdot\frac{22\pi}{12}\right)$

   $u=\frac{11\pi}{6}$

2. $\tan(157.5^\circ)$

   $\left(\frac{1}{2}\cdot 315\right)$

   $u=315^\circ$

3. $\sin\left(\frac{5\pi}{12}\right)$

   $\left(\frac{1}{2}\cdot\frac{10\pi}{12}\right)$

   $u=\frac{5\pi}{6}$

4. $\sec(112.5^\circ)$

   $\sec\left(\frac{1}{2}(225^\circ)\right)$