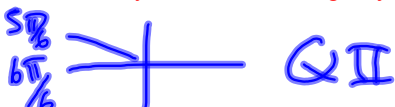


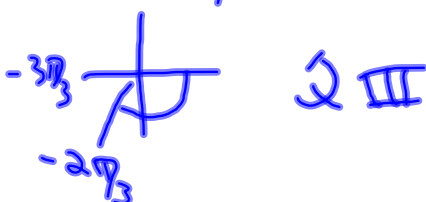
Thinking in radians

Name the quadrant and the reference angle for each angle below:

Try it without looking at your unit circle.

1. $\theta = \frac{5\pi}{6}$  Q II

2. $\theta = \frac{7\pi}{4}$  Q IV

3. $\theta = -\frac{2\pi}{3}$  Q III

Convert radians $\rightarrow$ degrees multiply by $\frac{180^\circ}{\pi}$

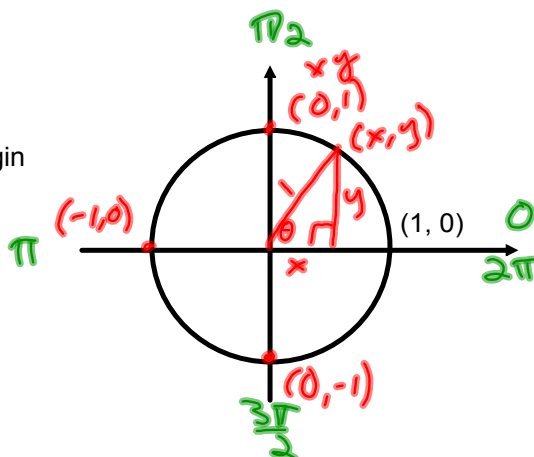
degrees $\rightarrow$ radians multiply by $\frac{\pi}{180^\circ}$

Practice: 45° to radians

$$\frac{45^\circ}{1} \cdot \frac{\pi}{180} = \frac{45\pi}{180} = \boxed{\frac{\pi}{4}}$$

Section 3.3 - Circular Functions

"unit circle" - circle of radius 1 centered at the origin



What is the equation for the unit circle?

$$(x-0)^2 + (y-0)^2 = 1^2$$

$$\boxed{x^2 + y^2 = 1}$$

Label the angles (in radians) where the axes intersect the unit circle.

We can define the trigonometric functions using the unit circle, which will be equivalent to our original definitions if $r = 1$ and (x, y) is a point on the unit circle.

$$\sin \theta = \frac{y}{1} = y \qquad \csc \theta = \frac{1}{y}$$

$$\cos \theta = \frac{x}{1} = x \qquad \sec \theta = \frac{1}{x}$$

$$\tan \theta = \frac{y}{x} \qquad \cot \theta = \frac{x}{y}$$

Based on the unit circle, $\sin \theta = y$ and $\cos \theta = x$.

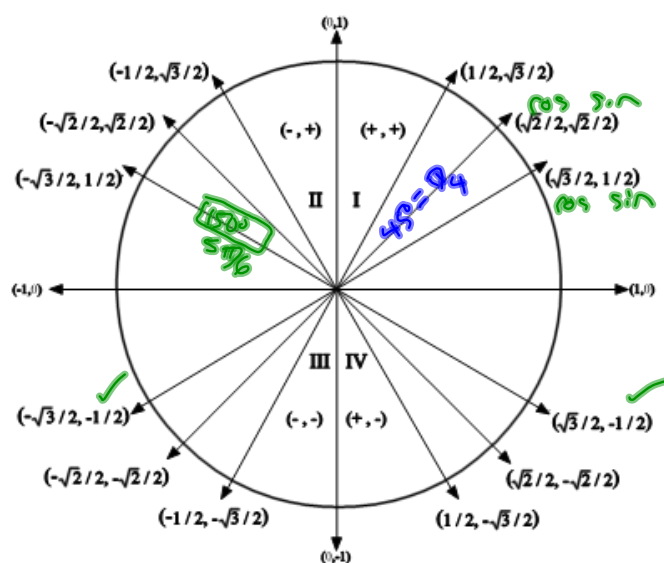
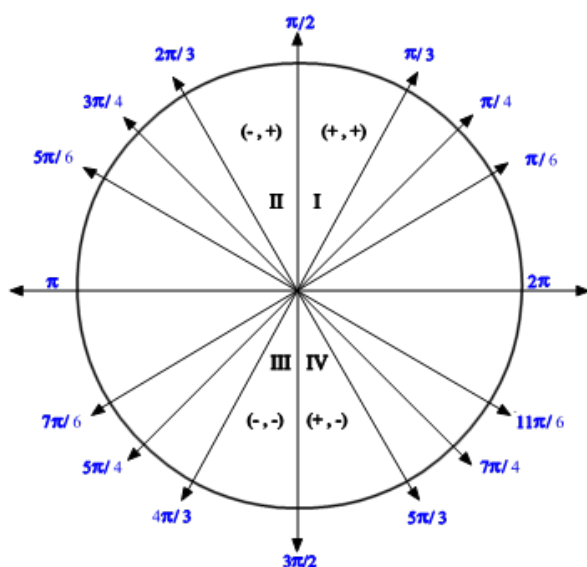
Find the all six functions of $\theta = 0, \pi/2, \pi, 3\pi/2$ and 2π .

$$\begin{array}{ll} \sin 0 = 0 & \csc 0 = \text{undef} \\ \cos 0 = 1 & \sec 0 = 1 \\ \tan 0 = \frac{0}{1} = 0 & \cot 0 = \text{undef} \end{array}$$

$\theta = \pi/2$ point $(0,1)$ on unit circle
 $x \quad y$

$$\begin{array}{ll} \sin \pi/2 = 1 & \csc \pi/2 = 1 \\ \cos \pi/2 = 0 & \sec \pi/2 = \text{undef} \\ \tan \pi/2 = \text{undef} & \cot \pi/2 = 0 \end{array}$$

Fill in the values of the points on the unit circle corresponding to these angles.



Use the unit circle to find all values of θ such that

a. $\cos \theta = 0$

b. $\sin \theta = -1/2$

$\theta = 90^\circ, 270^\circ$
 $\frac{\pi}{2}, \frac{3\pi}{2}$

$\theta = 210^\circ, 330^\circ$
 $\frac{7\pi}{6}, \frac{11\pi}{6}$

Because on a unit circle, $r = 1$, we can redefine the trigonometric functions as circular functions defined on the unit circle.

Circular function definitions:

$$\sin \theta = y$$

$$\cos \theta = x$$

$$\tan \theta = \frac{y}{x}$$

$$\csc \theta = \frac{1}{y}$$

$$\sec \theta = \frac{1}{x}$$

$$\cot \theta = \frac{x}{y}$$

Use the circular function definitions to find the six trigonometric functions of the angle whose terminal side intersects the unit circle at $(0.5403, 0.8415)$.

$$\sin \theta = .8415$$

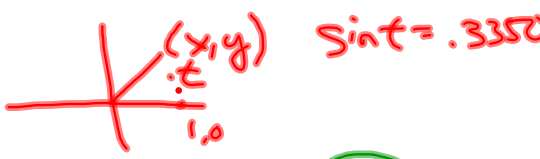
$$\cos \theta = .5403$$

$$\tan \theta = \frac{.8415}{.5403} \approx 1.5575$$

x y

Section 3.3 #48

← unit circle
If t is the distance from $(1, 0)$ to $(-0.9422, 0.3350)$ along the circumference of the unit circle, find $\sin t$, $\cos t$ and $\tan t$.



$\sin t = .3350$

$$\tan t = \frac{.3350}{-.9422} \approx \frac{y}{x}$$

The argument of $\sin(2x)$ is $2x$ and the argument of $f(x+1)$ is $x+1$.

Find the argument of each of the following functions.

$$\ln(2x-5)$$

$$\tan(t + \pi)$$

$$\cos(2B + C)$$

$$2x-5$$

$$t + \pi$$

$$2B+C$$

We have defined circular functions. These are functions of real numbers -- which, right now we interpret as an angle given in radians.

If they are functions of real numbers, what are their domains and ranges?

function	domain	range
$\sin t = \frac{y}{r}$	$(-\infty, \infty)$	$[-1, 1]$
$\cos t = \frac{x}{r}$	$(-\infty, \infty)$	$[-1, 1]$
$\tan t = \frac{y}{x}$	all real #s except $\frac{\pi}{2} + k\pi$	$(-\infty, \infty)$
$\csc t = \frac{1}{\sin t}$	all real #s except $k\pi$	$(-\infty, -1] \cup [1, \infty)$
$\sec t = \frac{1}{\cos t}$	all real #s except $\frac{\pi}{2} + k\pi$	$(-\infty, -1] \cup [1, \infty)$
$\cot t = \frac{x}{y}$	all real #s except $k\pi$	$(-\infty, \infty)$

k is any integer

