Finding the Mean and Standard Deviation by hand

$$Mean = \bar{x} = \frac{\sum x_i}{n}$$

$$Variance = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1} = s^2 \text{ or } \sigma^2 \text{ (Sample or population)}$$

$$Standard\ Deviation = \sqrt{\frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1}} = \sqrt{Variance} = s \text{ or } \sigma \text{ (Sample or population)}$$

Example: The heights in inches of 6 members of a family are below. Find the mean, variance, and the standard deviation of the heights:

Heights (in inches) – 70, 46, 57, 66, 68, 53

$$Mean = \frac{70 + 46 + 57 + 66 + 68 + 53}{6} = 60$$

$$\sum x_i^2 = (70^2 + 46^2 + 57^2 + 66^2 + 68^2 + 53^2) = 22054$$

$$\left(\sum x_i\right)^2 = (70 + 46 + 57 + 66 + 68 + 53)^2 = 129600$$

$$Variance = \frac{22054 - \frac{129600}{6}}{5} = 90.8$$

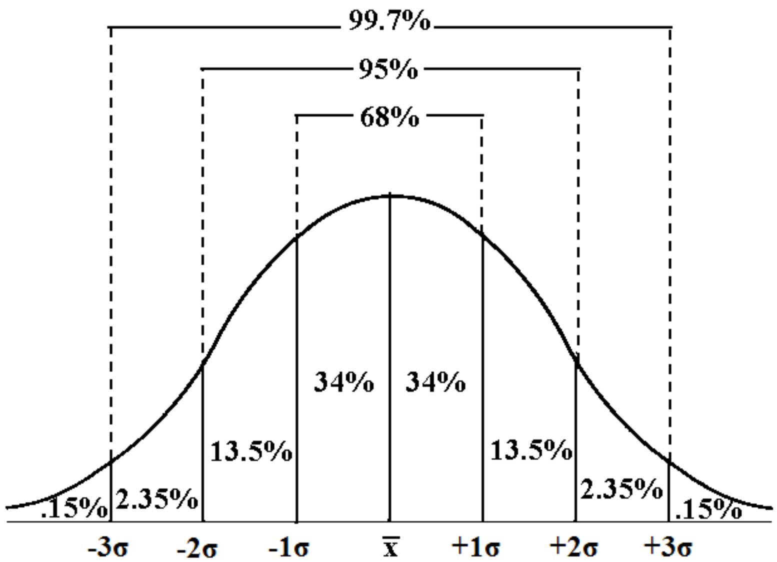
$$Standard\ Deviation = \sqrt{90.8} = 9.53$$

Chebyshev's Inequality

The percent of observations within k standard deviations of the mean is represented by the expression $\left(1 - \frac{1}{k^2}\right) \cdot 100\%$

Empirical Rule

68%-95%-99.7% -> Percent of observations we predict that will be between one, two, and three standard deviations from the mean



$$\frac{68\%}{2} = 34\%$$

$$\frac{95\% - 68\%}{2} = 13.5\%$$

$$\frac{99.7\% - 95\%}{2} = 2.35\%$$

$$\frac{100\% - 99.7\%}{2} = 0.15\%$$

Using Frequency to calculate the mean and standard deviation

* x_i is the midpoint of each level, and f_i is the frequency of each level

$$Mean = \bar{x} = \frac{\sum x_i f_i}{n}$$

$$Variance = \frac{\sum x_i^2 f_i - \frac{(\sum x_i f_i)^2}{n}}{\sum f_i - 1}$$

$$Standard\ Deviation = \sqrt{\frac{\sum x_i^2 f_i - \frac{(\sum x_i f_i)^2}{n}}{\sum f_i - 1}} = \sqrt{Variance}$$

The table below shows the ages of 1000 golfers who played golf this week at the local golf course. Use the table to find the mean, variance, and standard deviation of the golfers who played this past week:

Age	**Frequency**
0-9	15
10-19	75
20-29	107
30-39	165
40-49	255
50-59	243
60-69	127
70-79	13

To be able to solve this, we expand the table for our calculations:

Age	**Midpoint, x_i**	**Frequency, f_i**	**$x_i f_i$**	**x_i^2**	**$x_i^2 f_i$**
0-9	5	15	75	25	375
10-19	15	75	1125	225	16875
20-29	25	107	2675	625	66875
30-39	35	165	5775	1225	202125
40-49	45	255	11475	2025	516375
50-59	55	243	13365	3025	735075
60-69	65	127	8255	4225	536575
70-79	75	13	975	5625	73125

$\sum f_i = 1000, \ \sum x_i f_i = 43720, \ \sum x_i^2 f_i = 2147400$

$$Mean = \frac{43720}{1000} = 43.72$$

$$Variance = \frac{2147400 - \frac{43720^2}{1000}}{1000 - 1} = 236.198$$

$$Standard\ Deviation = \sqrt{236.198} = 15.37$$