CHAPTER 9

SIGNAL CONDITIONING

In this chapter the term signal conditioning represents the enormous variety of systems used to "condition" a sensor's output, or "signal," so that it can effectly serve as an input to a computer-controlled data acquistion system. In short, system herein referred to as a "signal conditioning" system fills the function supplied by the darker shaded block in the diagram below.

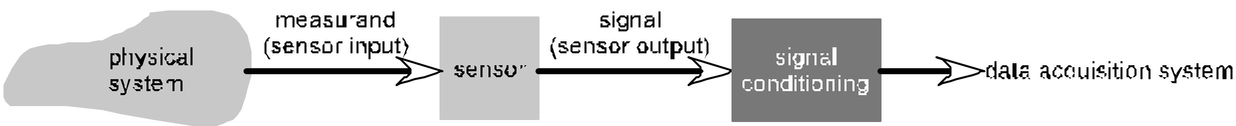


Figure 1: Overview of Measurement System

The desired output from the signal conditioning system, which is the input to the data acquisition system, is often a voltage with a range of a few volts—1 to 5 volts is a common range. A data acquisition system often includes sampling and analog-to-digital conversion. At this stage, the physical quantity being measured is in a digital format fit to be read by digital computer. Once read by a computer system, a variety of means are available to

What type of systems could be included in the "signal conditioning" block?

Suppose the sensor output is too small a voltage. In this case, the signal conditioning block would need to include amplification. For a sensor output that is too large a voltage, attentuation would become part of the signal conditioning block.

If the sensor output were in the form of an electric current, the signal conditioning block would need to include a conversion from current to voltage.

For a resistive sensor—one in which a change in resistance (ΔR) is the sensor's output, the signal conditioning (SC) block would include a ΔR to ΔV conversion. Similarly, when using a capacitive sensor, the SC block would include a ΔC to ΔV conversion, and, for an inductive sensor, a ΔL to ΔV conversion would be needed.

The output of the signal conditioning system must be able to supply the current required of the data acquisition system while, at the same time, maintaining the required voltage. That is, is must be able to supply the needed power and not have too high a Thevenin resistance. Such a situation would arise, for example, when using a piezoelectric accelerometer, in which case signal conditioning would include buffering.

A wide-variety of signal processing can be included in the signal processing block. Filtering is the type treated in this chapter and is important in ensuring that unwanted noise is not present in the signal input to the data acquisition system.

9.1 Introduction

Some type of conditioning is often required for the signals output from sensors. When the output is too small, amplification may be necessary. The presence of interfering noise may require filtering.

Frequently the form of the signal must be altered. A strain gage provides, as an output signal, a change in resistance, ΔR. This signal may need to be changed into a change of voltage.

These are examples of signal conditioning which may be required to interface between the raw sensor signals and data acquisition systems.

9.2 Amplifier Circuit Models

Amplifiers are modeled with the aid of dependent sources. Fig. 9.1 shows a voltage amplifier which amplifies a voltage from a non-ideal source (the amplifier input) and supplies a voltage (the amplifier output) to a load modeled as a resistance.

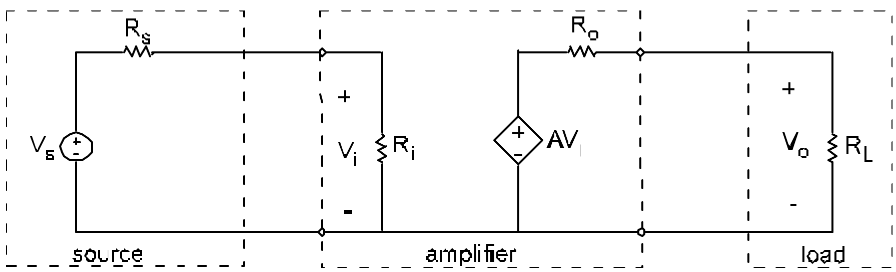


Figure 2: Voltage-Voltage Amplifier

The circuit can be analyzed by employing voltage division twice, once for the input circuit (the left loop) and once for the output circuit (the right loop).

$$V_i = \frac{R_i}{R_i + R_s} V_s \qquad\qquad V_o = \frac{R_L}{R_L + R_o} AV_i$$

Eliminating V_i,

$$V_o = \frac{R_L}{R_L + R_o} \frac{R_i}{R_i + R_s} AV_s$$

The overall gain can be found by taking the ratio of the output voltage to the input voltage.

$$\text{Overall Gain} = \frac{V_o}{V_s} = \frac{R_L}{R_L + R_o} \frac{R_i}{R_i + R_s} A$$

How might the amplifier be designed so that the overall gain is maximized for a given A? As one can see from the ratios above, if the amplifier input resistance, R_i, is much larger than the source resistance, R_s, and if the amplifier output resistance, R_o, is smaller than the load resistance, R_L, the overall gain is approximately equal to A.

$$\left.\frac{V_o}{V_s}\right|_{\substack{R_o \ll R_L \\ R_i \gg R_s}} \cong A$$

One can design amplifiers in which both the input and output are currents.

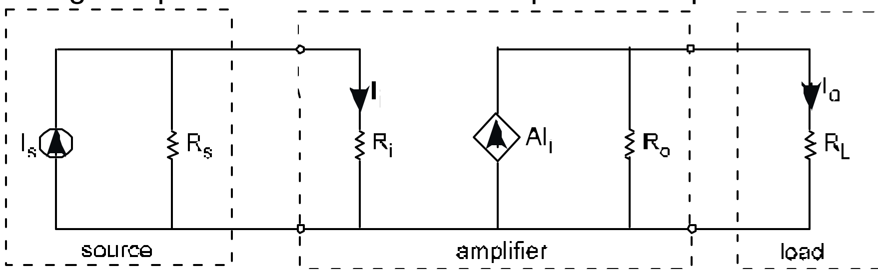


Figure 9.3: Current-Current Amplifier

The model in Fig. 9.2 can be analyzed using current division in both the input circuit and the output circuit.

$$I_i = \frac{R_s}{R_i + R_s} I_s \qquad\qquad I_o = \frac{R_o}{R_L + R_o} A I_i$$

Eliminating I_i,

$$I_o = \frac{R_o}{R_L + R_o} \frac{R_s}{R_i + R_s} A I_s$$

The overall gain is

$$\frac{I_o}{I_s} = \frac{R_o}{R_L + R_o} \frac{R_s}{R_i + R_s} A$$

How might the amplifier be designed so that the overall gain is maximized for a given A? For this case, the amplifier should have a low input resistance and a high output resistance.

9.3 Operational Amplifiers

9.3.1 *General*

Many of the building blocks used in circuits can be designed using operational amplifiers. An operational amplifier is modeled as a voltage amplifier.

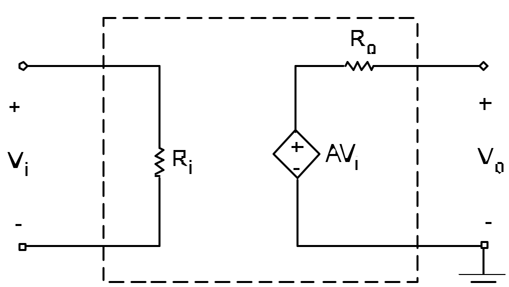


Figure 9.4: Op-Amp Circuit Model

Op-amps are realized as an integrated circuit (IC). The op-amp must be powered by an external power supply consisting of two voltages, one positive with respect to ground, and the other negative with respect to ground. Voltages are commonly between 9 to 18 volts.

The amplifier within in Fig. 9.3 is most often symbolized by a triangle. Fig. 9.4 shows the op-amp as a triangle together with its external power supplies.

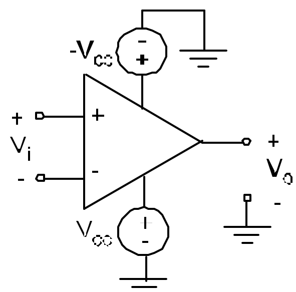


Figure 9.5: Op-Amp Circuit Symbol with External Supplies

Most often, the op-amp is drawn with the external power supplies implied. That is, they are often not explicitly shown but rather are assumed present.

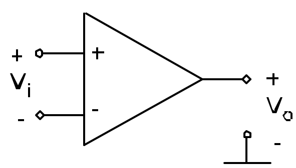


Figure 9.6: Op-Amp Circuit Symbol with Implied External Supplies

The external power supplies have current flowing in them, and, to account for this current, a line is often drawn from ground to the operational amplifier as shown in Fig. 9.6. The left representation, less used, emphasizes that there is a current path for output current to flow. Below the representation on the right will be used, but the reader should keep in mind the existence of this current path through the implied supply voltages.

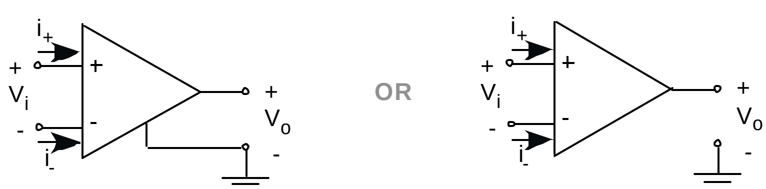
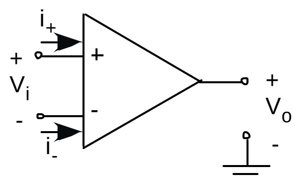


Figure 9.7: Op-Amp Circuit Symbol

9.3.2 *Ideal op-amp*

In this chapter, the ideal op-amp model will be used. The input resistance of an op-amp is very high with bipolar op-amps having input resistances above 10^6 MΩ and FET op-amps having input resistances around 10^{12} Ω. The output resistance of op-amps with feedback is quite low and is usually neglected (assumed to be zero). The voltage gain (A in Fig. 9.3) is very large, greater than 10^5 at DC. The voltages involved in op-amp circuits are small—typically less than ±18V.

What are the implications? Consider V_o to be 10 V, A to be 10^5, and R_i to be $10^7\ \Omega$. In this case, V_i would be 10 μV and the current input to the positive or negative op-amp inputs would be 1 pA. In most circuits, these small voltages and currents can safely be assumed negligible, and, in the *ideal op-amp* model, they are assumed zero. The resulting V_o is very close to the actual value (notice that, since $V_i = 0$, this requires A to be infinite). In the ideal op-amp model, R_o is assumed to be zero. This typically introduces very little error since the other resistances in op-amp circuits are typically greater than 2 KΩ.



$V_i = 0$

$i_+ = i_- = 0$ (or equivalently, $R_i = \infty$)

$R_o = 0$

Figure 9.8: Op-Amp Circuit Symbol

Note: The maximum possible output voltage (V_0) for the amplifier is limited by the supply voltages used. The maximum output voltage is typically about 1.5 V less than the supply voltages used. For example, if a ±15V power supply were used to power an op-amp chip, the maximum possible output voltage would be ±13.5 volts. The amplifier would amplify only so long as V_o remained between ±13.5 volts.

For larger inputs the amplifier output would remain at ±13.5 V. At this point, the amplifier is said to be saturated and the normal input-output relation does not hold.

There are literally hundreds of operational amplifiers available commercially, from those for application in high-speed systems to those intended for low-power power applications to those able to supply high power. In this discussion, it will be assumed a general purpose op-amp is being discussed.

Example 1

The ideal op-amp is a very versatile amplifier! The gain of the amplifier can be changed by merely adjusting the value of external resistors. The configuration shown in Fig. 9.9 a standard configuration, the inverting configuration.

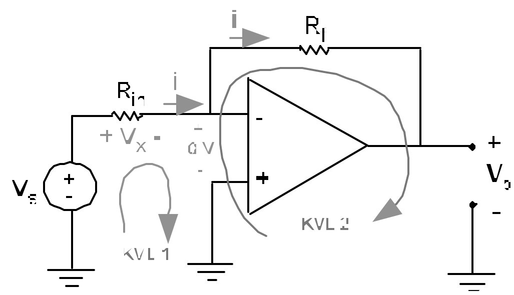


Figure 9.9: Inverting Amplifier I

Beginning the analysis with KVL 1, one obtains:

$$-V_s + V_x + 0 = 0 \quad \rightarrow \quad V_x = V_s \quad \rightarrow \quad i = V_s/R_{in}$$

Since the current into the negative op-amp input is zero (for the ideal op-amp model), the current through R_f is the same as that through R_{in}. Given this result from KCL, one use KVL about loop 2 (KVL 2).

$$-0V + iR_f + V_o = 0 \quad \rightarrow \quad V_o = -iR_f = -\frac{R_f}{R_{in}}V_s$$

The overall gain, H, is:

$$H = \frac{V_o}{V_s} = -\frac{R_f}{R_{in}}$$

Notice that the gain of the amplifier is controlled by the choice of R_f and R_{in}.

Note: For practical op-amp circuits, using general purpose op-amps, choose resistors so that the resistance seen by the op-amp output is greater than 1 KΩ. Resistors having resistances below this value can result in the op-amp being unable to supply the required current for proper circuit operation.

The fact that the performance of an op-amp circuit is determined by the external components (at least as long as the ideal op-amp model remains valid) makes the op-amp a very versatile tool for the designer and is a primary reason op-amp circuits are so widely used.

It is often good to look at a problem from more than one point-of-view. With this thought in mind, consider the same circuit, this time using nodal analysis.

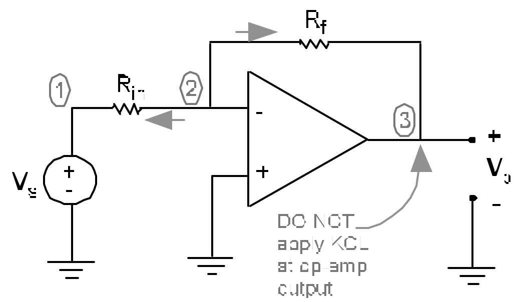


Figure 9.10: Inverting Amplifier II

The one special rule to remember when applying nodal analysis in circuits with ideal op-amps is that one may not write a KCL equation at the output of an op-amp due to the current paths associated with the external power supplies, which are not included in the ideal op-amp model. By keeping this in mind, nodal analysis can be an effective analysis technique for op-amp circuit. The equations read:

$V_1 = V_s$ (voltage source equation)

$V_2 = 0$ (op-amp model, the voltage difference across inputs is 0 V)

$V_3 = V_o$ (labeling, often the output would not be given a separate node voltage, V_o would be used from the start)

One KCL equation at node 2.

$$\frac{V_2 - V_1}{R_{in}} + \frac{V_2 - V_3}{R_f} = \frac{0 - V_s}{R_{in}} + \frac{0 - V_o}{R_f} = 0$$

$$\frac{V_o}{V_s} = -\frac{R_f}{R_{in}}$$

Example 2

Consider an op-amp amplifier powered by an external ±15 V power supply, and suppose the amplifier has been designed for an *overall gain* of -10. Since a ±15 V supply is used, the output is limited to ±13.5 V. Since the overall gain is -10, when the input is outside the range ±1.35 V, the output will enter saturation. This is illustrated in Fig. 9.11.

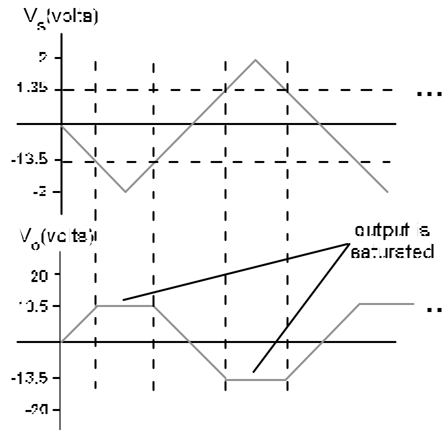


Figure 9.11: Input-Output Curves for an Amplifier Showing Output Saturation

9.4 Amplifiers and Buffers

There are a variety of amplifying circuits that can be obtained using one or more op-amps. The characteristics a few popular configurations will be explored.

9.4.1 *Inverting Configuration*

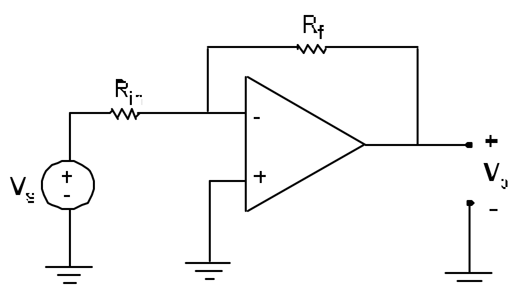


Figure 9.12: Inverting Amplifier

Important characteristics for the amplifier are its gain and its input resistance.

$$\text{Gain} = H = \frac{V_o}{V_s} = -\frac{R_f}{R_{in}} \qquad \text{Input Resistance} = \frac{V_s}{i_{in}} = R_{in}$$

1. Beginning with its gain, the first item to notice is that the gain is negative, which gives the amplifier its name, the inverting amplifier.
2. Next is the input resistance as seen by V_s which in this case is just R_{in}.

Note: In this case, the source is assumed to be an ideal voltage source. If it was not and its Thevenin resistance could not be neglected with respect to R_{in}, the voltage gain of the amplifier would be $H = -R_f/(R_s + R_{in})$ as shown in Fig. 9.13.

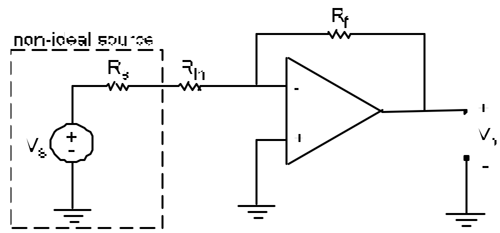


Figure 9.13: Inverting Amplifier with Non-Ideal Source

This could create a problem from two standpoints. First, R_S might not be accurately known, or might change with frequency or temperature, for example. Second, V_S might not be able to supply sufficient current at the input of the op-amp circuit.

3. Although most applications require the magnitude of the gain be larger than 1, it is possible with the inverting configuration to have an amplifier which attenuates the input. All that is required is to choose $R_f < R_{in}$.

9.4.2 *Non-Inverting Configuration*

The non-inverting amplifier is another canonical configuration in op-amp circuits.

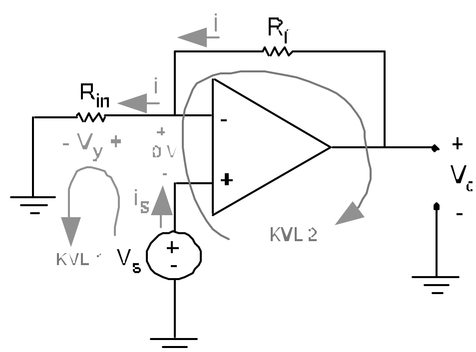


Figure 9.14: Non-Inverting Amplifier

From KVL 1, one obtains $V_y = V_S$. Next, by Ohm's law, $i = V_S/R_{in}$. Then, by KVL 2,

$$-V_s - R_f i + V_o = 0 \qquad \rightarrow \qquad -V_s - R_f \frac{V_s}{R_{in}} + V_o = 0$$

Taking the ratio of V_o/V_S, the amplifier gain can be obtained. Important characteristics for the amplifier are its gain and its input resistance.

$$\text{Gain} = H = \frac{V_o}{V_s} = 1 + \frac{R_f}{R_{in}} \qquad \text{Input Resistance} = \frac{V_s}{i_s} \cong 10^{12}\ \Omega$$

1. The gain is positive and, unlike the inverting amplifier, its magnitude cannot be smaller than one.

2. The input resistance is that of the op-amp input and so is very large. This would be desirable, for example if the source, V_S, could only source a very small current.

9.4.3 *Differential Amplifier*

The differential amplifier is designed to amplify the difference between two voltages.

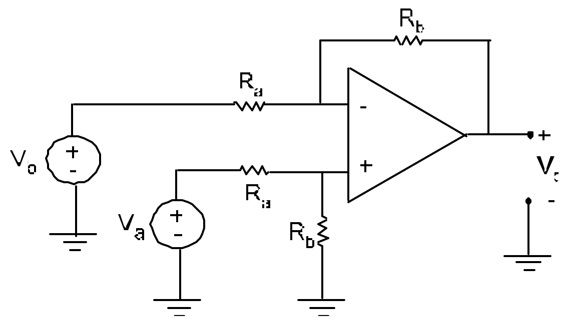


Figure 9.15: Differential Amplifier

The circuit will be analyzed in two ways. First, superposition (that is, V_a and V_b will be turned on in turn) will be used to find V_o in terms of V_a and V_b.

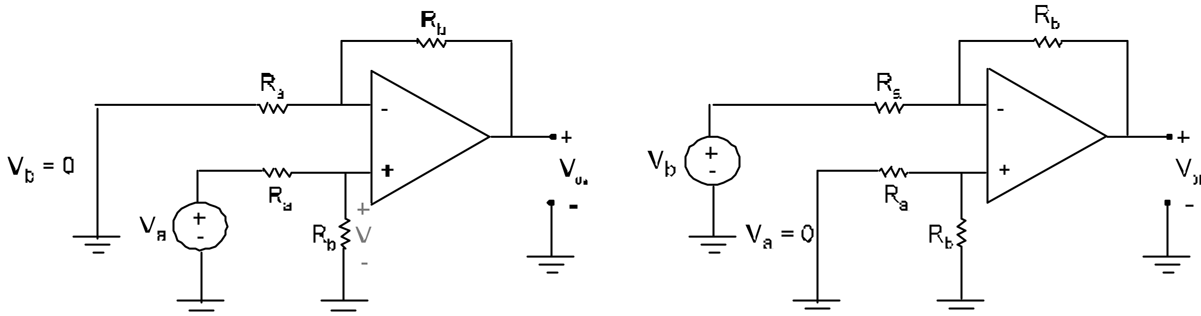


Figure 9.16: Analyzing the Differential Amplifier using Superposition

The circuit on the left in Fig. 9.16 shows the differential amplifier with $V_b = 0$. This circuit is the non-inverting amplifier with the voltage, V, at the positive input (this voltage can be obtained, for example, with voltage division).

$$V_{oa} = \left(1 + \frac{R_b}{R_a}\right)V = \left(1 + \frac{R_b}{R_a}\right)\frac{R_b}{R_a + R_b}V_a$$

The circuit on the right in Fig. 9.14 shows the differential amplifier with $V_a = 0$. This circuit is the inverting configuration.

$$V_{ob} = -\frac{R_b}{R_a}V_b$$

Note: For the circuit on the right the resistances at the positive input are in parallel and can be represented by one resistance. This equivalent resistance can have no current since the input resistance of the ideal op-amp is infinite. Since the resistance has zero current through it, by Ohm's law it has zero voltage across it. Therefore, the presence of R_a and R_b at the positive input has no impact on the behavior of the circuit on the right in Fig. 9.16.

The voltage present at the output voltage is the sum of V_{oa} and V_{ob}.

$$V_o = V_{oa} + V_{ob} = \left(1 + \frac{R_b}{R_a}\right)\frac{R_b}{R_a + R_b}V_a - \frac{R_b}{R_a}V_b$$

$$V_o = \frac{R_b}{R_a}V_a - \frac{R_b}{R_a}V_b = \frac{R_a}{R_b}(V_a - V_b)$$

Nodal analysis can also be employed to analyze the differential amplifier circuit.

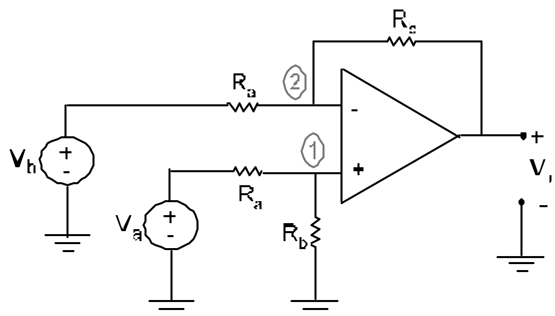


Figure 9.17: Analyzing the Differential Amplifier using Nodal Analysis

KCL at node 1 $\quad \dfrac{V_1 - V_a}{R_a} + \dfrac{V_1}{R_b} = 0 \quad \rightarrow \quad V_1 = \dfrac{R_b}{R_a + R_b}V_a$

KCL at node 2 $\quad \dfrac{V_2 - V_b}{R_a} + \dfrac{V_1 - V_o}{R_b} = 0$

Using ideal op-amp $V_1 = V_2$

These equations can be combined to eliminate V_1 and V_2. First since V_1 is related to V_a using KCL 1 and since $V_1 = V_2$ do to the ideal op-amp, KCL 2 becomes.

$$\frac{\left(\frac{R_b}{R_a + R_b}\right)V_a - V_b}{R_a} + \frac{\frac{R_b}{R_a + R_b}V_a - V_o}{R_b} = \frac{\left(\frac{R_b^2}{R_a + R_b}\right)V_a - R_bV_b + \frac{R_aR_b}{R_a + R_b}V_a - R_aV_o}{R_aR_b} = 0$$

Dividing out R_aR_b,

$$\left(\frac{R_b^2}{R_a + R_b}\right)V_a - R_bV_b + \frac{R_aR_b}{R_a + R_b}V_a - R_aV_o = 0$$

Obtaining a common denominator,

$$\left(\frac{R_b^2}{R_a + R_b}\right)V_a - \frac{(R_a + R_b)R_b}{R_a + R_b}V_b + \frac{R_aR_b}{R_a + R_b}V_a - R_aV_o = 0$$

Solving for V_o,

$$V_o = \frac{R_b}{R_a}V_a - \frac{R_b}{R_a}V_b = \frac{R_a}{R_b}(V_a - V_b)$$

Notice that if the sources, V_a and V_b, were not ideal sources, their resistances would affect the gain.

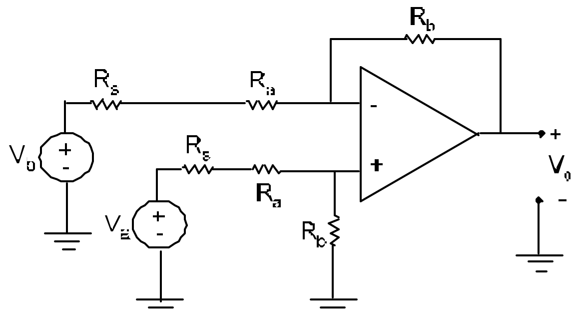


Figure 9.18: Differential Amplifier with Non-Ideal Sources

The resulting expression for the output voltage is

$$V_o = \frac{R_b}{R_a + R_s}(V_a - V_b)$$

Note: As long as the source resistances for V_a and V_b are equal the output voltage is a multiple of the voltage difference--any common signals on both sources will be rejected. The rejection of common signals is, of course, never perfect but is an important feature in differential amplifiers. Common signals can be due to noise of some other source which the engineer wishes to eliminate. The Common Mode Rejection Ratio (CMRR) is a measure of the circuit's ability to reject these common signals.

In dB, the CMRR is defined as

$$20 \log\left(\frac{\text{Gain}\big|_{\text{differential mode}}}{\text{Gain}\big|_{\text{common mode}}}\right)$$

If the source resistances differ the CMRR suffers a significant reduction. This is a significant limitation regarding the differential amplifier.

9.4.4 *Voltage Follower*

The voltage follower has a gain of one—the output voltage is equal to the input voltage. Its usefulness lies in the fact that its input resistance is that of the op-amp (very high), while its output resistance is that of the op-amp with feedback (very low).

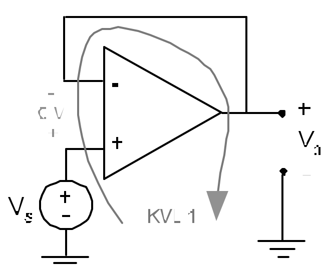


Figure 9.19: Voltage Follower

By applying Kirchoff's Voltage Law about KVL 1, it can be clearly seen that the device $V_o = V_S$. The voltage follower is typically used to "buffer" inputs. That is the voltage follower transforms the input impedance of an input to a very large value so that any circuit driving the input will not be required to provide much current.

> ***Note:*** The reader has been warned not to use resistors with resistances less than 1 KΩ in op-amp circuits. But, with the voltage follower, the output is directly connected to the negative input (R_f = 0!). Why is this permitted? The answer is that the op-amp output sees only the negative input, which offers a very high input resistance (and whatever effective resistance is connected to V_o). The effective resistance connected to V_o must not be too small.

Two op-amps, connected as voltage followers, can be used to buffer the inputs of the differential amplifier to produce an improved differential amplifier. The resulting configuration is an instrumentation amplifier—one with a fixed gain of R_b/R_a.

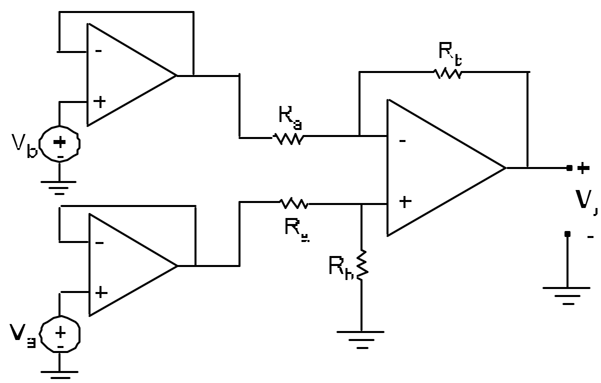


Figure 9.20: An Improved Differential Amplifier

9.4.5 *Comparator*

An op-amp using no feedback can be used to implement a comparator. A comparator is a circuit that compares an unknown voltage to a known voltage.

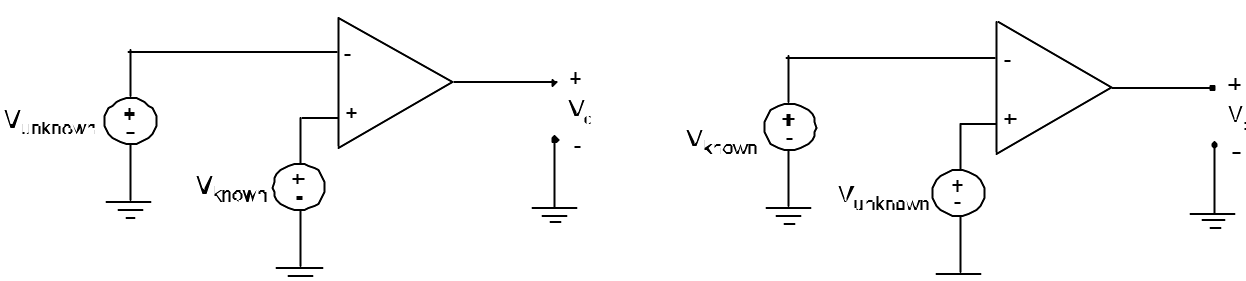


Figure 9.21: Op-Amp Comparators

Left Circuit $\begin{cases} V_o \cong V_{cc} - 1.5\text{ V} & \text{for } V_{unknown} < V_{known} \\ V_o \cong -V_{cc} + 1.5\text{ V} & \text{for } V_{unknown} > V_{known} \end{cases}$ Right Circuit $\begin{cases} V_o \cong V_{cc} - 1.5\text{ V} & \text{for } V_{unknown} > V_{known} \\ V_o \cong -V_{cc} + 1.5\text{ V} & \text{for } V_{unknown} < V_{known} \end{cases}$

9.5 Filtering

9.5.1 *Frequency Response Background*

The frequency response of a circuit is its steady-state response to a sinusoidal input as the frequency of the sinusoidal input varies. As an example, suppose the system input and output are both voltages, $v_i(t)$ and $v_o(t)$, respectively. In the time domain (sinusoidal steady state),

$$v_i(t) = V_i \cos(\omega t + \theta_i) \quad \text{and} \quad v_o(t) = V_o \cos(\omega t + \theta_o)$$

In the frequency domain, the corresponding phasor quantities are:

$$\boldsymbol{V}_i = V_i\angle\theta_i \quad \text{and} \quad \boldsymbol{V}_o = V_o\angle\theta_o$$

Note: Since filtering is used in many other areas than electric power, the convention used in power of always representing phasor quantities in RMS will not be followed here. In this section, phasor quantities will be in peak. That is, the magnitude of phasors will correspond to the sinusoidal amplitudes, not their RMS values.

Recall from Laplace analysis, the transfer function is obtained by taking the ratio between the output and input (assuming the initial conditions are zero).

$$H(s) \equiv \left.\frac{V_o(s)}{V_i(s)}\right|_{\text{initial conditions = 0}}$$

The system's frequency response is the sinusoidal steady-state relation between the input and output phasors and is found by substituting jω for s in the transfer function. The system frequency response is a phasor quantity relating the input and output phasors as a function of frequency. The system frequency response is a phasor where both the magnitude and phase are functions of ω, the frequency. Expressing

$$\boldsymbol{H} = \left.H(s)\right|_{s = j\omega} = \frac{\boldsymbol{V}_o}{\boldsymbol{V}_i}$$

Expressed in polar form,

$$H\angle\theta_H = \frac{V_o\angle\theta_o}{V_i\angle\theta_i}$$

The relationship between input and output is

$$V_o\angle\theta_o = H\angle\theta_H \; V_{in}\angle\theta_i$$

It is important for the reader to note that a system's frequency response is a phasor relation which relates magnitudes and phases.

$$V_o = HV_i \quad \text{and} \quad \theta_o = \theta_H + \theta_i$$

Example 3

As an example, use nodal analysis to find the transfer function of the system below.

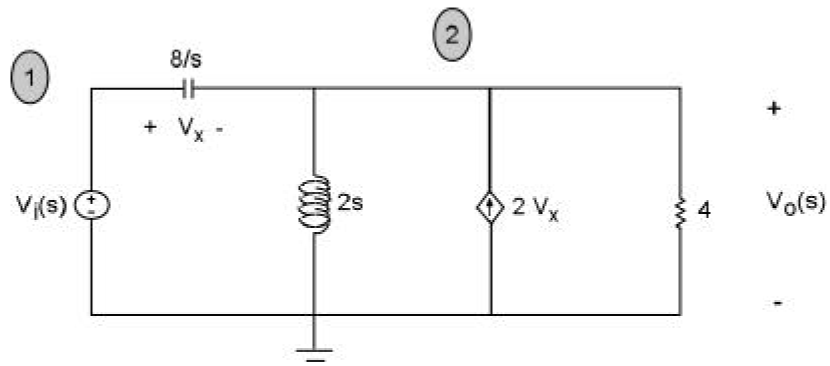


Figure 9.22: Transfer Function

The strategy will be to first perform nodal analysis on the circuit. Then, having found the node voltages (here V_1 and V_2), the output variable can be expresses as a linear combination of the node voltages and so can be express in terms of the input, V_i.

$$V_1 = \frac{N_1(s)}{D_1(s)} V_i \qquad V_2 = \frac{N_2(s)}{D_2(s)} V_i$$

In passing, it should be noted that $D_1 = D_2$.

In linear systems, any circuit variable can be expressed as a linear combination of the node voltages. Here the relation is trivial, $V_o = V_2 = 0V_1 + 1V_2$. The general case would be,

$$V_o = \sum_{i=1}^{N-1} a_i V_i$$

To proceed with this specific example, the nodal equations read:

Define control variable $\rightarrow$ $V_x = V_1 - V_2$

Nodal equations

Voltage source equation $\rightarrow$ $V_1 = V_i$

KCL at node 2 $\rightarrow$ $\dfrac{V_2 - V_1}{8/s} + \dfrac{V_2}{2s} - 2(V_1 - V_2) + \dfrac{V_2}{4} = 0$

Maple is a good tool for finding transfer functions. The required Maple code to find H(s) in this example is:

```
> restart;
> eqns:={v1=vi,
```

```
> (v2-v1)*s/8+v2/(2*s)-2*(v1-v2)+v2/4=0}:
> soln:=solve(eqns,{v1,v2}):
> assign(soln);
> vo:=v2:
> TF:=vo/vi;
```

$$TF := \frac{s\,(s + 16)}{s^2 + 18s + 4}$$

To obtain the system frequency response, substitute jω for s.

9.5.2 *Filters*

Filters are frequency selective systems. The interest here is often in the system's amplitude response. The phase response is usually of less interest.

$$\boldsymbol{H} = \frac{\boldsymbol{V}_o}{\boldsymbol{V}_i} \xrightarrow[\text{in polar form}]{} H\angle q_H = \frac{V_o\angle q_o}{V_i\angle q_i}$$

Where $H = V_o / V_i$ is the system magnitude response, and $q_H = q_o - q_i$ is the system phase response.

Suppose the input to a system includes signals at a variety of frequencies—some desired at the output, some not. A filter can be used to "filter out" undesired frequencies so that the desired frequencies at the output have a greater relative magnitude (relative to the amplitudes of the undesired frequencies).

Viewing the filter as an input/output relation, filters are classified by how the input and output magnitudes are related at different frequencies. Four basic types of filters will be considered here: *lowpass*, *highpass*, *bandpass,* and *notch*.

Low pass filters pass low frequencies and block high frequencies. Two things should be mentioned here. First, what is meant by "low" or "high" depends on the system under consideration. Second, a low pass filter does not block high frequencies entirely. For a given input with a range of frequencies, a low pass system will decrease the amplitudes of the high frequency components with respect to the amplitudes of the low frequency components.

Similarly, high pass filters pass high frequencies and block low frequencies; band pass filters pass only a band of frequencies while blocking both low and high; and notch filters pass both low and high frequencies while blocking a narrow range of frequencies.

Another way to classify filters is whether they are passive or active. Passive filters consist solely of passive components (R's, C's, L's, transformers, etc.) Active filters also use active components such as transistors or op-amps. Active filters can provide a gain greater than one whereas the gain greater than one is not possible with passive filters. Filters which utilize op-amps are active filters. Filters utilizing can amplify *and* filter since they receive the needed energy for amplification external power supplies.

Example 4

Consider an active filter using the inverting op-amp configuration. Fig. 9.23 shows the s-domain representation.

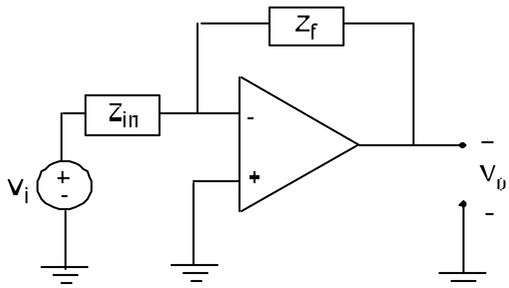


Figure 9.23: Active Filter

The reader should be able to use circuit analysis to find that the transfer function for this circuit is $H(s) = V_o(s)/V_i = -Z_f / Z_{in}$.

Example 5

The circuit in Fig. 9.24 is an active filter using the inverting configuration. It is a **low pass** filter with a break frequency, $\omega_b = 1/R_fC$ and a low frequency gain R_f/R_{in}.

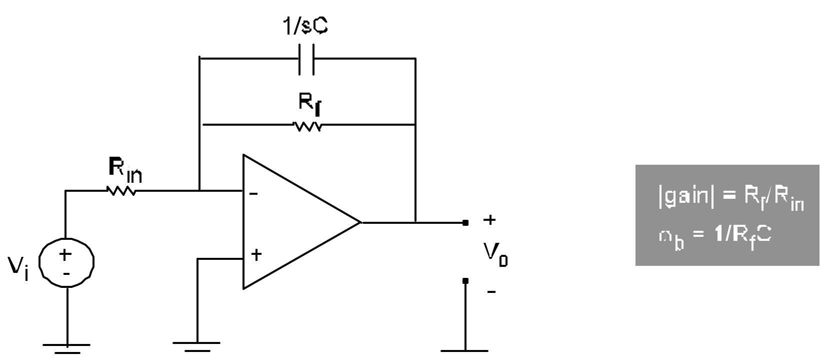


Figure 9.24: Low-Pass Active Filter

Analyzing this op-amp circuit, the transfer function is found.

$$Z_{in} = R_{in} \qquad Z_f = \frac{1}{\frac{1}{R_f} + sC} = \frac{R_f}{sR_fC + 1}$$

$$H(s) = -\frac{Z_f}{Z_{in}} = -\frac{\frac{R_f}{sR_fC+1}}{R_{in}} = -\frac{R_f}{R_{in}}\frac{1}{\frac{s}{1/R_fC}+1} \qquad \left(= K\frac{1}{\frac{s}{w_b}+1}\right)$$

Since, in filtering, the magnitude response is the important aspect of the frequency response, only the Bode magnitude plot is necessary to evaluate the filter's performance.

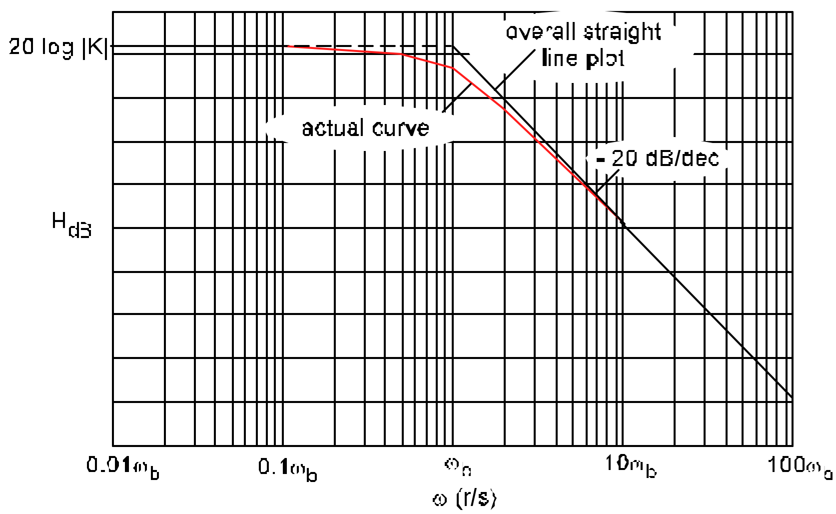


Figure 9.25: Bode Magnitude Plot of Low-Pass Filter

Note: In this case, the gain is actually negative, but it is customary to refer to the gain as being the magnitude of the frequency response. In this view a negative gain is one having a certain magnitude together with a 180° phase shift.

Example 6

Suppose a DC signal has been corrupted with a narrow periodic pulse. The periodic pulse, the noise, can cause system malfunctions.

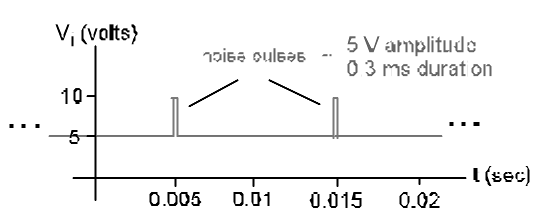


Figure 9.26: DC Signal with Noise Pulses

One approach to solve this difficulty would be to filter the noise out. Using a filter will not completely eliminate the noise but can reduce its magnitude with respect to the desired signal. A **low pass** filter is needed here since the signal is at DC ($\omega = 0$) and the noise is a pulse with a range of frequency components.

In order to proceed, the frequency content of the input signal needs to be found using Fourier analysis. In this way, the corner frequency can be placed at a lower frequency than the first component of the noise. In this case, this is easily done since the signal is at DC so the fundamental frequency, $\omega_o = 2\pi/T = 2\pi/0.01s = 628.3$ rad/s, will mark the beginning of the noise frequencies.

Designing the filter for $\omega_b = 10$ r/s and K =1, results in the output being

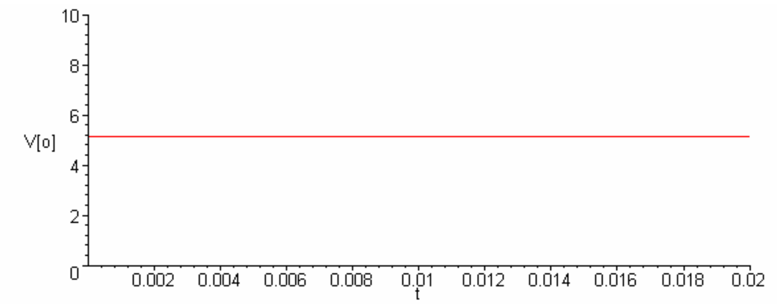


Figure 9.27: Filtered Output (w_b = 10 r/s)

The fact that the noise has not been completely eliminated is evident upon closer examination (note the scale in Fig. 9.28).

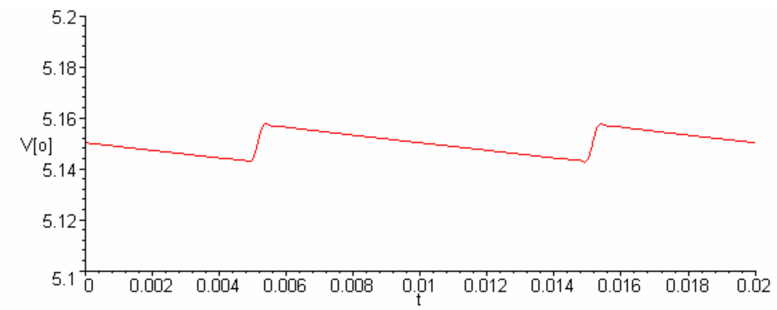


Figure 9.28: Close-up of Filtered Output (w_b = 10 r/s)

Although the noise has not been eliminated, it has been significantly reduced through filtering!

How would this circuit be designed? Starting with the two design equations

$$|K| = \frac{R_f}{R_{in}} = 1 \qquad\qquad w_b = \frac{1}{R_f C} = 10$$

it is evident that one of the three parameters must be chosen. Then, with this choice, the two design equations will determine the other two parameters.

Choosing C = 1 μF = 10^{-6} F constrains R_f and R_{in} to be 100 KΩ.

Suppose the filter had been designed for K = 1 and $\omega_b = 100$. What performance consequences would be anticipated? Since the break frequency is relatively close to the first fundamental frequency of the noise, one should expect to see more “noise” survive at the output. Fig. 9.29 shows the result

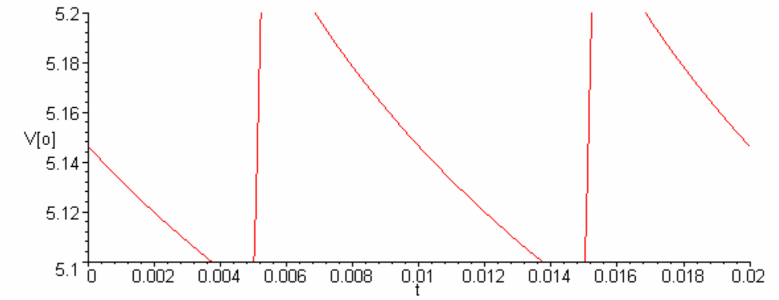


Figure 9.29: Close-up of Filtered Output (w_b = 100 r/s)

At this scale, the noise looks high. On a larger scale one can see that the noise may be acceptable.

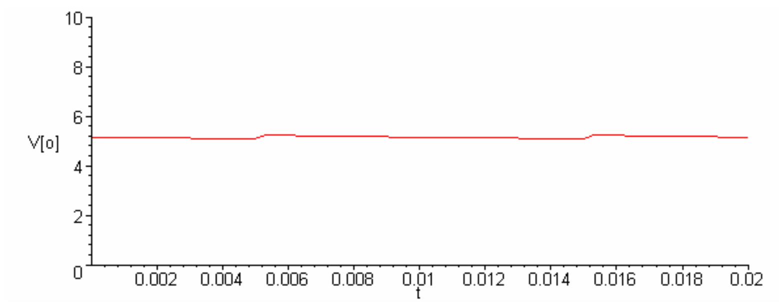


Figure 9.30: Filtered Output (w_b = 100 r/s)

For this filter, C = 0.1 µF and $R_f = R_{in} = 0$ would be a possible design.

A passive first-order **low pass** filter can be designed using one resistance and one capacitance. In Fig. 9.31, a voltage source is input to a first-order passive filter which feeds a resistive load.

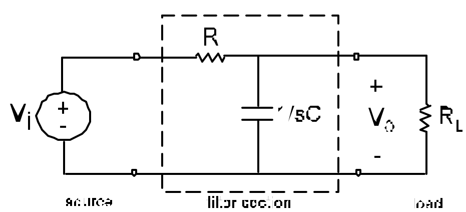


Figure 9.31: First-order Passive Low Pass Filter

The s-domain relation between V_i and V_s is

$$\frac{V_o}{V_i} = \frac{R_L/(sCR_L+1)}{\dfrac{R_L}{sCR_L+1}+R} = \frac{R_L}{R_L+R}\,\frac{1}{\dfrac{s}{(R_L+R)/R_LRCR}+1} \qquad \left(= K\,\frac{1}{\dfrac{s}{w_b}+1} \right)$$

What can one observe from this analysis? First, the low frequency gain, K, is always less than one. This must be true since this is a passive circuit. There is no external source of power by which V_i might be amplified.

$$\frac{R_L}{R_L + R} \leq 1$$

Second, both the gain and the break frequency, ω_b,

$$w_b = \frac{R_L + R}{R_L R C}$$

are affected by variations in the load resistance. One can imagine this might not be desired.

Consider the benefit achieved by placing an op-amp, configured as a buffer, into this first-order **low pass** filter.

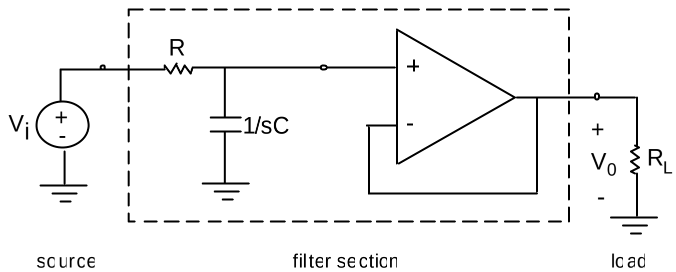


Figure 9.32: Active Filter

Here, the relation between V_i and V_s is

$$\frac{V_o}{V_i} = \frac{1/sC}{\frac{1}{sC} + R} = 1\frac{1}{\frac{s}{1/RC} + 1} \quad \left(= K \frac{1}{\frac{s}{w_b} + 1} \right)$$

Notice, both the gain, K, and the break frequency, ω_b, are now independent of the load resistance! This, of course assumes the load resistance is not too low for the op-amp output (< 1 kΩ).

9.5.3 *Filter Circuits*

A second-order **low pass** filter gives a faster drop-off past the break frequency than the first-order filter.

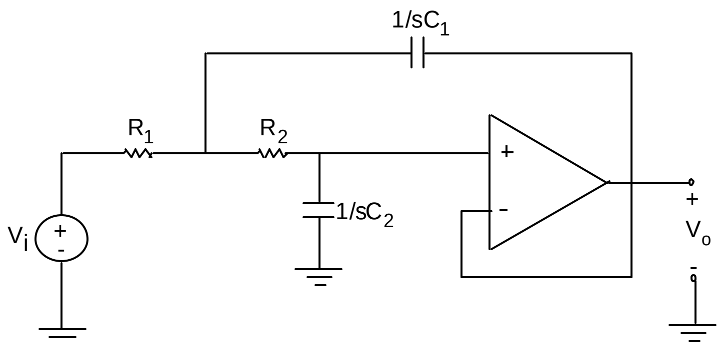


Figure 9.33: Second-order Active Low Pass Filter

This filter is the second-order Sallen-Key filter, and the relation between V_o and V_i is

$$\frac{V_o}{V_i} = \frac{1}{\frac{s^2}{1/R_1R_2C_1C_2} + C_2(R_1 + R_2)s + 1} = K\frac{1}{\frac{s^2}{w_n^2} + \frac{2V}{w_n}s + 1}$$

$$K = 1$$

$$w_n = \sqrt{\frac{1}{R_1R_2C_1C_2}}$$

$$\frac{2V}{w_n} = 2V\sqrt{R_1R_2C_1C_2} = C_2(R_1 + R_2) \qquad \rightarrow \qquad V = \frac{R_1 + R_2}{2}\sqrt{\frac{C_2}{R_1R_2C_1}}$$

Given an underdamped filter, the straight line Bode plot is

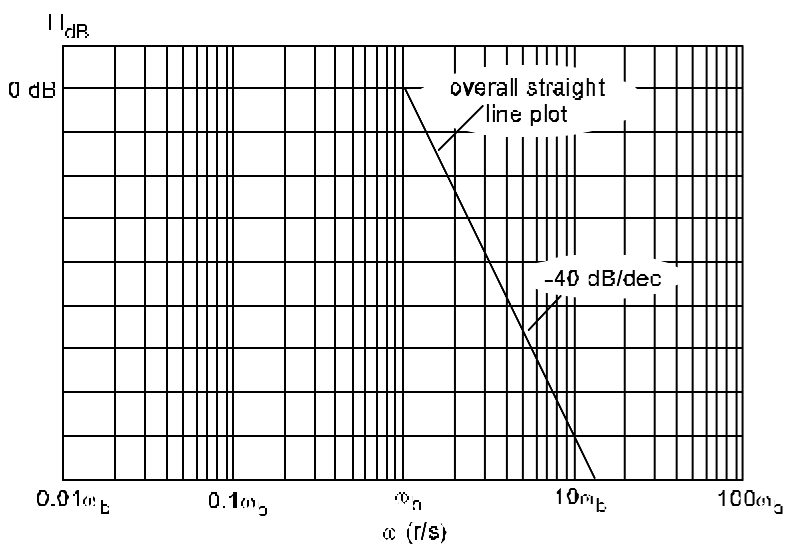


Figure 9.34: Straight Line Bode Magnitude Plot

A first-order passive **high-pass** filter is shown in Fig. 9.35.

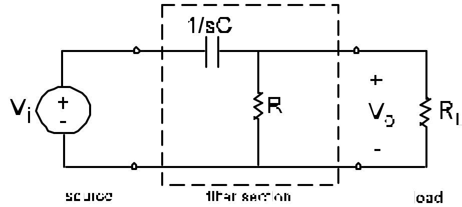


Figure 9.35: First-order Passive High Pass Filter

The relation between V_o and V_i is

$$\frac{V_o}{V_i} = \frac{sR_{\parallel}C}{\frac{s}{1/R_{\parallel}C} + 1}, \text{ where } R_{\parallel} = \frac{RR_L}{R + R_L}$$

At DC, the gain is zero (at DC, $s = j\omega = j0 = 0$) as one would expect in a high pass filter. Notice that the magnitude of the transfer function (that is, the gain) depends on the value of R_L as does the break frequency, $\omega = 1/R_{\parallel}C$. Using a voltage follower, similar to that found in Fig. 9.32 will allow the transfer function to become independent of R_L.

A **bandpass** filter blocks all signals aside from those in a band of frequencies. This band of frequencies is called the pass band

A magnitude plot of a 2nd-order bandpass filter (there are no first-order bandpass filters) is shown in Fig. 36. The plot assumes the system is underdamped ($\zeta < 1$). This plot is simply a magnitude plot, not a Bode magnitude plot in which 20 log H is plotted.

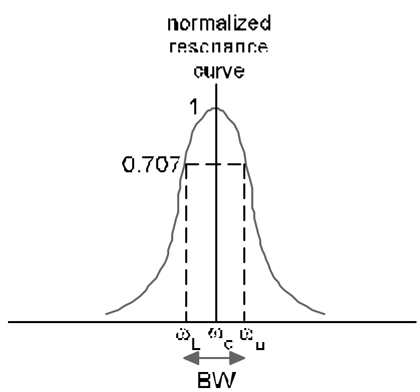


Figure 9.36: Normalized Resonance Peak

The Sallen-Key second-order bandpass filter is given is shown in Fig. 9.37.

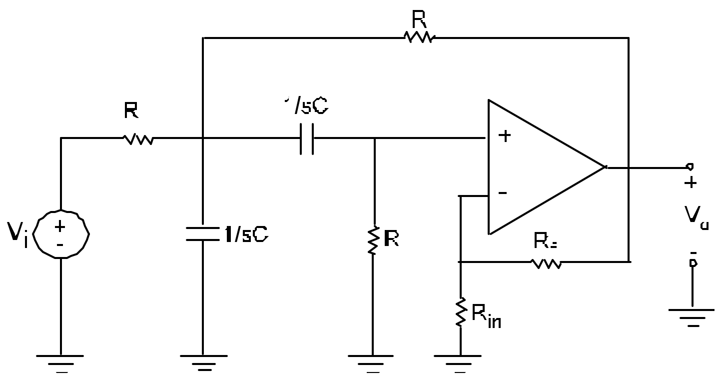


Figure 9.37: Bandpass Filter

The s-domain relationship between V_o and V_i is

$$\frac{V_o}{V_i} = \frac{R_{in} + R_f}{R_f} \frac{\frac{s}{RC}}{s^2 + \frac{4 - \left(\frac{R_{in} + R_f}{R_f}\right)}{RC} s + \left(\frac{\sqrt{2}}{RC}\right)^2}$$

The denominator can be written as

$$s^2 + \frac{4 - \left(\frac{R_{in} + R_f}{R_f}\right)}{RC} s + \left(\frac{\sqrt{2}}{RC}\right)^2 = s^2 + 2Vw_n s + w_n^2 = s^2 + BW\, s + w_n^2$$

With this filter design, the half-power bandwidth of the filter ($BW = \omega_u - \omega_L$) can be adjusted by varying the gain of the amplifier within the filter.

To a good approximation (to within 1% if $BW << \omega_o$) the corner frequencies are

$$w_u = w_o + \frac{BW}{2} \qquad\qquad w_L = w_o - \frac{BW}{2}$$

The sharpness of the pass band is a measure of the selectivity of the filter. The quality factor is a useful measure of the sharpness. In general, Q is defined as a ratio between maximum stored energy and the energy dissipated in a radian time.

$$Q = \frac{E_{stored\text{-}max}}{E_{dissipate\ in\ t = 1/w}}$$

In the context of filtering, more useful relations are

$$Q = \frac{w_o}{BW} = \frac{1}{2V} \qquad \left(= \frac{1}{4 - \frac{R_f + R_{in}}{R_{in}}} \text{ here} \right)$$

A high Q corresponds to a sharp resonant peak and therefore to a highly selective bandpass filter. The range of Q for this circuit is

$$Q \rightarrow \frac{1}{3} \qquad \text{as } \left.\frac{R_f + R_{in}}{R_{in}}\right|_{\text{for } R_{in} >> R_f} \rightarrow 1$$

$$Q \rightarrow \infty \quad \text{as } \frac{R_f + R_{in}}{R_{in}} \rightarrow 4$$

An alternate bandpass design, one in which the designer can independently choose the midband gain, lower break frequency (ω_L) and upper break frequency (ω_u) is discussed in Design Problem 6.7.1.

A notch filter refers to a filter in which the attenuation rapidly becomes large about a particular frequency. Such a filter might be used to remove a power line harmonic from signals. The Twin-T notch filter, shown in Fig. 9.38, has a notch frequency of 1/RC.

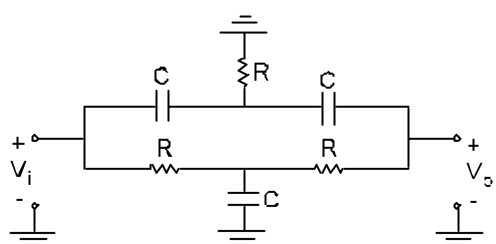


Figure 9.38: Twin-T Notch Filter

The components must be well matched in order to obtain high attenuation at the notch frequency.

Horowitz and Hill (*The Art of Electronics*, 2nd ed.) give a tunable notch filter.

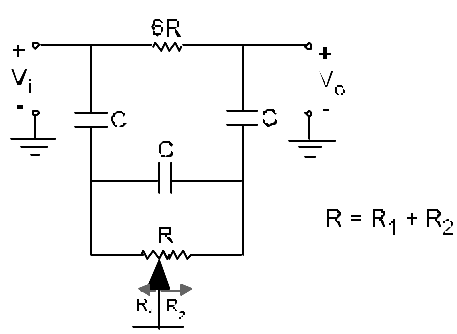


Figure 9.39: Tunable Notch Filter (adapted from Horowitz and Hill)

The notch frequency for the filter in Fig. 9.33 is $w_c = 1/C\sqrt{3R_1R_2}$.

9.6 Bridge Circuits

The uses of bridge circuits include the measurement of impedance and frequency.

9.6.1 *Impedance Measurement*

Consider the bridge circuits below.

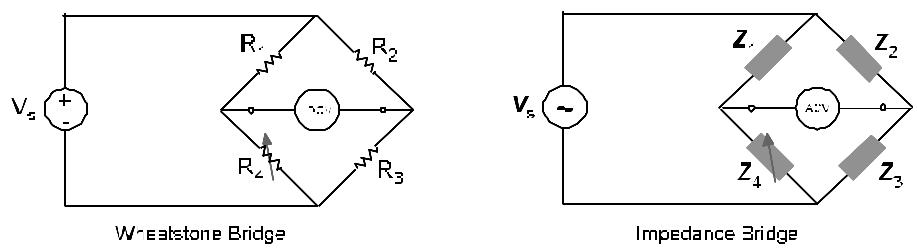


Figure 9.40: Null Detecting Bridge Circuits

The Wheatstone Bridge is a classic circuit. The idea is for three of the four resistors to be known and the fourth to be determines. For the presents, consider R_1 as the resistance to be measured, R_2 and R_3 has fixed known resistances, and R_4 as a known variable resistance. When the voltage measured by the DMM is zero (or null, thus the name null detector), voltage division gives

$$\frac{R_1}{R_4} = \frac{R_2}{R_3} \qquad \rightarrow \qquad R_1 = \frac{R_2}{R_3} R_4$$

The variable resistance, R_4, is varied until the null is achieved.

Likewise the impedance bridge can be used to determine an unknown impedance. If $\boldsymbol{Z}_1$ is the impedance to be measured, $\boldsymbol{Z}_2$ and $\boldsymbol{Z}_3$ are fixed impedances (that is, at a particular frequency), and $\boldsymbol{Z}_4$ is a known variable impedance. Again, when the measured voltage (magnitude and phase), voltage division gives,

$$\boldsymbol{Z}_1 = \frac{\boldsymbol{Z}_2}{\boldsymbol{Z}_3} \boldsymbol{Z}_4$$

There have been many ingenious uses of null detecting bridge circuits. The focus here, however, will be the use of bridge circuits in signal conditioning.

9.6.2 *Bridge Circuits in Signal Conditioning*

Suppose strain gages are used, together will a cantilever beam, to form a load cell to measure forces as illustrated in Fig. 9.41.

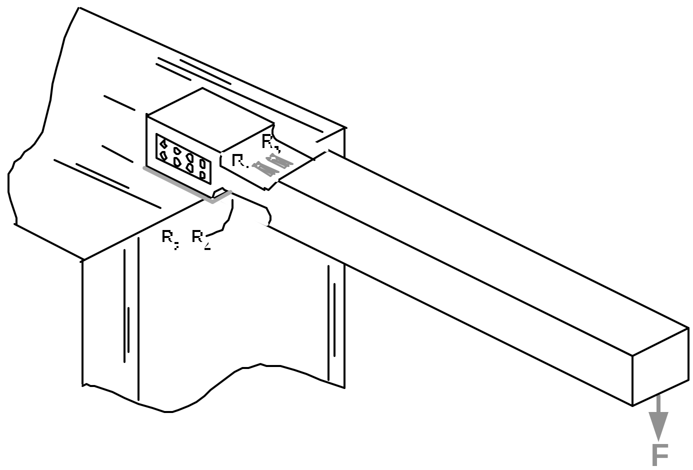


Figure 9.41: Load Cell

It is clear that the force, F, will place the strain gages R_1 and R_3 in tension and R_2 and R_4 in compression. R_1 and R_3 will therefore have resistances higher than nominal and R_2 and R_4 will have resistances lower than nominal. Now suppose these strain gages are connected as shown in Fig. 9.36.

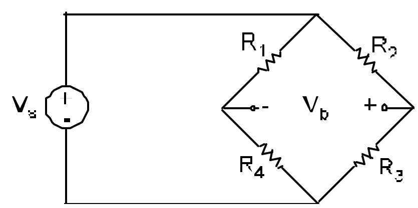


Figure 9.42: Wheatstone Bridge Signal Conditioning ($\Delta R \rightarrow \Delta V_b$)

By voltage division

$$V_b = \left(\frac{R_3}{R_2 + R_3} - \frac{R_4}{R_1 + R_4} \right) V_s$$

where R_1, R_3 = R + ΔR and R_2, R_4 = R - ΔR.

$$V_b = \left(\frac{R + \Delta R}{R + R} - \frac{R - \Delta R}{R + R} \right) V_s = \left(\frac{R + \Delta R - R + \Delta R}{2\,R} \right) V_s$$

$$V_b = \frac{\Delta R}{R} V_s$$

Note: This bridge is a four-arm active bridge, where all four bridge resistances are strain gages has distinct advantages over the one-arm active bridge, where only one bridge resistance is a strain gage, the other three being fixed "dummy" resistances with the strain gages nominal resistance. First, the four-arm active bridge is more sensitive. In the one-arm active bridge,

$$V_b = \frac{\Delta R}{4R} V_s$$

Second, the four-arm bridge is more robust with respect to temperature changes since all four strain gages are more likely to experience more similar changes in temperature than the one strain gage and three dummy resistances. Third, the four-arm bridge, through the placement of strain gages on the beam, can be made relatively insensitive to torsion.

What voltages should one expect for V_b? That is, for typical strain gages with typical strains, is V_b going to be 10 V?, 1 V? 1mV? or 1μV? For the four-arm bridge, the typical values below will give an idea of the size of voltages to be expected for V_b.

typical R (metallic strain gage) ~ 120 Ω

typical gage factor, S ~ 2

typical strain, ε ~ 0.001

typical V_s ~ 10 V

The above example shows how a bridge circuit can be used as signal conditioner, converting a change in resistance, ΔR, to a change in voltage, ΔV. In example 8.3, one type of accelerometer was shown to couple changes in capacitance to acceleration.

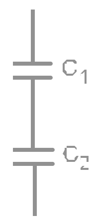


Figure 9.43: Circuit Model of Accelerometer

For this sensor, acceleration produces opposing changes in C_1 and C_2. That is, either C_1 increases and C_2 decreases or vice versa, depending on the sense of acceleration. The Twin-T bridge circuit can be used to convert the change in capacitance, ΔC into a change in voltage, ΔV.

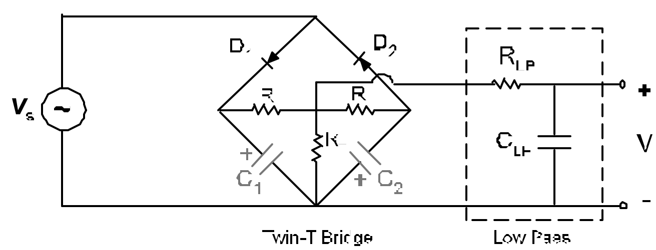


Figure 9.44: Twin-T Bridge Signal Conditioning (DC ® DV)

The AC source, V_s, powers the bridge. When the AC waveform is positive, D_1 conducts and charges C_1 with the polarity indicated. When the AC waveform is negative, D_2 conducts and charges C_2 with the polarity indicated. When one capacitor is charging the other is discharging through the resistors.

The key to the circuit's operation is to note that the capacitance of C_1 and C_2 will not effect their being fully charged during their charging time. This is due to the low impedance of the charging path. The discharge path, however, is a different matter.

The capacitors discharge through a resistive network and so will have an associated RC time constant which grows as C grows. For example, if an acceleration caused C_1 to increase and C_2 to decrease, then the output voltage, V (a DC voltage, by the way), would be positive.

9.7 Current Sources

Constant current sources are used measurement and signal conditioning. When using a DMM as an ohmmeter, for example, a current source sends a known current through the resistance under measurement. What the meter actually measures is voltages which, since the current is known, need only to be properly scaled to be a measure of the resistance.

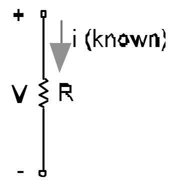


Figure 9.45: Measuring Resistance

Many DMMs can be used to perform either a 2-wire resistance measurement or a 4-wire measurement. In the 2-wire measurements, the current source and the voltmeter are connected to the same terminals. In the 4-wire measurements, the current source and voltmeter have separate connections.

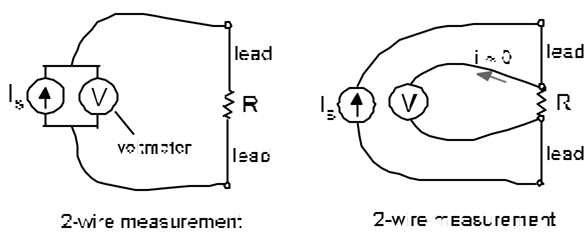


Figure 9.46: 2-Wire and 4-Wire Resistance Measurement

The difference in the two measurements can be appreciated by considering the effect that lead resistance would have on the voltmeter reading (and therefore on the resistance measurement). As is apparent from Fig. 9.46, if the lead resistances were not negligible, the voltage sensed with the 4-wire measurement would be I_sR while the voltage sensed in the 2-wire measurement would be $I_s(R + 2R_{lead})$, where R_{lead} is the resistance of each lead.

A circuit which can supply current to a floating load is shown in Fig. 9.47.

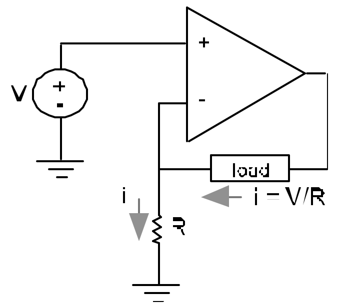


Figure 9.47: Current Source to Floating Load

The degree to which a constant current is supplied to the load depends upon the degree to which V and R remain constant. The voltage supplied will likely depend on the environment, especially on temperature. The resistance of R will change with temperature. If the aim is for a constant current through the load, it will be important to choose a resistor with a low temperature coefficient of resistance (TCR).

A current can be supplied to a non-floating load is shown in Fig. 9.48. As noted above, the performance of the current source depends on the degree to which V and R remain constant. Use of a precision voltage reference, such as REF 102 from Texas Instruments, will ensure the stability of V (temperature coefficient = 5 ppm/°C and long term drift stability of 10 ppm/1000 hrs). Resistors are widely available with TCRs of ±15 ppm/°C, with precision resistances available with TCRs 2.5 ppm/°C and lower.

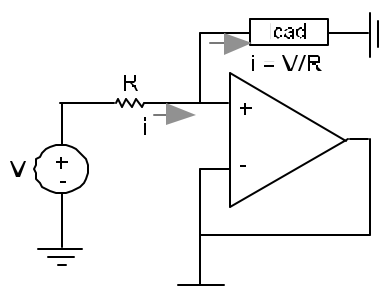


Figure 9.48: Current Source to Grounded Load

9.8 Limitations of Operational Amplifiers

Operational amplifiers are limited regarding what output currents and voltages they can supply. They have frequency and speed limitations which must be considered by circuit designers. Operational amplifiers are not perfectly balanced, and many op-amps have may need additional circuitry to adjust for input bias currents and output voltage offsets.

9.8.1 *Output Current and Voltage Limitations*

Operational amplifiers have definite limits to what currents they can supply. The equivalent resistance seen by the output pin of an op-amp must not fall to such a level that the op-amp is required to source too much current. Most op-amp outputs include short circuit protection which protects the op-amp from damage by limiting the output current available. In general purpose op-amps, output current is typically limited to 2-5 mA. If the maximum voltage at the output voltage were 10 V, for example, this limitation on output current would limit the equivalent resistance permissible at the output pin to be greater than 5 KΩ, if for example, the maximum current were 2 mA.

> ***Note:*** The limitation in output current is the reason for the guidelines given above warning against using resistances below 1 or 2 KΩ in op-amp circuits without good reasons. Circuit design should ensure that the feedback circuit does not require much output current, especially if the output current will be used for other purposes.

The maximum voltage which can be supplied by the op-amp is related to the external voltage supplied to power the op-amp. In general, the output voltage is limited to about 1.5 V below the supply voltage level. For example, if the supply voltage in a given application were ±10 V, the output voltage would be limited to ±8.5 V.

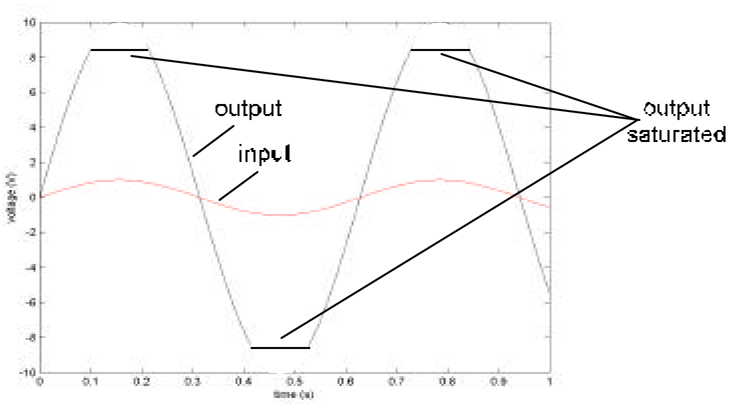


Figure 9.49: Op-Amp Output Saturation

If an op-amp based amplifier's input were large to force the output to reach 8.5 V, making the input larger would *not* result in the output likewise growing larger. The output would be said to be saturated and for larger inputs, the amplifier would not function as designed.

Example 7

Given the non-inverting amplifier shown in Fig. 9.50. The amplifier is strapped for a gain of 5 ($1 + R_f/R_{in}$). The input is a 1 KHz triangle wave with a peak-to-peak amplitude of 5 V so that the maximum amplitude of the input is ± 2.5 V. A gain of 5 would give a maximum output voltage of ±12.5 V, and, since op-amp is supplied with an external ±15 V supply, would not result in saturation due to voltage limitations.

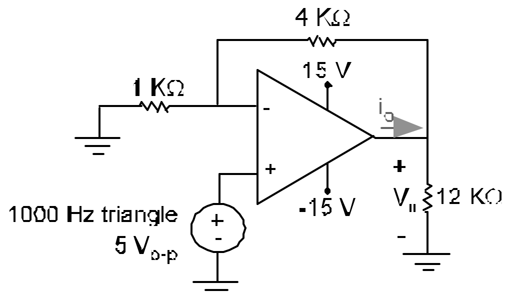


Figure 9.50: Non-Inverting Amplifier (gain = 5)

Assume, however, that the maximum output current is 2.5 V. Since the equivalent resistance at the op-amp output is 3 KΩ (the 12 KΩ load resistance and the 4 KΩ feedback resistance in parallel), the maximum permitted output voltage is ±7.5 V as shown in Fig. 9.51.

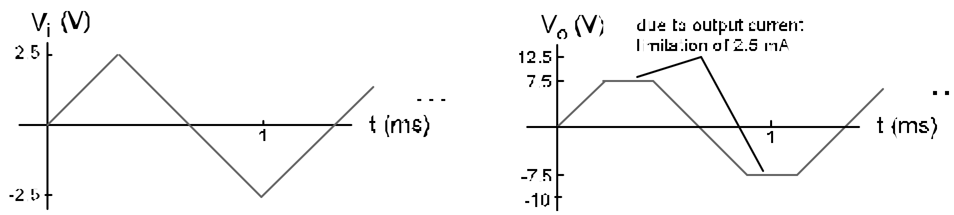


Figure 9.51: Output Waveform Distorted Due to Output Current Limitations

Notice that the fact the circuit does not perform as intended is due purely to poor design. The resistances chosen for R_f and R_{in} are too small. A gain of 4 would result if 30 KΩ were chosen for R_f and 7.5 KΩ for R_i. If these values were used the resistance seen at the op-am output would be 8.57 KΩ (30 KΩ in parallel with 12 KΩ). For V_o = ±12.5 V, the resulting current would be less than 2 mA, well within the capabilities of the op-amp.

9.8.2 *Frequency and Speed Limitations*

The ideal op-amp model is derived by assuming the open-loop gain of the op-amp is infinite. The actual gain is not infinite and decreases with frequency.

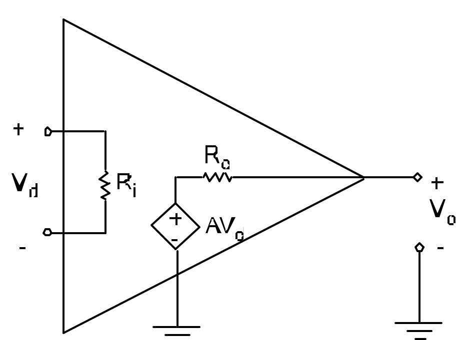


Figure 9.52: Op-Amp Circuit Model

Fig. 9.52 shows the model for the op-amp, including open-loop gain, A, and the input and output resistances.

The parameter which describes the behavior of A is referred to as the unity gain bandwidth which is the frequency at which the open-loop gain, A = 1. The gain acts like a first-order low pass filter with a break frequency usually between 1 and 20 Hz. Fig. 9.53 shows a high-speed op-amp with a unity gain BW of 3 MHz with a break frequency of 30 Hz. One can see that, for f = 30 KHz, A has been reduced 40 dB or 100.

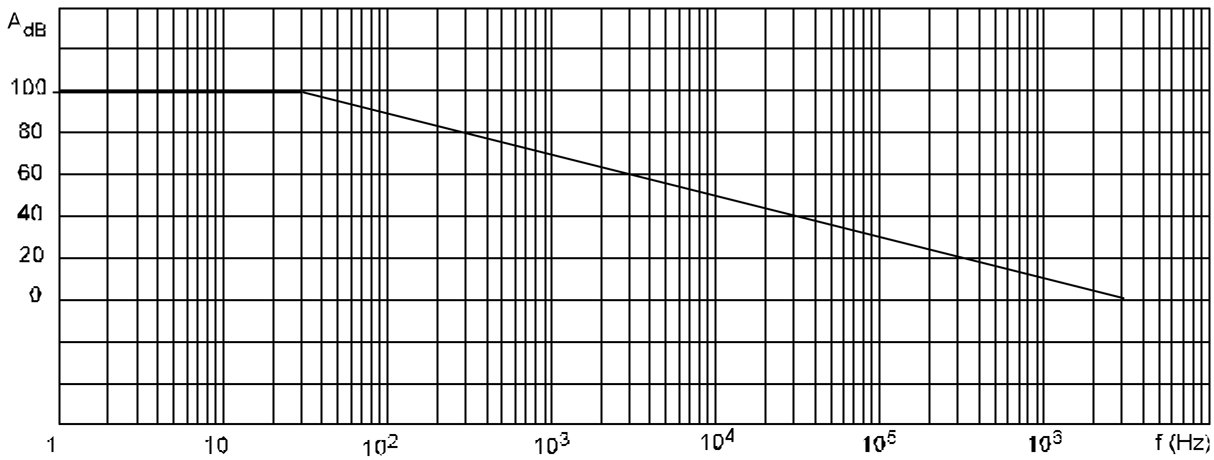


Figure 9.53: Op-Amp Unity Gain Bandwidth

Example 8

Consider using an op-amp amplifier, an inverting configuration strapped for a gain of 41 to amplify a 30 KHz, 40 mV peak-to-peak sinusoid. Use the inverting configuration with R_{in} = 2 KΩ and R_f = 82 KΩ with the model shown in Fig. 9.46. Assume an open-loop gain as shown in Fig. 9.53.

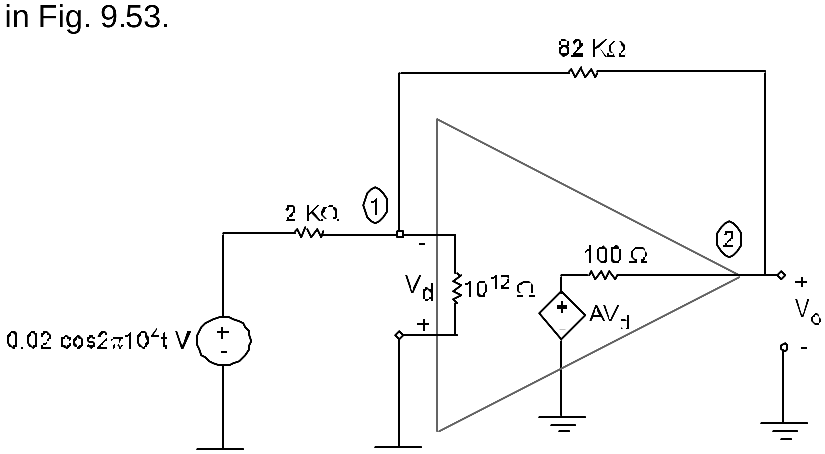


Figure 9.54: Amplifier with Non-Ideal Op-Amp

The nodal analysis equations are,

KCL at node 1 $$\frac{V_1 - V_s}{2000} + \frac{V_1 - V_2}{82000} + \frac{V_1}{10^{12}} = 0$$

KCL at node 2 $$\frac{V_2 - V_1}{82000} + \frac{V_1 + 100V_1}{100} = 0$$

Note: The voltage source equations are included in the KCL equations.

Using MAPLE to solve equations,

```
> restart;
> eqns:=(V1-Vs)/2000+(V1-V2)/82000+V1/10^12=0,(V2-V1)/82000+(V2+100*V1)/100=0:
> soln:=solve({eqns},{V1,V2}):
> assign(soln);
> Vo:=evalf(V2,4);
```

$$V_o := -28.86\ V_s$$

Notice that the gain is actually around -29 rather than -41 which is likely not acceptable (%error = -29.6%). Now consider the same amplifier, this time strapped for a gain of 6.8 rather than a gain of 40 (same circuit with $R_i = 10\ K\Omega$, $R_f = 68\ K\Omega$).

$V_o := -6.307\ V_s$ (percent error now -7.3%)

Exploring this once more, this time with the amplifier strapped for a gain of 2.2 (same circuit with $R_i = 10\ K\Omega$, $R_f = 24\ K\Omega$).

$V_o := -2.321\ V_s$ (percent error now -3.3%)

The point of this example is two-fold. First, the circuit designer must be aware of frequency effects when employing op-amp circuits. Second, to obtain gain at higher frequencies when using op-amps, one may need to employ multistage amplifiers. For example, at 30 KHz, one may wish to use two amplifiers in cascade, each strapped for a gain of 6.8 as indicated in Fig. 9.49. The actual gain then would be $0.927(6.8)^2 = 42.9$ for a percent error of 4.5%.

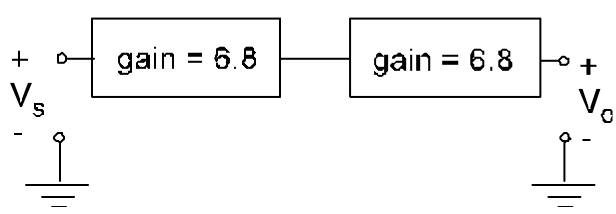


Figure 9.55: Amplifiers in Cascade

Another parameter that affects op-amp dynamical performance is referred to as slew rate. Op-amps are limited by the time rate of change which their outputs can achieve. The slew rate of an op-amp (usually gives in V/μs) gives the maximum rate of change the op-amp output can maintain. Whereas frequency limitations typically first affect op-amp gain, slew rate limitations can affect op-amp distort op-amp signals. A typical value for a general purpose op-amp would be around 10 V/μs with some high-speed op-amps offering slew rates of 100 V/μs and above.

Example 9

Suppose an op-amp voltage follower has a 5 V amplitude pulse train with a period of 10 μs at its input. Take the op-amp slew rate to be 10 V/μs.

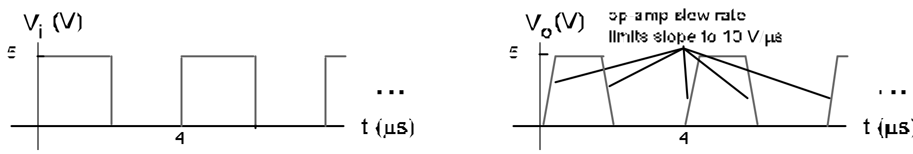


Figure 9.56: Signal Distortion due to Slew Rate Limitations

The distortions sinusoids suffer due to slew rate limitations are more subtle and harder to detect. The maximum slope for sinusoids is when they cross the time axis. Their rate of change is greatest when their value is small. What can happen is that their shape is "straightened out" in this region. The slope of the straight line being that imposed by slew rate limitations.

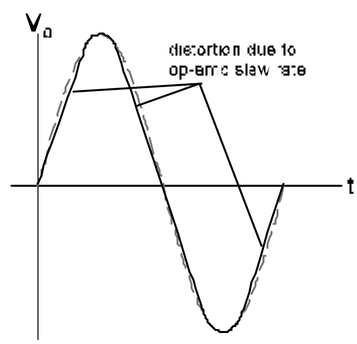


Figure 9.57: Sinusoidal Distortion due to Slew Rate Limitations

9.8.3 *Input Bias Currents and Input Voltage Offset*

Real op-amps have non-zero input currents. These small currents can cause the output to be non-zero if the input pins are not connected to equal impedances. For example,

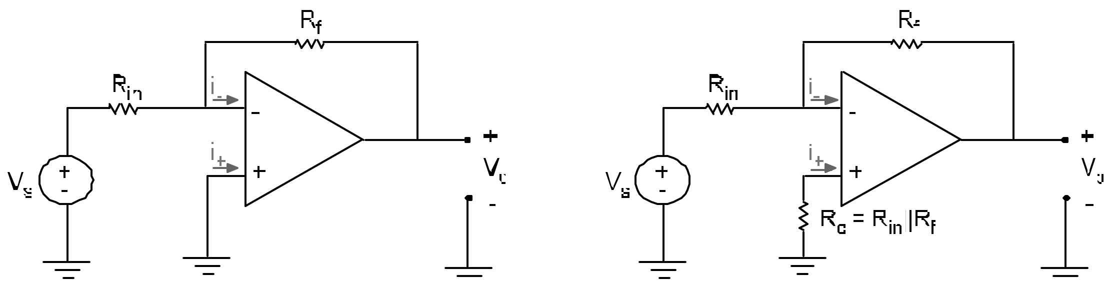


Figure 9.58: Input Bias Current Compensation

in the inverting amplifier shown in Fig. 9.58, if i_- and i_+ are not zero then V_d will have two components—that due to the source, V_s, and that due to the input bias currents. This effect, always present in real op-amps can be minimized with an input bias current resistance, R_c, shown in the circuit on the right. This resistance is sized so that the two input pins of the op-amp are connected to equal equivalent resistances.

For many purposes, for non-precision measurements or for applications involving large signals, this compensation is not necessary. For precision work, or for applications involving small signals, the small non-ideal behavior due to non-zero bias currents needs to be reduced.

Note: In real op-amps, the input currents are not equal. There is an input bias current offset. The above compensation will not completely eliminate errors due to non-equal bias currents, but will greatly reduce them.

Real op-amps also have input voltage offset. That is, even with the input pins tied together, there will be a non-zero voltage at the op-amp output. Some op-amps provide external pin connection, with which to compensate for input voltage offset. On the other hand, op-amps have improved markedly in recent years, and many op-amps do not provide any connections for compensation. The reader should consult more specialized texts for further information.

9.9 Summary

Signal Conditioning involves a wide range of topics. A sensor's signal could be in the form of a change in resistance and signal conditioning is required to transform this signal into one involving a change of voltage. A signal could be too small and require amplification. A signal could be corrupted with noise and require filtering to obtain a cleaner signal. A sensor may not be able to source much current and buffering may be required.

Several specific examples have been discussed in this chapter, and many of these signal conditioning schemes have been accomplished using the ideal operational amplifier. Circuit designers should be able to consider non-ideal behavior when designing signal conditioning circuitry.

9.10 Computer Tools and Other Resources

Maple is an excellent tool with which to calculate system transfer functions of linear, time-invariant lumped element systems.

9.10.1 *Electrical Circuit*

Maple is an excellent tool with which to determine transfer functions.

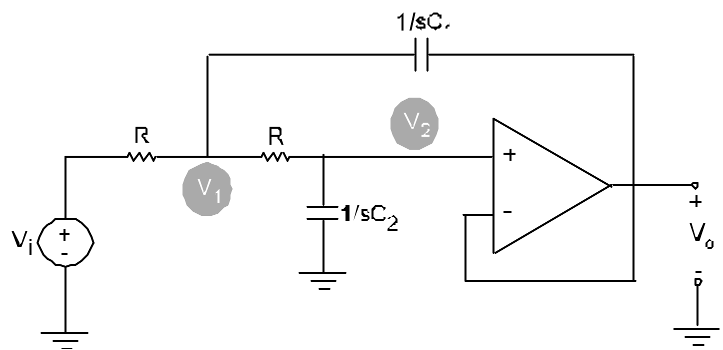


Figure 9.59: Input Bias Current Compensation

Performing nodal analysis (since the op-amp is a voltage follower, V_o is on the output and the inputs),

KCL at V_1: $$\frac{V_1 - V_i}{R} + \frac{V_1 - V_2}{R} + \frac{V_1 - V_o}{1/sC_1} = 0$$

KCL at V_2: $$\frac{V_2 - V_1}{R} + \frac{V_2}{1/sC_2} = 0$$

Ideal op-amp: $$V_2 = V_o$$

Using Maple to solve for V_o in terms of V_i,

```
> restart:
> eq1:=(V[1]-V[i])/R+(V[1]-V[2])/R+(V[1]-V[o])*s*C[1]=0:
> eq2:=(V[2]-V[1])/R+V[2]*s*C[2]=0:
> eq3:=V[2]=V[o]:soln:=solve({eq1,eq2,eq3},{V[1],V[2],V[o]}):
> assign(soln):
> V[o]:=collect(V[o],s):
> TF:=V[o]/V[i];
```

$$H := \frac{1}{R^2 C_1 C_2 s^2 + 2RC_2 s + 1}$$

In this case, the output is very simply related to one of the node voltages ($V_o = V_2$). In a more general case, however, one is still assured that the output can be expressed as a linear combination of node voltages.

9.10.2 *Mechanical System with Multiple Outputs and Multiple Outputs*

For systems with multiple inputs and multiple outputs (MIMO systems), superposition can be used to turn one input on at a time and the relation between one particular input and each respective output. For the MIMO case, the transfer function is a matrix.

In a two-input, two-output system, for example, the transfer function relationship would be,

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$\begin{aligned} X_1 &= H_{11}\, F_1 + H_{12}\, F_2 \\ X_2 &= H_{21}\, F_1 + H_{22}\, F_2 \end{aligned}$$

Example 10

For the system shown below, consider the two inputs to be the applied torques, T_1 and T_2, and the outputs as θ_1 and θ_2. J_1 and J_2 are rotational inertias, b_1 and b_2 represent rotational viscous dampers, and k_1 and k_2 represent the spring constants of the compliant shafts.

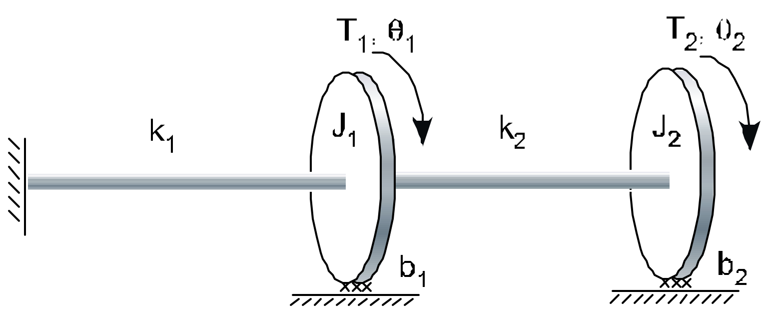


Figure 9.60: Fourth-Order Mechanical System

The equations of motion are,

$$\begin{aligned} J_1 s^2 \theta_1 &= T_1 - k_1\theta_1 + k_2(\theta_2 - \theta_1) - b_1 s\theta_1 \\ J_2 s^2 \theta_2 &= T_2 - k_2(\theta_2 - \theta_1) - b_2 s\theta_2 \end{aligned}$$

Using MAPLE,

```
> restart:
> eq1:=J[1]*s^2*theta1=T[1]-k[1]*theta1+k[2]*(theta2-theta1)-b[1]*s*theta1:
> eq2:=J[2]*s^2*theta2=T[2]-k[2]*(theta2-theta1)-b[2]*s*theta2:
> soln:=solve({eq1,eq2},{theta1,theta2}):
```

```
> assign(soln):
> H[11]:=subs({T[1]=1,T[2]=0},theta1);
> H[12]:=subs({T[1]=0,T[2]=1},theta1);
> H[21]:=subs({T[1]=1,T[2]=0},theta2);
> H[22]:=subs({T[1]=0,T[2]=1},theta2);
```

$$H_{11} = \frac{J_2 s^2 + b_2 s + k_2}{J_1 J_2 s^4 + (J_1 b_2 + J_2 b_1)s^3 + (b_1 b_2 + J_1 k_2 + J_2 k_1 + J_2 k_2)s^2 + (b_1 k_2 + b_2 k_1 + b_2 k_2)s + k_1 k_2}$$

$$H_{12} = \frac{k_2}{J_1 J_2 s^4 + (J_1 b_2 + J_2 b_1)s^3 + (b_1 b_2 + J_1 k_2 + J_2 k_1 + J_2 k_2)s^2 + (b_1 k_2 + b_2 k_1 + b_2 k_2)s + k_1 k_2}$$

$$H_{21} = \frac{k_2}{J_1 J_2 s^4 + (J_1 b_2 + J_2 b_1)s^3 + (b_1 b_2 + J_1 k_2 + J_2 k_1 + J_2 k_2)s^2 + (b_1 k_2 + b_2 k_1 + b_2 k_2)s + k_1 k_2}$$

$$H_{22} = \frac{J_1 s^2 + b_1 s + (k_1 + k_2)}{J_1 J_2 s^4 + (J_1 b_2 + J_2 b_1)s^3 + (b_1 b_2 + J_1 k_2 + J_2 k_1 + J_2 k_2)s^2 + (b_1 k_2 + b_2 k_1 + b_2 k_2)s + k_1 k_2}$$

9.11 Design Examples

9.11.1 *Variable Gain Instrumentation Amplifier*

In Fig. 9.15, an improved differential amplifier with high input impedance was obtained by applying voltage followers at the input. A variable gain amplifier, still with high input impedance is given in Fig. 9.55.

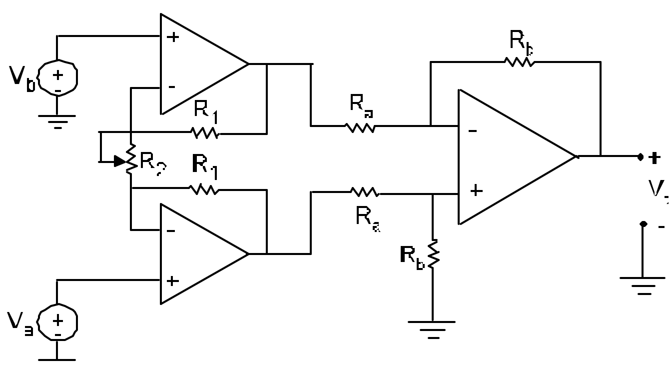


Figure 9.61: Variable Gain Instrumentation Amplifier

$$V_o = \left(\frac{2R_1}{R_2} + 1\right)\frac{R_b}{R_a}(V_a - V_b)$$

9.11.2 *Charge Amplifier*

Some sensors, such as a piezoelectric accelerometer can supply hardly any current. Fortunately there are op-amps available that have extremely low input bias currents of 10 fA and less (for example, the LMC 6081 and LMC 6042 from National Semiconductor). These amplifiers can easily serve as charge amplifiers for most piezoelectric amplifiers. The following circuit for an instrumentation amplifier with input currents of less than 100 fA appears on the LMC 6081 data sheet.

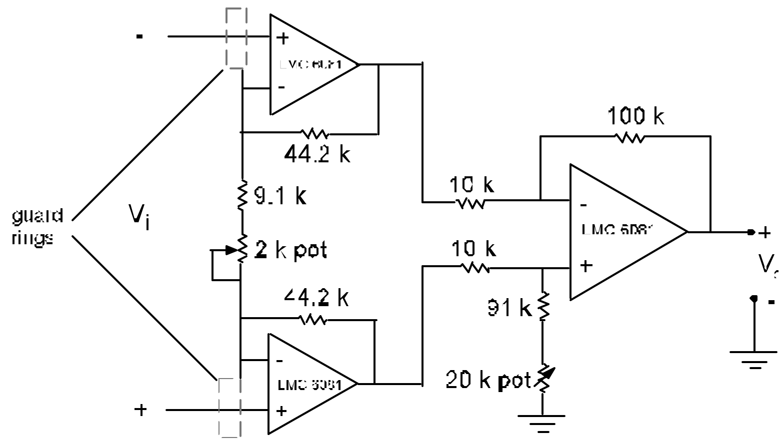


Figure 9.62: Amplifier on LMC 6081 Data Sheet

The guard rings do not appear on the data sheet schematic but are help lower parasitic input capacitance which is helpful when ultra-low input currents are needed.