PHYSICS

FOR SCIENTISTS AND ENGINEERS **A STRATEGIC APPROACH** 4/E

Chapter 10 Lecture

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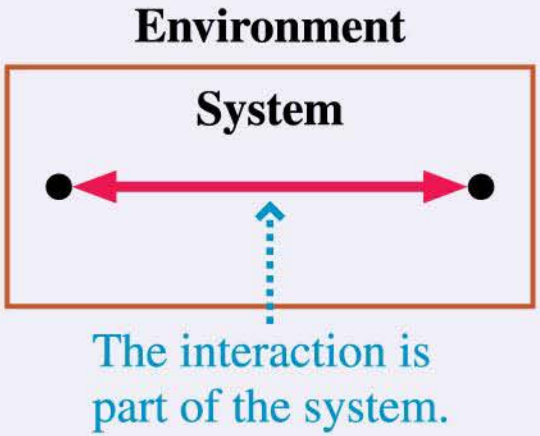
Chapter 10 Interactions and Potential Energy

IN THIS CHAPTER, you will develop a better understanding of energy and its conservation.

Chapter 10 Preview

How do interactions affect energy?

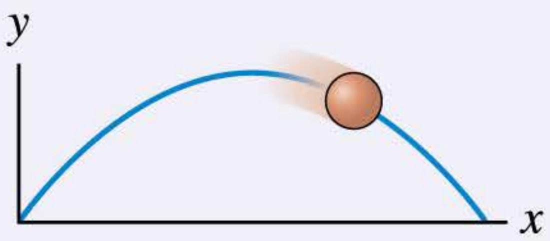
We continue our investigation of energy by allowing interactions to be part of the system, rather than external forces. You will learn that interactions can store energy within the system. Further, this interaction energy can be transformed—via the interaction forces—into kinetic energy.

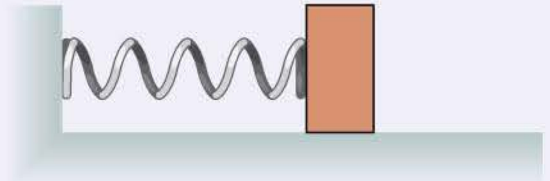


What is potential energy?

Interaction energy is usually called potential energy. There are many kinds of potential energy, each associated with *position*.

- Gravitational potential energy changes with height.
- Elastic potential energy changes with stretching.



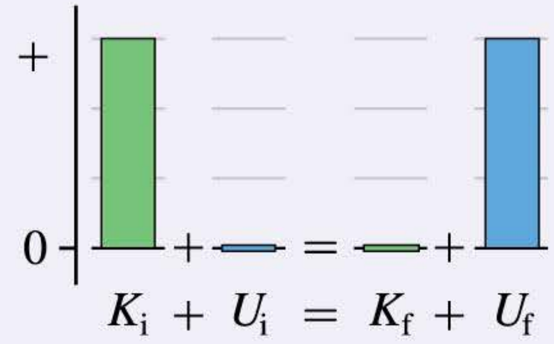


« LOOKING BACK Section 9.1 Energy overview

When is energy conserved?

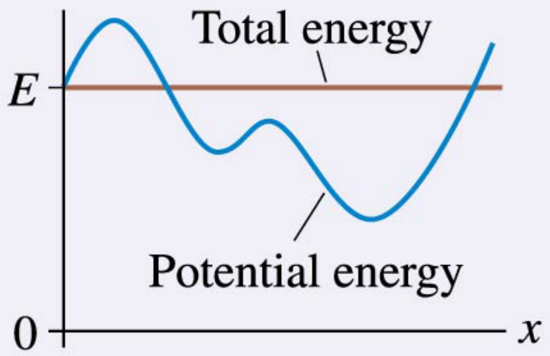
- If a system is isolated, its total energy is conserved.
- If a system both is isolated and has *no dissipative forces*, its mechanical energy, $K + U$, is conserved.

Energy bar charts are a tool for visualizing energy conservation.



What is an energy diagram?

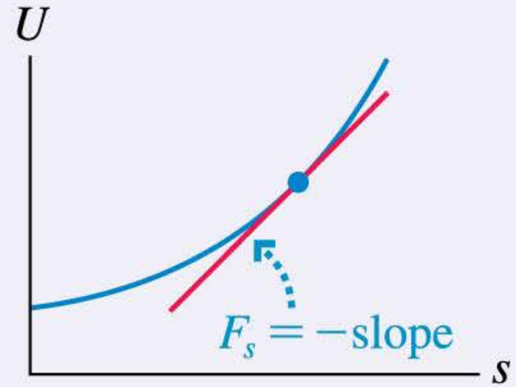
An energy diagram is a graphical representation of how the energy of a particle changes as it moves. Turning points occur where the total energy line crosses the potential-energy curve. And potential-energy minima are points of stable equilibrium.



How is force related to potential energy?

Only certain types of forces, called conservative forces, are associated with a potential energy. For these forces,

- The work done changes the potential energy by $\Delta U = -W$.
- Force is the negative of the slope of the potential-energy curve.



Where are we now in our study of energy?

Energy is a big topic, not one that can be presented in a single chapter. Chapters 9 and 10 are primarily about mechanical energy and the mechanical transfer of energy via work. And we've touched on thermal energy because it's unavoidable in realistic mechanical systems with friction. These are related by the energy principle:

$$\Delta E_{\text{sys}} = \Delta K + \Delta U + \Delta E_{\text{th}} = W_{\text{ext}}$$

Part V of this book, Thermodynamics, will expand our energy ideas to include heat and a deeper understanding of thermal energy. Then we'll add another form of energy—electric energy—in Part VI.

Chapter 10 Reading Questions

Reading Question 10.1

A particular interaction force does work W_{int} inside a system. The potential energy of the interaction is U. Which equation relates U and W_{int}?

A. $U = W_{\text{int}}$

B. $\Delta U = W_{\text{int}}$

C. $U = -W_{\text{int}}$

D. $\Delta U = -\mathrm{W}_{\text{int}}$

Reading Question 10.1

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A. $U = W_{\text{int}}$

B. $\Delta U = W_{\text{int}}$

C. $U = -W_{\text{int}}$

✔ **D.** $\boldsymbol{\Delta U = -W_{\text{int}}}$

Reading Question 10.2

Gravitational potential energy is

A. The area under a gravity force-versus-time graph.
B. The gravitation constant times mass-squared divided by distance-squared.
C. Mass times the acceleration due to gravity times vertical position.
D. ½ mass times speed-squared.
E. Velocity per unit mass.

Reading Question 10.2

Gravitational potential energy is

A. The area under a gravity force-versus-time graph.
B. The gravitation constant times mass-squared divided by distance-squared.
✔ **C. Mass times the acceleration due to gravity times vertical position.**
D. ½ mass times speed-squared.
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Reading Question 10.3

A method for keeping track of transformations between kinetic energy and gravitational potential energy, introduced in this chapter, is

A. Credit-debit tables.
B. Kinetic energy-versus-time graphs.
C. Energy bar charts.
D. Energy conservation pools.
E. Energy spreadsheets.

Reading Question 10.3

A method for keeping track of transformations between kinetic energy and gravitational potential energy, introduced in this chapter, is

A. Credit-debit tables.

B. Kinetic energy-versus-time graphs.

C. Energy bar charts.

D. Energy conservation pools.

E. Energy spreadsheets.

Reading Question 10.4

Mechanical energy is

A. The energy due to internal moving parts.
B. The energy of motion.
C. The energy of position.
D. The sum of kinetic energy plus potential energy.
E. The sum of kinetic, potential, thermal, and elastic energy.

Reading Question 10.4

Mechanical energy is

A. The energy due to internal moving parts.

B. The energy of motion.

C. The energy of position.

D. **The sum of kinetic energy plus potential energy.**

E. The sum of kinetic, potential, thermal, and elastic energy.

Reading Question 10.5

For conservative forces, Force can be found as being the negative of the derivative of

A. Impulse.
B. Kinetic energy.
C. Momentum.
D. Potential energy.
E. Work.

Reading Question 10.5

For conservative forces, Force can be found as being the negative of the derivative of

A. Impulse.

B. Kinetic energy.

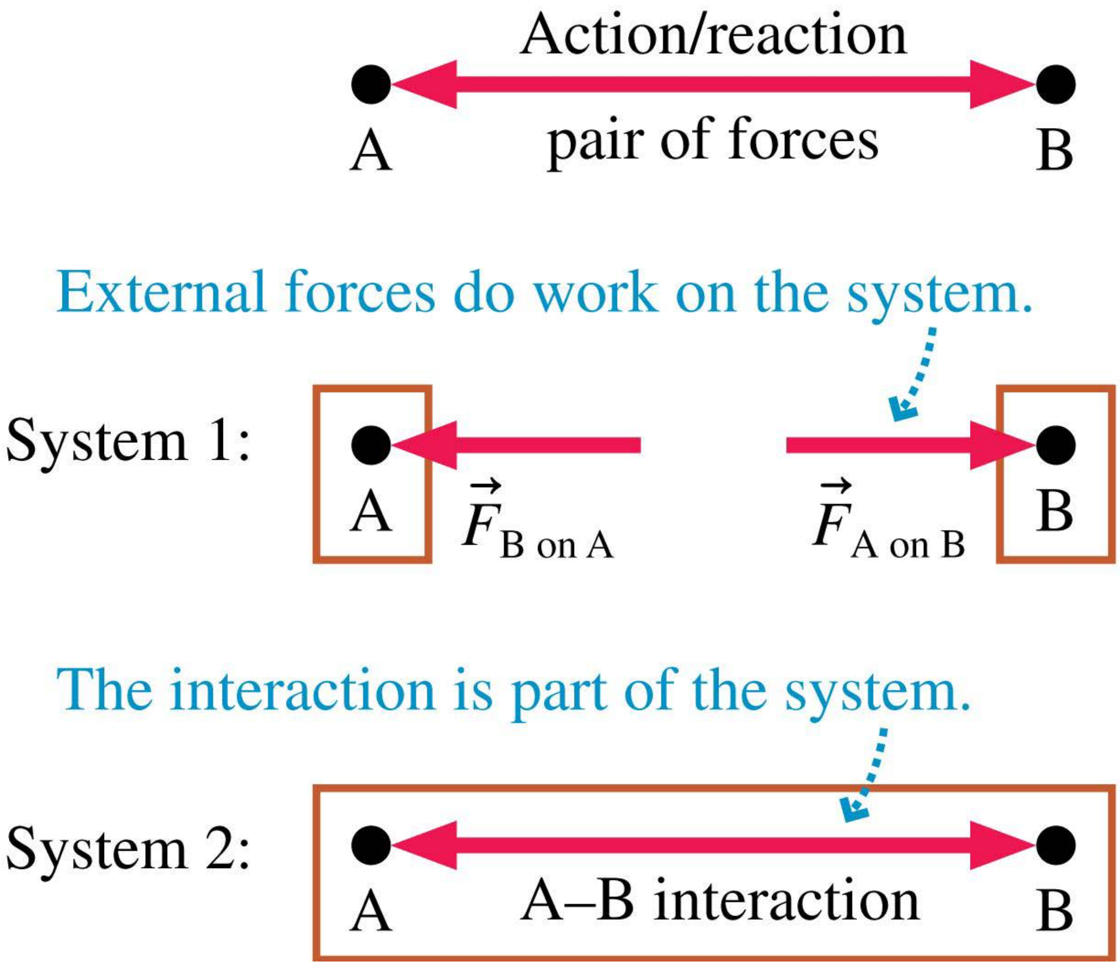
C. Momentum.

D. Potential energy.

E. Work.

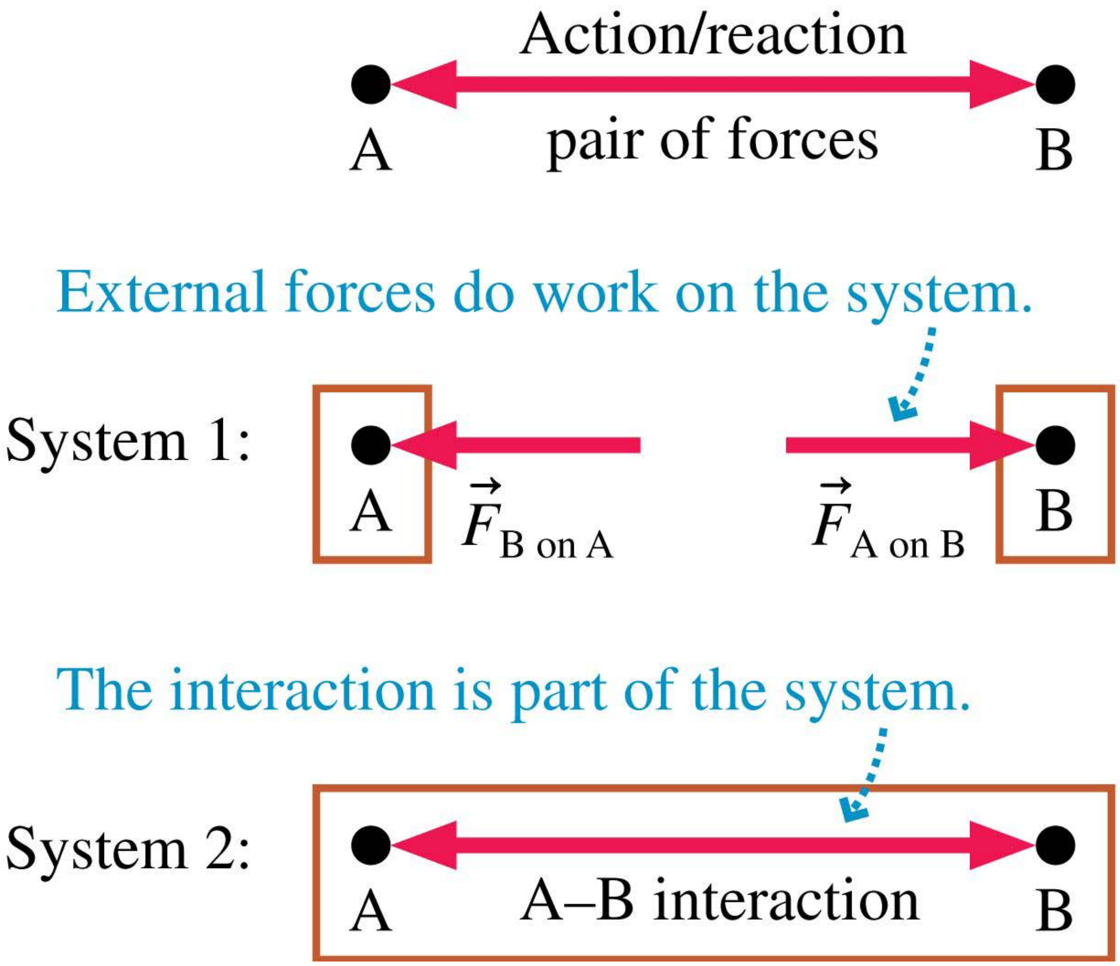
Chapter 10 Content, Examples, and QuickCheck Questions

Potential Energy



- Consider two particles A and B that interact with each other and nothing else.
- There are two ways to define a system.
- System 1 consists only of the two particles, the forces are external, and the work done by the two forces change the system's kinetic energy.

Potential Energy



- System 2 includes the interaction within the system.
- Since $W_{\text{ext}} = 0$, we must define an energy associated with the interaction, called the **potential energy**, U.
- When internal forces in the system do work, this changes the potential energy.

Potential Energy

- Consider two particles A and B that interact with each other and nothing else.
- If we define the system to include the interaction between the particles, then as these forces do work, the potential energy changes by

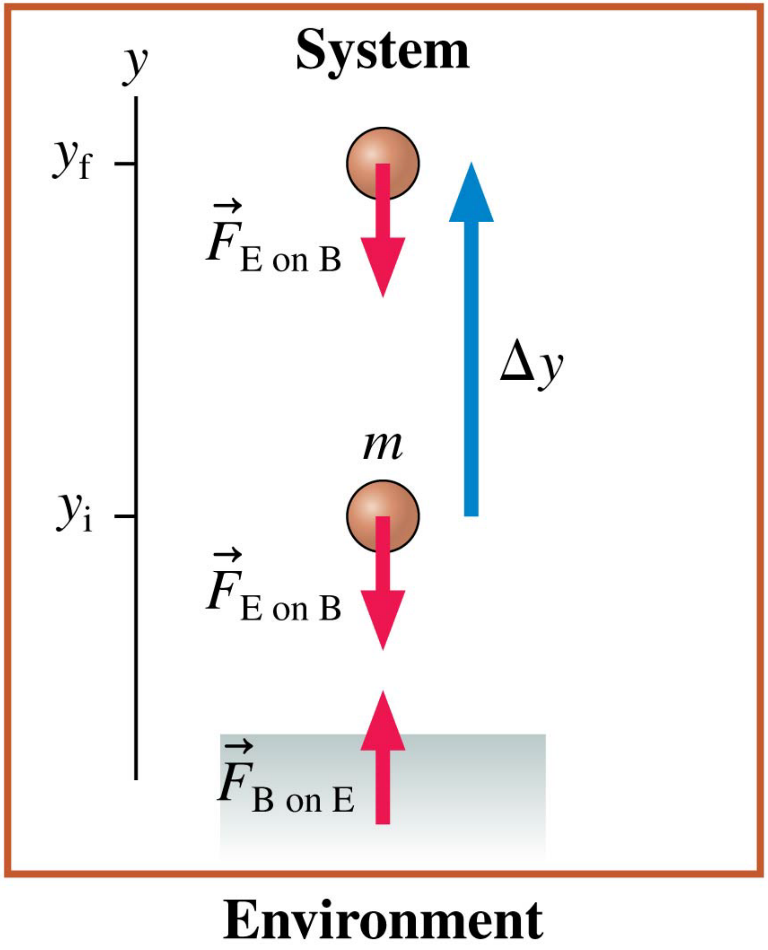
$$\Delta U = -(W_A + W_B) = -W_{int}$$

where W_{int} is the total work done inside the system by the interaction forces.

- The system's kinetic energy can increase if its potential energy decreases by the same amount.
- In effect, **the interaction stores energy inside the system** with the *potential* to be converted to kinetic energy hence the name *potential energy*.

Gravitational Potential Energy

- The figure shows a ball of mass m moving upward from an initial vertical position y_i to a final vertical position y_f.
- Let's define the system to be ball+earth, including their gravitational interaction.
- This introduces an energy of interaction, the gravitational potential energy, U_G, which changes by



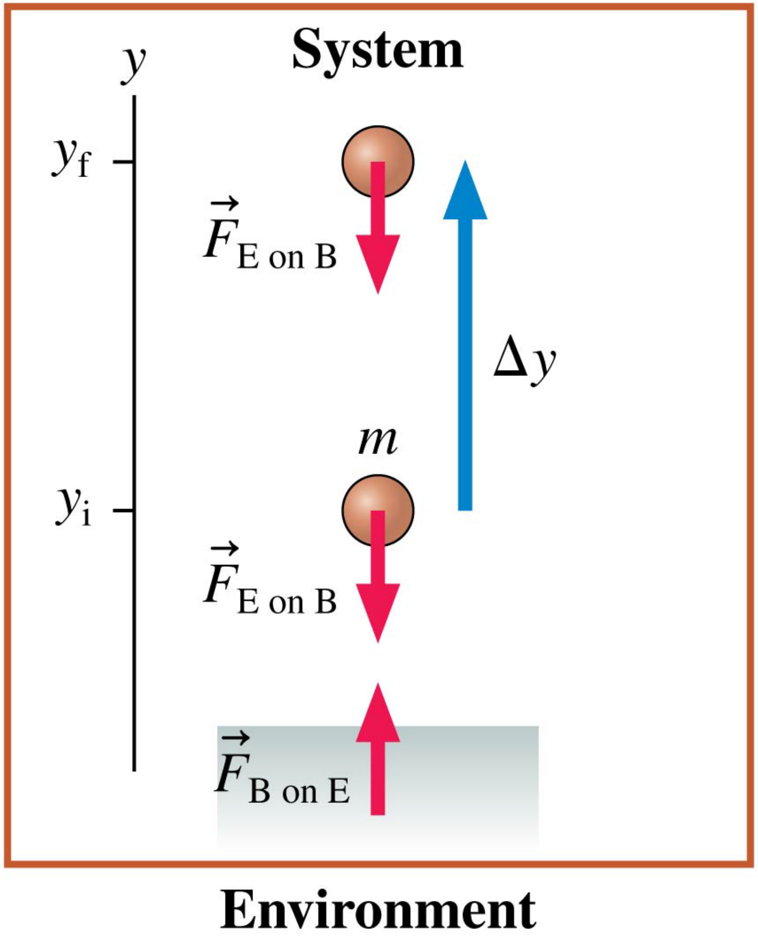
$$\Delta U_G = -(W_B + W_E)$$

where W_B is the work gravity does on the ball and W_E is the work gravity does on the earth.

Gravitational Potential Energy

- W_E is practically zero since the Earth has almost no displacement as the ball moves.
- The force of gravity on the ball is $(F_G)_y = -mg$.
- So, as the ball moves up a distance Δy, the gravitational potential energy changes by

$$\Delta U_G = -W_B = mg\,\Delta y$$



Gravitational Potential Energy

- Define **gravitational potential energy** as an energy of position:

$$U_G = mgy \qquad \text{(gravitational potential energy)}$$

- The sum $K + U_G$ is not changed when an object is in free fall. Its initial and final values are equal:

$$K_f + U_{Gf} = K_i + U_{Gi}$$

Example 10.1 Launching a Pebble

EXAMPLE 10.1 | **Launching a pebble**

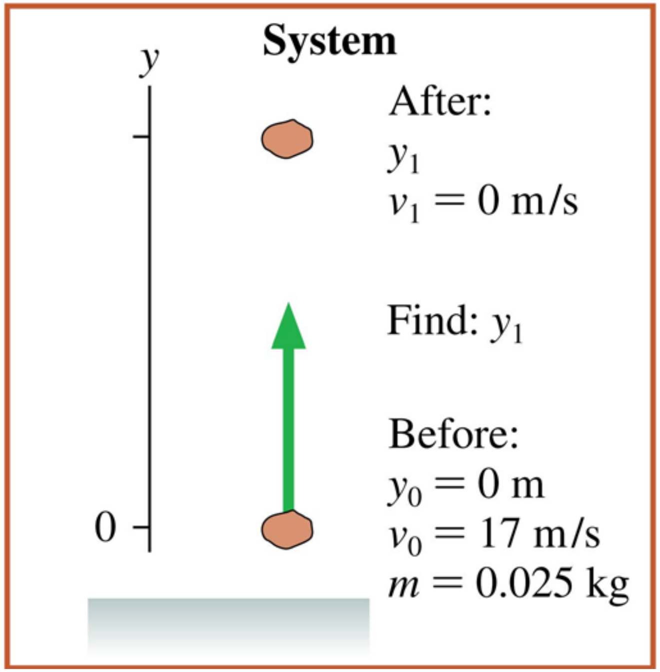
Rafael uses a slingshot to shoot a 25 g pebble straight up at 17 m/s. How high does the pebble go?

MODEL Let the system consist of both the earth and the pebble, which we model as a particle. Assume that air resistance is negligible. There are no external forces to do work, but the system does have gravitational potential energy.

Example 10.1 Launching a Pebble

EXAMPLE 10.1 | **Launching a pebble**

VISUALIZE FIGURE 10.3 is a before-and-after pictorial representation. The before-and-after representation will continue to be our primary visualization tool.



Example 10.1 Launching a Pebble

EXAMPLE 10.1 | **Launching a pebble**

SOLVE The energy principle for the pebble + earth system is

$$\Delta E_{sys} = \Delta K + \Delta U_G = W_{ext} = 0$$

That is, the system energy does not change at all. Instead, kinetic energy is transformed into potential energy without loss inside the system. In principle, the kinetic energy is that of the ball plus the kinetic energy of the earth. But as we just noted, the enormous mass difference means that the earth is effectively at rest while the pebble does all the moving, so the only kinetic energy we need to consider is that of the pebble. Thus we have

$$0 = \Delta K + \Delta U_G = (\tfrac{1}{2}mv_1^{\,2} - \tfrac{1}{2}mv_0^{\,2}) + (mgy_1 - mgy_0)$$

Example 10.1 Launching a Pebble

EXAMPLE 10.1 **Launching a pebble**

SOLVE We know that $v_1 = 0$ m/s and we chose a coordinate system in which $y_0 = 0$m, so we're left with

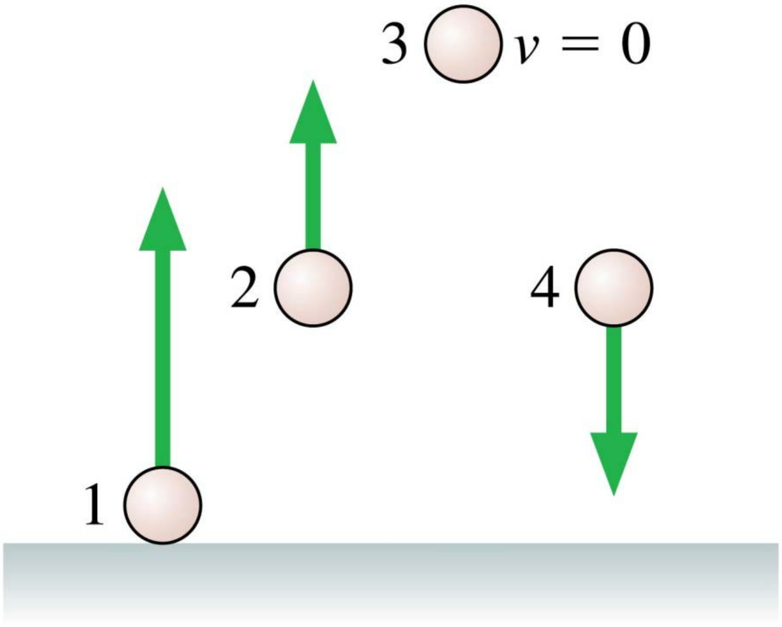
$$y_1 = \frac{v_0^2}{2g} = \frac{(17\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 15\text{ m}$$

The answer did not depend on the pebble's mass, which is not surprising after our earlier practice with free-fall problems.

ASSESS A height of 15 m45 ft seems reasonable for a slingshot.

QuickCheck 10.1

Rank in order, from largest to smallest, the gravitational potential energies of the balls.



A. $1 > 2 = 4 > 3$

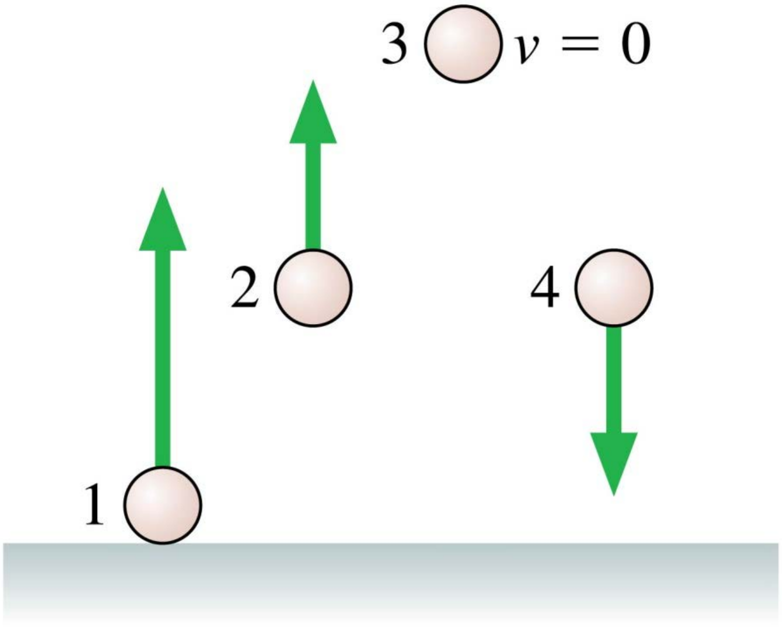
B. $1 > 2 > 3 > 4$

C. $3 > 2 > 4 > 1$

D. $3 > 2 = 4 > 1$

QuickCheck 10.1

Rank in order, from largest to smallest, the gravitational potential energies of the balls.



A. $1 > 2 = 4 > 3$

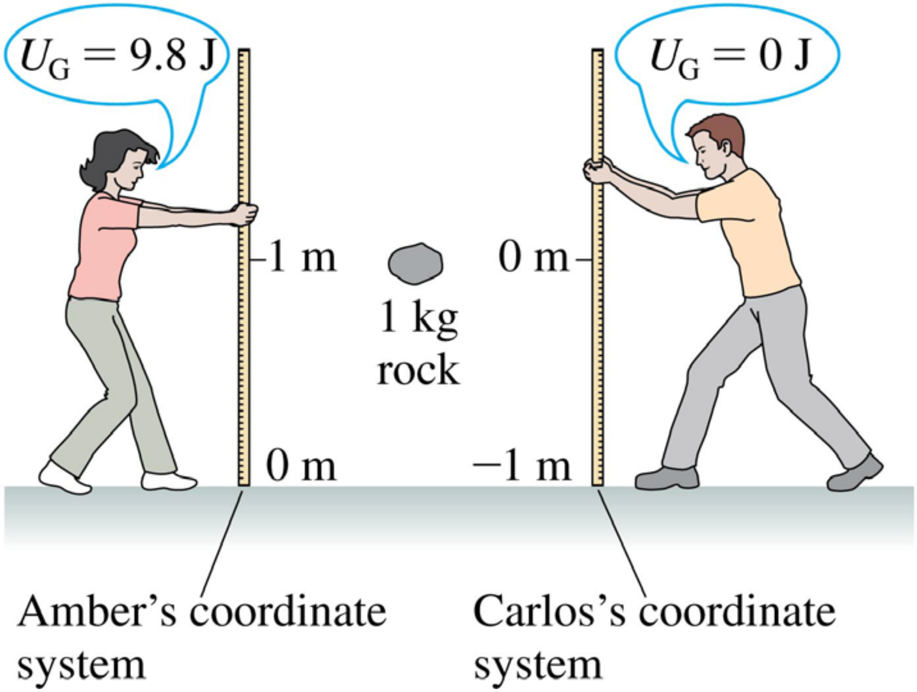
B. $1 > 2 > 3 > 4$

C. $3 > 2 > 4 > 1$

✔ **D. $3 > 2 = 4 > 1$**

The Zero of Potential Energy

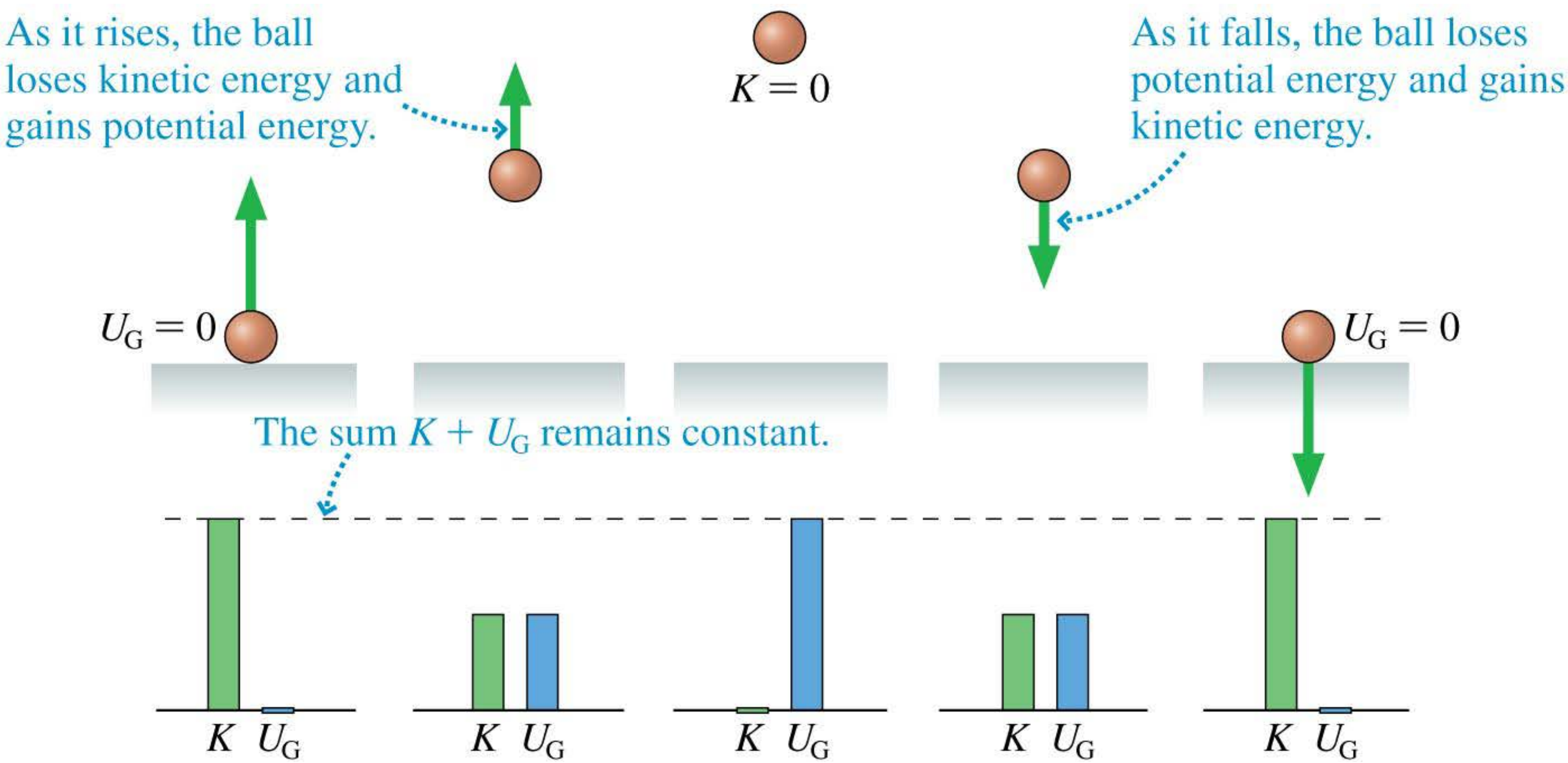
- Amber and Carlos use coordinate systems with different origins to determine the potential energy of a rock.
- No matter where the rock is, Amber's value of U_G will be equal to Carlos's value plus 9.8 J.



- If the rock moves, both will calculate exactly the *same* value for ΔU_G.
- In problems, only ΔU_G has physical significance, not the value of U_G itself.

Energy Bar Charts

- A ball is tossed up into the air.
- The simple bar charts below show how the sum of $K + U_G$ remains constant as the pebble rises and then falls.



Example 10.2 Dropping a Watermelon

EXAMPLE 10.2 | **Dropping a watermelon**

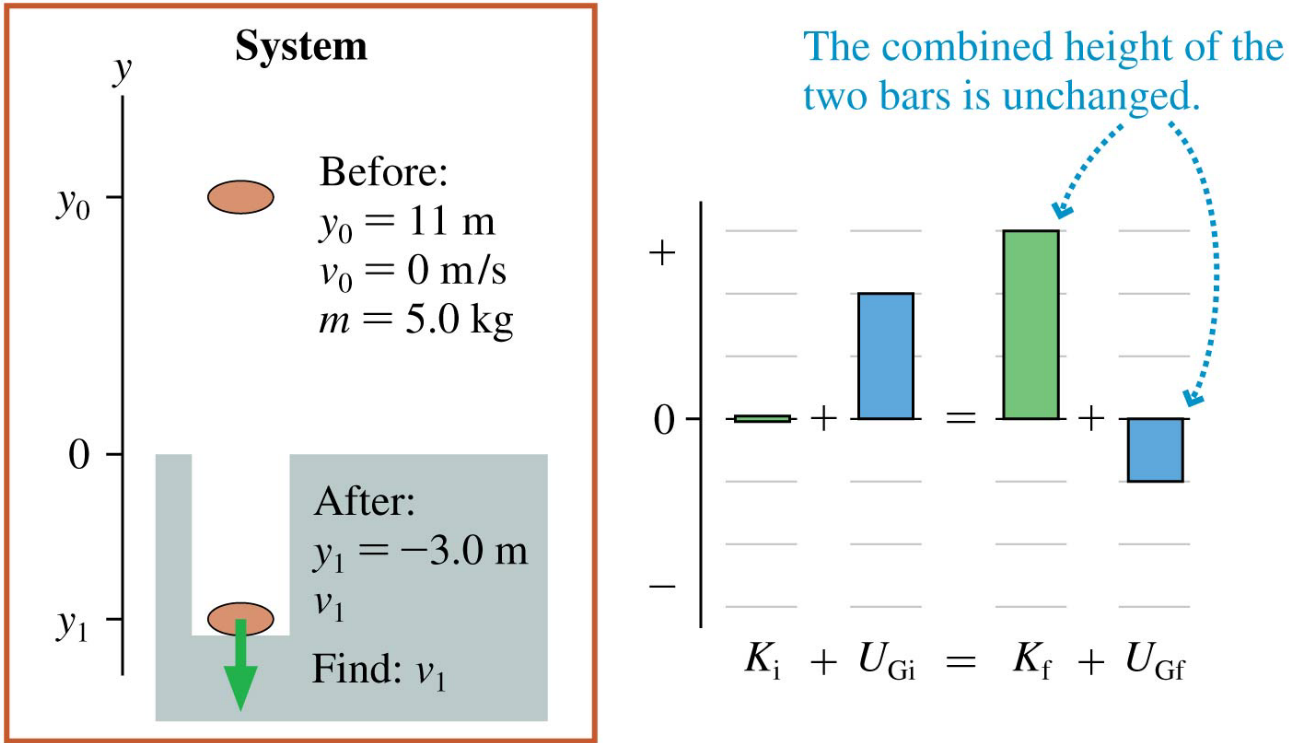
A 5.0 kg watermelon is dropped from a third-story balcony, 11 m above the street. Unfortunately, the water department forgot to replace the cover on a manhole, and the watermelon falls into a 3.0-m-deep hole. How fast is the watermelon going when it hits bottom?

MODEL Let the system consist of both the earth and the watermelon, which we model as a particle. Assume that air resistance is negligible. There are no external forces, and the motion is vertical, so the system's mechanical energy is conserved.

Example 10.2 Dropping a Watermelon

EXAMPLE 10.2 | **Dropping a watermelon**

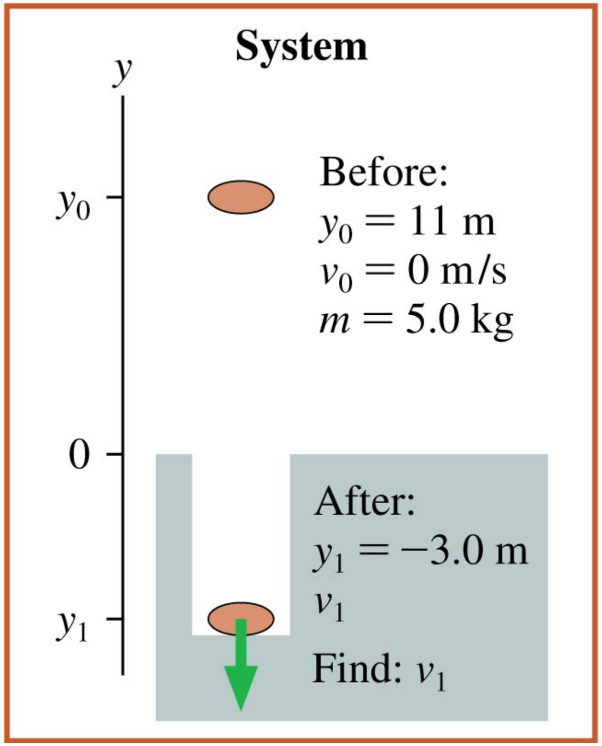
VISUALIZE **FIGURE 10.6** shows both a before-and-after pictorial representation and an energy bar chart. Initially the system has gravitational potential energy but no kinetic energy. Potential energy is transformed into kinetic energy as the watermelon falls.

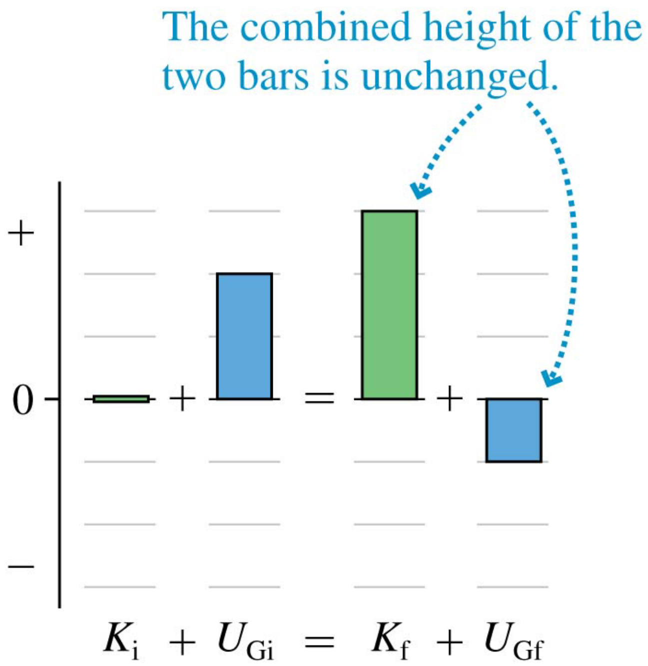


Example 10.2 Dropping a Watermelon

EXAMPLE 10.2 | **Dropping a watermelon**

VISUALIZE Our choice of the *y*-axis origin has placed the zero of potential energy at ground level, so the potential energy is negative when the watermelon reaches the bottom of the hole. Even so, the combined height of the two bars has not changed.





Example 10.2 Dropping a Watermelon

EXAMPLE 10.2 **Dropping a watermelon**

SOLVE The energy principle for the watermelon + earth system, written as a conservation statement, is

$$K_i + U_{Gi} = 0 + mgy_0 = K_f + U_{Gf} = \frac{1}{2}mv_1^2 + mgy_1$$

Solving for the impact speed, we find

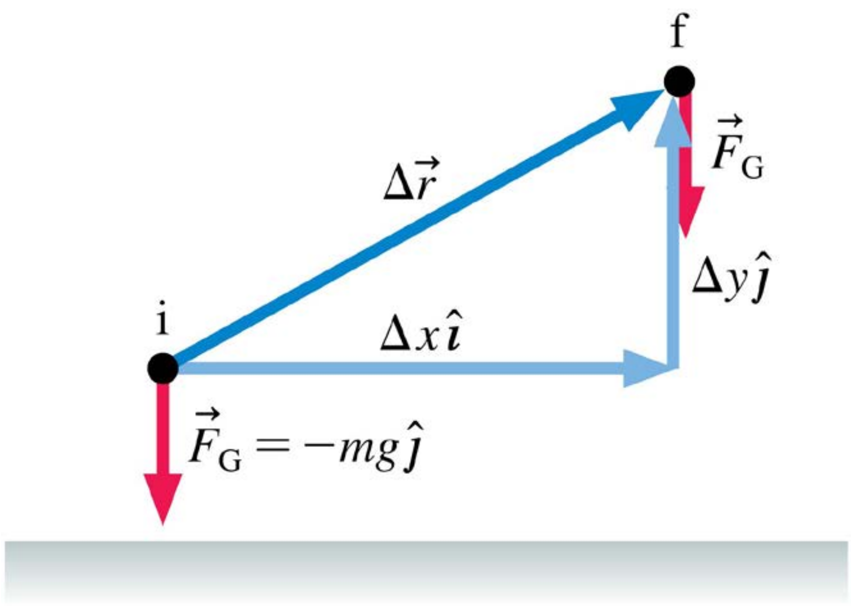
$$\begin{aligned} v_1 &= \sqrt{2g(y_0 - y_1)} \\ &= \sqrt{2(9.80\ \text{m/s}^2)(11.0\ \text{m} - (-3.0\ \text{m}))} \\ &= 17\ \text{m/s} \end{aligned}$$

Example 10.2 Dropping a Watermelon

EXAMPLE 10.2 | **Dropping a watermelon**

ASSESS A speed of 17 m/s ≈ 35 mph seems reasonable for the watermelon after falling ≈ 4 stories. In thinking about this problem, you might be concerned that, once below ground level, potential energy continues being transformed into kinetic energy even though the potential energy is "less than none." Keep in mind that the actual value of U is not relevant because we can place the zero of potential energy anywhere we wish, so a negative potential energy is just a number with no implication that it's "less than none." There's no "storehouse" of potential energy that might run dry. As long as the interaction acts, potential energy can continue being transformed into kinetic energy.

Gravitational Potential Energy



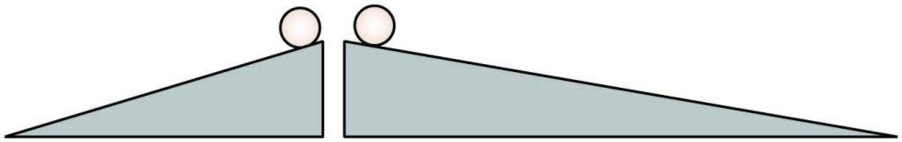
- The figure shows a particle of mass m moving at an angle while acted on by gravity.
- The work done by gravity is

$$W_{\text{by grav}} = \vec{F}_G \cdot \Delta\vec{r} = (F_G)_x(\Delta r_x) + (F_G)_y(\Delta r_y) = 0 + (-mg)(\Delta y)$$
$$= -mg\,\Delta y$$

- Because the force of gravity has no x-component, the work depends only on the vertical displacement Δy.
- Consequently, **the change in gravitational potential energy depends only on an object's vertical displacement.**

QuickCheck 10.2

Starting from rest, a marble first rolls down a steeper hill, then down a less steep hill of the same height. For which is it going faster at the bottom?



A. Faster at the bottom of the steeper hill.

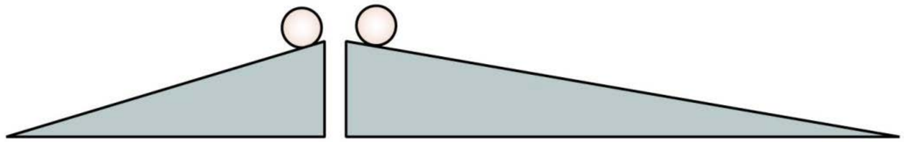
B. Faster at the bottom of the less steep hill.

C. Same speed at the bottom of both hills.

D. Can’t say without knowing the mass of the marble.

QuickCheck 10.2

Starting from rest, a marble first rolls down a steeper hill, then down a less steep hill of the same height. For which is it going faster at the bottom?



A. Faster at the bottom of the steeper hill.

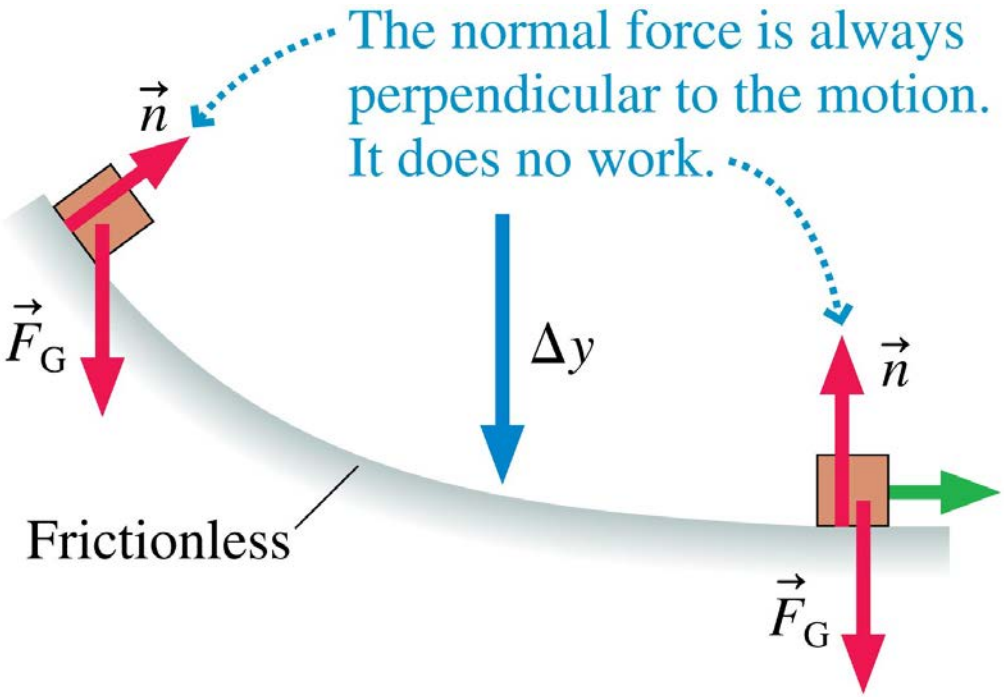
B. Faster at the bottom of the less steep hill.

✔ **C. Same speed at the bottom of both hills.**

D. Can't say without knowing the mass of the marble.

Gravitational Potential Energy

- The figure shows an object sliding down a curved, frictionless surface.
- The change in gravitational potential energy of the object + earth system depends only on Δy, the distance the object descends, not on the shape of the curve.



- The normal force is always perpendicular to the box’s instantaneous displacement, so it does no work.

QuickCheck 10.3

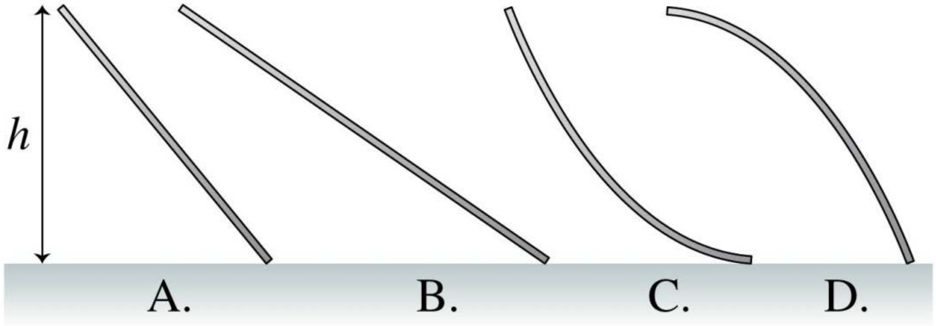
A small child slides down the four frictionless slides A–D. Rank in order, from largest to smallest, her speeds at the bottom.

A. $v_D > v_A > v_B > v_C$

B. $v_D > v_A = v_B > v_C$

C. $v_C > v_A > v_B > v_D$

D. $v_A = v_B = v_C = v_D$



QuickCheck 10.3

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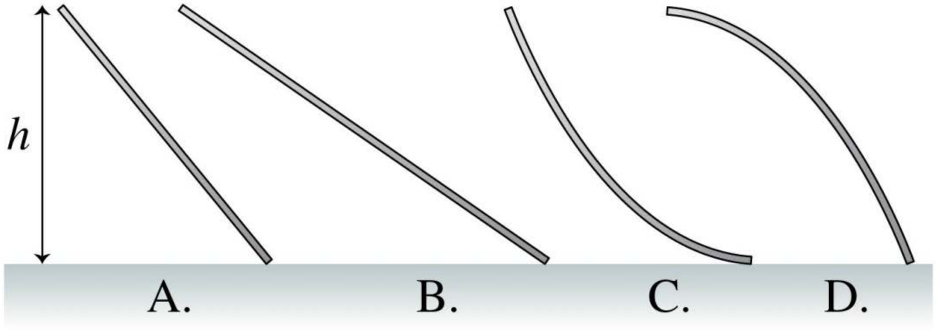
A. $v_D > v_A > v_B > v_C$

B. $v_D > v_A = v_B > v_C$

C. $v_C > v_A > v_B > v_D$

✔ **D.** $\boldsymbol{v_A = v_B = v_C = v_D}$



Example 10.3 The Speed of a Sled

EXAMPLE 10.3 The speed of a sled

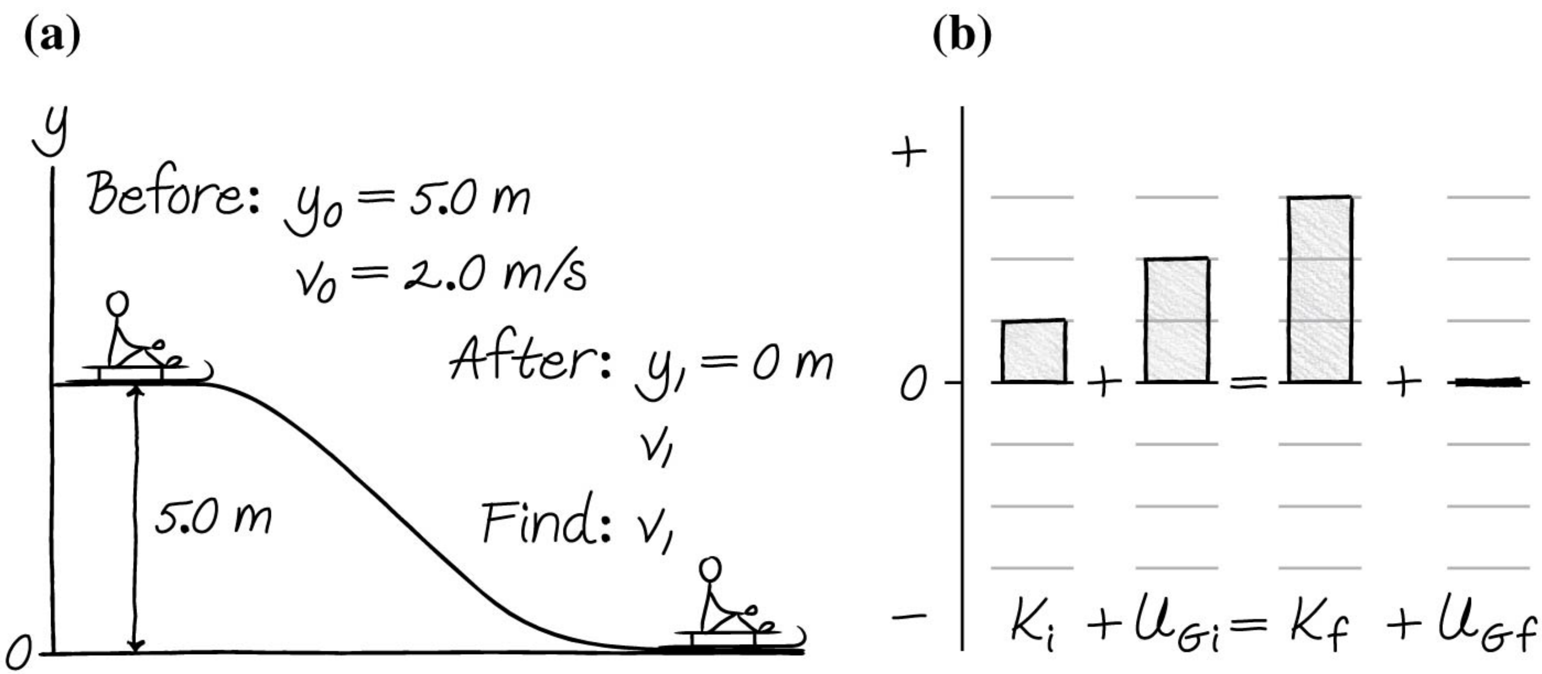
Christine runs forward with her sled at 2.0 m/s. She hops onto the sled at the top of a 5.0-m-high, very slippery slope. What is her speed at the bottom?

MODEL Let the system consist of the earth and the sled, which we model as a particle. Because the slope is "very slippery," we'll assume that friction is negligible. The slope exerts a normal force on the sled, but it is always perpendicular to the motion and does not affect the energy.

Example 10.3 The Speed of a Sled

EXAMPLE 10.3 | **The speed of a sled**

VISUALIZE **FIGURE 10.9a** shows a before-and-after pictorial representation. We are not told the angle of the slope, or even if it is a straight slope, but the change in potential energy depends only on the vertical distance Christine descends and *not* on the shape of the hill. **FIGURE 10.9b** is an energy bar chart in which we see an initial kinetic *and* potential energy being transformed into entirely kinetic energy as Christine goes down the slope.



Example 10.3 The Speed of a Sled

EXAMPLE 10.3 | **The speed of a sled**

SOLVE The energy analysis is just like in Example 10.2; the fact that the object is moving on a curved surface hasn't changed anything. The energy principle, written as a conservation statement, is

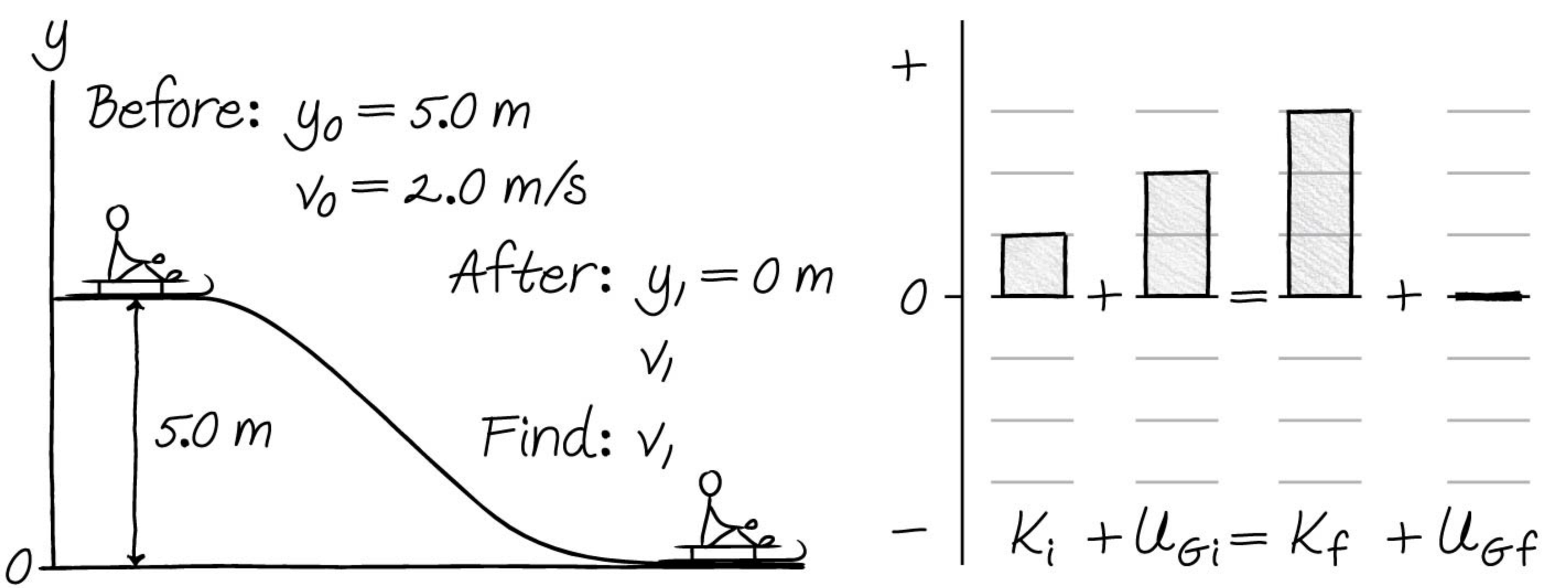
$$K_i + U_{Gi} = \tfrac{1}{2}mv_0^{\,2} + mgy_0$$
$$= K_f + U_{Gf} = \tfrac{1}{2}mv_1^{\,2} + 0$$

Her speed at the bottom is

$$v_1 = \sqrt{v_0^{\,2} + 2gy_0}$$
$$= \sqrt{(2.0\ \text{m/s})^2 + 2(9.80\ \text{m/s}^2)(5.0\ \text{m})}$$
$$= 10\ \text{m/s}$$

ASSESS 10 m/s ≈ 20 mph is fast but believable for a 5 m ≈ 15 ft descent.

(a) **(b)**



Motion with Gravity and Friction

- Friction increases the thermal energy of the system—defined to include both objects—by $\Delta E_{th} = f_k \Delta s$.
- For a system with both gravitational potential energy and friction, the energy principle becomes

$$K_i + U_{Gi} = K_f + U_{Gf} + \Delta E_{th}$$

- The mechanical energy $K + U_G$ is *not* conserved if there is friction.
- Because $\Delta E_{th} > 0$ (friction always makes surfaces hotter, never cooler), the final mechanical energy is less than the initial kinetic energy.
- Some fraction of the initial kinetic and potential energy is transformed into thermal energy during the motion.

QuickCheck 10.4

A child is on a playground swing, motionless at the highest point of his arc. What energy transformation takes place as he swings back down to the lowest point of his motion?

A. $K \rightarrow U_G$
B. $U_G \rightarrow K$
C. $E_{th} \rightarrow K$
D. $U_G \rightarrow E_{th}$
E. $K \rightarrow E_{th}$

QuickCheck 10.4

A child is on a playground swing, motionless at the highest point of his arc. What energy transformation takes place as he swings back down to the lowest point of his motion?

A. $K \rightarrow U_G$

✔ **B.** $\boldsymbol{U_G \rightarrow K}$

C. $E_{th} \rightarrow K$

D. $U_G \rightarrow E_{th}$

E. $K \rightarrow E_{th}$

QuickCheck 10.5

A skier is gliding down a gentle slope at a constant speed. What energy transformation is taking place?

A. $K \rightarrow U_G$
B. $U_G \rightarrow K$
C. $E_{th} \rightarrow K$
D. $U_G \rightarrow E_{th}$
E. $K \rightarrow E_{th}$

QuickCheck 10.5

A skier is gliding down a gentle slope at a constant speed. What energy transformation is taking place?

A. $K \rightarrow U_G$

B. $U_G \rightarrow K$

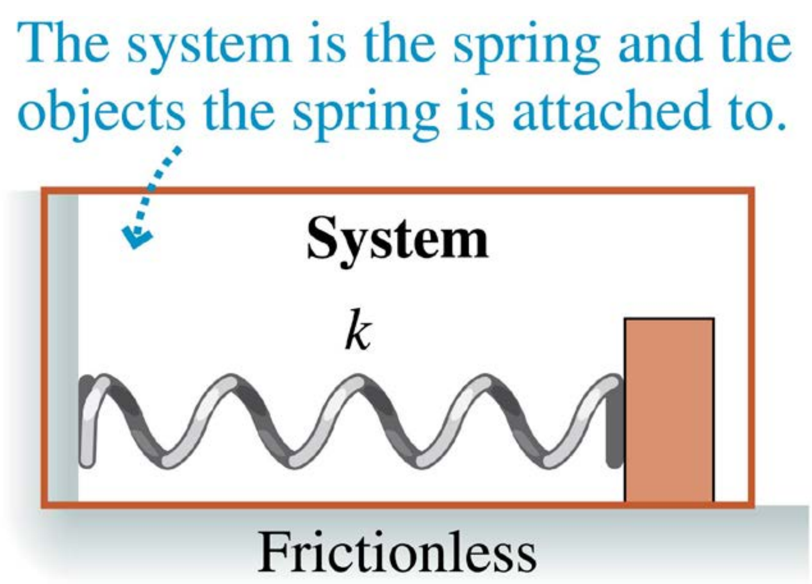
C. $E_{th} \rightarrow K$

✔ **D.** $\boldsymbol{U_G \rightarrow E_{th}}$

E. $K \rightarrow E_{th}$

Elastic Potential Energy

- The figure shows a spring exerting a force on a block while the block moves on a frictionless, horizontal surface.
- Let's define the system to be block + spring + wall.
- The spring is the interaction between the block and the wall.



- Because the interaction is inside the system, it has an interaction energy, the elastic potential energy, given by

$$U_{Sp} = \frac{1}{2}k(\Delta s)^2 \qquad \text{(elastic potential energy)}$$

where Δs is the displacement of the spring from its equilibrium length.

QuickCheck 10.6

A spring-loaded gun shoots a plastic ball with a launch speed of 2.0 m/s. If the spring is compressed twice as far, the ball's launch speed will be

A. 1.0 m/s
B. 2.0 m/s
C. 2.8 m/s
D. 4.0 m/s
E. 16.0 m/s

QuickCheck 10.6

A spring-loaded gun shoots a plastic ball with a launch speed of 2.0 m/s. If the spring is compressed twice as far, the ball's launch speed will be

A. 1.0 m/s
B. 2.0 m/s
C. 2.8 m/s
✔ **D. 4.0 m/s**
E. 16.0 m/s

Conservation of energy: $\frac{1}{2}mv^2 = \frac{1}{2}k(\Delta x)^2$

Double $\Delta x \rightarrow$ double v

QuickCheck 10.7

A spring-loaded gun shoots a plastic ball with a launch speed of 2.0 m/s. If the spring is replaced with a new spring having twice the spring constant (but still compressed the same distance), the ball's launch speed will be

A. 1.0 m/s
B. 2.0 m/s
C. 2.8 m/s
D. 4.0 m/s
E. 16.0 m/s

QuickCheck 10.7

A spring-loaded gun shoots a plastic ball with a launch speed of 2.0 m/s. If the spring is replaced with a new spring having twice the spring constant (but still compressed the same distance), the ball's launch speed will be

A. 1.0 m/s

B. 2.0 m/s

✔ **C. 2.8 m/s**

D. 4.0 m/s

E. 16.0 m/s

Conservation of energy: $\frac{1}{2}mv^2 = \frac{1}{2}k(\Delta x)^2$

Double k → increase v by square root of 2

Including Gravity

- For a system that has gravitational interactions, elastic interactions, and friction, but no external forces that do work, the energy principle is

$$K_{\mathrm{i}} + U_{\mathrm{Gi}} + U_{\mathrm{Sp\,i}} = K_{\mathrm{f}} + U_{\mathrm{Gf}} + U_{\mathrm{Sp\,f}} + \Delta E_{\mathrm{th}}$$

- This is looking a bit more complex as we have more and more energies to keep track of, but the message is both simple and profound:

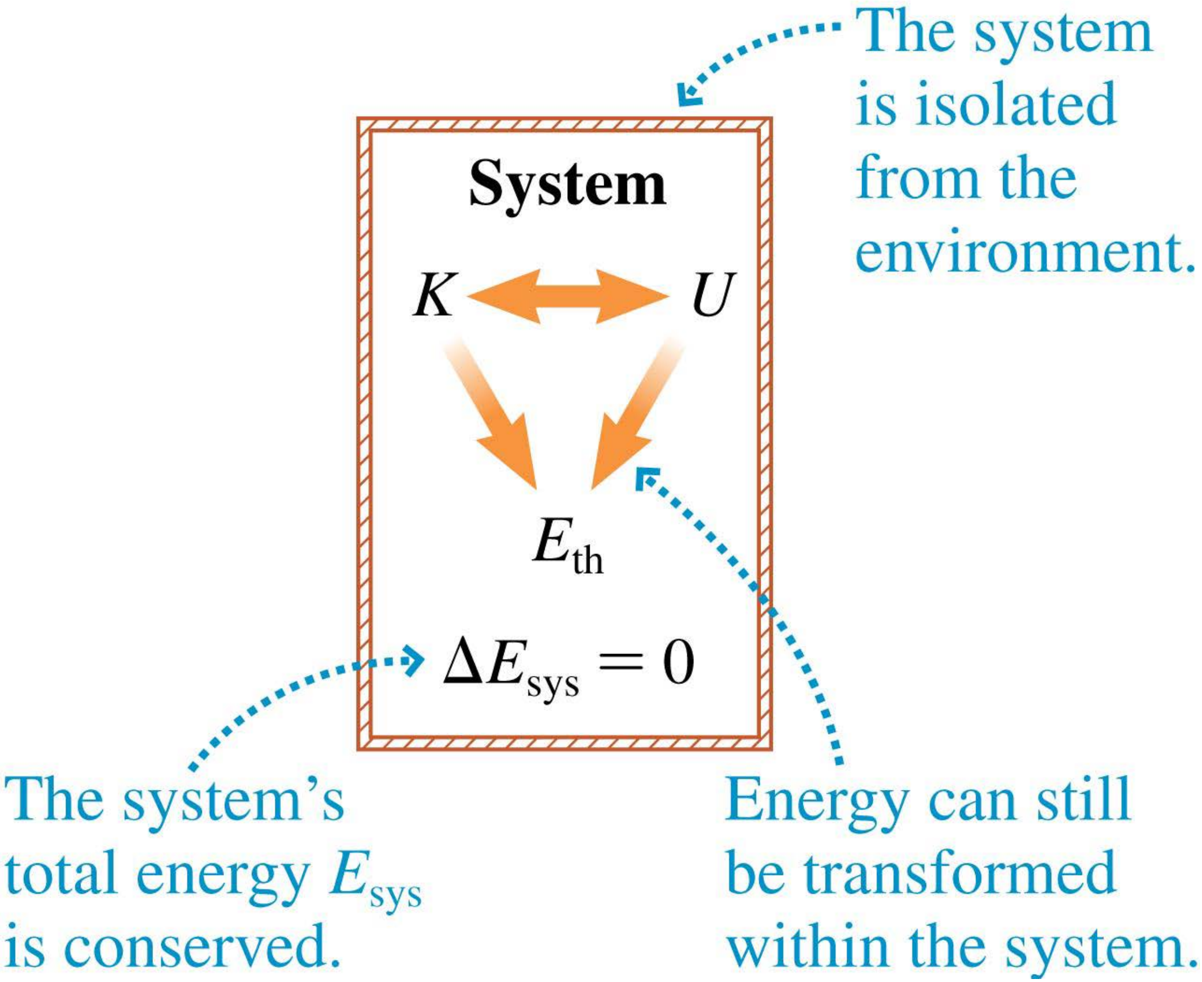
  For a system that has no other interactions with its environment, the total energy of the system does not change.
- It can be transformed in many ways by the interactions, but the total does not change.

Conservation of Energy

Law of conservation of energy The total energy $E_{sys} = E_{mech} + E_{th}$ of an isolated system is a constant. The kinetic, potential, and thermal energy within the system can be transformed into each other, but their sum cannot change. Further, the mechanical energy $E_{mech} = K + U$ is conserved if the system is both isolated and nondissipative.

The Basic Energy Model

- When a system is isolated, E_{sys}, the total energy of the system, is constant.



Problem-Solving Strategy: Energy-Conservation Problems

PROBLEM-SOLVING STRATEGY 10.1 MP

Energy-conservation problems

MODEL Define the system so that there are no external forces or so that any external forces do no work on the system. If there's friction, bring both surfaces into the system. Model objects as particles and springs as ideal.

VISUALIZE Draw a before-and-after pictorial representation and an energy bar chart. A free-body diagram may be needed to visualize forces.

SOLVE If the system is both isolated and nondissipative, then the mechanical energy is conserved:

$$K_i + U_i = K_f + U_f$$

where K is the total kinetic energy of all moving objects and U is the total potential energy of all interactions within the system. If there's friction, then

$$K_i + U_i = K_f + U_f + \Delta E_{th}$$

where the thermal energy increase due to friction is $\Delta E_{th} = f_k \Delta s$.

ASSESS Check that your result has correct units and significant figures, is reasonable, and answers the question.

Exercise 14

Example 10.7 The Speed of a Pendulum

EXAMPLE 10.7 | **The speed of a pendulum**

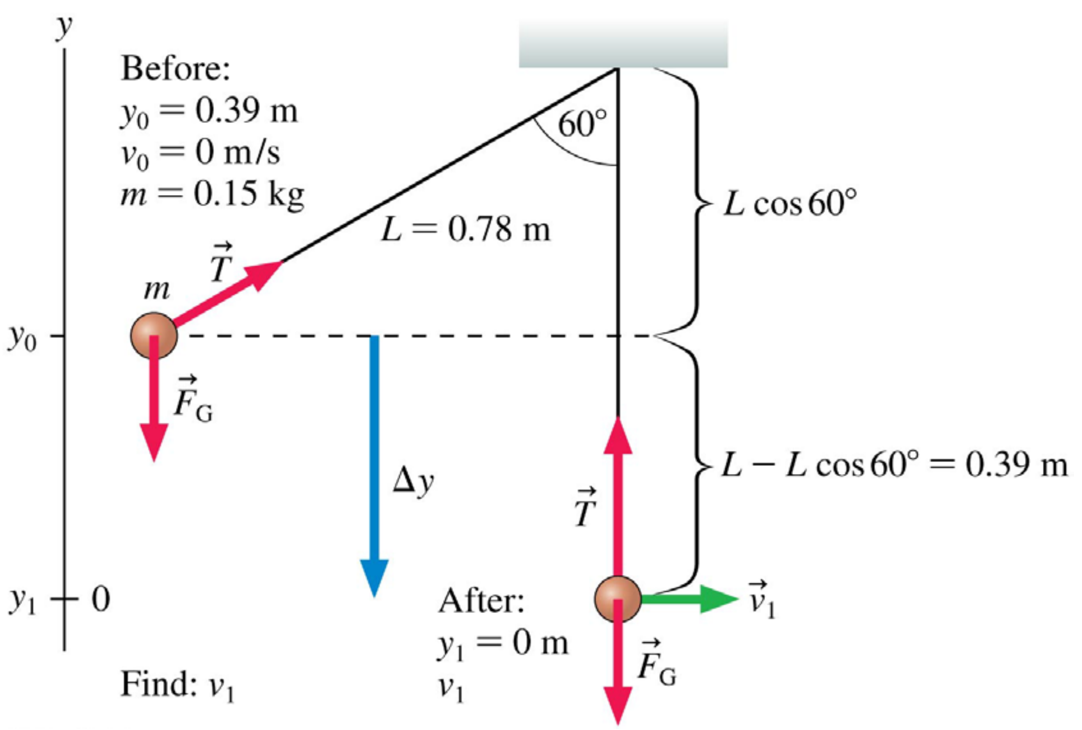
A pendulum is created by attaching one end of a 78-cm-long string to the ceiling and tying a 150 g steel ball to the other end. The ball is pulled back until the string is 60° from vertical, then released. What is the speed of the ball at its lowest point?

MODEL Let the system consist of the earth and the ball. The tension force, like a normal force, is always perpendicular to the motion and does no work, so this is an isolated system with no friction. Its mechanical energy is conserved.

Example 10.7 The Speed of a Pendulum

EXAMPLE 10.7 | **The speed of a pendulum**

VISUALIZE **FIGURE 10.16** shows a before-and-after pictorial representation, where we've placed the zero of potential energy at the lowest point of the ball's swing. Trigonometry is needed to determine the ball's initial height.



Example 10.7 The Speed of a Pendulum

EXAMPLE 10.7 **The speed of a pendulum**

SOLVE Conservation of mechanical energy is

$$K_i + U_{Gi} = 0 + mgy_0 = K_f + U_{Gf} = \frac{1}{2}mv_1^2 + 0$$

Thus the ball's speed at the bottom is

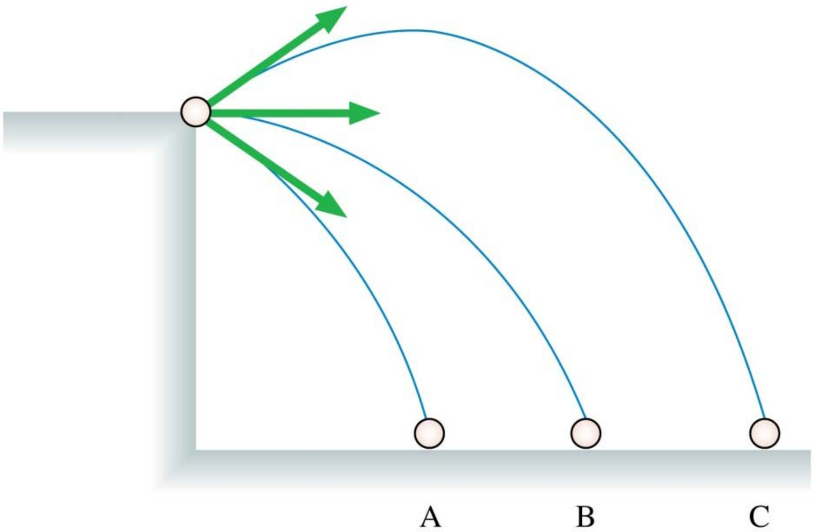
$$v_1 = \sqrt{2gy_0} = \sqrt{2(9.80\ \text{m/s}^2)(0.39\ \text{m})} = 2.8\ \text{m/s}$$

The speed is exactly the same as if the ball had simply fallen 0.39 m.

ASSESS To solve this problem directly from Newton's laws of motion requires advanced mathematics because of the complex way the net force changes with angle. But we can solve it in one line with an energy analysis!

QuickCheck 10.8

Three balls are thrown from a cliff with the same speed but at different angles. Which ball has the greatest speed just before it hits the ground?



A. Ball A

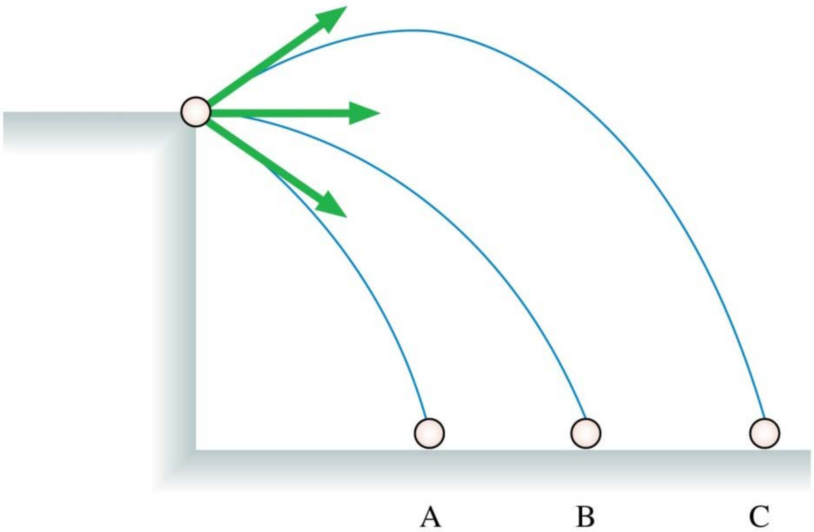
B. Ball B

C. Ball C

D. All balls have the same speed.

QuickCheck 10.8

Three balls are thrown from a cliff with the same speed but at different angles. Which ball has the greatest speed just before it hits the ground?



A. Ball A

B. Ball B

C. Ball C

✔ **D. All balls have the same speed.**

QuickCheck 10.9

A hockey puck sliding on smooth ice at 4 m/s comes to a 1-m-high hill. Will it make it to the top of the hill?

A. Yes

B. No

C. Can’t answer without knowing the mass of the puck.

D. Can’t say without knowing the angle of the hill.

QuickCheck 10.9

A hockey puck sliding on smooth ice at 4 m/s comes to a 1-m-high hill. Will it make it to the top of the hill?

A. Yes

✔ **B. No**

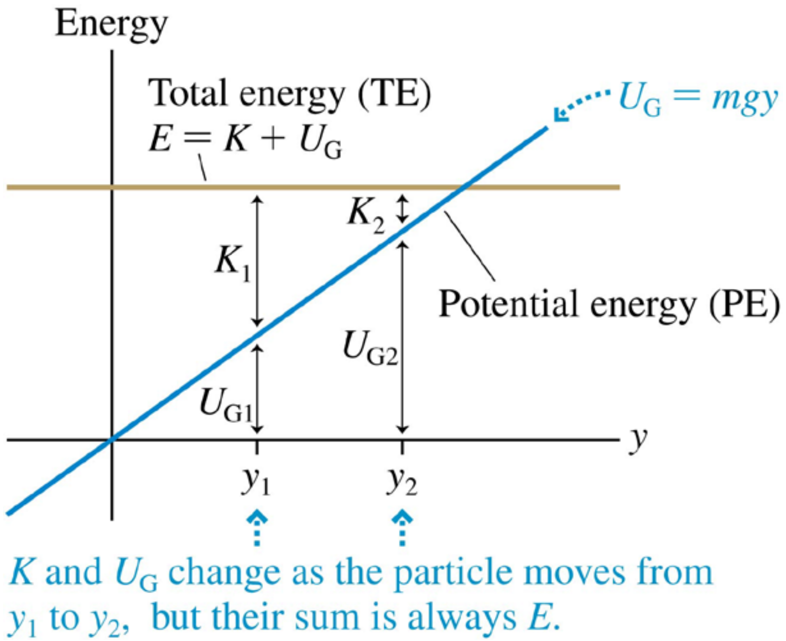
$$\frac{1}{2}mv^2 = mgy \text{ requires } v^2 = 2gy \approx 20\ \text{m}^2/\text{s}^2$$

C. Can't answer without knowing the mass of the puck.

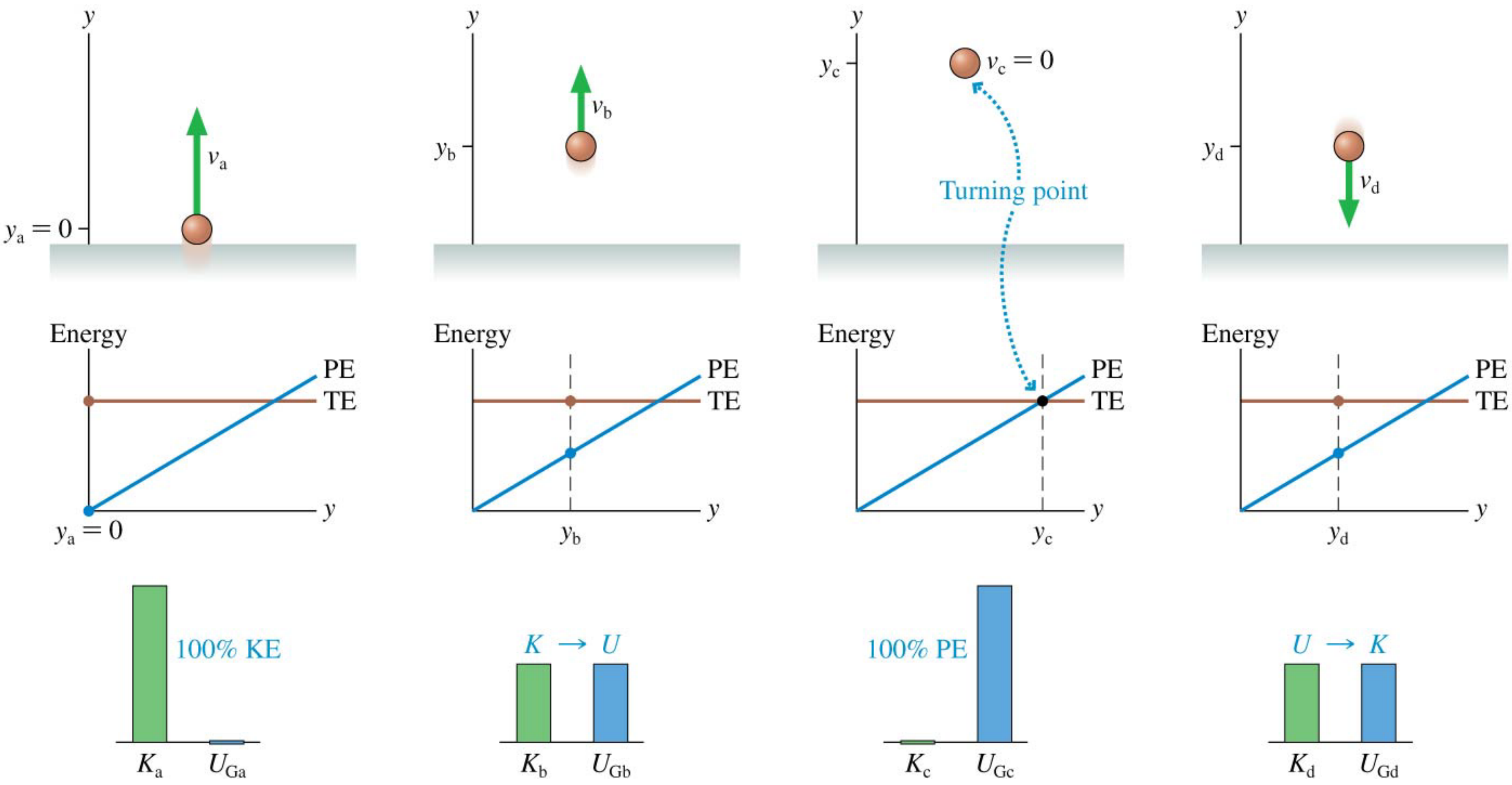
D. Can't say without knowing the angle of the hill.

Energy Diagrams

- Potential energy is a function of position.
- Functions of position are easy to represent as graphs.
- A graph showing a system's potential energy and total energy as a function of position is called an **energy diagram**.
- Shown is the energy diagram of a particle in free fall.
- Gravitational potential energy is a straight line with slope mg and zero y-intercept.
- Total energy is a horizontal line, since mechanical energy is conserved.



A Four-Frame Movie of a Particle in Free Fall

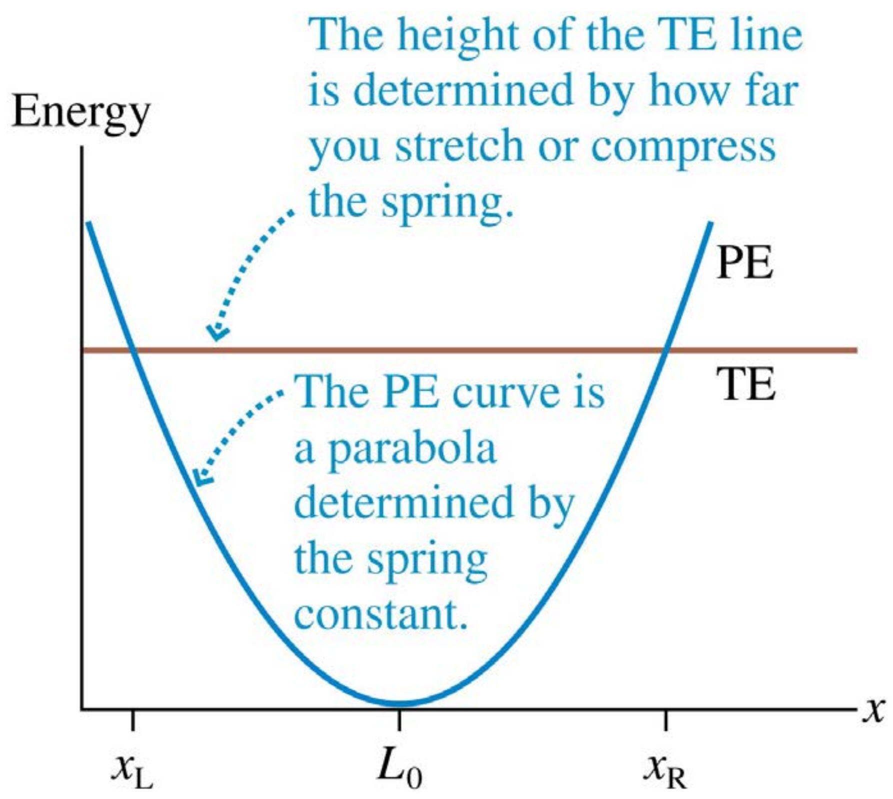


Energy Diagrams

- Shown is the energy diagram of a mass on a horizontal spring.
- The potential energy (PE) is the parabola:

$$U_{Sp} = ½k(x - L_0)^2$$

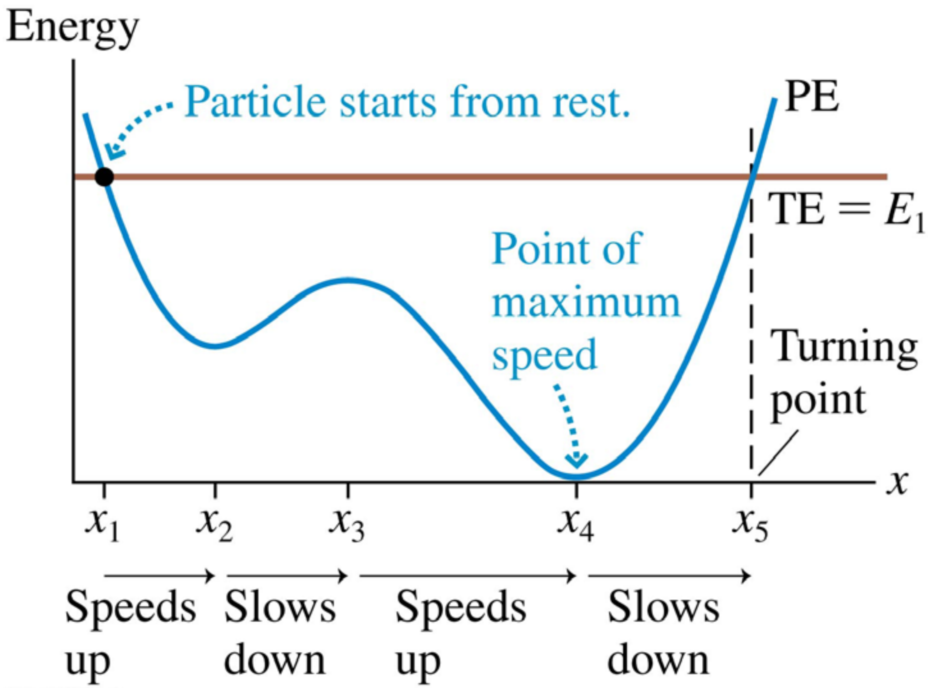
- The PE curve is determined by the spring constant; you can't change it.



- You can set the total energy (TE) to any height you wish simply by stretching the spring to the proper length at the beginning of the motion.

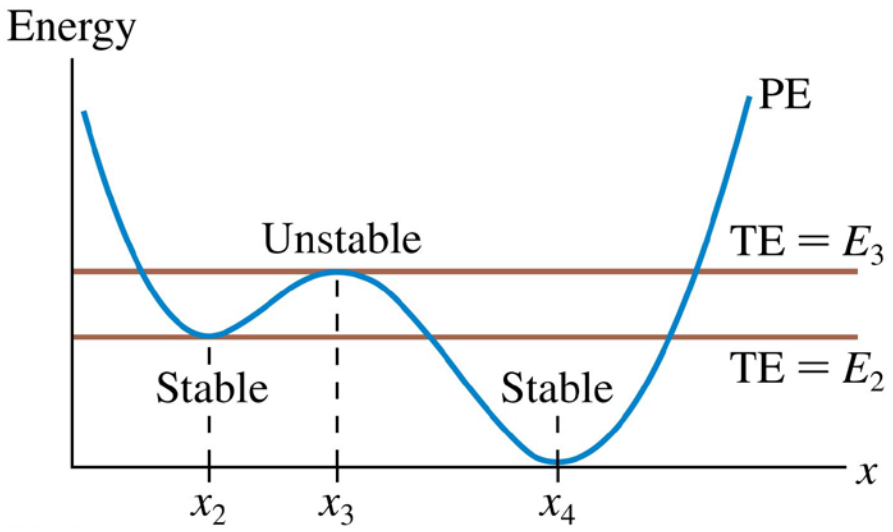
Energy Diagrams

- Shown is a more general energy diagram.
- The particle is released from rest at position x_1.
- Since K at x_1 is zero, the total energy $\mathrm{TE} = U$ at that point.
- The particle speeds up from x_1 to x_2.
- Then it slows down from x_2 to x_3.
- The particle reaches maximum speed as it passes x_4.
- When the particle reaches x_5, it turns around and reverses the motion.



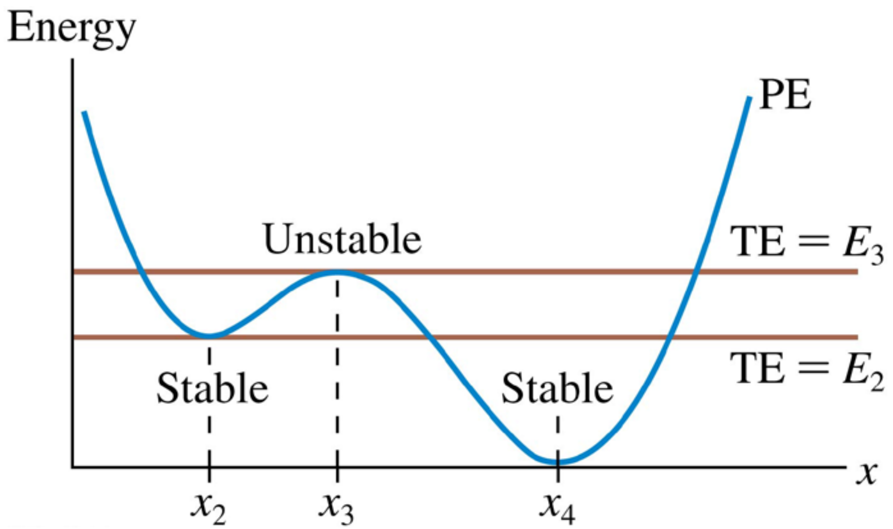
Equilibrium Positions: Stable

- Consider a particle with the total energy E_2 shown in the figure.
- The particle can be at rest at x_2, but it cannot move away from x_2: This is *static equilibrium*.



- If you disturb the particle, giving it a total energy slightly larger than E_2, it will oscillate very close to x_2.
- An equilibrium for which small disturbances cause small oscillations is called a point of **stable equilibrium**.

Equilibrium Positions: Unstable



- Consider a particle with the total energy E_3 shown in the figure.
- The particle can be at rest at x_3, and it does not move away from x_3: This is *static equilibrium*.
- If you disturb the particle, giving it a total energy slightly larger than E_3, it will speed up as it moves away from x_3.
- An equilibrium for which small disturbances cause the particle to move away is called a point of **unstable equilibrium**.

Tactics: Interpreting an Energy Diagram

TACTICS BOX 10.1

Interpreting an energy diagram

1. The distance from the axis to the PE curve is the particle's potential energy. The distance from the PE curve to the TE line is its kinetic energy. These are transformed as the position changes, causing the particle to speed up or slow down, but the sum $K + U$ doesn't change.
2. A point where the TE line crosses the PE curve is a turning point. The particle reverses direction.
3. The particle cannot be at a point where the PE curve is above the TE line.
4. The PE curve is determined by the properties of the system—mass, spring constant, and the like. You cannot change the PE curve. However, you can raise or lower the TE line simply by changing the initial conditions to give the particle more or less total energy.
5. A minimum in the PE curve is a point of stable equilibrium. A maximum in the PE curve is a point of unstable equilibrium.

Exercises 15–17

Example 10.8 Balancing a Mass on a Spring

EXAMPLE 10.8 | **Balancing a mass on a spring**

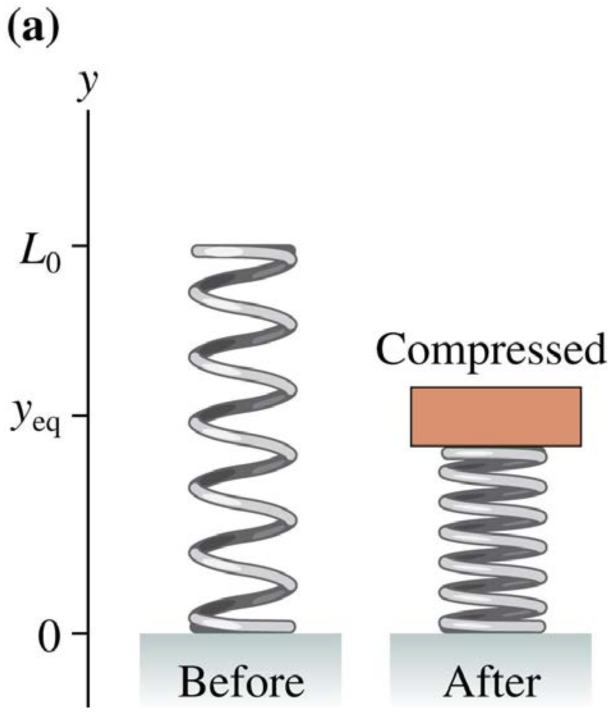
A spring of length L_0 and spring constant k is standing on one end. A block of mass m is placed on the spring, compressing it. What is the length of the compressed spring?

MODEL Assume an ideal spring obeying Hooke's law. The block + earth + spring system has both gravitational potential energy U_G *and* elastic potential energy U_{Sp}. The block sitting on top of the spring is at a point of stable equilibrium (small disturbances cause the block to oscillate slightly around the equilibrium position), so we can solve this problem by looking at the energy diagram.

Example 10.8 Balancing a Mass on a Spring

EXAMPLE 10.8 | **Balancing a mass on a spring**

VISUALIZE **FIGURE 10.22a** is a pictorial representation. We've used a coordinate system with the origin at ground level, so the displacement of the spring is $y - L_0$.

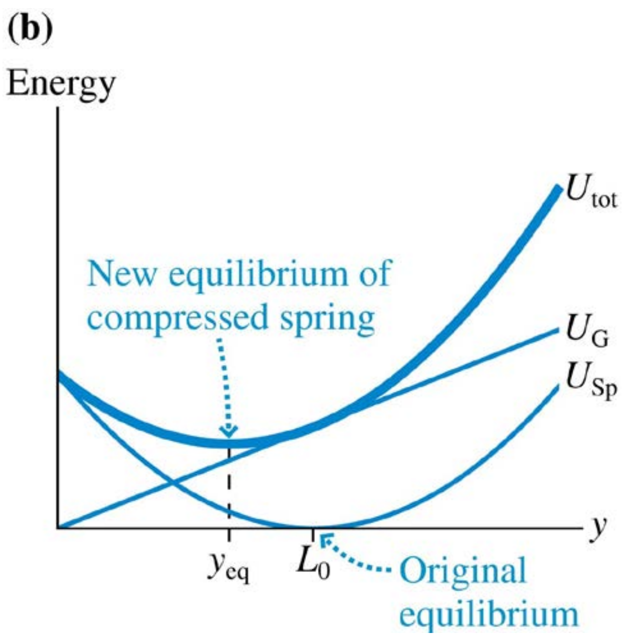


Example 10.8 Balancing a Mass on a Spring

EXAMPLE 10.8 | **Balancing a mass on a spring**

SOLVE **FIGURE 10.22b** shows the two potential energies separately and also shows the total potential energy:

$$U_{\text{tot}} = U_G + U_{Sp}$$
$$= mgy + \tfrac{1}{2}k(y - L_0)^2$$



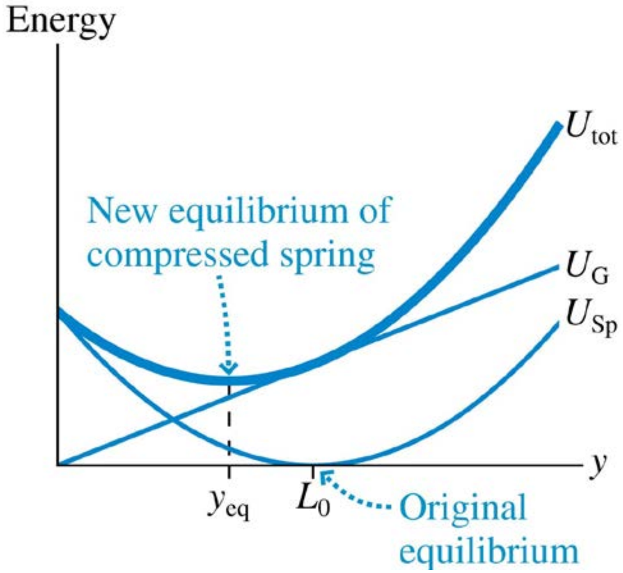
Example 10.8 Balancing a Mass on a Spring

EXAMPLE 10.8 **Balancing a mass on a spring**

SOLVE The equilibrium position (the minimum of U_{tot}) has shifted from L_0 to a smaller value of y, closer to the ground. We can find the equilibrium by locating the position of the minimum in the PE curve. You know from calculus that the minimum of a function is at the point where the derivative (or slope) is zero. The derivative of U_{tot} is

$$\frac{dU_{tot}}{dy} = mg + k(y - L_0)$$

(b)



Example 10.8 Balancing a Mass on a Spring

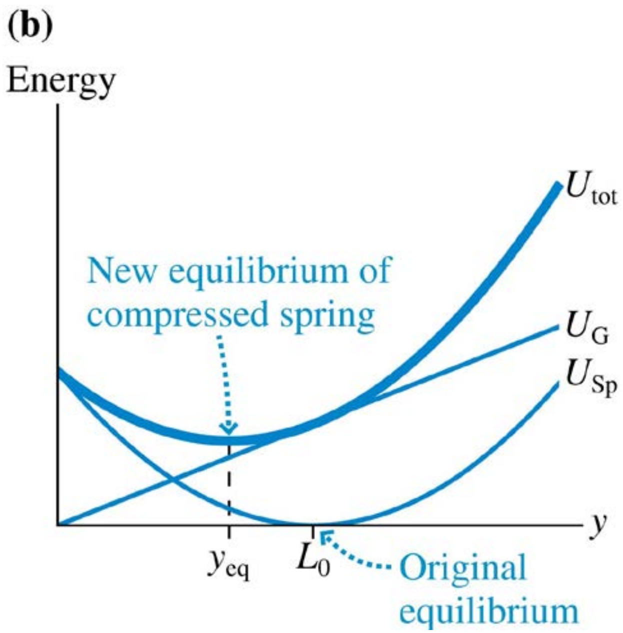
EXAMPLE 10.8 Balancing a mass on a spring

SOLVE The derivative is zero at the point y_{eq}, so we can easily find

$$mg + k(y_{eq} - L_0) = 0$$

$$y_{eq} = L_0 - \frac{mg}{k}$$

The block compresses the spring by the length mg/k from its original length L_0, giving it a new equilibrium length $L_0 - mg/k$.



Force and Potential Energy

- We can find the potential energy of an interaction by calculating the work the interaction force does inside the system.
- Can we reverse this procedure?
- That is, if we know a system's potential energy, can we find the interaction force?
- Suppose that an object undergoes a *very small* displacement Δs.
- During this small displacement, the system's potential energy changes by

$$\Delta U = -W_{\text{int}} = -F_s \Delta s$$

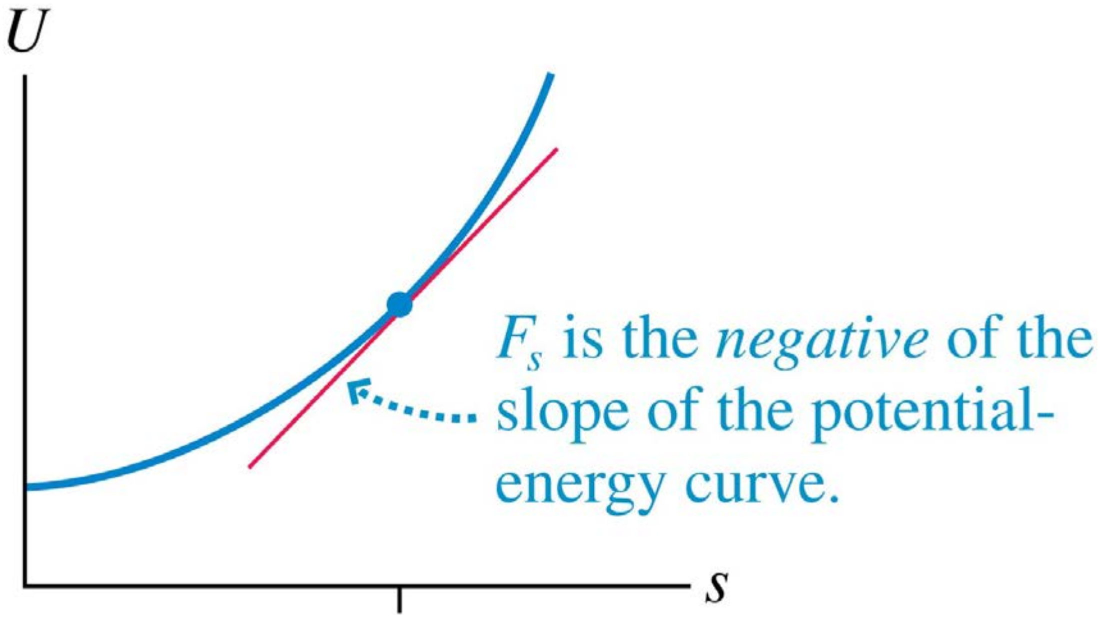
which we can rewrite as

$$F_s = -\frac{\Delta U}{\Delta s}$$

Finding Force from Potential Energy

- In the limit $\Delta s \to 0$, we find that the force at position s is

$$F_s = \lim_{\Delta s \to 0}\left(-\frac{\Delta U}{\Delta s}\right) = -\frac{dU}{ds}$$

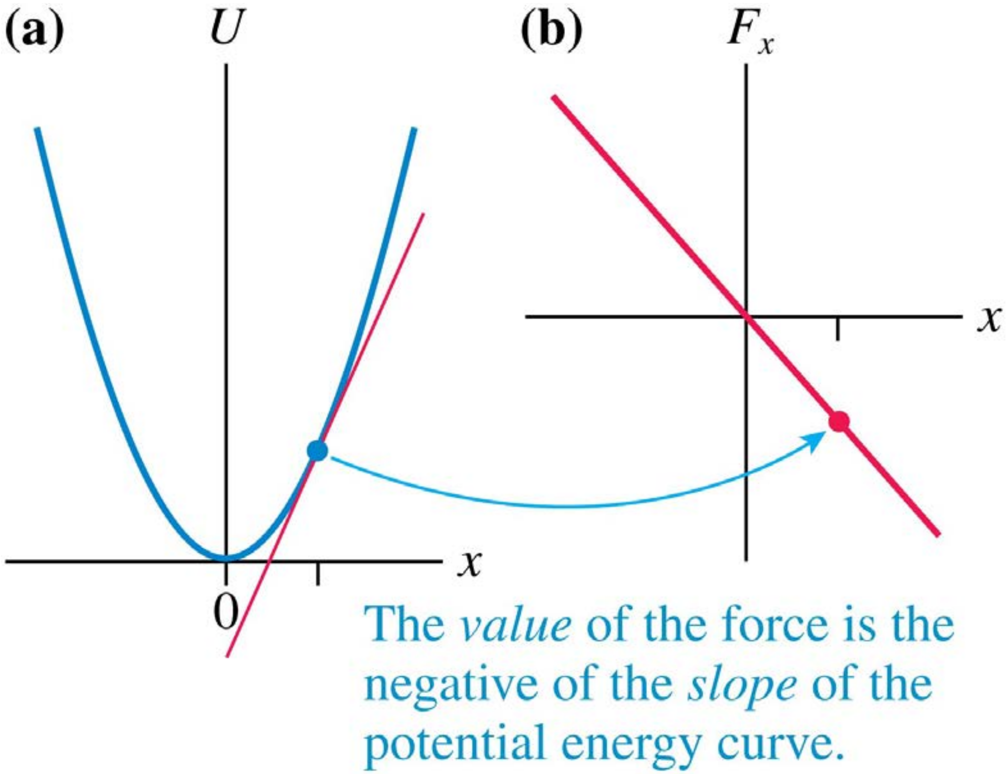


- The force on the object is the negative of the derivative of the potential energy with respect to position:

$$F_s = -\frac{dU}{ds} = \text{the negative of the slope of the PE curve at } s$$

Finding Force from Potential Energy

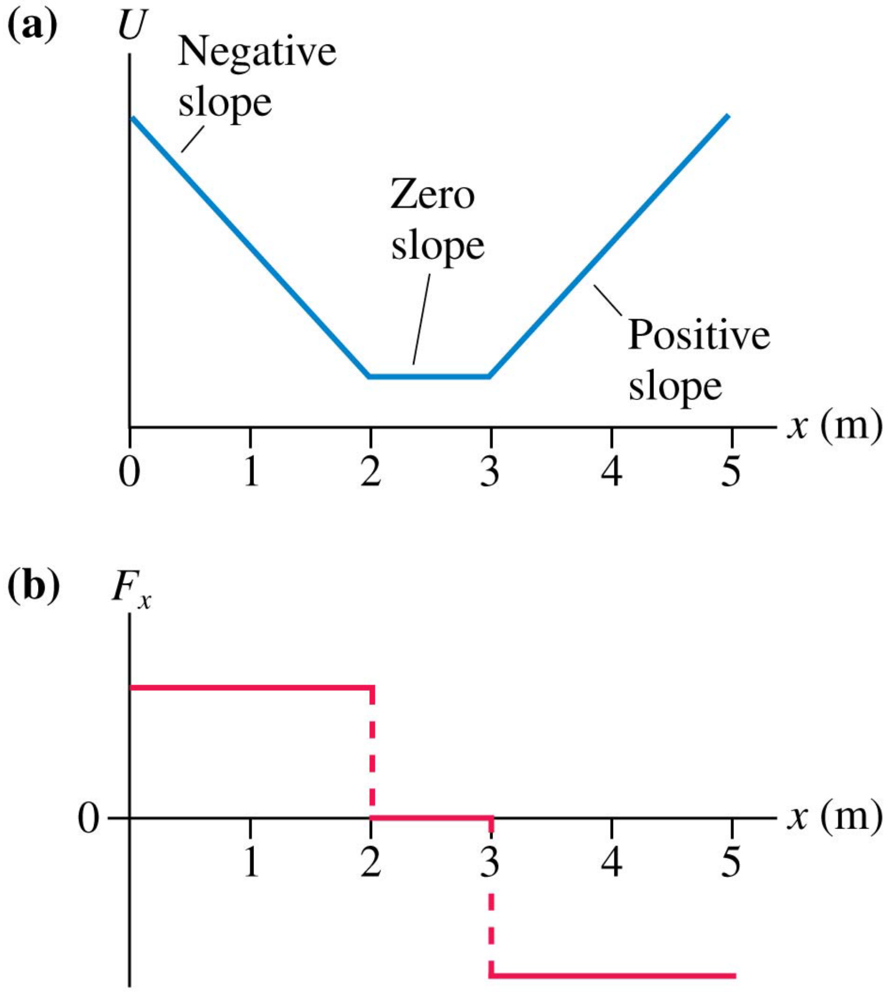
- Figure (a) shows the potential-energy curve for a horizontal spring with $x_{eq} = 0$.
- The force on an object attached to the spring is $F_x = -kx$.
- Figure (b) shows the corresponding F-versus-x graph.



- At each point, the *value* of F is equal to the negative of the *slope* of the U-versus-x graph.

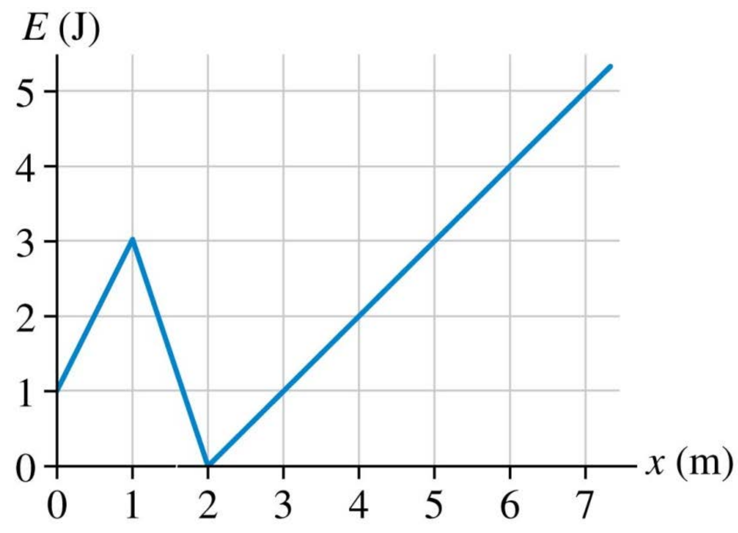
Finding Force from Potential Energy

- Figure (a) is a more general potential-energy diagram.
- Figure (b) is the corresponding F-versus-x graph.
- Where the slope of U is negative, the force is positive.
- Where the slope of U is positive, the force is negative.
- Where the slope is zero, the force is zero.



QuickCheck 10.10

A particle with the potential energy shown is moving to the right. It has 1.0 J of kinetic energy at $x = 1.0 \text{ m}$. In the region $1.0 \text{ m} < x < 2.0 \text{ m}$, the particle is



A. Speeding up.

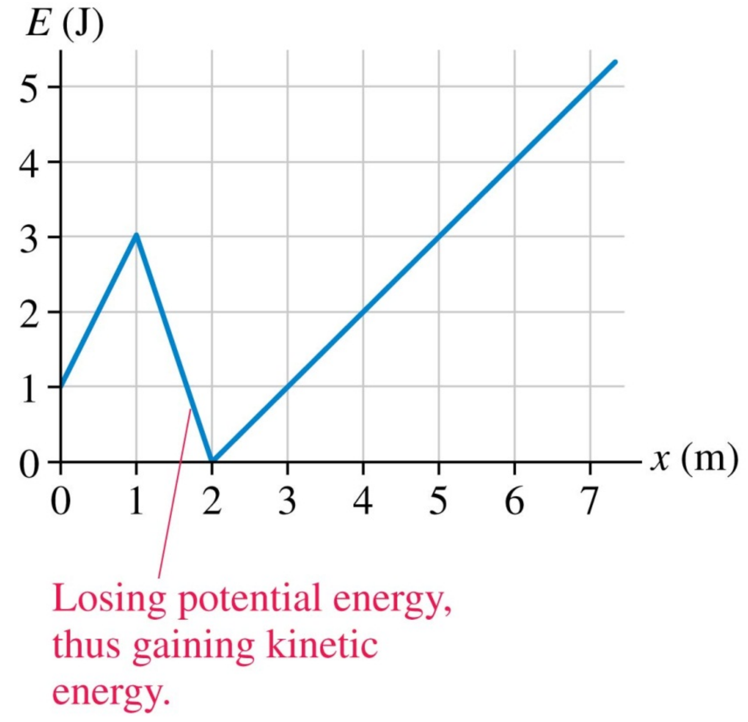
B. Slowing down.

C. Moving at constant speed.

D. I have no idea.

QuickCheck 10.10

A particle with the potential energy shown is moving to the right. It has 1.0 J of kinetic energy at $x = 1.0$ m. In the region $1.0 \text{ m} < x < 2.0 \text{ m}$, the particle is



A. **Speeding up.**

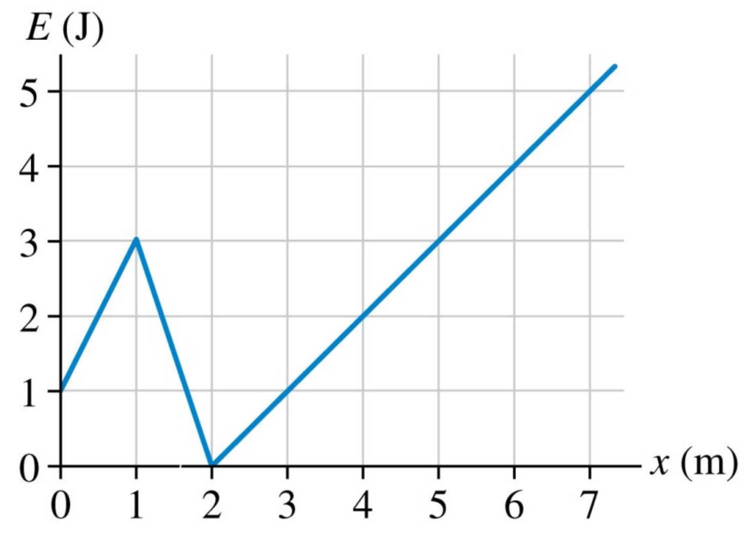
B. Slowing down.

C. Moving at constant speed.

D. I have no idea.

QuickCheck 10.11

A particle with the potential energy shown is moving to the right. It has $1.0\ \text{J}$ of kinetic energy at $x = 1.0\ \text{m}$. Where is the particle's turning point?



A. $1.0\ \text{m}$

B. $2.0\ \text{m}$

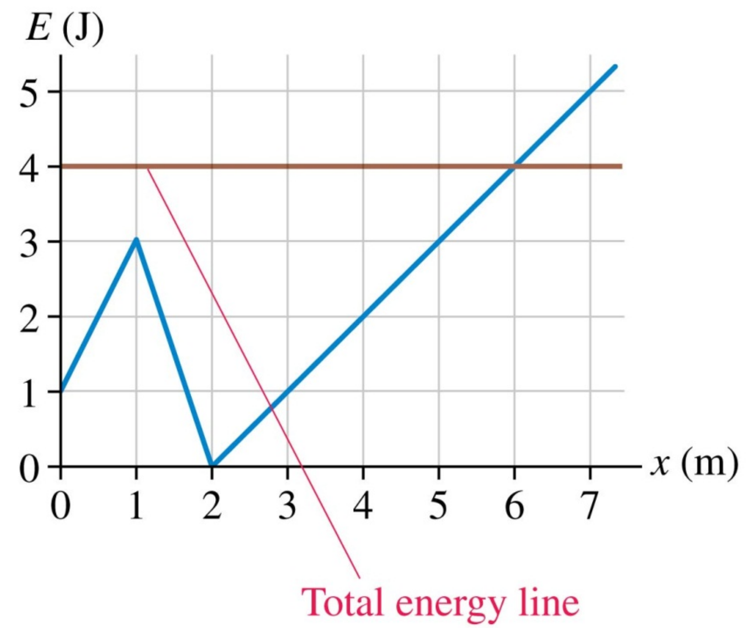
C. $5.0\ \text{m}$

D. $6.0\ \text{m}$

E. It doesn't have a turning point.

QuickCheck 10.11

A particle with the potential energy shown is moving to the right. It has 1.0 J of kinetic energy at $x = 1.0$ m. Where is the particle's turning point?



A. 1.0 m

B. 2.0 m

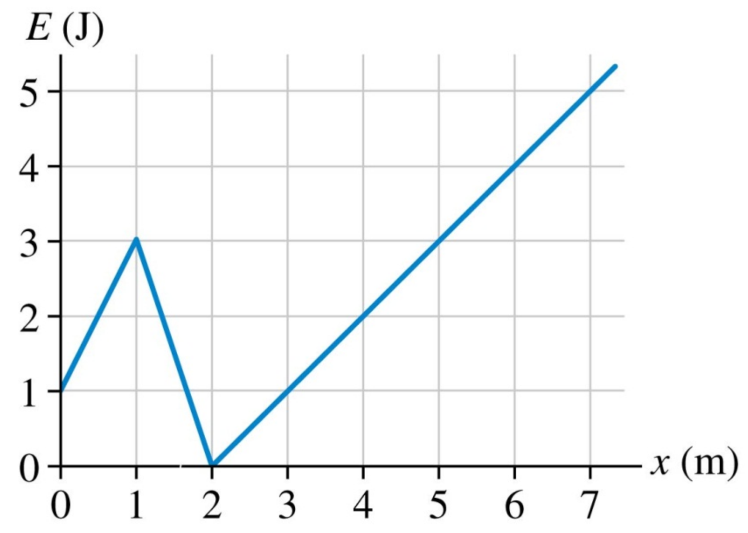
C. 5.0 m

✔ **D. 6.0 m**

E. It doesn't have a turning point.

QuickCheck 10.12

A particle with this potential energy could be in stable equilibrium at $x =$



A. 0.0 m

B. 1.0 m

C. 2.0 m

D. Either A or C

E. Either B or C

QuickCheck 10.12

A particle with this potential energy could be in stable equilibrium at x =

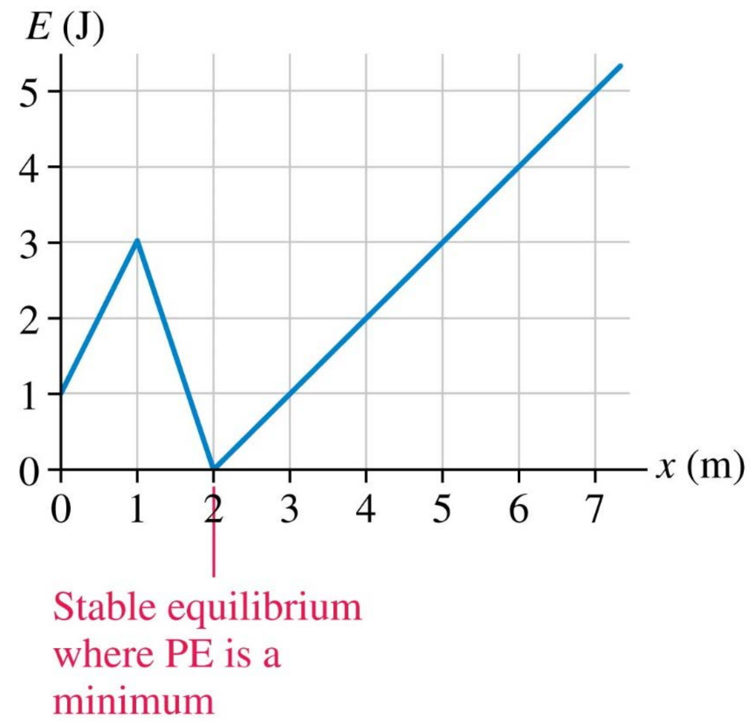
A. 0.0 m

B. 1.0 m

✔ **C. 2.0 m**

D. Either A or C.

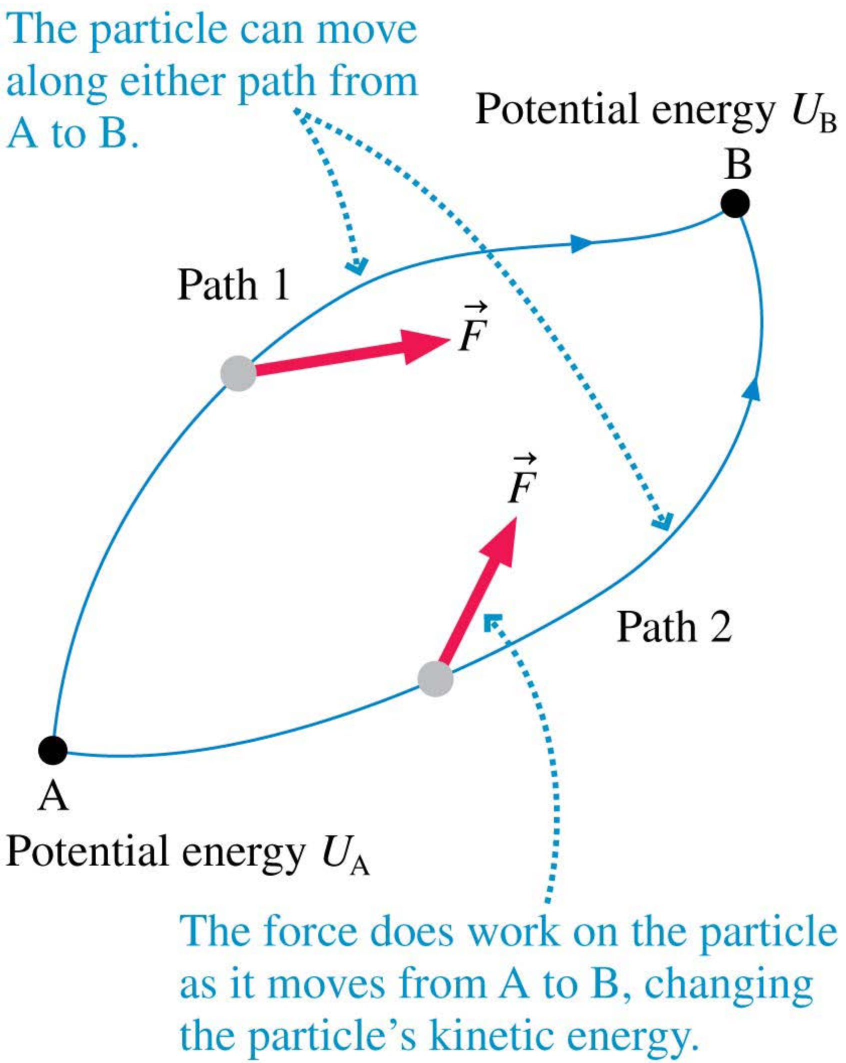
E. Either B or C.



Conservative Forces

- The figure shows a particle that can move from A to B along either path 1 or path 2 while a force $\vec{F}$ is exerted on it.
- If there is a potential energy associated with the force, this is a conservative force.
- The work done by $\vec{F}$ as the particle moves from A to B is independent of the path followed:

$$\Delta U = -W_c(\mathrm{i} \rightarrow \mathrm{f})$$

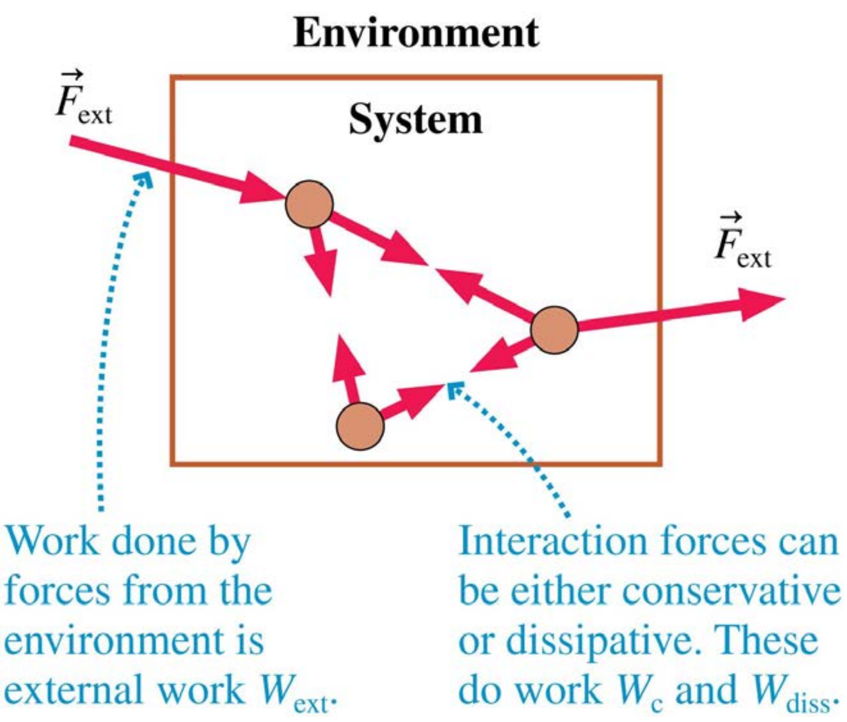


Nonconservative Forces

- If an object slides up and down a slope with friction, then it returns to the same position with *less* kinetic energy.
- Part of its kinetic energy is transformed into gravitational potential energy as it slides up, but part is transformed into thermal energy that lacks the “potential” to be transformed back into kinetic energy.
- A force for which we cannot define a potential energy is called a **nonconservative force**.
- Friction and drag, which transform mechanical energy into thermal energy, are nonconservative forces, so there is no “friction potential energy.”
- Similarly, forces like tension and thrust are nonconservative.

The Energy Principle Revisited

- The figure shows a system of three objects that interact with each other and are acted on by external forces from the environment.
- In the previous section we found that forces can be conservative, doing work W_c, or nonconservative, doing work W_{nc}.



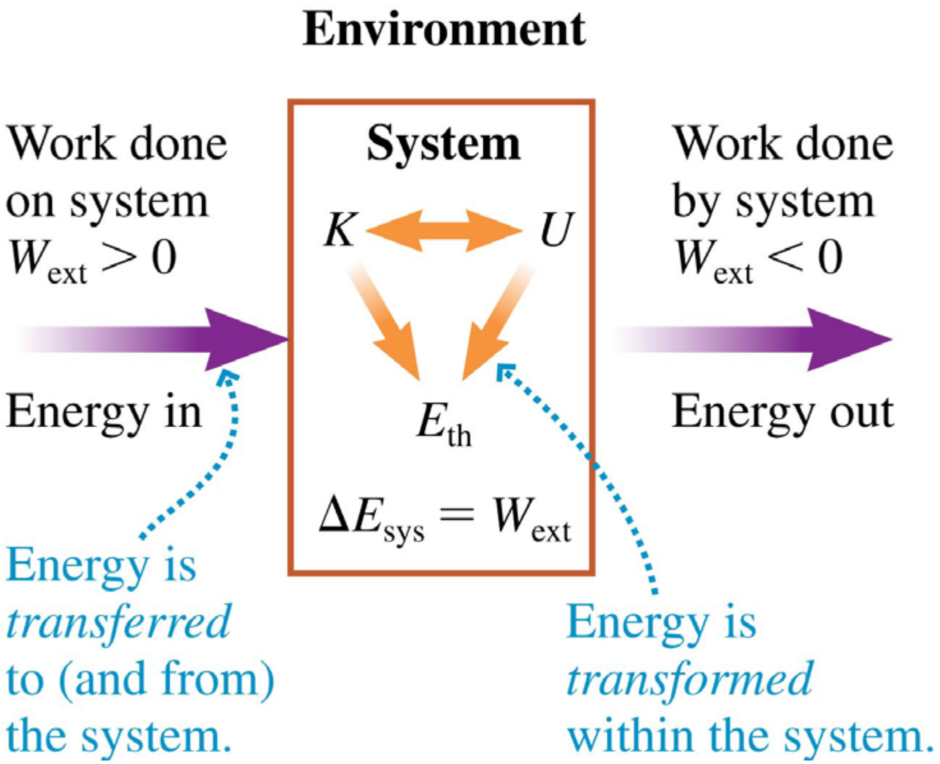
- Now lets make a further distinction by dividing the nonconservative forces into dissipative forces and external forces.
- Dissipative forces, like friction and drag, transform mechanical energy into thermal energy

The Energy Principle Revisited

- With these terms, the energy principle becomes

$$\Delta K + \Delta U + \Delta E_{th} = \Delta E_{mech} + \Delta E_{th} = \Delta E_{sys} = W_{ext}$$

- An isolated system is a system on which no work is done by external forces.
- **Thus the total energy E_{sys} of an isolated system is conserved.**



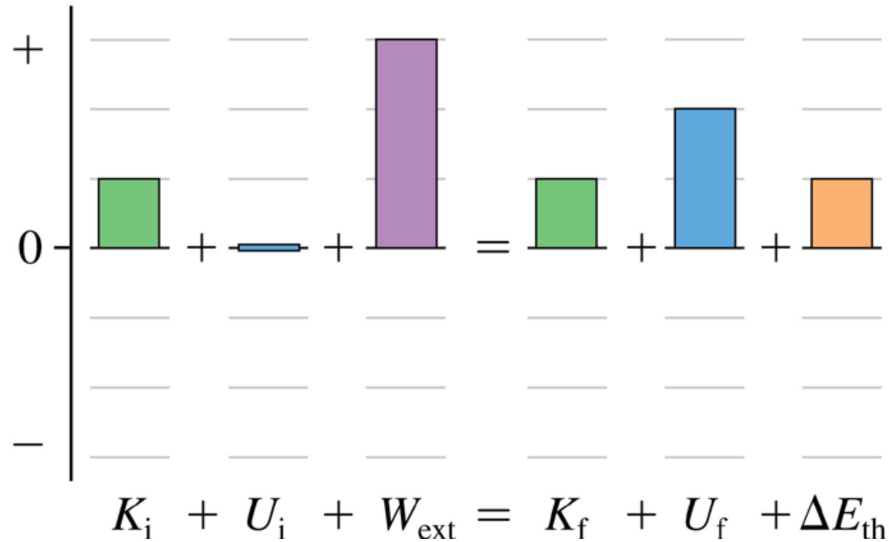
Example 10.10 Hauling Up Supplies

EXAMPLE 10.10 | **Hauling up supplies**

A mountain climber uses a rope to drag a bag of supplies up a slope at constant speed. Show the energy transfers and transformations on an energy bar chart.

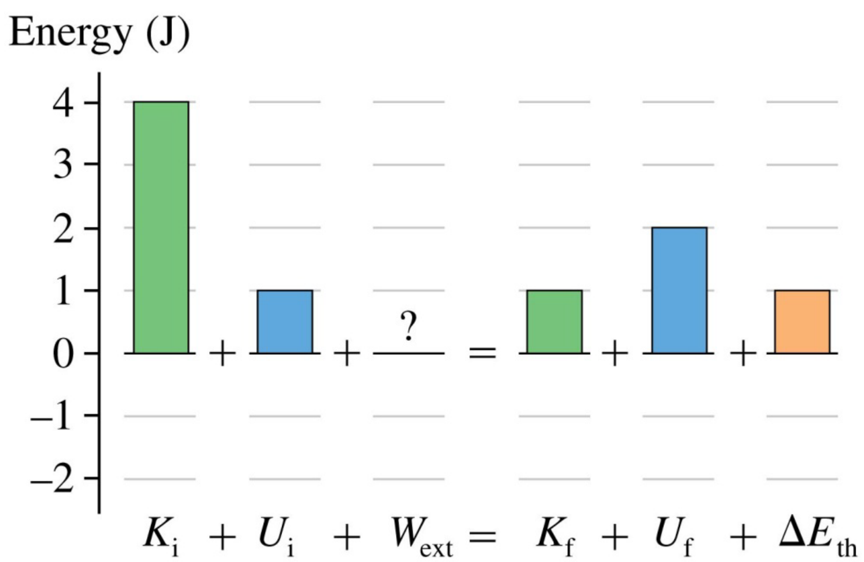
MODEL Let the system consist of the earth, the bag of supplies, and the slope.

SOLVE The tension in the rope is an external force that does work on the bag of supplies. This is an energy transfer into the system. The bag has kinetic energy, but it moves at a steady speed and so K is not *changing*. Instead, the energy transfer into the system increases both gravitational potential energy (the bag is gaining height) and thermal energy (the bag and the slope are getting warmer due to friction). The overall process is $W_{ext} \rightarrow U + E_{th}$. This is shown in **FIGURE 10.29**.



QuickCheck 10.13

How much work is done by the environment in the process represented by the energy bar chart?



A. –2 J

B. –1 J

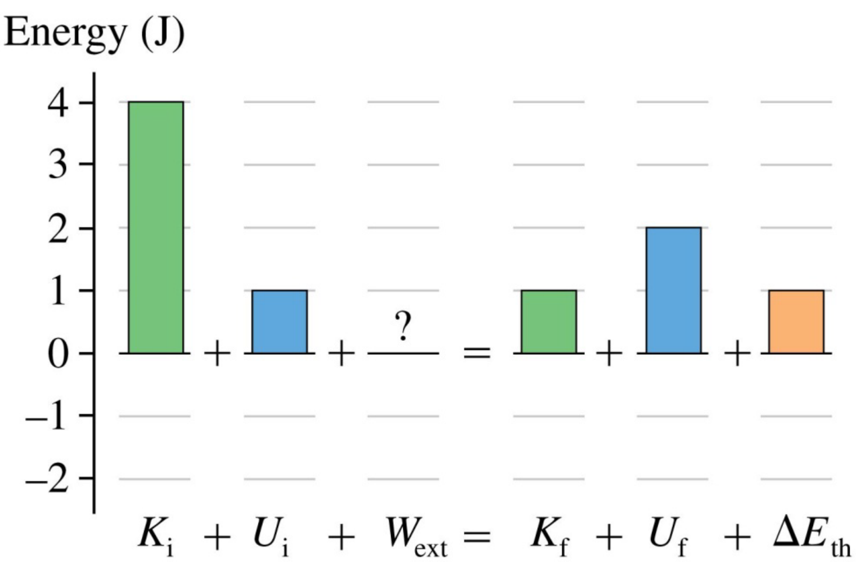
C. 0 J

D. 1 J

E. 2 J

QuickCheck 10.13

How much work is done by the environment in the process represented by the energy bar chart?



A. –2 J

B. –1 J

C. 0 J

D. 1 J

E. 2 J

The system started with 5 J but ends with 4 J. 1 J must have been transferred from the system to the environment as work.

Chapter 10 Summary Slides

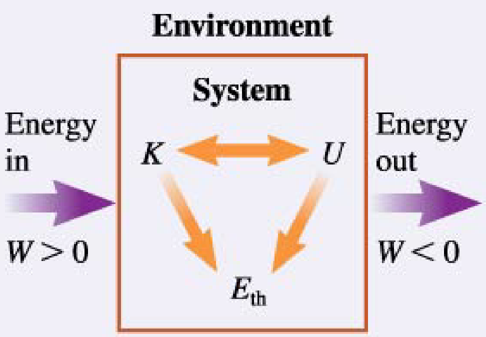
General Principles

The Energy Principle Revisited

- Energy is *transformed* within the system.
- Energy is *transferred* to and from the system by work W.

Two variations of the energy principle are

$$\Delta E_{sys} = \Delta K + \Delta U + \Delta E_{th} = W_{ext}$$
$$K_i + U_i + W_{ext} = K_f + U_f + \Delta E_{th}$$



Basic energy model

General Principles

Solving Energy Problems

MODEL Define the system.

VISUALIZE Draw a before-and-after pictorial representation and an energy bar chart.

SOLVE Use the energy principle:

$$K_i + U_i + W_{ext} = K_f + U_f + \Delta E_{th}$$

ASSESS Is the result reasonable?

General Principles

Law of Conservation of Energy

- **Isolated system:** $W_{ext} = 0$. The total system energy $E_{sys} = K + U + E_{th}$ is conserved. $\Delta E_{sys} = 0$.
- **Isolated, nondissipative system:** $W_{ext} = 0$ and $W_{diss} = 0$. The **mechanical energy** $E_{mech} = K + U$ is conserved: $K_i + U_i = K_f + U_f$.

Important Concepts

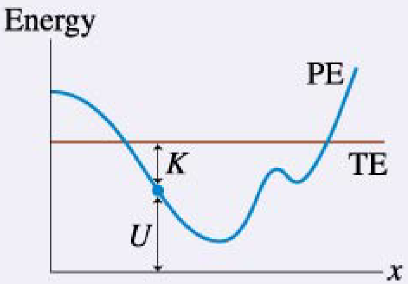
Potential energy, or *interaction energy,* is energy stored inside a system via interaction forces. The energy is stored in *fields*.

- Potential energy is associated only with **conservative forces** for which the work done is independent of the path.
- Work W_{int} by the interaction forces causes $\Delta U = -W_{int}$.
- Force $F_s = -dU/ds = -$(slope of the potential energy curve).
- Potential energy is an energy of the system, not an energy of a specific object.

Important Concepts

Energy diagrams show the potential-energy curve PE and the total mechanical energy line TE.

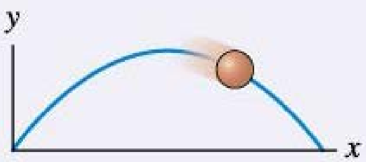
- From the axis to the curve is U. From the curve to the TE line is K.
- **Turning points** occur where the TE line crosses the PE curve.
- Minima and maxima in the PE curve are, respectively, positions of **stable** and **unstable equilibrium.**



Applications

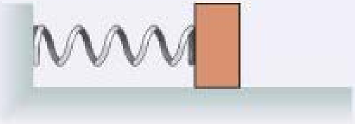
Gravitational potential energy is an energy of the earth + object system:

$$U_G = mgy$$



Elastic potential energy is an energy of the spring + attached objects system:

$$U_{Sp} = \frac{1}{2}k(\Delta s)^2$$



Applications

Energy bar charts show the energy principle in graphical form.