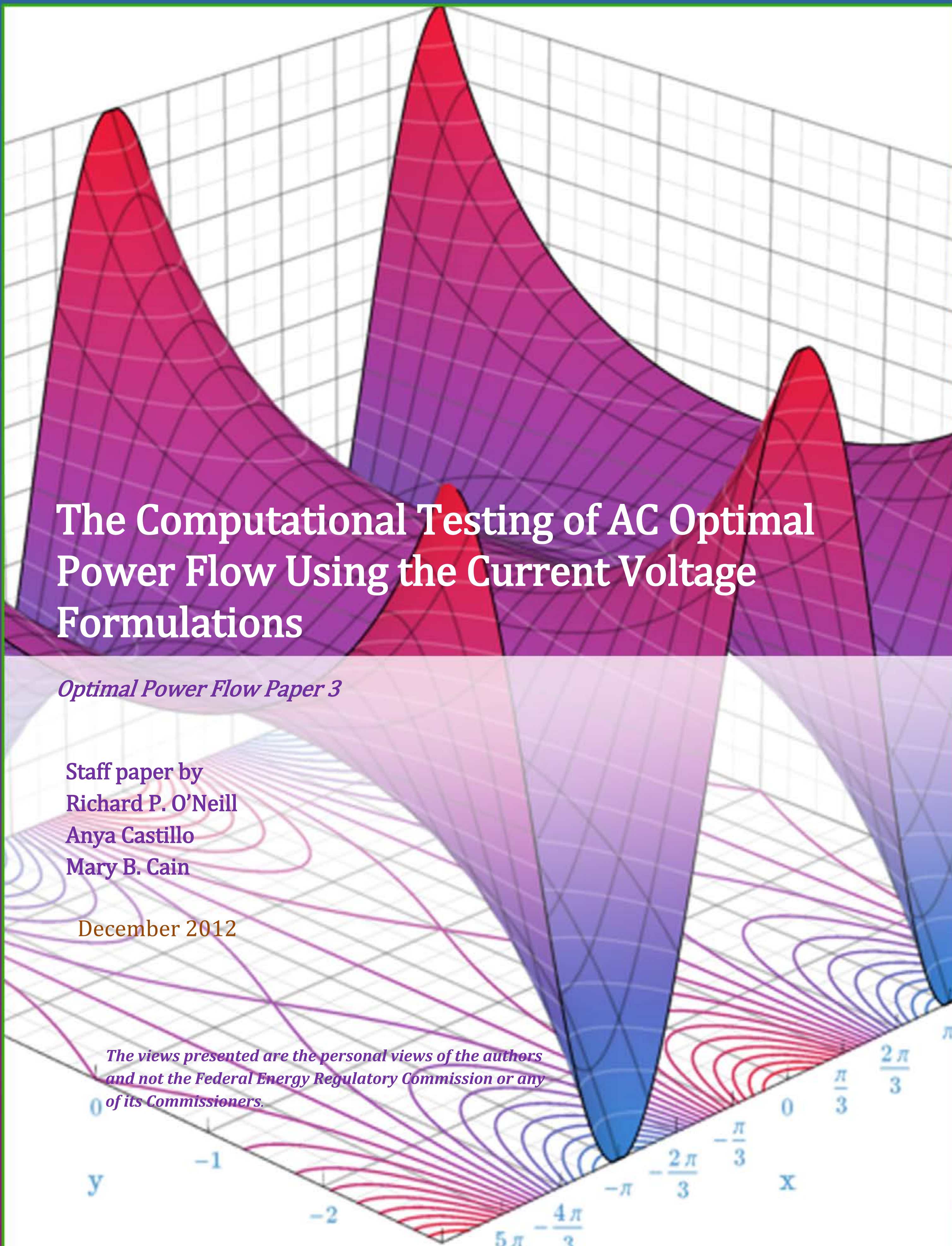

The Computational Testing of AC Optimal Power Flow Using the Current Voltage Formulations

Optimal Power Flow Paper 3

Staff paper by
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Abstract and Executive Summary

Solving the iterative linear approximation of the current voltage (IV) ACOPF is compared to solving the ILIV-ACOPF with several nonlinear solvers. In general, the linear approximation approach is more robust and faster than several of the commercial nonlinear solvers. For the nonlinear solvers, the default parameters are used. The accuracy of the nonlinear solvers is higher when the optimal solution is found. On several starting points, the nonlinear solvers failed to converge or contained positive relaxation variables above the threshold value. We find that the flat start is superior to the average of the random starts. The iterative linear program approximation is more robust and consistent across starting points. The iterative linear program approach finds a near feasible near optimal in almost all problems and all starting points. Using the linear approximation approach opens up the possibility of solving the AC optimal topology and AC optimal unit commitment problems faster as a mixed integer linear program (MIP). The ranked performance of the nonlinear solvers changes depending on the test problem.

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1. Introduction

After hundreds of papers over the last half century on formulating and solving the AC optimal power flow (ACOPF) and many attempts to solve the ACOPF commercially, in practice, approximate 'DC' models with surrogate AC constraints are still widely used. Cain et al (2012)presented a literature review of the history and various formulations of the ACOPF. The ACOPF is a continuous-variable nonconvex program (see Lesieutre and Hiskens, 2005; Hiskens, 2001). For nonconvex problems, most solvers terminate at a local optimal solution or fail to converge. Local non-global optimal solutions could cost the electric power industry billions of dollars annually because of unrealized gains from suboptimal solutions. We often take it on faith that the local solution obtained is also global or at least a good one. In this paper, we study small- to medium-size problems to better understand the nature and structure of the problem.

Several small problems are well-studied or at least well-tested. They have helped demonstrate that there are high and low voltage solutions to the power flow equations on small examples. However, little is known about the high and low voltage solutions to the ACOPF with more than several buses. The hypothesis is that the low voltage solution is more expensive or is constrained to be infeasible. Although academic papers usually address optimal solutions, in practice, we settle for near-optimal solutions that can be obtained in a specified time window. Due to nonconvexities, there is no guarantee that the software will find an optimal solution. Robustness is the ability to find an optimal solution or at least a feasible solution. Castillo et al. present a literature review of computational studies and test problem performance (Castillo 2012).

Today, most papers offering algorithms, that solve the ACOPF faster and more robustly, test these algorithms on the standard IEEE/MATPOWER test set and/or some proprietary problems not generally available. Schecter et al (2012) present an analysis of the standard test problems. Even though they appear often in the literature, we know relatively little about these problems or how representative they are of actual electricity networks. For example, are these test sets easier or harder to solve relative to the broader universe of actual ACOPFs? Test results on proprietary problems require greater trust in the results since they are seldom if ever reproduced by independent parties. Since we have no proprietary problems, in this paper, we test using generally available test problems.

To compare nonlinear solvers, one can consider several dimensions including the quality of the optimal solution, time to solution, and solution robustness, that is, the number of instances a 'good' feasible solution is found. In this paper, we test the performance of linear approximations of ACOPF-IV formulation against the nonlinear ACOPF-IV; both formulations are presented in a companion paper (O'Neill et al, 2012).

Finally, we note that all models here approximate a physical system. A steady-state power-flow model approximates a larger dynamic model. For example, power flow models generally do not model frequency excursions and any dynamic constraints or contingencies from offline studies, which are achieved by adjusting static constraints. Moreover, a feature of the ACOPF is that many of the problem constraints are soft, that is, are not absolute. Thermal constraints or constraints that often contain significant engineering safety margins, such as the voltage stability constraints, are a couple examples of 'soft' inequality constraints. As it is common accepted practice, penalty slack variables are assigned to these inequalities. Such relaxation slack variables are given high objective function costs since exact costs and bounds are hard to quantify.

2. Test Design

Approach. We begin by generating a starting point. To test the linear formulation, we solve the nonlinear IV formulation with quadratic costs (OPF2) to benchmark the solver against the generally accepted optimal solution. Next, we solve the nonlinear IV formulation with a linear ten-step objection function (OPF) approximating the quadratic costs to benchmark the linear objective function and examine how the software performs with a linear objective function. We then solve a series of linearized IV formulations (OPFlin). At each step, employing the previous optimal solution as the new starting point for linearization, we re-linearize the first order approximation and add new constraints that further improve or bound the approximations. When any one of three criteria (explained below) is satisfied, the algorithm stops. Finally, we make an additional run through the nonlinear IV formulation with the linear objective function starting with the optimal solution to the last iteration of the linear IV formulation (OPFp). The GAMS code to test the solvers is unchanged except for the names of the solvers.

Starting Points. We solve each test problem with ten different starting points. The first starting point is a 'flat' start (in polar coordinates, voltage magnitude = 1 and voltage angle = 0) for all buses. The remaining nine points are uniformly random points interior to the voltage magnitude constraints. The starting value for current is $\underline{I} = Y\underline{V}$ where $\underline{V}$ is the generated point. This $\underline{I}$, $\underline{V}$ point satisfies the current flow equations but not necessarily the generator and load constraints. This approach helps test the robustness of the solver. In practice, choosing a good starting point based on experience is most likely a better strategy, but cannot be easily tested. But in a contingency, choosing a starting point using the last available solution may not be a good starting point.

Test Problems. We define small test problems (for example, 14-bus, 30-bus, 57-bus) as problems that include between 10 to 99 buses. They are helpful because they solve fast and allow for a better understanding of what is happening in the solution process at the nodal and branch level. We define medium size problems (for example, 118-bus, 300-bus) as problems with 100 to 999 buses. These problems are difficult to analyze at the nodal and branch level, but solve relatively quickly. Most vertically integrated utility transmission networks fit in this category. Large problems are between 1,000 and 9,999 buses and most large system operators fit in this category. We define very large problems as 10,000 or more buses; large ISOs like PJM and MISO fit in this category.

We solve each of the five now standard IEEE/MATPOWER test problems (14-bus, 30-bus, 57-bus, 118-bus, and 300-bus). Each test problem contains the same voltage magnitude constraints at each bus and contains no constraints on the transmission element flows or voltage angles. We scale each objective function. Summary parameters, the widely accepted quadratic optimal value, and the step function approximation of the objective function for each test problems are in Table 1.

Table 1. Test problem parameters and optimal solutions

Nodes	Branches	Generators		demand	V^{max}	V^{min}	Best Known Optimal Value	
		Number	Capacity				Step Function	Quadratic
14	20	5	7.724	2.590	1.06	0.94	82.76	80.81
30	41	6	326.80	42.42	1.10	0.95	5.918	5.745
57	80	7	326.78	235.26	1.06	0.94	423.7	417.4
118	186	54	99.66	42.42	1.06	0.94	1311	1297
300	411	69	19.76	12.51	1.06	0.94	7488	7197

Additional information on the test problems and solutions is available at the MATPOWER site (http://www.pserc.cornell.edu/matpower/). For the quadratic formulation, the local optimal solutions are proved to be global by solving a semidefinite program (SDP) dual formulation and finding a solution without a duality gap (see Lavaei and Low). For the linear solvers, the BTHETA formulation is solved and presented for contrast. Since we test more nonlinear solvers than linear solvers, the linear solvers are used more than once in the testing procedure.

Hardware. The problems were solved on a DELL Latitude laptop with an Intel® CoreTM i5-540M (2.53GHz, 3M cache) and 4GB memory. Minor differences in solution times were recorded when the problems were run at different times of day and with and without network connection but the differences were small enough to be considered negligible.

Solvers. The nonlinear programs used solvers MINOS 5.51, SNOPT 7.2-4, IPOPT version 3.8, CONOPT version 3.14V and KNITRO version 7.0.0. Three of the solvers have overlapping authors/developers see table 2.

Table 2. Solvers and developers

	CONOPT	SNOPT	MINOS	IPOPT	KNITRO
Arne Drud	X	X			
Walter Murray		X	X		
Philip Gill		X	X		
Bruce Murtagh			X		
Michael Saunders			X		
Carl Laird				X	
Andreas Wächter				X	
R.H. Byrd					X
J. Nocedal					X
R.A. Waltz					X

Linear programs used GUROBI version 4.0.0 and CPLEX version 12.2.0.1. Their authors/developers also overlap since Robert Bixby, Zonghao Gu and Edward Rothberg were principal developers on both.

We used the default settings for all solvers. In today's market, a generic solver is valued, in part, by its ability to internally choose the best strategy for solving a problem with default settings while internally choosing solution strategies dynamically. Asking the user to set numerous parameters is viewed as a weakness, because it requires a level of expertise that should not be a prerequisite for using the solver. Nevertheless, expert tuning is important in commercial installations where the problem is solved repetitively. In general, a nonlinear solver spends most of its time to get close to the optimal solution. For example, the reduced gradient generally goes from 10^{-2} to 10^{-8} (or less) in less than 10% of the iterations.

Problem Formulations. The problems were formulated in GAMS version 23.6 starting with code written by Michael Ferris and modified by Richard O'Neill. For all formulations, relaxation variables with high penalties, 10^7, are added to the upper and lower bounds for P, Q and V to produce feasible solutions, albeit at times with penalties active in the objective function. In actual dispatch markets, penalty slack variables are inserted for various reasons, including soft constraints. Often 'near' feasible solutions are usable. Even though most commercial applications use penalty parameters, the linear IV formulation produced better results than the nonlinear solvers.

As a quadratic approximation to c(S), we use

$$C_2(P \bullet P) + C_1 P + a[C_2(Q^+ \bullet Q^+) + C_2(Q^- \bullet Q^-) + C_1(Q^+ + Q^-)].$$

Generally, $0 < a < .1$. For a linear approximation, a step function approximation can be used. Here a is set to 0.1.

Performance Measures. For power system dispatch, robustness is important to operations. An important metric of power systems operation software is a good feasible solution within a specified time window. For the test problems, we can break this into two dimensions: CPU time to converge, that is, the fraction of the time that a 'good' usable local optimal solution was found, and robustness, that is, the ability to find a good usable solution.

The linear program is a further approximation of the nonlinear program. Therefore, we develop a nonlinear 'feasibility' test to measure the 1-norm 'distance' of the linear program optimal solution variables from a nonlinear feasible solution.

Let $V^{r*}, V^{j*}, I^{r*}, I^{j*}, P^*, Q^*$ be an optimal solution to the linear program. By the formulation, $V^{r*}, V^{j*}, I^{r*}, I^{j*}$ is a feasible solution to the flow equations. Let

$$\underline{V}^m = (V^{r*}\bullet V^{r*} + V^{j*}\bullet V^{j*})^{1/2}$$

$$\underline{P} = V^{r*}\bullet I^{r*} + V^{j*}\bullet I^{j*}$$

$$\underline{Q} = V^{j*}\bullet I^{r*} - V^{r*}\bullet I^{j*}$$

$\underline{P}$ and $\underline{Q}$ are the implied injections from the flow equations and $\underline{V}^m$ is the voltage magnitude. We calculate feasible normalized deviation metrics for $V, P,$ and Q as follows. For voltage magnitude feasibility excursions, for each bus, n, set $v^f_n = 0$.

if $\underline{v}^m_n > v^{max}_n$, $v^f_n = \underline{v}^m_n/v^{max}_n$.

if $\underline{v}^m_n < v^{min}_n$, $v^f_n = -\underline{v}^m_n/v^{min}_n$.

Therefore if $v^f_n = 0$, the voltage is within its magnitude bounds. if $v^f_n > 0$, the voltage exceeds its maximum level. If $v^f_n < 0$, the voltage is less than its minimum level.
We now sum the absolute value of the deviations and normalize by the number of buses, obtaining the total infeasibility voltage metric:

$$v^d = \sum_n abs(v^f_n)/n.$$

The maximum infeasibility voltage metric is:

$$v^{dmax} = max_n \{abs(v^f_n)\}.$$

For nodal real power imbalances, feasibility excursions, for each bus, n, set $p^f_n = 0$.

If $\underline{p}_n > p^{max}_n$, $p^f_n = abs(\underline{p}_n/p^{max}_n)$.

If $\underline{p}_n \neq p^{max}_n$ and $p^{max}_n = p^{min}_n$, $p^f_n = abs(\underline{p}_n/p^{flow}_n)$

where $p^{flow}_n = \sum_k abs(p^{flow}_{kn})/2$.

If $\underline{p}_n = p^{max}_n$ and $p^{max}_n = p^{min}_n$, $p^f_n = 0$.

If $\underline{p}_n < p^{min}_n$, $p^f_n = -abs(\underline{p}_n/p^{min}_n)$.

If $p^f_n = 0$, the real power is within its magnitude bounds. If $p^f_n > 0$, the real power exceeds its maximum level or $\underline{p}_n \neq p^{max}_n$ and $p^{max}_n = p^{min}_n$. If $p^f_n < 0$, the real power is less than its minimum level.
We now sum the absolute value of the deviations and divide by the number of buses, obtaining the total infeasibility real power metric:

$$p^d = \sum_n abs(p^f_n)/n.$$

The total maximum infeasibility real power metric is:

$p^{dmax} = max_n \{abs(p^f_n)\}$.

We define q^f_n, q^d, and q^{dmax} in the same way.

V^d, P^d and Q^d measure the total infeasibility implied by the nonlinear equations.

V^{dmax}, P^{dmax} and Q^{dmax} measure the maximum infeasibility implied by the nonlinear equations.

We test solvers with and without relaxation slack variables. Relaxation slack variables are removed by setting the upper bound on the slack variables to zero. We also add an additional norm for active relaxation slack variables:

$$Tepsinorm = \sum_n (p^{minepsi}{}_n + p^{maxepsi}{}_n)/p^{dsum} + \sum_n (q^{minepsi}{}_n + sq^{maxepsi}{}_n)/q^{dsum} + \sum_n (v^{minepsi} + v^{maxepsi}{}_n)$$

If $Tepsinorm$ is less than epsilon, we consider the solution acceptable and remove the penalty values from the objective function. If not, it is recorded as a feasibility violation, that is, $TEPSIN = TEPSIN + 1$.

Parameter Setting. The relaxation penalty was set at 10^7. MaxLin is the maximum number of calls to the linear solver. It is set at 10. feasum is the sum of normalized infeasibilities. It is set to 0.05. maxfeas is the maximum normalized infeasibilty. It is set to 0.01. The LP terminates when any of the three conditions are met. Tepsi is the infeasibility test based on the magnitude of the penalty slacks.

3. Results for Each Test Problem

We start by presenting summary results for each test problem. More detailed results are in the appendix

14-bus problem. Without relaxation slack variables, all solvers with the exception of KNITRO found a near global optimal solution for each starting point. On average, SNOPT, CONOPT and MINOS were the fastest solvers. GUROBI was competitive with the nonlinear solvers on the OPF. Comparing OPF and OPFp, using the LP approximation final solution as the starting point did not lower cpu time. The flat start does better than the average random starting point.

With relaxation slack variables, all solvers declare a local optimal solution for each starting point, but only CONOPT and IPOPT find the global optimal value. The other solvers find local optimal solutions near the global optimal value. On average MINOS is the fastest solver followed by CONOPT. SNOPT is erratic, at times, working well and at times, appearing to get hung up. Overall, since CONOPT is the second fastest and always finds a global optimum, CONOPT outperforms the other nonlinear solvers.

For the linear solvers, GUROBI outperforms CPLEX in cpu time. The optimal solution values are within 1 percent of the best nonlinear solvers, converging in 4 to 7 linear program passes. GUROBI and CPLEX are faster with relaxation slack variables.

We compare a CONOPT OPF solution to a GUROBI OPFlin linear approximation solution. The OPF solution and OPFlin solution have only minor differences in nodal prices for real and reactive power. The objective function values differ by less than one percent. The voltage magnitudes and dual variables from the bus equations are similar, but the actual generator dispatch (real and reactive) differs somewhat. This is not surprising since the surface near the optimal solution is relatively flat (see Schecter and O'Neill, 2013).

A standard test for robustness is increasing demand. The maximum real power generation is 7.724 and the fixed real demand is 2.59. The maximum possible demand increase factor is 2.98. We solve the 14-bus problem with demand multiplied by 1.5, 2 and 2.5. The problems require increasing CPU time. When demand is 2.5 times greater, the penalty slacks are greater than the set threshold. In 2x and 2.5x demand, CONOPT did not converge on two starting points, but the GUROBI solutions were robust.

30-bus problem. Without relaxation variables, all solvers with the exception of KNITRO find the same optimal solution for all starting points. On average, SNOPT, CONOPT and MINOS were the fastest solvers and solve OPFp faster than OFP2 and OPF. MINOS, CONOPT and IPOPT solve faster than GUROBI solved OPFlin.

With relaxation slack variables, all solvers declare a local optimal solution, but only CONOPT and IPOPT find the global optimal value. The other solvers find optimal solutions near the global optimal value. On average, SNOPT is the fastest solver followed by CONOPT. CONOPT outperforms the other nonlinear solvers on the OPFp where SNOPT appears to get hung up on some problems. KNITRO is very erratic at times working well and at times appearing to get hung up. Overall, CONOPT outperforms the other nonlinear solvers in finding the global optimal solution.

For the linear solvers, GUROBI outperforms CPLEX. The optimal solution values are within 1 percent of the best nonlinear solvers. GUROBI is not as fast as SNOPT, but finds a better solution on OPF. CONOPT outperforms GUROBI. The linear solvers do not to have problems with relaxation slack variables.

We compare the OPF CONOPT solution to the GUROBI OPFlin solution and find differences in P, Q and V and nodal prices for real and reactive power. The actual generator dispatch is reasonably close and the objective function value differs by less than one percent. We solve the 30-bus problem with demand multiplied by 1.2, 1.4 and 1.6. The results are not markedly different.

57-bus Problem. Without relaxation slack variables, all solvers except KNITRO find the global optimal value for all starting points. SNOPT, CONOPT and MINOS are the fastest nonlinear solvers. GUROBI is faster on OPFlin than any nonlinear solver. Without relaxation slack variables for OPF2, CONOPT performs better on the flat start than any random starting point. For OPF2 on three starting points, CONOPT got hung up. For the OPF, CONOPT performs better on the flat start than on most random starting points.

With relaxation slack variables, only CONOPT and IPOPT find the global optimal value. The other solvers find optimal solutions near the global optimal value. SNOPT is the slowest solver and gets hung up on some problems. CONOPT outperforms the other nonlinear solvers on the OPF2 and OPF, but CONOPT fails to converge on one problem, and has higher optimal values. For linear solvers, CPLEX times bound the GUROBI times. The optimal solution values are within 1 percent of the best nonlinear solvers. With demand increased by 1.5, 2 and 2.5, minor differences are observed.

We compare the CONOPT OPF solution to the GUROBI OPFlin solution and find only minor differences in P, Q and V and nodal prices for real and reactive power. The voltages and dual variables are similar but the actual generator dispatch is different, and the objective function value differs by less than one percent.

118-bus Problem. Without relaxation slack variables, all solvers except KNITRO find the global optimal value for all starting points. SNOPT, CONOPT and MINOS are the fastest solvers. CPLEX and GUROBI are faster on OPFlin than any nonlinear solver, except MINOS. OPFp did not solve faster than the OPF.

With relaxation slack variables, all nonlinear solvers have trouble. Only CONOPT and KNITRO find the optimal solution for the OPF2. CONOPT, IPOPT and KNITRO found the optimal solution to the OPF. With relaxation slack variables, CONOPT found optimal values near the optimal value and performed better on the flat start than the average random starting point for OPF2 and OPF.

With relaxation slack variables, all nonlinear solvers have trouble solving the problems. Only CONOPT and KNITRO find the optimal solution for the OPF2. CONOPT, IPOPT and KNITRO found the optimal solution to the OPF. For linear solvers, GUROBI and CPLEX have similar performance with relaxation slack variables. The optimal solution values are within 2 percent of the best nonlinear solver performance. With demand increased by 1.2 and 1.4, OPFlin is more robust.

300-bus problem. Without relaxation slack variables, all solvers declare an infeasible problem. With relaxation slack variables, all solvers except KNITRO declare a local optimal solution, but occasionally fail to remove the penalty slacks. Only CONOPT and IPOPT find the global optimal value.

For linear solvers, CPLEX outperforms, GUROBI. The optimal solution values are within 3 percent of the best nonlinear solvers. All linear approximations took the full 10 passes and did not meet the other convergence criteria. The linear solvers were more robust than the nonlinear solvers. With demand increased by 1.2 and 1.4, the problems require more CPU time, the penalty slacks are greater than the set threshold and convergence becomes more difficult.

4. Summary of Results and Conclusions.

In general, the IV linear approximation solves faster than the nonlinear solvers used in this study. The iterative linear program approach appears to find a near feasible and near optimal solution in almost all problems and starting points. The accuracy of the nonlinear solvers is better when the optimal solution is found. On several starting points, the nonlinear solvers failed to converge or contained positive relaxation variables above the threshold. The flat start is superior to random starts. OPF solves faster than OPF2. The ranked performance of the nonlinear solvers changes depending on the test problem.

The linear program approximation is more robust and consistent across starting points. In some instances, the linear solution variables do not match closely to the nonlinear solution variables, but are close to feasible solutions. This is not surprising since the surface near the optimal is relatively flat (see Schecter and O'Neill). Even the crude implementation of the iterative linear program approach is faster than the nonlinear solvers. Using the linear approximation approach opens up the possibility of solving the AC optimal topology and AC optimal unit commitment problems faster using MIP algorithm.

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Appendix

Solution Statistics. The summary statistics are over the set of starting points. They include

OPTVAVG is the average optimal solution.
CPUAVG is the average cpu time.
CPUMAX is the maximum cpu time.,
CONVRG for non-linear solvers is the number that reported convergence over the ten starting points. For linear solvers is the number of linear programs solved over the ten starting points.
OPTVSDV is the standard deviation from the average optimal solution.
CPUSDV is the standard deviation from the average cpu time.
OPTVCV is the coefficient of variation (standard deviation/mean) of optimal values
CPUCV is the coefficient of variation (standard deviation/mean) of CPU times
ITER is the number of different starting points.
MINOPF is the minimum objective function value found.
OPTVAL is the optimal value returned by the solver.
OPTM is the objective solution value less the penalty value.
Pflow is the real power flow through the bus
Qflow is the reactive power flow through the bus

In tables with the dual variables, they are from the bus equations,

$$\underline{P} = V_r^* \bullet I_r^* + V_j^* \bullet I_j^*$$
$$\underline{Q} = V_j^* \bullet I_r^* - V_r^* \bullet I_j^*$$

14-bus problem

All solvers with the exception of KNITRO found the global optimal solution for each starting point, see Table 14.1. On average, SNOPT , CONOPT and MINOS were the fastest solvers. Comparing OPF and OPFp, the LP approximation starting point did seem to help. GUROBI was competitive with the nonlinear solvers on the OPF.

Table 14.1. Solution Statistics for the 14-Bus Problem for the Nonlinear Solvers Without Relaxation Slack Variables.

			date	2012	329		
type	optvavg	optvcv	cpuavg	cpucv	cpumax	CONVRG	Tepsin
OPF2							
CONOPT	80.8150	0.0000	0.0520	0.1360	0.1090	10	0
MINOS	80.8150	0.0000	0.0690	0.0800	0.0940	10	0
SNOPT	80.8150	0.0000	0.0750	0.1340	0.1250	10	0
KNITRO	81.8780	0.0020	0.2530	0.2040	0.7330	10	0
IPOPT	80.8150	0.0000	0.3700	0.0660	0.5000	10	0
OPF							
SNOPT	82.7620	0.0000	0.0390	0.0850	0.0620	10	0
CONOPT	82.7620	0.0000	0.0510	0.1150	0.0940	10	0
MINOS	82.7620	0.0000	0.0520	0.0960	0.0780	10	0
IPOPT	82.7620	0.0000	0.6080	0.1460	1.3100	10	0
KNITRO	84.0920	0.0050	2.5850	0.7350	20.6080	10	
OPFp							
MINOS	82.7620	0.0000	0.0440	0.1580	0.0780	10	0
CONOPT	82.7620	0.0000	0.0470	0.2230	0.1100	10	0
SNOPT	82.7620	0.0000	0.0760	0.2070	0.2030	10	0
IPOPT	82.7620	0.0000	0.3560	0.0580	0.4840	10	0
KNITRO	83.0780	0.0020	0.3650	0.0680	0.5150	10	0
Btheta							
GUROBI	76.5910	0.0000	0.0030	0.0400	0.0040	10	0
GUROBI	76.5910	0.0000	0.0040	0.0550	0.0050	10	0
GUROBI	76.5910	0.0000	0.0040	0.0820	0.0060	10	0
CPLEX	76.5910	0.0000	0.0180	0.0300	0.0210	10	0
CPLEX	76.5910	0.0000	0.0190	0.1170	0.0400	10	0
OPFlin							
GUROBI	82.2900	0.0030	0.0410	0.3260	0.0090	60	0
GUROBI	82.2900	0.0030	0.0410	0.3700	0.0060	60	0
GUROBI	82.2900	0.0030	0.0410	0.3800	0.0060	60	0
CPLEX	82.3300	0.0040	0.0970	0.3130	0.0180	51	0
CPLEX	82.3300	0.0040	0.1070	0.3100	0.0350	51	0

Table 14.1a. Linear Program Passes for GUROBI by Starting Point With and Without Relaxation Slack Variables

Starting point	1	2	3	4	5	6	7	8	9	10	Total
Without	4	4	5	9	5	8	7	8	6	4	60
With	5	9	7	9	8	8	8	8	9	9	80

With relaxation slack variables, all solvers declare a local optimal solution for each starting point, but only CONOPT and IPOPT find the global optimal value see Table 14.1r. The other solvers find local optimal solutions near the global optimal value. On average MINOS is the fastest solver followed by CONOPT. SNOPT is erratic, at times working well and at times appearing to get hung up. Overall, since it is the second fastest and always finds a global optimum, CONOPT outperforms the remaining nonlinear solvers.

Table 14.1r. Solution Statistics for the 14-Bus Problem with the Nonlinear Solvers With Relaxation Slack Variables.

type	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVRG	TEPSIN	MINOPT
OPF2								
MINOS	82.279	0.002	0.052	0.234	0.094	10	0	81.849
CONOPT	80.815	0.000	0.083	0.065	0.109	10	0	80.815
KNITRO	81.148	0.001	0.197	0.261	0.670	10	0	81.009
IPOPT	80.815	0.000	0.432	0.062	0.562	10	0	80.815
SNOPT	83.051	0.005	14.137	0.714	104.17	10	0	81.267
OPF								
MINOS	85.077	0.002	0.042	0.129	0.063	10	0	84.382
CONOPT	82.762	0.000	0.056	0.152	0.109	10	0	82.762
KNITRO	85.585	0.015	0.170	0.053	0.234	10	0	83.042
IPOPT	82.762	0.000	0.446	0.058	0.546	10	0	82.762
SNOPT	84.249	0.002	28.238	0.947	281.89	10	0	83.346
OPFp								
MINOS	83.540	0.001	0.008	0.317	0.016	10	0	82.894
CONOPT	82.762	0.000	0.020	0.307	0.062	10	0	82.762
KNITRO	84.767	0.018	0.136	0.173	0.328	10	0	82.954
IPOPT	82.762	0.000	0.237	0.039	0.281	10	0	82.762
SNOPT	82.954	0.001	44.842	0.948	448.17	10	0	82.762

For the linear solvers, GUROBI outperforms CPLEX in cpu time. The optimal solution values are within 1 percent of the best nonlinear solvers. Generally, converging in 4 to 7 linear program passes (see table 14.2r). GUROBI and CPLEX are faster with relaxation slack variables.

Table 14.2r. Solution statistics for the 14-bus problem with the linear solvers with relaxation slack variables.

				Date	2012	315
type	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVRG
BTHETA						
GUROBI	76.591	0.000	0.002	0.050	0.002	10
GUROBI	76.591	0.000	0.002	0.050	0.002	10
GUROBI	76.591	0.000	0.002	0.153	0.004	10
CPLEX	76.591	0.000	0.009	0.049	0.011	10
CPLEX	76.591	0.000	0.010	0.111	0.020	10
OPFlin						
GUROBI	82.402	0.003	0.022	0.306	0.003	56
GUROBI	82.402	0.003	0.022	0.281	0.003	56
GUROBI	82.402	0.003	0.024	0.332	0.004	56
CPLEX	82.368	0.003	0.063	0.125	0.008	62
CPLEX	82.368	0.003	0.065	0.214	0.008	62

Table 14.3 reports the test results run on a different day. The notable difference is in the reported times.

Table 14.3. Solution statistics at a different time with relaxation slack variables.

	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVRG	TEPSIN
OPF2							
MINOS	82.279	0.002	0.054	0.205	0.109	10	0
CONOPT	80.815	0	0.067	0.147	0.109	10	0
KNITRO	81.48	0.004	0.194	0.187	0.484	10	0
IPOPT	80.815	0	0.495	0.049	0.624	10	0
SNOPT	83.051	0.005	16.429	0.734	125.612	10	0
OPF							
CONOPT	82.762	0	0.053	0.178	0.094	10	0
MINOS	85.077	0.002	0.059	0.127	0.094	10	0
KNITRO	125.766	0.263	0.222	0.158	0.483	10	0
IPOPT	82.773	0	0.516	0.061	0.655	10	0
SNOPT	84.249	0.002	32.032	0.947	319.848	10	0
OPFp							
MINOS	83.54	0.001	0.016	0.444	0.078	10	0
CONOPT	82.762	0	0.033	0.227	0.063	10	0
KNITRO	87.747	0.033	0.135	0.074	0.171	10	0
IPOPT	82.762	0	0.251	0.068	0.359	10	0
SNOPT	82.967	0.001	46.272	0.948	462.496	10	0
OPFlin							
GUROBI	82.407	0.003	0.022	0.247	0.004	57	0
CPLEX	82.368	0.003	0.063	0.175	0.02	62	0
BTHETA							
GUROBI	76.591	0	0.002	0.097	0.002	10	0
CPLEX	76.591	0	0.011	0.131	0.018	10	0

CONOPT. With relaxation slack variables, the OPF2 formulation had 211 rows, 239 columns 884 non-zeroes, 145 non-constant first derivatives with 365 evaluation operations. The OPF and OPFp formulations had 225 rows, 379 columns, 1,139 non-zeroes, 140 non-constant first derivatives non-zeroes with 322 evaluation operations. The difference is due to the linear objective function. With and without relaxation slack variables for all starting points, CONOPT found the optimal solutions of 80.81 for OPF2 and 82.76 for OPF and OPFp. CONOPT performed considerably faster on OPF and OPFp. The flat starting point did not improve the process.

Table 14.4. CONOPT performance on the 14-bus with and without relaxation slack variables.

	OPF2		OPF		OPFp	
	without	with	without	With	without	with
start	iterations	iterations	iterations	iterations	iterations	iterations
1	53	122	29	33	21	20
2	47	93	38	42	27	29
3	50	117	28	27	35	24
4	50	112	35	38	42	19
5	67	87	29	39	25	28
6	38	129	36	56	20	24
7	39	64	28	33	21	20
8	50	86	35	30	31	36
9	48	103	35	33	21	20
10	50	102	31	29	23	18
average	49	102	33	36	27	24

KNITRO. With relaxation slack variables, the OPF2 formulation had 238 variables, 210 constraints (28 linear equalities, 42 nonlinear equalities, 140 linear inequalities), 794 nonzeros in the Jacobian, and 89 nonzeros in the Hessian OPF and OPFp 378 variables, 224 constraints, (42 linear equalities, 42 nonlinear equalities, 140 linear inequalities) 948 nonzeros in the Jacobian and 84 nonzeros in the Hessian. The flat start does better than the average random starting point.

Table 14.4. KNITRO performance on the 14-bus without relaxation slack variables.

start	OPF2		OPF		OPFp	
point	iterations	opt-valu	iterations	opt-valu	iterations	opt-valu
1	144	80.815	102	82.762	84	82.762
2	169	80.815	153	82.762	90	82.762
3	166	80.815	133	82.762	87	82.762
4	202	80.815	143	82.762	86	82.762
5	159	80.815	126	82.762	87	82.762
6	222	80.815	151	82.762	91	82.762
7	184	80.815	92	82.762	89	82.762
8	189	80.815	158	82.762	91	82.762
9	172	80.815	99	82.762	89	82.762
10	162	80.815	132	82.762	91	82.762
avg	177	80.815	129	82.762	89	82.762

With relaxation slack variables for one random starting point KNITRO declared convergence with an objective value of 169.80 with relaxation slack variables active at very low levels and an objective value of 99.014 when the penalty values are removed. KNITRO also had trouble with the reactive power formulation of $Q = Q^+ - Q^-$ not driving one member of each pair to zero in the optimal solution.

Table 14.4. KNITRO performance on the 14-bus with relaxation slack variables.

Start	OPF2		OPF		OPFp	
point	Iteration	value	Iteration	value	Iteration	value
1	20	8.101	24	8.788	27	18.640
2	18	8.142	21	8.304	21	8.304
3	21	8.100	26	9.391	24	8.304
4	20	8.101	25	14.210	19	8.304
5	24	8.147	25	8.304	42	11.020
6	14	9.721	43	169.800	22	8.304
7	30	8.110	32	8.310	21	8.304
8	21	8.101	25	8.304	23	8.304
9	36	8.137	24	8.304	21	8.304
10	24	8.101	24	8.305	24	8.470
avg	23	8.276	27	25.202	24	9.626

SNOPT. With relaxation variables, the OPF2 formulation had 211 rows, 239 columns, 884 non-zeroes, and 145 non-constant first derivatives with 365 evaluation operations. The OPF and OPFp formulations had 225 rows, 379 columns, 1,139 non-zeroes, and 140 with 322 evaluation operations. The flat start does better than the average random point. With relaxation slack variables, SNOPT occasionally got hung up on the OPF2 and OPF (see Table 14.6).

Table 14.6 SNOPT performance on the 14-bus with and without relaxation slack variables.

	OPF2				OPF				OPFp			
	without		with		without		with		without		with	
start	iters	optval	Iters	optval	iters	optval	Iters	optval	iters	optval	Iters	optval
1	159	80.82	7	84.03	85	82.76	6	76.59	63	82.76	2	82.89
2	183	80.82	>e5	84.03	106	82.76	4	83.34	71	82.76	2	82.88
3	233	80.82	8	84.16	149	82.76	>e6	83.79	66	82.76	5	82.76
4	175	80.82	5	84.28	141	82.76	8	81.27	70	82.76	3	83.05
5	214	80.82	8	81.27	116	82.76	8	84.29	67	82.76	5	82.76
6	199	80.82	8	85.30	135	82.76	5	83.56	66	82.76	5	82.76
7	190	80.82	8	84.01	149	82.76	5	84.01	66	82.76	6	82.76
8	200	80.82	8	81.27	154	82.76	6	84.11	77	82.76	5	82.76
9	222	80.82	9	81.47	100	82.76	4	84.19	67	82.76	5	87.45
10	215	80.82	8	84.07	100	82.76	4	83.75	64	82.76	5	82.76
avg	199	80.82	8	83.39	124	82.76	6	82.93	68	82.76	5	83.43

MINOS. Without relaxation slack variables, the flat start does better than the average random point. With relaxation slack variable, MINOS was fast, but never found the optimal value.

Table 14.7 MINOS performance on the 14-bus with and without relaxation slack variables.

	OPF2				OPF				OPFp			
	without		with		without		with		without		with	
start	iters	optval	Iters	optval	iters	optval	Iters	optval	iters	optval	Iters	optval
1	18	80.82	6	82.35	29	82.76	6	85.61	16	82.76	6	83.78
2	29	80.82	7	82.34	46	82.76	6	85.03	15	82.76	6	83.82
3	28	80.82	6	81.85	42	82.76	6	84.38	24	82.76	5	83.65
4	23	80.82	6	81.87	55	82.76	7	84.88	37	82.76	5	83.88
5	25	80.82	6	81.87	95	82.76	6	85.30	15	82.76	4	82.90
6	19	80.82	6	81.85	19	82.76	6	86.11	27	82.76	5	83.94
7	20	80.82	6	81.87	28	82.76	6	84.63	22	82.76	5	83.79
8	32	80.82	6	82.48	34	82.76	7	84.89	16	82.76	5	83.68
9	44	80.82	5	82.13	35	82.76	5	85.22	16	82.76	4	82.89
10	28	80.82	6	82.23	35	82.76	6	84.73	30	82.76	5	83.40
avg	28	80.82	6	82.08	43	82.76	6	85.08	22	82.76	5	83.57

GUROBI. GUROBI produces a BTHETA formulation with 247 rows, 503 columns and 1,117 non-zeroes. GUROBI presolve resulted in 23 rows, 63 columns, 85 nonzeros. GUROBI found an optimal solution in 13 iterations with an objective value of 7.659 in less than .005s. GUROBI produces an OPFlin formulation with 394 rows, 379 columns and 1,365 non-zeroes. Depending on the starting point, takes 4 to 9 passes of the linear program.

Table 14.8. Linear Approximation Nonzeros, Iterations and Optimal Value by LP Pass and Starting Point for the 14-Bus Using GUROBI with Relaxation Slack Variables

pass	non-zeros	iters	optval	non-zeros	iters	optval	non-zeros	iters	optval
	start point 1			2			3		
1	1281	56	77.22	1303	64	79.41	1303	62	78.93
2	1359	71	81.69	1363	67	78.91	1363	65	78.47
3	1391	71	82.46	1387	62	77.33	1387	63	77.52
4	1419	76	82.34	1417	69	78.99	1417	71	80.95
5	1447	70	82.30	1445	71	82.29	1445	70	82.48
6	1475	74	82.33	1473	71	82.56	1473	86	82.56
	start point 4			5			6		
1	1303	56	78.28	1303	62	79.13	1303	63	78.48
2	1363	68	77.57	1363	62	78.32	1363	60	78.17
3	1391	70	79.98	1387	67	77.49	1387	58	77.56
4	1419	73	82.27	1417	73	81.81	1417	66	81.82
5	1447	75	82.54	1445	70	82.60			
6	1475	70	82.52	1473	69	82.57			
7	1503	68	82.32	1501	71	82.63			
	start point 7			8			9		
1	1303	59	77.98	1303	62	79.53	1303	57	78.13
2	1363	61	77.56	1363	62	78.96	1363	67	77.63
3	1391	68	82.07	1387	68	78.42	1391	71	82.32
4	1419	76	82.04	1413	58	77.89	1419	68	82.60
5				1439	67	78.81	1447	66	82.57
6				1469	66	82.25	1475	83	82.62
7							1503	71	82.55
8							1531	77	82.62
	start point 10								
1	1303	56	77.75						
2	1363	71	78.77						
3	1391	70	82.72						
4	1419	77	82.64						
5	1447	81	82.61						
6	1475	73	82.55						
7	1503	84	82.62						
8	1531	70	82.55						

CPLEX. CPLEX produces a BTHETA formulation with 247 rows, 503 columns, and 1,117 non-zeroes that was reduced to 23 rows, 64 columns, and 86 nonzeros. Almost identical to GUROBI, At CPLEX-defined iteration 1, CPLEX found an optimal solution with a value of 76.59. CPLEX is very similar to GUROBI.

Table 14.9. Linear approximation nonzeros, iterations and optimal value by LP pass and starting point for the 14-bus using CPLEX

pass	non-zeros	iterations	optval	non-zeros	iterations	optval	non-zeros	iterations	optval
start point 1				2			3		
1	1281	87	77.22	1303	75	79.41	1303	75	78.93
2	1359	81	81.69	1363	93	78.91	1363	85	78.47
3	1391	90	82.65	1387	105	77.33	1387	102	77.52
4	1419	87	82.54	1417	85	78.99	1413	143	78.71
5	1447	91	82.62	1445	80	82.16	1439	189	82.18
6	1475	82	82.56	1473	83	82.57	1465	101	82.49
start point 4				5			6		
1	1303	73	78.28	1303	80	79.13	1303	70	78.48
2	1363	77	77.57	1363	80	78.32	1363	70	78.17
3	1391	86	81.61	1387	102	77.49	1387	120	77.56
4	1419	102	82.66	1417	117	81.81	1417	185	81.82
5	1447	85	82.66	1445	107	82.60			
6	1475	79	82.52	1473	91	82.57			
7	1503	84	82.28	1501	84	82.63			
8				1529	80	82.55			
start point 7				8			9		
1	1303	81	77.98	1303	77	79.53	1303	71	78.13
2	1363	83	77.56	1363	84	78.96	1363	86	77.63
3	1391	86	82.07	1387	115	78.42	1391	92	82.29
4	1419	79	82.06	1413	76	77.89	1419	83	82.45
5	1447	106	82.36	1439	124	78.81	1447	86	82.42
6				1469	118	82.25			
start point 10									
1	1303	76	77.75						
2	1363	89	78.77						
3	1391	85	82.72						

Comparing the OPF to the linear approximation. We compare the OPF solution to the OPFlin solution and find only minor differences in P, Q and V and nodal prices for real and reactive power. We examine a CONOPT solution and linear approximation using GUROBI. The objective function values differ by less than one percent. In tables 14.10 and 14.11, the voltage magnitudes and dual variables from the bus equations are similar but the actual generator dispatch (real and reactive) differs somewhat.

Table 14.10 Solution Information from The CONOPT Solver With Objective Value Of 82.76

bus	Voltage		generation		load		Lambda ($/MVA-hr)	
no.	Mag	Ang	P	Q	P	Q	P	Q
	(pu)	(deg)	(MW)	(MVAR)	(MW)	(MVAR)		
1	1.06	0	1.826	0.1	0	0	35.733	2.536
2	1.04	-3.73	0.42	0.5	0.217	0.127	37.672	2.924
3	0.984	-9.19	0.3	0	0.942	0.19	40.637	3.951
4	0.999	-8.134	0	0	0.478	-0.039	40.092	3.685
5	1.004	-6.936	0	0	0.076	0.016	39.396	3.665
6	1.032	-12.038	0.033	0.047	0.112	0.075	40.1	4.001
7	1.02	-10.596	0	0	0	0	40.412	4.042
8	1.025	-9.631	0.1	0.032	0	0	40.277	4.001
9	1.017	-12.504	0	0	0.295	0.166	40.682	4.238
10	1.012	-12.729	0	0	0.09	0.058	40.883	4.389
11	1.018	-12.523	0	0	0.035	0.018	40.658	4.287
12	1.016	-12.937	0	0	0.061	0.016	40.94	4.285
13	1.011	-13.002	0	0	0.135	0.058	41.18	4.449
14	0.996	-13.8	0	0	0.149	0.05	42.002	4.726
totals			2.679	0.679	2.59	0.735	losses	0.089

Table 14.11 Solution Information from Linear Approximation Using GUROBI With Objective Value Of 82.65

bus	Voltage		generation		load		Lambda ($/MVA-hr)	
no.	Mag	Ang	P	Q	P	Q	P	Q
	(pu)	(deg)	(MW)	(MVAR)	(MW)	(MVAR)		
1	1.073	0	1.662	0.1	0	0	35.631	2.358
2	1.053	-3.41	0.42	0.5	0.217	0.127	37.539	2.868
3	0.996	-9.145	0.184	0	0.942	0.19	40.3	3.961
4	1.011	-7.202	0	0	0.478	-0.039	40.192	3.627
5	1.016	-6.087	0	0	0.076	0.016	39.487	3.618
6	1.033	-10.221	0.1	0	0.112	0.075	40.274	3.989
7	1.023	-8.164	0	0	0	0	40.593	3.983
8	1.02	-5.321	0.3	0.003	0	0	40.523	4.001
9	1.022	-10.449	0	0	0.295	0.166	40.872	4.144
10	1.016	-10.722	0	0	0.09	0.058	41.111	4.29
11	1.021	-10.614	0	0	0.035	0.018	40.881	4.219
12	1.017	-11.122	0	0	0.061	0.016	41.208	4.203
13	1.013	-11.173	0	0	0.135	0.058	41.477	4.346
14	0.998	-11.866	0	0	0.149	0.05	42.356	4.525
totals			2.666	0.603	2.59	0.735	losses	0.076

Increasing Demand. A standard test for robustness is increasing demand. The maximum real power generation is 7.724 and the fixed real demand is 2.59. The maximum possible demand increase factor is 2.98. We solve the 14-bus problem with demand multiplied by 1.5, 2 and 2.5. The results are in table 14.12. The problems require increasing CPU time. When demand is 2.5 times greater, the penalty slacks are greater than the set threshold. In 2x and 2.5x demand, CONOPT did not converge on two starting points. The GUROBI solutions were robust.

Table 14.12. Solution Statistics for 14-Bus Problem With Increased Demand Using CONOPT and GUROBI With Relaxation Slack Variables

type	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVRG	TEPSIN
OPF2	80.815	0.000	0.067	0.145	0.125	10	0
OPF	82.762	0.000	0.061	0.133	0.094	10	0
OPFlin	81.584	0.018	0.013	0.321	0.003	35	0
OPFp	82.762	0.000	0.041	0.220	0.094	10	0
BTHETA	76.591	0.000	0.002	0.100	0.003	10	
1.5*demand							
OPF2	133.668	0.000	0.080	0.156	0.125	10	0
OPF	137.152	0.000	0.078	0.063	0.094	10	0
OPFlin	135.965	0.003	0.015	0.368	0.004	39	0
OPFp	137.152	0.000	0.048	0.221	0.109	10	0
BTHETA	128.607	0.000	0.002	0.097	0.002	10	
2*demand							
OPF2	217.458	0.000	0.094	0.094	0.140	9	1
OPF	222.310	0.000	0.069	0.141	0.125	10	0
OPFlin	212.053	0.017	0.031	0.480	0.003	72	0
OPFp	222.310	0.000	0.078	0.185	0.125	10	0
BTHETA	181.925	0.000	0.002	0.085	0.002	10	
2.5*demand							
OPF2	268.552	0.000	0.081	0.119	0.125	8	10
OPF	273.322	0.000	0.090	0.068	0.125	10	10
OPFlin	272.321	0.001	0.019	0.363	0.004	45	10
OPFp	273.322	0.000	0.058	0.184	0.110	10	10
BTHETA	236.973	0.000	0.002	0.070	0.002	10	

30-bus problem

Without relaxation variables, all solvers with the exception of KNITRO find the same optimal solution for all starting points see table 30.1. MINOS, CONOPT and IPOPT are faster than the GUROBI solution. On average, SNOPT, CONOPT and MINOS were the fastest solvers. OPFp solves faster than OFP2 and OPF .

Table 30.1. Solution Statistics for the 30-Bus Problem Without Relaxation Slack Variables.

type	optvavg	optvcv	cpuavg	cpucv	cpumax	CONVRG	Tepsin
OPF2							
MINOS	5.745	0.000	0.103	0.072	0.141	10	0
CONOPT	5.745	0.000	0.117	0.087	0.156	10	0
SNOPT	5.745	0.000	0.136	0.067	0.172	10	0
IPOPT	5.745	0.000	0.532	0.061	0.734	10	0
KNITRO	7.565	0.105	0.674	0.221	1.716	10	0
OPF							
SNOPT	5.918	0.000	0.059	0.062	0.078	10	0
CONOPT	5.918	0.000	0.075	0.125	0.140	10	0
MINOS	5.918	0.000	0.083	0.097	0.110	10	0
IPOPT	5.918	0.000	0.722	0.076	1.030	10	0
KNITRO	10.018	0.083	1.247	0.382	5.585	10	0
OPFp							
SNOPT	5.918	0.000	0.024	0.104	0.032	10	0
MINOS	5.918	0.000	0.027	0.119	0.047	10	0
CONOPT	5.918	0.000	0.034	0.214	0.078	10	0
IPOPT	5.918	0.000	0.431	0.087	0.656	10	0
KNITRO	8.180	0.106	0.546	0.224	1.576	10	0
OPFlin							
GUROBI	5.914	0.002	0.126	0.312	0.013	83	0
GUROBI	5.914	0.002	0.129	0.292	0.013	83	0
GUROBI	5.914	0.002	0.133	0.320	0.013	83	0
CPLEX	5.914	0.002	0.233	0.289	0.031	87	0
CPLEX	5.914	0.002	0.235	0.300	0.029	87	0

Table 30.1a. LP Passes By Starting Point for the 30-Bus Using GUROBI
With And Without Relaxation Slack Variables

Start point	1	2	3	4	5	6	7	8	9	10	total
Without	2	10	10	10	10	7	9	7	9	9	83
with	5	9	7	9	8	8	8	8	9	9	80

With relaxation slack variables, all solvers declare a local optimal solution, but only CONOPT and IPOPT find the global optimal value see Table 30.1r. The other solvers find optimal solutions near the global optimal value. On average SNOPT is the fastest solver followed by CONOPT. CONOPT outperforms the other nonlinear solvers on the OPFp where SNOPT appears to get hung up on some problems. KNITRO is very erratic at times working well and at times appearing to get hung up. Overall, CONOPT outperforms the other nonlinear solvers in finding the global optimal solution.

Table 30.1r. Solution Statistics for the 30-Bus Problem with The Nonlinear Solvers.

type	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVRG	TEPSIN	MINOPT
OPF2								
SNOPT	6.722	0.010	0.067	0.099	0.109	10	0	6.319
CONOPT	5.745	0.000	0.175	0.098	0.296	10	0	5.745
MINOS	6.803	0.008	0.204	0.376	0.858	10	0	6.459
KNITRO	7.472	0.086	0.626	0.216	1.575	10	0	5.807
IPOPT	5.745	0.000	1.019	0.084	1.700	10	0	5.745
OPF								
SNOPT	6.216	0.007	0.056	0.149	0.094	10	0	6.042
CONOPT	5.918	0.000	0.097	0.138	0.188	10	0	5.918
MINOS	6.683	0.010	0.173	0.255	0.468	10	0	6.298
KNITRO	620.6	0.918	0.726	0.162	1.404	10	0	6.492
IPOPT	5.919	0.000	201.272	0.628	1000.2	8	0	5.918
OPFp								
CONOPT	5.918	0.000	0.040	0.189	0.078	10	0	5.918
KNITRO	15.492	0.166	0.377	0.117	0.624	10	0	6.495
MINOS	6.727	0.017	0.512	0.345	1.840	10	0	6.141
IPOPT	5.919	0.000	0.772	0.070	1.154	10	0	5.918
SNOPT	5.928	0.001	87.056	0.743	675.1	10	0	5.918

For linear solvers, GUROBI outperforms CPLEX see table 30.2. The optimal solution values are within 1 percent of the best nonlinear solvers. GUROBI is not as fast as SNOPT, but finds a better solution on OPF. CONOPT outperforms GUROBI. The linear solvers appear not to have problems with relaxation variables.

Table 30.2. Solution Statistics for the 30-Bus Problem with the Linear Solvers.

type	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVRG	TEPSIN
OPFlin							
GUROBI	5.909	0.004	0.167	0.185	0.020	80	0
GUROBI	5.909	0.004	0.185	0.191	0.028	80	0
GUROBI	5.909	0.004	0.189	0.216	0.019	80	0
CPLEX	5.910	0.004	0.300	0.202	0.030	88	0
CPLEX	5.910	0.004	0.308	0.181	0.045	88	0
BTHETA							
GUROBI	5.659	0.000	0.008	0.026	0.009	10	0
GUROBI	5.659	0.000	0.009	0.075	0.013	10	0
GUROBI	5.659	0.000	0.009	0.048	0.011	10	0
CPLEX	5.659	0.000	0.024	0.017	0.028	10	0
CPLEX	5.659	0.000	0.028	0.110	0.056	10	

Table 30.3 reports the test results run at a different day. The only notable difference is in the reported times.

Table 30.3 Solution Statistics for the 30-Bus Problem with Relaxation Slack Variables From Another Day.

	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVRG	TEPSIN
OPF2							
SNOPT	6.713	0.01	0.055	0.221	0.109	10	0
CONOPT	5.745	0	0.1	0.136	0.172	10	0
MINOS	6.865	0.009	0.151	0.385	0.655	10	0
KNITRO	8.492	0.118	0.462	0.337	1.716	10	0
IPOPT	5.746	0	0.616	0.081	0.936	10	0
OPF							
SNOPT	6.211	0.007	0.05	0.139	0.094	10	0
CONOPT	5.918	0	0.069	0.107	0.109	10	0
MINOS	6.718	0.017	0.165	0.348	0.546	10	0
KNITRO	8.974	0.243	0.441	0.186	0.936	10	0
IPOPT	5.918	0	200.61	0.63	1000.153	8	0
OPFp							
CONOPT	5.918	0	0.075	0.111	0.11	10	0
KNITRO	12.104	0.111	0.188	0.089	0.28	10	0
MINOS	6.893	0.014	0.329	0.22	0.796	10	0
IPOPT	5.92	0	0.37	0.071	0.514	10	0
SNOPT	6.091	0.011	10.103	0.636	56.893	10	0
BTHETA							
GUROBI	5.659	0	0.004	0.043	0.004	10	
CPLEX	5.659	0	0.011	0.037	0.014	10	
OPFlin							
GUROBI	5.801	0.005	0.022	0.109	0.01	29	0
CPLEX	5.811	0.006	0.056	0.441	0.013	38	0

CONOPT. The OPF2 formulation had 451 rows, 511 columns, 1,891 non-zeroes, and 306 non-constant first derivatives with 739 evaluation operations. The OPF and OPFp formulations had 481 rows, 811 columns, 2,395 non-zeroes, and 300 non-constant first derivatives with 690 evaluation operations. With relaxation slack variables for the flat start (starting point 1), CONOPT found an optimal solution equal to 5.745 with a reduced gradient of 5.7E-09 in 69 iterations. For the first random start (starting point 2), CONOPT found an optimal solution equal to 5.745 with a reduced gradient of 6.6E-10 in 104 iterations. For the second random start (starting point 3), CONOPT found an optimal solution equal to 5.745 with a reduced gradient of 2.3E-8 in 110 iterations.

CONOPT found an optimal solution of 5.745 for OPF2 and 5.918 for OPF and OPFp. The flat start does better than the average random point for OPF2 and OPF.

Table 30.4. CONOPT Performance on the 30-Bus With and Without Relaxation Slack Variables.

	OPF2		OPF		OPFp	
start point	without iterations	with iterations	without iterations	with iterations	without iterations	with iterations
1	45	69	20	20	28	39
2	108	90	89	120	17	35
3	48	92	83	100	9	37
4	121	103	95	52	10	45
5	52	40	95	60	12	46
6	178	126	84	42	11	52
7	40	148	40	38	12	154
8	118	95	64	42	26	26
9	146	212	24	35	10	60
10	133	118	74	52	29	46
avg	99	109	67	56	16	54

KNITRO. KNITRO does not return iterations. For several starting points, KNITRO declared convergence with an objective value of with relaxation slack variables active and solutions far off the optimal value when the penalty values are removed (see Table 30.5).

Table 30.5. KNITRO Performance on the 30-Bus With and Without Relaxation Slack Variables.

	OPF2			OPF			OPFp		
start	without	with		without	with		without	with	
point	optval	optval	optm	optval	optval	optm	optval	optval	optm
1	11.21	80.338	5.807	10.042	45.228	29.64	12.592	23.35	14.691
2	11.124	6.18	6.18	12.034	6.495	6.495	11.654	6.495	6.495
3	5.753	6.176	6.176	11.786	18.537	18.537	6.09	73.662	26.125
4	63.74	411.272	5.889	6.159	21260	6023.5	12.39	6.495	6.495
5	5.751	17.702	7.653	11.772	6.495	6.495	6.089	6.496	6.496
6	5.758	11.695	11.07	6.173	6.492	6.492	6.089	70.23	25.913
7	5.817	26.614	11.583	11.079	6.5	6.5	6.148	96.968	24.631
8	5.766	6.18	6.18	12.068	95.50	95.503	6.089	6.495	6.495
9	5.752	38.626	7.529	13.256	6.508	6.508	6.02	21.51	20.674
10	11.726	30.805	6.654	6.21	6.495	6.495	8.639	16.905	16.905
avg	13.240	63.559	7.472	10.058	2145.8	620.62	8.180	32.861	15.492

SNOPT. SNOPT without relaxation slack variables perform better on the flat start than any random starting point. With different starting points, SNOPT declared many different solution values optimal on OPF2 and OPF (see Table 30.6). SNOPT perform better on the flat start than on any starting point for OPF2 and OPF. With different starting points, SNOPT declared many different solution values optimal on any starting point for OPF2 and OPF see table 30.6. SNOPT found the optimal solutions of 5.745 for OPF2 and 5.918 for OPF and OPFp.

Table 30.6 SNOPT Performance on the 30-Bus With and Without Relaxation Slack Variables

	OPF2	OPF2		OPF	OPF		OPFp	OPFp	
start	without	with		without	with		without	with	
point	iters	iters	optval	iters	iters	optval	iters	iters	optval
1	193	92	6.707	115	84	6.042	113	123	5.918
2	430	207	6.889	229	168	6.101	141	105	5.989
3	347	237	6.757	277	204	6.094	92	65	5.918
4	398	223	6.840	237	159	6.337	95	112	5.918
5	353	174	6.527	196	158	6.372	97	120	5.918
6	485	212	6.319	289	292	6.34	104	86	5.932
7	318	247	6.962	222	190	6.102	123	71	5.933
8	443	169	6.862	238	134	6.254	129	95	5.918
9	368	171	6.471	223	177	6.117	89	66	5.918
10	364	206	6.885	232	172	6.406	90	98	5.918
avg	370	194	6.722	226	174	6.217	107	94	5.928

MINOS. Without relaxation slack variables, MINOS always found the optimal value and performs better on the flat start than any random starting point on OPF2 and OPF. With relaxation slack variables, MINOS never found the optimal value for any problem for any starting point. MINOS found the optimal solutions of 5.745 for OPF2 and 5.918 for OPF and OPFp.

Table 30.7 MINOS Performance on the 30-Bus Without Relaxation Slack Variables

	OPF2			OPF			OPFp		
start	without	with		without	with		without	with	
point	iters	iters	optval	iters	iters	optval	iters	iters	optval
1	258	534	6.966	142	187	6.496	153	3846	6.358
2	308	180	6.864	231	251	6.665	139	217	6.141
3	356	1299	6.603	251	763	6.838	146	1895	6.677
4	346	2462	6.954	185	1346	7.014	156	1799	7.150
5	381	196	6.898	225	215	6.729	137	6052	6.875
6	358	592	6.727	262	235	6.298	162	588	7.067
7	285	164	6.459	191	1315	6.929	139	684	6.734
8	258	170	6.861	210	189	6.415	159	755	7.038
9	341	217	6.706	229	207	6.657	161	212	6.215
10	268	256	6.989	215	826	6.788	139	536	7.014
avg	316	607	6.803	214	553	6.683	149	1658	6.727

GUROBI. The BTHETA formulation had 517 rows, 1,553 columns and 3,283 non-zeroes. GUROBI presolve resulted in 41 rows, 120 columns, 160 nonzeros and found an optimal solution in 23 iterations with objective value of 5.659. The first pass of OPFlin had 842 rows 811 columns and 2,881 non-zeroes. GUROBI finds the linear IV solution value is within 1 percent of the OPF solution value.

Table 30.8r. Linear Approximation Nonzeros, Iterations and Optimal Value by LP Pass and Starting Point for the 30-Bus Using GUROBI with Relaxation Slack Variables

pass	nonzeros	iters	optval	nonzeros	iters	optval	nonzeros	iters	optval
start point 1				2			3		
1	2701	232	5.774	2749	145	5.802	2749	168	5.798
2	2859	157	5.880	2875	142	5.803	2875	163	5.793
3	2923	139	5.842	2931	156	5.803	2931	154	5.785
4	2983	151	5.855	2989	163	5.782	2989	167	5.786
5	3043	152	5.835	3047	183	5.914	3051	164	5.905
6				3105	168	5.918	3111	167	5.917
7				3167	176	5.917	3171	156	5.917
8				3227	183	5.918			
9				3287	173	5.918			
start point 4				5			6		
1	2749	160	5.793	2749	144	5.816	2749	165	5.794
2	2875	147	5.792	2875	148	5.811	2875	136	5.790
3	2931	147	5.787	2931	155	5.792	2931	150	5.787
4	2989	160	5.831	2989	166	5.787	2989	163	5.800

5	3051	177	5.870	3047	168	5.905	3051	171	5.899
6	3111	177	5.917	3105	168	5.917	3111	178	5.917
7	3171	162	5.917	3163	180	5.918	3171	186	5.917
8	3231	169	5.918	3221	173	5.918	3231	172	5.918
9	3285	175	5.918						
start point 7				8			9		
1	2749	145	5.810	2749	155	5.806	2749	186	5.816
2	2875	152	5.807	2875	149	5.801	2875	141	5.814
3	2931	153	5.795	2931	145	5.789	2931	173	5.804
4	2989	166	5.789	2989	172	5.793	2989	154	5.788
5	3051	166	5.906	3051	169	5.903	3047	163	5.907
6	3111	172	5.917	3111	171	5.917	3105	171	5.887
7	3171	183	5.917	3171	163	5.917	3167	173	5.918
8	3231	164	5.918	3231	172	5.918	3227	171	5.918
start point 10									
1	2749	188	5.796						
2	2875	154	5.793						
3	2931	151	5.787						
4	2989	170	5.840						
5	3051	170	5.851						
6	3111	183	5.915						
7	3171	155	5.907						
8	3231	176	5.918						
9	3291	159	5.917						

CPLEX. CPLEX produced essentially the same as GUROBI.

Comparing the OPF to the linear approximation. We compare the CONOPT OPF solution to the GUROBI OPFlin solution and find differences in P, Q and V and nodal prices for real and reactive power. The actual generator dispatch is different, but the objective function value differs by less than one percent.

Table 30.10 Solution Information from CONOPT With Objective Value Of 5.918

bus	Voltage		generation		load		Lambda($/MVA-hr)	
no.	Mag (pu)	Ang (deg)	P (MW)	Q (MVAR)	P (MW)	Q (MVAR)	P	Q
1	1.05	0	0.444	0	0	0	3.76	0.175
2	1.048	-0.785	0.56	0.389	0.217	0.127	3.795	0.177
3	1.027	-1.907	0	0	0.024	0.012	3.866	0.22
4	1.023	-2.269	0	0	0.076	0.016	3.887	0.228
5	1.029	-2.218	0	0	0	0	3.862	0.208
6	1.017	-2.68	0	0	0	0	3.905	0.235
7	1.013	-2.993	0	0	0.228	0.109	3.926	0.242
8	1.005	-3.076	0	0	0.3	0.3	3.928	0.261
9	1.03	-3.825	0	0	0	0	3.929	0.202
10	1.037	-4.414	0	0	0.058	0.02	3.94	0.185
11	1.03	-3.825	0	0	0	0	3.929	0.202
12	1.011	-3.548	0	0	0.112	0.075	3.911	0.271
13	1.011	-2.292	0.16	0	0	0	3.899	0.271
14	1.003	-4.097	0	0	0.062	0.016	3.97	0.286
15	1.008	-3.937	0	0	0.082	0.025	3.948	0.275
16	1.015	-4.234	0	0	0.035	0.018	3.95	0.247
17	1.025	-4.578	0	0	0.09	0.058	3.96	0.213
18	1.006	-4.855	0	0	0.032	0.009	4.009	0.263
19	1.008	-5.188	0	0	0.095	0.034	4.026	0.251
20	1.014	-5.046	0	0	0.022	0.007	4.008	0.236
21	1.05	-4.602	0	0	0.175	0.112	3.934	0.13
22	1.058	-4.546	0.25	0.608	0	0	3.919	0.106
23	1.023	-2.741	0.18	0	0.032	0.016	3.866	0.266
24	1.028	-3.444	0	0	0.087	0.067	3.923	0.234
25	1.017	-1.667	0	0	0	0	3.869	0.287
26	0.999	-2.092	0	0	0.035	0.023	3.945	0.338
27	1.019	-0.298	0.33	0	0	0	3.805	0.291
28	1.017	-2.525	0	0	0	0	3.891	0.247
29	0.999	-1.55	0	0	0.024	0.009	3.925	0.325
30	0.987	-2.428	0	0	0.106	0.019	4.008	0.339
totals	1.924	0.997	1.892	1.072	real	power	loss	0.032

Table 30.11 Solution Information from Linear Approximation Using GUROBI Objective Value of 5.88.

bus	Voltage		generation		load		Lambda($/MVA-hr)	
no.	mag(pu)	ang(deg)	P(MW)	Q(MVAR)	P(MW)	Q(MVAR)	P	Q
1	1.08	0	0.4	0.331	0	0	3.738	0.202
2	1.068	-0.726	0.56	0.6	0.217	0.127	3.775	0.199
3	1.029	-1.947	0	0	0.024	0.012	3.853	0.206
4	1.018	-2.329	0	0	0.076	0.016	3.875	0.206
5	1.034	-2.345	0	0	0	0	3.845	0.202
6	1.007	-2.77	0	0	0	0	3.893	0.205
7	1.009	-3.191	0	0	0.228	0.109	3.913	0.206
8	0.993	-3.285	0	0	0.3	0.3	3.921	0.22
9	0.983	-4.337	0	0	0	0	3.904	0.173
10	0.97	-5.189	0	0	0.058	0.02	3.909	0.156
11	0.983	-4.337	0	0	0	0	3.904	0.173
12	0.971	-4.426	0	0	0.112	0.075	3.909	0.24
13	0.968	-2.774	0.183	0	0	0	3.9	0.244
14	0.959	-5.177	0	0	0.062	0.016	3.966	0.232
15	0.96	-4.865	0	0	0.082	0.025	3.948	0.227
16	0.964	-5.17	0	0	0.035	0.018	3.936	0.205
17	0.963	-5.485	0	0	0.09	0.058	3.934	0.173
18	0.951	-5.984	0	0	0.032	0.009	3.994	0.195
19	0.949	-6.374	0	0	0.095	0.034	4.003	0.179
20	0.954	-6.149	0	0	0.022	0.007	3.983	0.173
21	0.967	-5.079	0	0	0.175	0.112	3.89	0.108
22	0.97	-4.851	0.25	0	0	0	3.869	0.09
23	0.963	-3.29	0.18	0	0.032	0.016	3.887	0.241
24	0.957	-4.012	0	0	0.087	0.067	3.916	0.198
25	0.965	-2.109	0	0	0	0	3.875	0.265
26	0.946	-2.749	0	0	0.035	0.023	3.957	0.284
27	0.979	-0.562	0.33	0	0	0	3.814	0.291
28	1.003	-2.612	0	0	0	0	3.884	0.219
29	0.959	-2.163	0	0	0.024	0.009	3.927	0.281
30	0.948	-3.276	0	0	0.106	0.019	4.001	0.265
totals	1.903	0.931	1.892	1.072	real	power	loss	0.011

Increasing Demand. A standard test for robustness is increasing demand. We solve the 30-bus problem with demand multiplied by 1.2, 1.4 and 1.6. The results are in Table 30.12. The problems require increasing CPU time. For OPFlin, CONVRG is the number of linear programs solved over the ten starting points.

Table 30.12. Solution Statistics for 30-Bus Problem with Increased Demand Using CONOPT and GUROBI with Relaxation Slack Variables.

type	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVRG	TEPSIN
OPF2	5.745	0.000	0.105	0.087	0.141	10	0
OPF	5.918	0.000	0.076	0.154	0.140	10	0
OPFlin	5.801	0.005	0.021	0.104	0.009	29	0
OPFp	5.918	0.000	0.058	0.197	0.109	10	0
BTHETA	5.659	0.000	0.004	0.081	0.006	10	
1.200*demand							
OPF2	7.274	0.000	0.103	0.105	0.171	10	0
OPF	7.504	0.000	0.046	0.111	0.078	10	0
OPFlin	7.366	0.010	0.027	0.289	0.010	34	0
OPFp	7.504	0.000	0.041	0.224	0.093	10	0
BTHETA	7.133	0.000	0.003	0.046	0.004	10	
RatioNlLp	1.701						
1.400*demand							
OPF2	8.912	0.000	0.112	0.076	0.156	10	0
OPF	9.208	0.000	0.058	0.136	0.109	10	0
OPFlin	9.050	0.013	0.030	0.286	0.010	38	0
OPFp	9.208	0.000	0.064	0.126	0.094	9	1
BTHETA	8.706	0.000	0.004	0.067	0.006	10	
1.600*demand							
OPF2	10.719	0.000	0.097	0.113	0.140	9	1
OPF	11.085	0.000	0.058	0.195	0.140	10	0
OPF lin	10.851	0.013	0.030	0.215	0.010	36	0
OPFp	11.085	0.000	0.075	0.125	0.109	10	0
BTHETA	10.415	0.000	0.004	0.071	0.006	10	

57-bus Problem

The results with different nonlinear solvers are in Table 57.1. Without relaxation slack variables, all solvers except KNITRO find the global optimal value for all starting points. SNOPT, CONOPT and MINOS are the fastest nonlinear solvers. GUROBI is faster on OPF than any nonlinear solver.

Table 57.1. Solution Statistics for the 57-Bus Problem without Relaxation Slack Variables.

type	optvavg	optvcv	cpuavg	cpucv	cpumax	CONVRG	Tepsin
OPF2							
SNOPT	417.378	0.000	0.374	0.064	0.500	10	0
CONOPT	417.378	0.000	0.451	0.060	0.592	10	0
MINOS	417.378	0.000	0.484	0.242	1.498	10	0
IPOPT	417.378	0.000	0.951	0.161	1.887	10	0
KNITRO	420.574	0.003	1.789	0.404	8.222	10	0
OPF							
CONOPT	423.750	0.000	0.242	0.108	0.390	10	0
SNOPT	423.750	0.000	0.315	0.072	0.406	10	0
MINOS	423.750	0.000	0.454	0.212	1.092	10	0
IPOPT	423.750	0.000	1.878	0.245	6.069	10	0
KNITRO	430.938	0.003	2.440	0.125	4.820	10	0
OPFp							
SNOPT	423.750	0.000	0.106	0.028	0.125	10	0
MINOS	423.750	0.000	0.118	0.038	0.140	10	0
CONOPT	423.750	0.000	0.242	0.108	0.390	10	0
IPOPT	423.750	0.000	0.777	0.036	0.905	10	0
KNITRO	426.248	0.004	1.718	0.185	3.744	10	0
Btheta							
GUROBI	410.582	0.000	0.019	0.042	0.024	10	0
GUROBI	410.582	0.000	0.020	0.054	0.027	10	0
CPLEX	410.582	0.000	0.043	0.030	0.053	10	0
CPLEX	410.582	0.000	0.044	0.042	0.057	10	0
CPLEX	410.582	0.000	0.044	0.023	0.048	10	0
OPFlin							
GUROBI	420.021	0.003	0.193	0.323	0.054	58	0
GUROBI	420.021	0.003	0.196	0.317	0.056	58	0
CPLEX	420.377	0.002	0.203	0.262	0.045	52	0
CPLEX	420.377	0.002	0.208	0.277	0.040	52	0
CPLEX	420.377	0.002	0.213	0.292	0.061	52	0

With relaxation slack variables, only CONOPT and IPOPT find the global optimal value. The other solvers find optimal solutions near the global optimal value. SNOPT is the slowest solver and had difficulty on some starting points. CONOPT outperforms the other nonlinear solvers on the OPF2 and OPF but CONOPT fails to converge on one problem, but has higher optimal values. KNITRO is very erratic at times working well and at times appearing to get hung up.

Table 57.1r. Solution Statistics for the 57-Bus Problem with Relaxation Slack Variables.

type	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVRG	TEPSIN	MINOPT
OPF2								
CONOPT	456.718	0.054	0.817	0.13	1.279	9	2	417.378
KNITRO	419.15	0.001	1.321	0.149	2.886	10	0	417.849
IPOPT	417.378	0	2.861	0.085	4.18	10	0	417.378
MINOS	420.322	0.001	2.961	0.61	17.176	10	0	418.442
SNOPT	420.322	0.001	5.749	0.608	33.01	10	0	418.442
OPF								
CONOPT	423.75	0	0.445	0.122	0.671	10	0	423.75
KNITRO	610.439	0.242	2.012	0.208	4.462	10	0	424.827
IPOPT	423.758	0	2.718	0.073	4.15	10	0	423.75
MINOS	428.053	0.001	100.128	0.947	999.998	9	0	426.227
SNOPT	428.12	0.001	100.268	0.946	999.998	9	0	426.205
OPFp								
MINOS	425.004	0	0.064	0.171	0.141	10	0	424.249
CONOPT	423.75	0	0.226	0.087	0.343	10	0	423.75
KNITRO	425.211	0	0.85	0.05	1.06	10	0	424.824
IPOPT	423.758	0	2.348	0.178	6.022	10	0	423.75
SNOPT	425.222	0.001	44.557	0.947	444.712	10	0	423.965

For linear solvers, CPLEX times bound the GUROBI times see Table 57.2. The optimal solution values are within 1 percent of the best nonlinear solvers. Both linear solvers are faster and more robust than the nonlinear solvers.

Table 57.2. Solution Statistics for the 57-Bus Problem with the Linear Solvers.

type	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVERG	TEPSIN
OPFlin							
CPLEX	420.588	0.003	0.153	0.223	0.043	56	0
GUROBI	420.878	0.003	0.278	0.171	0.046	50	0
GUROBI	420.878	0.003	0.280	0.185	0.044	50	0
GUROBI	420.878	0.003	0.288	0.171	0.043	50	0
CPLEX	420.588	0.003	0.384	0.257	0.102	56	0
BTHETA							
CPLEX	410.582	0.000	0.019	0.031	0.024	10	0
GUROBI	410.582	0.000	0.020	0.032	0.025	10	0
GUROBI	410.582	0.000	0.021	0.045	0.027	10	0
GUROBI	410.582	0.000	0.024	0.060	0.030	10	0
CPLEX	410.582	0.000	0.047	0.049	0.063	10	0

Table 57.1a. Linear Approximation Nonzeros, Iterations and Optimal Value By LP Pass and Starting Point for the 57-Bus Using GUROBI With And Without Relaxation Slack Variables.

Start	1	2	3	4	5	6	7	8	9	10	Total
Without	3	6	5	5	7	6	9	6	5	6	58
With	3	6	5	5	5	5	6	5	5	5	50

Table 57.3 reports the test results run at a different day. The only notable difference is in the reported times.

Table 57.3. Solution Statistics for the 57-Bus Problem with Different Solvers at Another Time Without Relaxation Slack Variables.

type	optvavg	optvcv	cpuavg	cpucv	cpumax	CONVRG	Tepsin
OPF2							
SNOPT	417.378	0.000	0.374	0.064	0.500	10	0
CONOPT	417.378	0.000	0.451	0.060	0.592	10	0
MINOS	417.378	0.000	0.484	0.242	1.498	10	0
IPOPT	417.378	0.000	0.951	0.161	1.887	10	0
KNITRO	420.574	0.003	1.789	0.404	8.222	10	0
OPF							
CONOPT	423.750	0.000	0.242	0.108	0.390	10	0
SNOPT	423.750	0.000	0.315	0.072	0.406	10	0
MINOS	423.750	0.000	0.454	0.212	1.092	10	0
IPOPT	423.750	0.000	1.878	0.245	6.069	10	0
KNITRO	430.938	0.003	2.440	0.125	4.820	10	0
OPFp							
SNOPT	423.750	0.000	0.106	0.028	0.125	10	0
MINOS	423.750	0.000	0.118	0.038	0.140	10	0
CONOPT	423.750	0.000	0.242	0.108	0.390	10	0
IPOPT	423.750	0.000	0.777	0.036	0.905	10	0
KNITRO	426.248	0.004	1.718	0.185	3.744	10	0
Btheta							
GUROBI	410.582	0.000	0.019	0.042	0.024	10	0
GUROBI	410.582	0.000	0.020	0.054	0.027	10	0
CPLEX	410.582	0.000	0.043	0.030	0.053	10	0
CPLEX	410.582	0.000	0.044	0.042	0.057	10	0
CPLEX	410.582	0.000	0.044	0.023	0.048	10	0
OPFlin							
GUROBI	420.021	0.003	0.193	0.323	0.054	58	0
GUROBI	420.021	0.003	0.196	0.317	0.056	58	0
CPLEX	420.377	0.002	0.203	0.262	0.045	52	0
CPLEX	420.377	0.002	0.208	0.277	0.040	52	0
CPLEX	420.377	0.002	0.213	0.292	0.061	52	0

CONOPT. The OPF2 formulation had 856 rows, 970 columns, 3,587 non-zeroes, and 577 non-constant first derivatives with 1,370 evaluation operations occupying 4 Mb. CONOPT has 0 pre-triangular equations and 73 post-triangular equations. For the flat start with relaxation slack variables, at iteration 42 CONOPT found a feasible solution and at iteration 166 found an optimal solution with an optimal value of 4.174E2 with a reduced gradient of 1.4E-08. With relaxation slack variables for starting point 2, at iteration 343 CONOPT found an optimal solution value of 417.4 .

The OPF and OPFp formulations had 913 rows, 1,540 columns, 4,505 non-zeroes, and 570 non-constant first derivatives with 1,311 function evaluations. CONOPT found 50 pre-triangular equations and 138 post-triangular equations. At iteration 15, CONOPT found a feasible solution and at 44 an optimal solution of 4.237E2. At starting point 2, CONOPT found a feasible solution at iteration 100 and at 151 found an optimal solution of 4.237E2 for OPFp.

Without relaxation slack variables for OPF2, CONOPT performs better on the flat start than any random starting point. For the OPF, CONOPT performs better on the flat start than most random starting points. Without relaxation variables, CONOPT finds the optimal values of 417.378 for OPF2 and 423.750 for OPF and OPFp. On three points, CONOPT got hung up on OPF2.

Table 57.4. CONOPT Performance on the 57-Bus without Relaxation Slack Variables.

	OPF2				OPF			OPFp		
start	without	with			without	with		without	with	
point	iters	iters	optval	optml	iters	iters	optval	iters	iters	optval
1	81	166	4.17E2	417.4	48	44	423.8	77	50	423.8
2	259	343	4.17E2	417.4	153	151	423.8	34	38	423.8
3	186	110	1.36E8	521.5	267	73	423.8	56	53	423.8
4	222	262	4.17E2	417.4	64	59	423.8	99	84	423.8
5	212	562	4.17E2	417.4	95	258	423.8	33	128	423.8
6	206	116	4.17E2	417.4	46	343	423.8	29	46	423.8
7	143	559	4.17E2	417.4	151	144	423.8	42	108	423.8
8	182	430	1.37E8	594.4	189	171	423.8	54	156	423.8
9	151	305	1.37E8	594.4	56	153	423.8	57	104	423.8
10	132	483	4.17E2	417.4	70	93	423.8	34	64	423.8
avg	177	334	4.10E7	463.2	114	149	423.8	52	83	423.8

KNITRO. Without relaxation slack variables, KNITRO finds local optima near the global optimal with the exception of the OPF problem. With relaxation slack variables, KNITRO finds local optima near the global optimal.

Table 57.4. KNITRO Performance on the 57-Bus With and Without Relaxation Slack Variables.

	OPF2	OPF2		OPF	OPF		OPFp	OPFp	
start	without	with		without	with		without	with	
point	optval	optval	optm	optval	optval	optm	optval	optval	optm
1	417.5	418.2	418.2	429.1	424.8	424.8	424.0	426.1	426.1
2	421.0	421.4	421.4	437.9	474.2	474.2	423.9	425.9	425.9
3	423.9	419.8	419.8	435.6	438.6	438.6	423.9	426.3	426.3
4	417.4	418.7	418.7	430.1	454.5	454.5	423.9	424.8	424.8
5	424.4	420.8	420.8	426.5	424.8	424.8	424.0	424.8	424.8
6	498.0	418.2	418.2	434.6	2009.2	2001.1	430.0	424.8	424.8
7	439.6	418.2	418.2	424.0	424.8	424.8	423.9	424.8	424.8
8	417.4	420.2	420.2	435.6	424.9	424.9	423.9	424.8	424.8
9	421.6	481.7	417.8	424.7	424.8	424.8	445.2	424.8	424.8
10	417.4	418.2	418.2	432.8	857.5	611.7	424.0	424.8	424.8
avg	429.8	425.5	419.2	431.1	635.8	610.4	426.7	425.2	425.2

SNOPT. Without relaxation slack variables, SNOPT performs better on the flat start than any random starting point for OPF2 and OPF. With relaxation slack variables, SNOPT performs better on the flat start than any random starting point for OPF2, OPF and OPFp. Without relaxation variables, SNOPT found the optimal solutions of 417.4 for OPF2 and 423.8 for OPF and OPFp.

Table 57.6 SNOPT Performance on the 57-Bus With and Without Relaxation Slack Variables.

	OPF2			OPF			OPFp		
start	without	with		without	with		without	with	
point	iters	iters	optval	iters	iters	optval	iters	iters	optval
1	658	729	421.6	298	450	426.2	268	136	424.3
2	1029	890	419.3	859	728	427.8	283	285	425.0
3	1403	1072	418.4	971	957	427.9	255	355	426.2
4	1132	6490	419.7	617	777	426.6	303	222	425.6
5	1287	1107	423.2	825	1235	430.9	258	1379	425.4
6	1053	1140	419.8	866	1004	427.3	240	226	424.7
7	1137	1081	419.9	970	896	428.6	273	407	424.0
8	1040	5182	419.7	961	824	430.1	257	208	425.2
9	1102	1419	420.1	854	4367	426.7	253	281	427.2
10	1107	1299	421.6	896	877	427.7	234	249	424.6
avg	1095	2041	420.3	812	1212	428.0	262	375	425.2

MINOS. Without relaxation slack variables, MINOS always found the optimal value and perform better on the flat start than the average random starting point for OPF2 and OPF. With relaxation slack variables, MINOS found optimal values near the the optimal value and perform better on the flat start than the average random starting point for OPF2 and OPF. Without relaxation variables, MINOS found the optimal solutions of 417.4 for OPF2 and 423.8 for OPF and OPFp.

Table 57.7 MINOS Performance on the 57-Bus With and Without Relaxation Slack Variables.

	OPF2			OPF			OPFp		
start	without	with		without	with		without	with	
point	iters	iters	optval	iters	iters	optval	iters	iters	optval
1	1386	4217	422.9	415	337	426.9	356	2609	428.2
2	1287	3193	418.4	1829	925	425.9	344	2609	430.6
3	1052	969	422.6	1128	5972	428.4	313	490	426.2
4	3738	1241	419.9	2178	2552	427.0	346	907	431.6
5	1285	2599	420.1	1103	1286	430.3	358	549	424.9
6	1080	3429	420.8	833	8550	426.1	372	520	427.7
7	932	772	418.8	1080	3139	428.2	328	2550	426.9
8	1492	1106	418.5	2627	924	428.1	388	1512	429.3
9	2035	709	420.0	1000	851	427.3	327	496	425.9
10	1062	1499	419.8	1457	2378	428.3	348	3147	430.9
avg	1535	1973	420.2	1365	2691	427.7	348	1539	428.2

IPOPT. Without relaxation slack variables, IPOPT always found the optimal value and does not perform better on the flat start than the average random starting point for OPF2 and OPF. With relaxation slack variables, IPOPT found the optimal value and perform better on the flat start than the average random starting point for OPF2 and OPF. With and without relaxation variables, IPOPT found the optimal solutions of 417.4 for OPF2 and 423.8 for OPF and OPFp.

Table 57.7 IPOPT Performance on the 57-Bus With and Without Relaxation Slack Variables.

	OPF2		OPF		OPFp		
start	without	with	without	with	without		with
point	iters	iters	iters	iters	iters	optval	iters
1	33	48	44	65	21		39
2	19	57	40	66	23		35
3	58	75	128	49	19		37
4	51	80	41	56	19		45
5	19	97	28	65	20		46
6	26	41	38	55	22		52
7	29	75	33	45	23		154
8	21	51	46	53	20		26
9	21	83	25	70	23		60
10	21	64	25	71	20		46
avg	30	67	45	60	21		54

GUROBI. The BTHETA formulation had 82 rows, 4,489 columns, and 9,272 non-zeroes occupying 5 Mb. Presolve time took 0.02s and resulted in 85 rows, 176 columns, 260 nonzeros. GUROBI solved in 46 iterations and 0.02 seconds objective of 4.106e 2. OPFlin had 1,598 rows, 1,540 columns and 5,451 non-zeroes. Presolve resulted in 612 rows, 731 columns, and 2750 nonzeros. For OPFlin, the progression of the linear optimal values is not monotonic because the problem has non-nonconvex constraints, but the terminal values are within a 1 percent range.

Table 57.8. Linear Approximation Nonzeros, Iterations and Optimal Value By LP Pass and Starting Point for the 57-Bus Using GUROBI With Relaxation Slack Variables.

pass	nonzeros	iters	optval	nonzeros	iter	optval	nonzeros	iters	optval
	start-point 1			2			3		
1	5109	410	413.92	5187	307	416.87	5187	287	417.02
2	5413	384	421.68	5417	360	416.75	5417	314	417.25
3	5527	315	421.93	5531	378	416.21	5527	394	416.28
4				5645	339	415.05	5639	408	416.61
5				5759	319	422.17	5751	340	417.74
6				5873	342	421.38			
	start-point 4			start-point 5			start-point 6		
1	5187	292	416.67	5187	282	416.85	5187	286	416.89
2	5417	352	414.85	5417	333	417.54	5417	320	417.14
3	5531	307	414.56	5527	322	417.59	5527	378	416.42
4	5645	304	420.74	5639	397	414.98	5639	361	415.44
5	5759	303	421.10	5755	424	422.42	5751	343	421.53
	start-point 7			start-point 8			start-point 9		
1	5187	279	417.39	5187	293	416.84	5187	303	417.37
2	5417	320	417.68	5417	322	417.24	5417	304	416.39
3	5527	396	417.34	5527	350	416.56	5527	354	415.13
4	5639	347	415.09	5639	393	415.15	5639	363	418.27
5	5751	344	422.25	5751	402	421.32	5751	385	420.93
6	5863	346	420.65						
	start-point 10								
1	5187	315	417.23						
2	5417	315	416.76						
3	5527	376	415.60						
4	5639	357	415.14						
5	5751	364	419.78						

Increasing Demand. A standard test for robustness is increasing demand. The maximum possible demand increase is 2.98. We solve the 57-bus problem with demand increased by 1.5, 2 and 2.5. The results are in table 57.9.

Table 57.9. Solution Statistics for 57-Bus Problem with Increased Demand Using CONOPT And GUROBI .

type	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVRG	TEPSIN
OPF2	456.718	0.054	0.530	0.156	1.045	9	2
OPF	423.750	0.000	0.267	0.146	0.484	10	0
OPFlin	417.658	0.008	0.086	0.249	0.031	39	0
OPFp	423.750	0.000	0.136	0.152	0.234	10	0
BTHETA	410.582	0.000	0.009	0.025	0.010	10	
1.2*demand							
OPF2	565.941	0.000	0.479	0.124	0.811	9	1
OPF	575.347	0.000	0.304	0.143	0.608	10	0
OPFlin	532.646	0.008	0.103	0.554	0.016	45	0
OPFp	575.347	0.000	0.187	0.086	0.266	10	0
BTHETA	519.450	0.000	0.009	0.037	0.011	10	
1.4*demand							
OPF2	741.424	0.000	0.382	0.160	0.749	8	2
OPF	753.168	0.000	0.248	0.122	0.421	10	0
OPFlin	705.462	0.014	0.077	0.239	0.032	36	0
OPFp	753.168	0.000	0.133	0.134	0.234	10	0
BTHETA	678.164	0.000	0.009	0.034	0.011	10	
1.6*demand							
OPF2	885.914	0.000	0.627	0.132	1.232	9	1
OPF	899.774	0.000	0.365	0.130	0.718	10	0
OPFlin	899.339	0.000	0.097	0.131	0.024	38	0
OPFp	899.774	0.000	0.183	0.068	0.250	10	0
BTHETA	885.907	0.000	0.008	0.035	0.011	10	

Comparing the OPF to the linear approximation. We compare the CONOPT OPF solution to the GUROBI OPFlin solution and find only minor differences in P, Q and V and nodal prices for real and reactive power. The voltages and dual variables are similar but the actual generator dispatch is not as close, but the objective function value differs by less than one percent.

Table 57.10 Solution Information from CONOPT with Objective Value Of 423.7.

bus	Voltage		generation		load		Lambda		Bus flow	
no.	Mag (pu)	Ang (deg)	P (MW)	Q (MVAR)	P (MW)	Q (MVAR)	P	Q	Pflow	Qflow
1	1.012	0.000	1.439	0.884	0.550	0.170	42.338	2.008	1.357	0.794
2	0.999	1.029	0.915	0.000	0.030	0.880	41.900	2.538	0.885	0.827
3	1.003	-1.286	0.420	0.413	0.410	0.210	42.899	2.025	0.479	0.291
4	1.005	-1.182	0.000	0.000	0.000	0.000	42.872	2.083	0.386	0.081
5	1.013	-0.145	0.000	0.000	0.130	0.040	42.320	2.165	0.268	0.030
6	1.022	0.777	0.806	0.000	0.750	0.020	41.700	2.129	0.522	0.072
7	1.023	1.311	0.000	0.000	0.000	0.000	41.507	2.265	0.773	0.273

8	1.043	4.173	4.400	0.964	1.500	0.220	40.532	2.002	2.900	0.777
9	1.002	-0.337	1.000	0.000	1.210	0.260	42.306	2.345	1.711	0.684
10	0.984	-3.666	0.000	0.000	0.050	0.020	43.741	2.307	0.388	0.551
11	0.983	-2.406	0.000	0.000	0.000	0.000	43.475	2.451	0.443	0.180
12	0.992	-3.445	3.690	0.427	3.770	0.240	43.769	2.003	0.235	0.262
13	0.977	-3.242	0.000	0.000	0.180	0.023	43.889	2.209	0.682	0.423
14	0.970	-3.605	0.000	0.000	0.105	0.053	43.971	1.966	0.534	0.838
15	0.988	-2.628	0.000	0.000	0.220	0.050	43.485	2.066	1.006	0.532
16	0.992	-3.914	0.000	0.000	0.430	0.030	43.955	2.036	0.430	0.029
17	0.994	-2.868	0.000	0.000	0.420	0.080	43.534	2.122	0.475	0.061
18	1.025	-5.421	0.000	0.000	0.272	0.098	43.223	2.166	0.323	0.109
19	0.988	-6.794	0.000	0.000	0.033	0.006	45.365	2.815	0.049	0.021
20	0.976	-6.968	0.000	0.000	0.023	0.010	45.799	3.020	0.023	0.035
21	1.015	-6.646	0.000	0.000	0.000	0.000	45.897	2.723	0.007	0.026
22	1.015	-6.584	0.000	0.000	0.000	0.000	45.838	2.699	0.039	0.028
23	1.014	-6.596	0.000	0.000	0.063	0.021	45.917	2.739	0.063	0.030
24	1.017	-6.051	0.000	0.000	0.000	0.000	46.219	3.141	0.176	0.445
25	1.000	-10.967	0.000	0.000	0.063	0.032	46.871	3.577	0.145	0.051
26	0.976	-5.550	0.000	0.000	0.000	0.000	46.250	3.210	0.176	0.415
27	1.012	-3.154	0.000	0.000	0.093	0.005	43.961	2.861	0.274	0.034
28	1.032	-1.832	0.000	0.000	0.046	0.023	42.628	2.644	0.325	0.064
29	1.049	-0.965	0.000	0.000	0.170	0.026	41.600	2.381	0.680	0.290
30	0.979	-11.549	0.000	0.000	0.036	0.018	48.086	4.255	0.080	0.046
31	0.950	-12.334	0.000	0.000	0.058	0.029	49.836	5.237	0.058	0.029
32	0.960	-11.704	0.000	0.000	0.016	0.008	48.910	4.888	0.069	0.017
33	0.957	-11.743	0.000	0.000	0.038	0.019	49.084	4.975	0.038	0.019
34	0.966	-7.748	0.000	0.000	0.000	0.000	48.395	4.371	0.069	0.021
35	0.973	-7.528	0.000	0.000	0.060	0.030	47.973	4.137	0.129	0.063
36	0.982	-7.271	0.000	0.000	0.000	0.000	47.350	3.816	0.154	0.104
37	0.990	-7.116	0.000	0.000	0.000	0.000	46.895	3.490	0.186	0.135
38	1.016	-6.545	0.000	0.000	0.140	0.070	45.670	2.615	0.368	0.257
39	0.988	-7.142	0.000	0.000	0.000	0.000	46.985	3.547	0.031	0.022
40	0.979	-7.263	0.000	0.000	0.000	0.000	47.393	3.928	0.024	0.023
41	1.006	-6.570	0.000	0.000	0.063	0.030	43.825	3.055	0.227	0.067
42	0.974	-8.192	0.000	0.000	0.071	0.044	46.303	3.798	0.096	0.044
43	1.019	-3.640	0.000	0.000	0.020	0.010	43.571	2.623	0.147	0.149
44	1.019	-6.019	0.000	0.000	0.120	0.018	45.218	2.564	0.271	0.026
45	1.035	-4.160	0.000	0.000	0.000	0.000	43.555	2.306	0.275	0.244
46	1.060	-5.186	0.000	0.000	0.000	0.000	43.982	0.918	0.429	0.850
47	1.034	-6.401	0.000	0.000	0.297	0.116	44.875	1.725	0.424	0.243
48	1.029	-6.435	0.000	0.000	0.000	0.000	45.127	2.040	0.156	0.176
49	1.038	-6.439	0.000	0.000	0.180	0.085	44.850	2.086	0.331	0.333
50	1.024	-6.496	0.000	0.000	0.210	0.105	45.444	2.678	0.210	0.105
51	1.052	-4.905	0.000	0.000	0.180	0.053	43.779	2.295	0.338	0.585
52	1.017	-2.485	0.000	0.000	0.049	0.022	44.088	3.077	0.176	0.032
53	1.007	-3.142	0.000	0.000	0.200	0.100	45.118	3.254	0.200	0.068
54	1.027	-2.478	0.000	0.000	0.041	0.014	43.988	2.893	0.116	0.043

55	1.057	-1.482	0.000	0.000	0.068	0.034	42.380	2.330	0.187	0.296
56	0.974	-8.915	0.000	0.000	0.076	0.022	47.263	3.457	0.113	0.029
57	0.969	-9.569	0.000	0.000	0.067	0.020	47.903	3.476	0.067	0.020
totals	12.670	2.688	12.508	3.364	real-power-loss 0.162					

Table 57.11. Solution Information from Linear Approximation Using GUROBI
Objective Value of 421.9 and Third Pass

bus	Voltage		generation		load		Lambda		Bus flow	
no.	Mag (pu)	Ang (deg)	P (MW)	Q (MVAR)	P (MW)	Q (MVAR)	P	Q	Pflow	Qflow
1	1.033	0.000	1.152	1.192	0.550	0.170	41.974	2.008	1.115	1.175
2	1.012	1.125	0.800	0.000	0.030	0.880	41.529	2.466	0.786	0.945
3	0.992	-0.270	0.420	0.000	0.410	0.210	42.745	1.486	0.497	0.317
4	0.988	0.237	0.000	0.000	0.000	0.000	42.863	1.392	0.513	0.208
5	0.987	2.001	0.000	0.000	0.130	0.040	42.582	1.246	0.314	0.102
6	0.992	3.308	1.000	0.000	0.750	0.020	42.091	1.122	0.673	0.146
7	0.985	3.879	0.000	0.000	0.000	0.000	41.782	1.236	0.762	0.266
8	0.997	7.057	4.577	0.000	1.500	0.220	40.778	1.016	3.047	0.175
9	0.986	1.474	1.000	0.000	1.210	0.260	42.466	1.604	1.841	0.725
10	0.997	-2.532	0.000	0.000	0.050	0.020	43.816	1.846	0.414	0.735
11	0.980	-1.065	0.000	0.000	0.000	0.000	43.575	1.863	0.477	0.259
12	1.031	-2.654	3.690	1.399	3.770	0.240	43.809	2.003	0.289	1.241
13	0.987	-2.232	0.000	0.000	0.180	0.023	43.930	1.808	0.756	0.780
14	0.977	-2.698	0.000	0.000	0.105	0.053	43.983	1.537	0.548	0.855
15	0.993	-1.853	0.000	0.000	0.220	0.050	43.377	1.699	0.898	0.617
16	1.026	-3.287	0.000	0.000	0.430	0.030	43.919	2.019	0.434	0.050
17	1.023	-2.527	0.000	0.000	0.420	0.080	43.360	2.113	0.426	0.067
18	1.009	-4.202	0.000	0.000	0.272	0.098	43.119	1.421	0.324	0.106
19	0.977	-5.783	0.000	0.000	0.033	0.006	45.382	2.063	0.048	0.013
20	0.969	-6.052	0.000	0.000	0.023	0.010	45.867	2.317	0.023	0.035
21	1.014	-5.639	0.000	0.000	0.000	0.000	45.971	2.026	0.009	0.030
22	1.015	-5.595	0.000	0.000	0.000	0.000	45.916	2.028	0.049	0.079
23	1.014	-5.587	0.000	0.000	0.063	0.021	46.002	2.039	0.064	0.077
24	1.003	-4.684	0.000	0.000	0.000	0.000	46.382	2.014	0.173	0.484
25	0.988	-9.822	0.000	0.000	0.063	0.032	46.870	2.313	0.147	0.047
26	0.960	-4.175	0.000	0.000	0.000	0.000	46.439	2.108	0.174	0.434
27	0.983	-1.203	0.000	0.000	0.093	0.005	44.166	1.740	0.273	0.021
28	0.999	0.376	0.000	0.000	0.046	0.023	42.837	1.542	0.324	0.023
29	1.014	1.420	0.000	0.000	0.170	0.026	41.805	1.293	0.683	0.273
30	0.968	-10.471	0.000	0.000	0.036	0.018	48.144	2.908	0.081	0.042
31	0.941	-11.384	0.000	0.000	0.058	0.029	49.982	3.817	0.059	0.029
32	0.954	-10.852	0.000	0.000	0.016	0.008	49.034	3.791	0.071	0.022
33	0.952	-10.893	0.000	0.000	0.038	0.019	49.219	3.860	0.038	0.019
34	0.966	-6.752	0.000	0.000	0.000	0.000	48.667	3.463	0.071	0.027
35	0.973	-6.539	0.000	0.000	0.060	0.030	48.218	3.293	0.132	0.069
36	0.982	-6.287	0.000	0.000	0.000	0.000	47.553	3.040	0.157	0.113
37	0.991	-6.141	0.000	0.000	0.000	0.000	47.059	2.760	0.188	0.146

38	1.018	-5.600	0.000	0.000	0.140	0.070	45.733	2.006	0.382	0.313
39	0.989	-6.164	0.000	0.000	0.000	0.000	47.156	2.809	0.030	0.025
40	0.980	-6.273	0.000	0.000	0.000	0.000	47.609	3.149	0.023	0.025
41	1.004	-5.379	0.000	0.000	0.063	0.030	43.862	2.435	0.234	0.068
42	0.972	-7.085	0.000	0.000	0.071	0.044	46.444	3.013	0.099	0.044
43	1.017	-2.347	0.000	0.000	0.020	0.010	43.647	2.018	0.152	0.149
44	1.022	-5.120	0.000	0.000	0.120	0.018	45.235	2.024	0.264	0.017
45	1.039	-3.353	0.000	0.000	0.000	0.000	43.434	1.935	0.270	0.247
46	1.066	-4.314	0.000	0.000	0.000	0.000	43.957	0.491	0.440	0.867
47	1.039	-5.509	0.000	0.000	0.297	0.116	44.891	1.215	0.428	0.268
48	1.033	-5.522	0.000	0.000	0.000	0.000	45.158	1.501	0.169	0.213
49	1.044	-5.513	0.000	0.000	0.180	0.085	44.860	1.576	0.342	0.339
50	1.033	-5.510	0.000	0.000	0.210	0.105	45.541	2.120	0.212	0.107
51	1.066	-3.837	0.000	0.000	0.180	0.053	43.815	1.761	0.361	0.603
52	0.984	-0.360	0.000	0.000	0.049	0.022	44.325	1.964	0.177	0.022
53	0.975	-1.142	0.000	0.000	0.200	0.100	45.363	2.138	0.201	0.078
54	1.002	-0.634	0.000	0.000	0.041	0.014	44.181	1.968	0.119	0.064
55	1.037	0.246	0.000	0.000	0.068	0.034	42.496	1.578	0.192	0.286
56	0.972	-7.872	0.000	0.000	0.076	0.022	47.385	2.599	0.115	0.032
57	0.967	-8.573	0.000	0.000	0.067	0.020	48.027	2.575	0.068	0.022
totals	12.639	2.591	12.508	3.364	real-power-loss 0.131					

118-bus Problem

Without relaxation slack variables, all solvers except KNITRO find the global optimal value for all starting points see Table 118.1. SNOPT, CONOPT and MINOS are the fastest solvers. CPLEX and GUROBI are faster on OPF than any nonlinear solver but MINOS. OPFp did not solve faster than the OPF.

Table 118.1. Solution statistics for the 118-bus problem with different nonlinear solvers without relaxation slack variables.

	optvavg	optvcv	cpuavg	cpucv	cpumax	CONVRG	Tepsin
OPF2							
SNOPT	1297	0	2.90	0.06	3.9	10	0
MINOS	1297	0	3.06	0.10	5.0	10	0
IPOPT	1297	0	3.19	0.14	6.5	10	0
CONOPT	1297	0	5.19	0.09	8.9	10	0
KNITRO	1298	0.001	173.97	0.52	755.1	8	0
OPF							
MINOS	1311	0	0.87	0.15	1.8	10	0
SNOPT	1311	0	1.52	0.05	1.8	10	0
CONOPT	1311	0	1.75	0.05	2.2	10	0
IPOPT	1311	0	4.63	0.12	7.7	10	0
KNITRO	1315	0.001	202.69	0.62	1000.1	8	2
OPFp							
MINOS	1311	0	1.09	0.35	3.5	9	0
CONOPT	1311	0	1.21	0.11	2.1	9	0
IPOPT	1311	0	3.22	0.31	12.5	9	0
SNOPT	1311	0	3.47	0.38	12.7	7	0
KNITRO	1319	0.002	277.41	0.48	1000.1	7	3
OPFlin							
CPLEX	1293	0.006	0.91	0.32	0.1	82	0
CPLEX	1293	0.006	0.92	0.33	0.1	82	0
CPLEX	1293	0.006	0.93	0.35	0.1	82	0
GUROBI	1292	0.007	1.15	0.28	0.2	89	0
GUROBI	1292	0.007	1.16	0.30	0.1	89	0
Btheta							
GUROBI	1261	0	0.08	0.04	0.1	10	0
GUROBI	1261	0	0.08	0.05	0.1	10	0
CPLEX	1261	0	0.14	0.06	0.2	10	0
CPLEX	1261	0	0.14	0.04	0.2	10	0
CPLEX	1261	0	0.14	0.02	0.2	10	0

Without relaxation slack variables, the results with different nonlinear solvers are in table 118.1r. With relaxation slack variables, all nonlinear solvers have trouble solving the problems. Only CONOPT and KNITRO find the optimal solution for the OPF2. CONOPT, IPOPT and KNITRO found the optimal solution to the OPF.

Table 118.1r. Solution Statistics for the 118-Bus Problem with Different Nonlinear Solvers with Relaxation Slack Variables.

type	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVRG	TEPSIN	MINOPF
OPF2								
SNOPT	1339.832	0.001	1.652	0.056	2.137	10	0	1330.587
MINOS	1694.146	0.052	3.424	0.195	9.079	5	9	1342.095
CONOPT	1807.122	0.049	8.441	0.103	14.227	9	9	1296.607
IPOPT	1955.824	0.038	10.906	0.113	20.327	10	10	1510.022
KNITRO	1370.009	0.049	129.387	0.722	1000.044	9	2	1297.639
OPF								
MINOS	1394.883	0.004	2.788	0.176	6.178	10	0	1365.595
CONOPT	1467.377	0.056	6.204	0.335	25.053	10	3	1311.498
IPOPT	1311.517	0	9.076	0.037	10.764	10	0	1311.499
SNOPT	1395.91	0.005	10.143	0.857	92.571	10	0	1361.567
KNITRO	3682.3	0.544	140.969	0.691	1000.044	8	1	1311.498
OPFp								
SNOPT	1380.287	0.038	1.256	0.487	6.957	10	1	1320.675
CONOPT	1327.655	0.012	1.911	0.232	5.99	10	1	1313.965
MINOS	1392.89	0.021	2.442	0.121	3.729	10	1	1334.394
KNITRO	1419.399	0.04	4.271	0.305	15.694	10	1	1313.963
IPOPT	1366.839	0.038	5.022	0.204	13.977	10	1	1311.499

For linear solvers, GUROBI and CPLEX have similar performance with relaxation slack variables see Table 118.2. The optimal solution values are within 2 percent of the best nonlinear solver performance.

Table 118.2. Solution Statistics for the 118-Bus Problem with the Linear Solvers with Relaxation Slack Variables.

type	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVRG	TEPSIN
OPFlin							
GUROBI	1293	0.006	1.89	0.41	0.15	86	0
GUROBI	1293	0.006	1.89	0.41	0.15	86	0
CPLEX	1295	0.006	1.89	0.43	0.17	82	0
GUROBI	1293	0.006	1.95	0.41	0.16	86	0
CPLEX	1295	0.006	1.97	0.45	0.18	82	0
BTHETA							
GUROBI	1261	0	0.09	0.03	0.10	10	0
GUROBI	1261	0	0.09	0.03	0.11	10	0
GUROBI	1261	0	0.09	0.03	0.11	10	0
CPLEX	1261	0	0.13	0.01	0.14	10	0
CPLEX	1261	0	0.15	0.07	0.21	10	0

When run at different times, the results were similar with the exception of cpu times.

Table 118.3. Solution Statistics for the 118-Bus Problem with Relaxation Slack Variables at A Different Time.

	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVRG	TEPSIN
OPF2							
SNOPT	1,342	0.00	0.69	0.05	0.83	10	0
CONOPT	1,797	0.04	3.39	0.06	4.52	10	9
IPOPT	1,931	0.04	3.91	0.06	4.96	10	10
MINOS	1,768	0.04	3.97	0.58	25.52	9	9
KNITRO	1,348	0.03	30.11	0.66	212.05	9	2
OPF							

MINOS	1,429	0.03	1.31	0.17	2.47	10	1
CONOPT	1,573	0.06	2.99	0.19	6.35	10	5
IPOPT	1,312	0.00	3.57	0.06	4.70	10	0
SNOPT	1,392	0.01	4.17	0.86	38.22	10	0
KNITRO	7,349	0.76	50.71	0.93	497.64	9	1
OPFp							
SNOPT	1,381	0.04	0.43	0.55	2.65	10	1
CONOPT	1,352	0.03	0.86	0.22	2.61	10	1
MINOS	1,393	0.02	1.06	0.12	1.95	10	1
KNITRO	410,600	0.95	1.46	0.30	5.15	10	1
IPOPT	1,367	0.04	1.86	0.17	4.63	10	1
BTHETA							
GUROBI	1,261	0.00	0.04	0.02	0.04	10	0
CPLEX	1,261	0.00	0.06	0.02	0.07	10	0
OPFlin							
GUROBI	1,289	0.01	0.51	0.63	0.06	62	0
CPLEX	1,288	0.01	0.62	0.62	0.08	61	0

CONOPT. The OPF2 formulation had 1,771 rows, 2,007 columns, 7,705 non-zeroes, and 1,234 non-constant first derivatives with 3,149 evaluation operations. CONOPT found 138 post-triangular equations. The results are presented in Table 118.4. At iteration 411, CONOPT finds a feasible solution and at iteration 710 found the optimal solution of 8.427E7. At iteration 805, CONOPT found a feasible solution and at iteration 1262, found an optimal solution with value of 1.889E8 with the following message: The error in the optimal objective function value. Hessian is less than the minimal tolerance on the objective. With another starting point, at iteration 931, CONOPT found a feasible solution and at iteration 1357, an optimal solution of 1.297E3 Reduced gradient 4.8E-08. The OPF and OPFp formulations had 1,889 rows, 3,187 columns, 9,961 non-zeroes, and 1,180 non-constant first derivatives with 2,714 evaluation operations. CONOPT found 64 pre-triangular equations and 212 post-triangular equations.

With relaxation slack variables, at iteration 216, CONOPT found a feasible solution and at iteration 291 found a local optimal solution with a value of 1.311E3. For starting point 2, at iteration 126, CONOPT found a feasible solution and at iteration 225 found an optimal solution with a value of 1.311E3. For starting point of 3, at iteration 185, CONOPT found a feasible solution and at iteration 780 found an optimal solution with a value of 7.044E7 with a reduced gradient of .5E-09. For OPFp, at iteration 91, CONOPT found a feasible solution and at iteration 151, CONOPT found an optimal solution with a value of 1.312E3 with a reduced gradient less than tolerance.

With relaxation slack variables, CONOPT found optimal values near the optimal value and performed better on the flat start than the average random starting point for OPF2 and OPF. Without relaxation variables, CONOPT found the optimal solutions of 1297 for OPF2 and 1311 for OPF. The performance with and without relaxation slack variables are in Table 118.3.

Table 118.3. CONOPT Performance on the 118-Bus With and Without Relaxation Slack Variables.

	OPF2				OPF				OPFp				
star	without	with			without	with			without		with		
pt	iter	iter	opt	opt	iter	iter	opt	opt	iter	opt	iter	opt	opt
1	69	710	8E	198	20	198	1E	131	11	131	11	1E	131
2	58	121	3E	167	30	402	1E	155	92	131	21	1E	131
3	56	131	1E	129	30	313	1E	131	19	131	25	1E	131
4	51	980	1E	202	38	664	7E	197	24	165	52	2E	147
5	53	107	1E	151	32	342	1E	131	15	131	24	1E	131
6	60	170	6E	171	22	311	1E	131	33	131	38	1E	131
7	43	122	1E	218	27	395	1E	131	13	131	16	1E	131
8	56	133	1E	204	43	359	1E	131	20	131	37	1E	131
9	80	611	2E	192	19	360	1E	131	88	131	13	1E	131
10	75	135	9E	195	21	103	7E	196	13	131	16	1E	131
avg	60	115	8E	183	28	438	2E	146	16	134	25	2E	132

KNITRO. The performance with and without relaxation slack variables are in Table 118.4.

Table 118.4. KNITRO Performance on the 118-Bus without Relaxation Slack Variables.

start point	OPF2 without optval	OPF2 with optval	optm	OPF without optval	OPF with optval	optm	OPFp without optval	OPFp with optval	optm
1	1297	1.3E3	1298	1338	1.3E3	1314	1312	1.3E3	1314
2	1297	1.3E3	1300	1317	1.3E3	1314	1312	1.3E3	1314
3	1297	1.3E3	1299	1314	1.3E3	1314	1312	1.3E3	1314
4	1324	1.3E3	1298	1312	2.0E4	18600	1312	1.5E7	1859
5	1297	8.6E7	1853	1312	1.5E8	64250	1312	1.9E3	1571
6	1297	1.2E8	1941	1312	3.3E+10	3.3E+10	1312	1.3E3	1314
7	1297	1.3E3	1298	1312	1.6E3	1375	1312	2.8E3	1566
8	1297	1.3E3	1299	1312	2.9E3	2909	1313	1.3E3	1314
9	1297	1.3E3	1298	1320	1.3E3	1314	1312	1.3E3	1314
10	1298	1.3E3	1298	1312	1.3E3	1320	1312	1.3E3	1314
avg	1300	2.0E7	1418	1316	3.3E9	3.3E9	1312	1.5E6	1419

SNOPT. Without relaxation slack variables, SNOPT always found the optimal value OPF2 of 1297, and OPF of 1311. SNOPT performed better on the flat start than each random starting point for OPF2, OPF and OPFp. With relaxation slack variables, SNOPT had difficulty (see Table 118.5).

Table 118.5 SNOPT Performance on the 118-Bus With and Without Relaxation Slack Variables.

	OPF2			OPF			OPFp				
start	without	with		without	with		without		with		
point	iter	iter	opt	iter	iter	opt	iter	opt	iter	opt	optm
1	1768	1433	1342	1396	927	1362	691	1311	489	1321	1322
2	3838	2462	1336	2434	1476	1399	808	1311	717	1329	1329
3	3327	2980	1347	2509	1703	1438	651	1311	570	1326	1326
4	3652	3029	1331	2612	1818	1430	17839	2048	8802	3E7	1878
5	3189	2853	1335	2365	1992	1385	883	1311	629	1326	1326
6	3911	2701	1348	2357	1923	1375	11621	1311	760	1322	1322
7	3535	2949	1339	2639	1862	1389	5565	1704	643	1321	1321
8	3460	3359	1341	2580	1886	1403	931	1311	506	1326	1326
9	3306	2569	1338	2626	1714	1376	799	1311	700	1326	1326
10	4804	2589	1342	2537	1735	1403	9647	1828	775	1328	1328
avg	3479	2692	1340	2406	1704	1396	4944	1476	1459	3E6	1380

MINOS. Without relaxation slack variables, MINOS always found the optimal value of 1297 for OPF2 and 1311 for OPF and OPFp. MINOS performs better on the flat start than the average random starting point. With relaxation slack variables, MINOS had dificulty (see Table 118.6).

Table 118.6 MINOS Performance on the 118-Bus With and Without Relaxation Slack Variables.

		OPF2				OPF				OPFp		
start	withou		with		withou	with		without			With	
poin	iter	iter	opt	opt	iter	iter	opt	iter	opt	iter	opt	omp
1	1765	234	3E	1677	792	605	140	665	131	404	137	137
2	2082	242	4E	1846	882	294	140	675	131	283	136	136
3	1808	580	1E	1342	938	281	137	690	131	400	138	138
4	3609	301	4E	1735	847	181	141	839	199	268	3E7	167
5	2875	895	1E	1609	921	498	139	710	131	139	134	134
6	4157	291	4E	2013	2137	234	136	703	131	340	135	135
7	2835	256	3E	1809	997	159	136	683	131	149	133	133
8	1874	330	5E	1864	877	462	141	711	131	154	134	134
9	2792	347	3E	1696	1907	193	140	712	131	318	136	136
10	2244	249	3E	1745	917	225	141	692	131	328	139	139
avg	2604	373	3E	1734	1122	313	139	209	138	278	3E6	139

IPOPT. Without relaxation slack variables, IPOPT always found the optimal value and perform better on the flat start than the average random starting point for OPF2 and OPF (see Table 118.7).

Table 118.7 IPOPT Performance on the 118-Bus without Relaxation Slack Variables.

start	OPF2		OPF		OPFp	
point	iterations	optval	iterations	optval	iterations	optval
1	38	1296.607	60	1311.498	26	1311.498
2	84	1296.607	83	1311.498	22	1311.498
3	57	1296.607	50	1311.498	23	1311.498
4	28	1296.607	28	1311.498	167	4651.809
5	42	1296.607	43	1311.498	23	1311.498
6	55	1296.607	77	1311.498	47	1311.498
7	39	1296.607	80	1311.498	23	1311.498
8	38	1296.607	72	1311.498	28	1311.498
9	31	1296.607	46	1311.498	28	1311.498
10	29	1296.607	51	1311.498	26	1311.498
avg	44	1296.607	59	1311.498	41	1645.529

With relaxation slack variables, IPOPT had difficulty (see Table 118.8).

Table 118.8 IPOPT Performance on the 118-Bus with Relaxation Slack Variables.

start	OPF2			OPF			OPFp		
point	iters	optval	optm	iters	optval	optm	iters	optval	optm
1	76	1.08E8	2086.1	97	1.31E3	1311.5	39	1.31E3	1311.6
2	82	4.10E7	1847.1	96	1.31E3	1311.5	47	1.31E3	1311.5
3	100	4.10E7	1847.1	81	1.31E3	1311.5	41	1.31E3	1311.5
4	185	6.01E7	2026.7	94	1.31E3	1311.5	120	3.11E7	1864.8
5	110	1.48E8	2250.0	96	1.31E3	1311.5	38	1.31E3	1311.5
6	81	6.95E7	1917.3	112	1.31E3	1311.5	71	1.31E3	1311.5
7	102	5.81E7	1686.8	107	1.31E3	1311.5	39	1.31E3	1311.5
8	160	4.54E8	2306.5	93	1.31E3	1311.5	59	1.31E3	1311.5
9	82	1.45E7	1510.0	100	1.31E3	1311.5	44	1.31E3	1311.5
10	91	2.74E8	2080.7	98	1.31E3	1311.5	46	1.31E3	1311.5

GUROBI. The BTHETA formulation had 2,137 rows, 16,525 columns and 34,195 non-zeroes. GUROBI Presolve reduced the problem size to 199 rows, 497 columns, 695 nonzeros in 0.03s. GUROBI solved in 103 iterations and 0.05 seconds with an optimal objective of 1.261e3. OPFlin started with 3070 rows, 3187 columns and 11273 non-zeroes. presolve resulted in 1240 rows, 1961 columns and 6046 non-zeroes. OPFlin pass 3 had 3,306 rows, 3,187 columns and 12,085 non-zeroes and presolve 1240 rows, 1961 columns and 6046 non-zeroes.

Table 118.9. Linear Approximation LP Passes By Starting Point for The 118-Bus Using GUROBI without Relaxation Slack Variables.

start	1	2	3	4	5	6	7	8	9	10
Without	3	10	6	10	10	10	10	10	10	10

Table 118.10. Linear Approximation Nonzeros, Iterations and Optimal Value by LP Pass and Starting Point for the 118-Bus Using GUROBI.

pass	Nonzeros	iters	optval	Nonzeros	iters	optval	Nonzeros	iters	optval
start point 1				2			3		
1	11273	711	1266	11377	691	1280	11377	692	1282
2	11849	578	1297	11845	597	1269	11845	702	1271
3	12085	536	1304	12083	578	1273	12079	610	1275
4				12319	567	1284	12317	626	1281
5				12555	595	1276	12553	620	1283
6				12791	666	1296	12789	619	1305
7				13027	647	1289			
8				13263	671	1287			
9				13499	615	1279			
10				13735	800	1291			
start point 4				5			6		
1	11377	708	1283	11377	679	1283	11377	700	1278
2	11849	800	1270	11849	599	1274	11845	673	1268
3	12069	653	1278	12085	625	1276	12083	693	1277
4	12305	1021	1299	12321	625	1288	12315	636	1299
5	12537	978	1291	12557	624	1285	12553	628	1276
6	12769	1275	1282	12793	605	1280	12785	700	1294
7	13001	987	1296	13029	656	1291	13019	686	1286
8	13233	1517	1287	13265	600	1281	13253	711	1285
9	13461	735	1291	13501	723	1294	13491	665	1296
10	13703	689	1296	13737	666	1287	13727	692	1286
start point 7				8			9		
1	11377	673	1280	11377	660	1281	11377	694	1286
2	11845	659	1270	11849	610	1275	11841	761	1269
3	12079	696	1276	12085	1014	1269	12073	623	1271
4	12317	615	1290	12321	654	1275	12313	713	1280
5	12553	664	1272	12557	603	1302	12545	988	1282
6	12789	691	1287	12793	644	1290	12783	600	1286
7	13025	601	1278	13029	632	1285	13019	657	1273
8	13261	639	1298	13265	645	1279	13255	658	1297
9				13501	1418	1290	13491	572	1298
10				13737	705	1283			
start point 10									
1	11377	684	1281						
2	11845	712	1277						
3	12083	612	1288						
4	12319	629	1278						
5	12555	713	1293						
6	12791	619	1284						
7	13027	764	1293						
8	13263	640	1282						
9	13499	723	1290						
10	13735	721	1283						

Increased Demand. With demand increased by 1.2 and 1.4, the results are in table 118.11. OPFlin is more robust.

Table 118.11. Solution Statistics for 118-Bus Problem with Relaxation Slack Variables with Increased Demand Using CONOPT and GUROBI.

type	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVRG	TEPSIN
OPF2	1797	0.044	3.65	0.06	4.65	10	9
OPF	1573	0.059	3.23	0.19	6.69	10	5
OPFlin	1289	0.007	0.53	0.60	0.07	62	0
OPFp	1352	0.028	0.93	0.21	2.61	10	1
BTHETA	1261	0.000	0.04	0.02	0.04	10	0
1.2*demand							
OPF2	1888	0.030	4.74	0.18	9.86	8	9
OPF	1814	0.043	2.18	0.24	6.07	10	3
OPFlin	1630	0.007	0.64	0.57	0.07	69	0
OPFp	1707	0.027	0.99	0.35	4.23	10	1
BTHETA	1601	0.000	0.04	0.02	0.04	10	0
1.4*demand							
OPF2	2141	0.024	3.85	0.12	7.13	10	6
OPF	2142	0.030	2.58	0.17	5.54	10	3
OPFlin	1978	0.006	0.82	0.40	0.07	84	0
OPFp	2059	0.020	1.37	0.22	3.84	10	1
BTHETA	1946	0.000	0.04	0.03	0.04	10	0

300-bus problem

Without relaxation slack variables, all solvers declare an infeasible problem. The results with different nonlinear solvers with relaxation slack variables are in table 300.1r. All solvers (except KNITRO) declare a local optimal solution, but occasionally fail to remove the penalty slacks. Only CONOPT and IPOPT find the global optimal value.

Table 300.1r. Solution Statistics for the 300-Bus Problem with the Nonlinear Solvers with Relaxation Variables.

type	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVRG	TEPSIN	MINOPT
OPF2								
SNOPT	7307	0.007	14.1	0.147	28.3	10	2	7203
CONOPT	7229	0.004	24.5	0.061	30	10	1	7197
MINOS	7305	0.004	32.5	0.315	129	10	1	7271
IPOPT	7345	0.008	34.7	0.13	73	10	4	7197
KNITRO	7550	0.018	529.3	0.25	1000.3	5	6	7205
OPF								
MINOS	7574	0.003	6.4	0.121	12.2	10	1	7531
CONOPT	7488	0	6.6	0.086	11.1	10	0	7488
SNOPT	7671	0.006	14.7	0.201	28.5	10	4	7551
IPOPT	7488	0	16	0.209	34.5	10	0	7488
KNITRO	14300	0.271	713.6	0.184	1000.1	3	8	7537
OPFp								
MINOS	7732	0.009	4.9	0.192	9.2	10	5	7503
SNOPT	7588	0.007	8.1	0.25	17.1	10	1	7499
IPOPT	7776	0.027	8.3	0.091	13.1	10	2	7488
CONOPT	7731	0.01	8.7	0.227	18.3	10	5	7488
KNITRO	90890	0.842	6940	0.916	67220	7	5	7494

For linear solvers, CPLEX outperforms GUROBI see table 300.2. The optimal solution values are within 3 percent of the best nonlinear solvers. All linear approximation took the full 10 passes and did not meet the other convergence criteria. The linear solvers were more robust than the nonlinear solvers.

Table 300.2. Solution Statistics for the 300-Bus Problem with the Linear Solvers.

type	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVRG	TEPSIN
OPFlin							
CPLEX	7528	0.012	9.9	0.338	0.4	100	0
CPLEX	7528	0.012	10	0.328	0.5	100	0
GUROBI	7615	0.042	16.2	0.161	0.5	100	0
GUROBI	7615	0.042	16.4	0.146	0.5	100	0
GUROBI	7615	0.042	16.6	0.163	0.4	100	0
GUROBI	7615	0.042	16.6	0.148	0.4	100	0
BTHETA							
GUROBI	7195	0	0.3	0.011	0.3	10	0
GUROBI	7195	0	0.3	0.012	0.3	10	0
CPLEX	7195	0	0.3	0.022	0.4	10	0
GUROBI	7195	0	0.3	0.009	0.3	10	0
GUROBI	7195	0	0.3	0.034	0.4	10	0
CPLEX	7195	0	0.3	0.027	0.4	10	0

Table 300.3. Solution Statistics for the 300-Bus Problem from Another Day.

	optvavg	optvcv	cpuavg	cpucv	cpumax	CONVRG	Tepsin
OPF2							
CONOPT	7291	0.009	22.97	0.081	31.9	10	2
MINOS	7277	0.000	34.00	0.296	128.8	10	0
SNOPT	7266	0.005	13.14	0.129	28.9	10	1
IPOPT	7420	0.008	70.15	0.157	123.2	10	6
KNITRO	7360	0.010	592.33	0.247	1000.2	5	6
KNITRO	7360	0.010	613.38	0.226	1000.1	5	6
OPF							
CONOPT	7488	0.000	6.58	0.081	8.7	10	0
MINOS	7580	0.003	6.43	0.108	11.4	9	1
SNOPT	7696	0.006	17.13	0.186	31.2	10	5
IPOPT	7488	0.000	32.86	0.274	107.8	10	0
KNITRO	52080	0.659	572.12	0.230	1000.1	5	7
KNITRO	8507	0.047	7260.10	0.855	66150.0	3	7
OPFp							
CONOPT	7682	0.010	9.86	0.282	28.8	10	4
MINOS	7733	0.009	5.34	0.203	10.2	10	5
SNOPT	7589	0.007	8.21	0.254	17.1	10	1
IPOPT	7776	0.027	18.83	0.217	41.4	10	2
KNITRO	31810	0.698	275.20	0.483	1000.0	8	5
OPFlin							
CPLEX	7520	0.013	7.76	0.681	0.4	77	0
GUROBI	7608	0.043	12.42	0.555	0.5	77	0
BTHETA							
GUROBI	7195	0.000	0.33	0.014	0.4	10	
CPLEX	7195	0.000	0.34	0.015	0.4	10	

CONOPT. The OPF2 formulation had 4,501 rows, 5,101 columns, 18,938 non-zeroes, and 3,069 non-constant first derivatives with 7,455 evaluation operations and occupied 29 Mb. CONOPT had 410 post-triangular equations. With relaxation slack variables, CONOPT found a feasible solution at 1020 iterations and an optimal solution at iteration 2404 with a reduced gradient of 2.9E-08. For starting point 2, CONOPT found an optimal solution of 3.219E7 at iteration 2110.

The OPF and OPFp formulations with relaxation slack variables had 4,801 rows, 8,101 columns, 24,059 non-zeroes, and 3,000 non-constant first derivatives with 6,900 evaluation operations occupying 29 Mb with 231 pre-triangular equations and 716 post-triangular equations. For OPF, at iteration 601, CONOPT found optimal solution of 3.219E7 with a reduced gradient 4.0E-09 and with active penalty slacks. For OPFp, at 126, CONOPT found a feasible solution and at iteration 459, an optimal solution of 3.219E7 with reduced gradient of 1.1E-08 with active penalty slacks.

Table 300.4. CONOPT Performance on the 300-Bus Problem.

start		OPF2			OPF			OPFp	
point	iters	optval	optm	iters	optval	optm	iters	optval	optm
1	2404	3.2E7	7197	644	3.2E7	7488	393	3.2E7	7488
2	1992	3.2E7	7197	518	3.2E7	7488	1507	4.8E7	7866
3	3084	5.3E7	7511	1088	3.2E7	7488	186	3.2E7	7488
4	2943	3.2E7	7197	454	3.2E7	7488	1432	8.7E7	7950
5	3248	3.2E7	7197	732	3.2E7	7488	1705	1.2E8	8026
6	3143	3.2E7	7197	850	3.2E7	7488	2115	4.1E7	7923
7	3025	3.2E7	7197	887	3.2E7	7488	523	3.2E7	7488
8	2977	3.2E7	7197	742	3.2E7	7488	434	3.2E7	7488
9	3654	3.2E7	7197	862	3.2E7	7488	374	3.2E7	7488
10	1713	3.2E7	7197	780	3.2E7	7488	1699	9.2E7	8100
avg	2818	3.4E7	7228	756	3.2E7	7488	1037	5.5E7	7731

KNITRO With relaxation slack variables, KNITRO performed poorly.

Table 300.4. KNITRO Performance on the 300-Bus.

start point	OPF2 optval	OPF2 optm	OPF optval	OPF optm	OPFp optval	OPFp optm
1	3.22E7	7205	3.22E7	7537	3.22E7	7494
2	3.22E7	7236	9.14E7	12160	4.88E7	586700
3	4.40E14	1.10E9	1.40E13	9.48E7	3.22E7	11400
4	3.51E8	7765	8.35E7	490500	8.69E7	8270
5	6.63E8	7833	5.60E7	235400	1.12E8	8404
6	1.37E9	7867	5.01E7	492000	1.04E8	200900
7	4.30E13	5927000	1.00E13	255300	3.22E7	7494
8	4.14E7	7522	3.22E7	23440	3.22E7	7494
9	3.22E7	7230	6.62E7	11920	3.22E7	7535
10	9.63E8	7919	3.58E8	189700	7.06E7	8152
avg	4.83E13	1.11E8	2.40E12	9.65E6	5.83E7	8.54E4

SNOPT

Table 300.6 SNOPT Performance on the 300-bus.

start point	OPF2 iters	OPF2 optval	OPF2 optm	OPF iters	OPF optval	OPF optm	OPFp iters	OPFp optval	OPFp optm
1	15478	3.2E7	7203	6307	3.2E7	7551	2302	3.2E7	7499
2	38329	5.6E7	7647	9966	3.2E7	7566	23643	3.2E7	7602
3	16815	3.2E7	7240	19742	3.2E7	7579	2988	3.2E7	7506
4	19695	3.2E7	7231	18787	3.2E7	7583	8845	3.2E7	7551
5	18051	3.2E7	7209	29142	5.6E7	7850	21500	3.2E7	7564
6	43060	5.6E7	7603	39031	7.3E7	7895	18142	3.2E7	7538
7	14882	3.2E7	7213	38342	5.1E7	7823	3066	3.2E7	7500
8	13259	3.2E7	7250	7478	3.2E7	7554	2893	3.2E7	7505
9	14162	3.2E7	7226	10238	3.2E7	7554	3061	3.2E7	7505
10	14195	3.2E7	7243	39834	4.1E7	7758	17940	9.2E7	8114
avg	20793	3.7E7	7307	21887	4.1E7	7671	10438	3.8E7	7588

MINOS

Table 300.7 MINOS Performance on the 300-bus.

start	OPF2			OPF			OPFp		
point	iters	optval	optm	iters	optval	optm	iters	optval	optm
1	24650	3.2E7	7277	5172	3.2E7	7537	2224	3.2E7	7503
2	24140	3.2E7	7271	6564	3.2E7	7540	8228	4.8E7	7858
3	128759	3.2E7	7278	8076	3.2E7	7613	4362	3.2E7	7526
4	18723	3.2E7	7287	6151	3.2E7	7560	5931	8.7E7	7973
5	23902	3.2E7	7273	5412	3.2E7	7545	9107	1.2E8	8074
6	26338	3.2E7	7278	12799	4.1E7	7794	9113	1.0E8	8059
7	23924	3.2E7	7278	7768	3.2E7	7531	3332	3.2E7	7528
8	27414	3.2E7	7272	7279	3.2E7	7545	2414	3.2E7	7513
9	19071	3.2E7	7274	8209	3.2E7	7537	2561	3.2E7	7514
10	19975	4.1E7	7567	5376	3.2E7	7541	9800	4.3E7	7767
avg	33690	3.3E7	7306	7281	3.3E7	7574	5707	5.7E7	7732

IPOPT

Table 300.8 IPOPT Performance on the 300-bus.

start	OPF2			OPF			OPFp		
point	iters	optval	optm	iters	optval	optm	iters	optval	optm
1	161	3.2E7	7197	127	3.2E7	7488	73	3.2E7	7488
2	381	3.2E7	7197	148	3.2E7	7488	124	3.2E7	7488
3	475	1.8E8	7627	331	3.2E7	7488	102	3.2E7	7488
4	325	5.8E7	7493	100	3.2E7	7488	76	8.7E7	9671
5	350	6.3E7	7520	95	3.2E7	7488	81	3.2E7	7488
6	453	3.2E7	7197	483	3.2E7	7488	126	1.0E8	8189
7	772	3.2E7	7197	71	3.2E7	7488	86	3.2E7	7488
8	500	3.2E7	7197	78	3.2E7	7488	74	3.2E7	7488
9	339	1.9E8	7630	106	3.2E7	7488	71	3.2E7	7488
10	409	3.2E7	7197	324	3.2E7	7488	74	3.2E7	7488
avg	417	6.8E7	7345	186	3.2E7	7488	89	4.5E7	7776

GUROBI. The BTHETA formulation had 5,155 rows, 96,519 columns, 194,899 non-zeroes and occupied 32 Mb. in 0.22s GUROBI Presolve resulted in 383 rows, 1089 columns, 1471 non-zeros. GUROBI solved in 196 iterations and 0.28 seconds with an optimal objective of 3.219e7 with active penalty slacks.

For GUROBI, OPFlin had 8,402 rows 8,101 columns and 29,105 non-zeroes occupying 27 Mb. Presolve resulted in 2906 rows, 4126 columns and 13385 nonzeroes. During the solution process, the following warning was issued Markowitz tolerance tightened to 0.03125. The sequence of optimal values is not monotonic due to linear approximations to the nonconvex constraints.

Table 300.9. Linear Approximation Nonzeros, Iterations and Optimal Value by LP Pass and Starting Point for the 300-Bus Using GUROBI.

pass	nonzeros	iters	optval	optm	nonzeros	iters	optval	optm
Start 1					Start 2			
1	27305	2575	3.22E7	7247	27611	3189	3.22E7	7297
2	28813	2680	3.22E7	7444	28855	5352	3.22E7	7333
3	29439	7004	3.22E7	7471	29449	9296	3.59E7	7931
4	30039	5601	3.22E7	7474	30031	7152	3.33E7	7718
5	30639	3988	3.22E7	7470	30627	6902	3.26E7	7933
6	31239	4742	3.22E7	7474	31215	4284	3.26E7	8171
7	31839	4339	3.22E7	7465	31813	4434	3.22E7	7670
8	32435	5001	3.22E7	7475	32415	5061	3.45E7	7918
9	33023	4668	3.22E7	7473	33017	3834	3.22E7	7384
10	33533	2674	3.22E7	7474	33613	3052	3.22E7	7572
Start 3					Start 4			
1	27611	2175	3.22E7	7297	27611	2519	3.22E7	7253
2	28859	5221	3.22E7	7321	28855	7615	3.22E7	7443
3	29459	4625	3.22E7	7320	29445	5499	3.22E7	7398
4	30059	2838	3.22E7	7388	30039	4080	3.22E7	7353
5	30659	3625	3.22E7	7452	30641	2644	3.22E7	7445
6	31259	4212	3.22E7	7462	31239	4839	3.22E7	7360
7	31859	5865	3.22E7	7470	31837	4633	3.22E7	7493
8	32459	5503	3.22E7	7466	32435	8338	3.22E7	7495
9	33059	2819	3.22E7	7471	33021	4817	3.22E7	7471
10	33659	4149	3.22E7	7465	33613	3771	3.22E7	7495
Start 5					Start 6			
1	27611	3721	3.22E7	7288	27611	2070	3.22E7	7284
2	28851	4615	3.22E7	7432	28855	5346	3.23E7	8027
3	29451	4606	3.42E7	7957	29437	7343	3.40E7	7965
4	29941	4902	3.25E7	7821	29925	6543	3.37E7	8042
5	30509	5235	3.22E7	7569	30483	9820	3.46E7	8073
6	31101	4919	3.25E7	7523	31067	6179	3.24E7	8154
7	31687	7197	3.28E7	8049	31649	7690	3.60E7	8012
8	32267	5424	3.24E7	8129	32259	4593	3.23E7	8083
9	32843	7576	3.23E7	8015	32859	4309	3.25E7	8406
10	33389	6546	3.23E7	7740	33447	4149	3.35E7	8496
Start 7					Start 8			
1	27611	4455	3.22E7	7284	27611	5215	3.22E7	7293
2	28859	3698	3.22E7	7347	28859	6209	3.22E7	7348
3	29459	4057	3.22E7	7375	29459	4506	3.22E7	7353
4	30059	3434	3.22E7	7324	30057	2792	3.22E7	7399
5	30659	2623	3.22E7	7406	30659	4018	3.22E7	7449
6	31259	2716	3.22E7	7464	31259	2948	3.22E7	7475
7	31859	2512	3.22E7	7465	31859	7125	3.22E7	7467
8	32459	5434	3.22E7	7473	32459	7940	3.22E7	7474
9	33059	4714	3.22E7	7466	33059	5049	3.22E7	7467
10	33657	4016	3.22E7	7474	33659	3519	3.22E7	7473
Start 9					Start 10			
1	27611	2803	3.22E7	7271	27611	2340	3.22E7	7266

2	28859	4538	3.22E7	7366	28855	3313	3.22E7	7403
3	29459	3458	3.22E7	7377	29453	5261	3.23E7	7762
4	30059	6661	3.22E7	7356	30047	3017	3.22E7	7484
5	30659	5502	3.22E7	7336	30647	5591	3.36E7	8198
6	31255	5052	3.22E7	7425	31221	6963	3.34E7	8154
7	31857	4591	3.22E7	7449	31799	5930	3.22E7	7421
8	32457	2256	3.22E7	7460	32393	5451	3.27E7	7632
9	33057	2230	3.22E7	7470	32945	7055	3.22E7	7359
10	33657	3081	3.22E7	7469	33563	3904	3.22E7	7497

Increasing Demand. With demand increased by 1.2 and 1.4, the results are in table 300.12. When demand is 1.2 or 1.4 times greater, the penalty slacks are greater than the set threshold and convergence becomes more difficult.

Table 300.12. Solution Statistics for 300-Bus Problem with Increased Demand Using CONOPT and GUROBI.

type	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVRG	TEPSIN
OPF2	7291.214	0.009	23.447	0.071	31.637	10	2
OPF	7488.069	0.000	6.830	0.077	8.923	10	0
OPFlin	7607.635	0.043	11.670	0.545	0.463	77	0
OPFp	7681.821	0.010	9.607	0.252	23.962	10	4
BTHETA	7195.254	0.000	0.320	0.015	0.348	10	0
1.2*demand							
OPF2	9403.831	0.000	20.247	0.069	29.157	10	10
OPF	9817.016	0.000	9.805	0.093	14.555	9	10
OPFlin	9783.840	0.010	18.072	0.675	0.680	76	9
OPFp	9826.498	0.000	7.839	0.164	13.385	8	10
BTHETA	9189.562	0.000	0.334	0.015	0.367	10	0
1.4*demand							
Type	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVRG	TEPSIN
OPF2	10360.000	0.000	27.110	0.072	36.130	7	10
OPF	10640.000	0.000	11.906	0.057	14.196	10	10
OPFlin	10750.000	0.022	25.323	0.228	0.963	100	10
OPFp	10640.000	0.000	8.731	0.078	13.931	10	10
BTHETA	11720.000	0.000	0.328	0.022	0.377	10	0