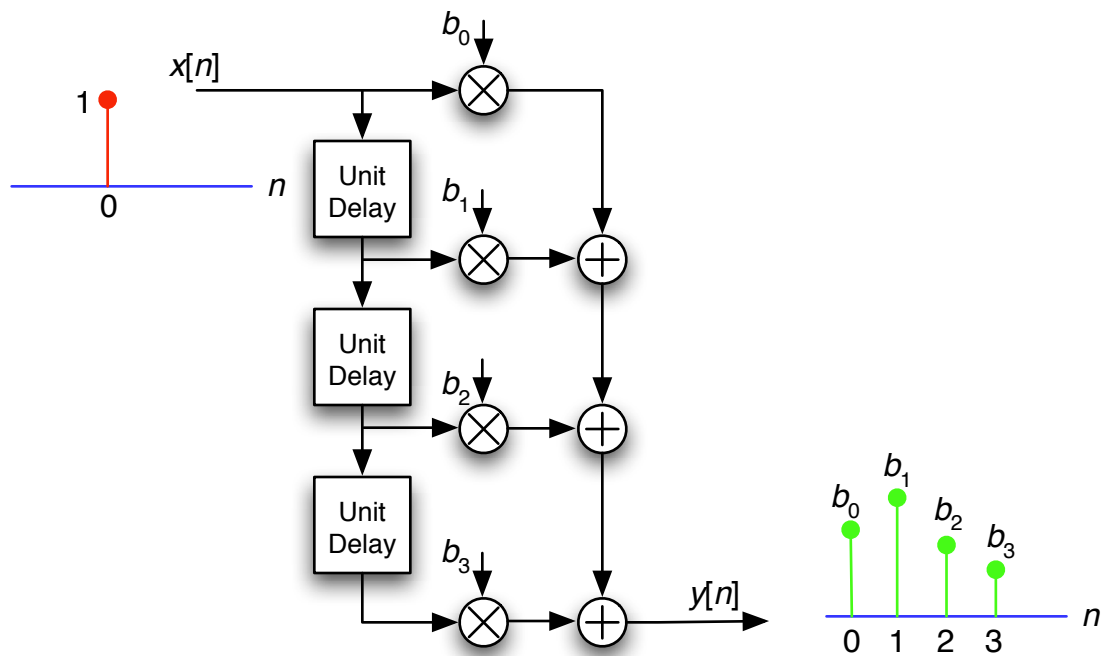


Introduction to Signals and Systems

ECE 2610 Lecture Notes
Spring 2011



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Introduction and Course Overview

Introduction

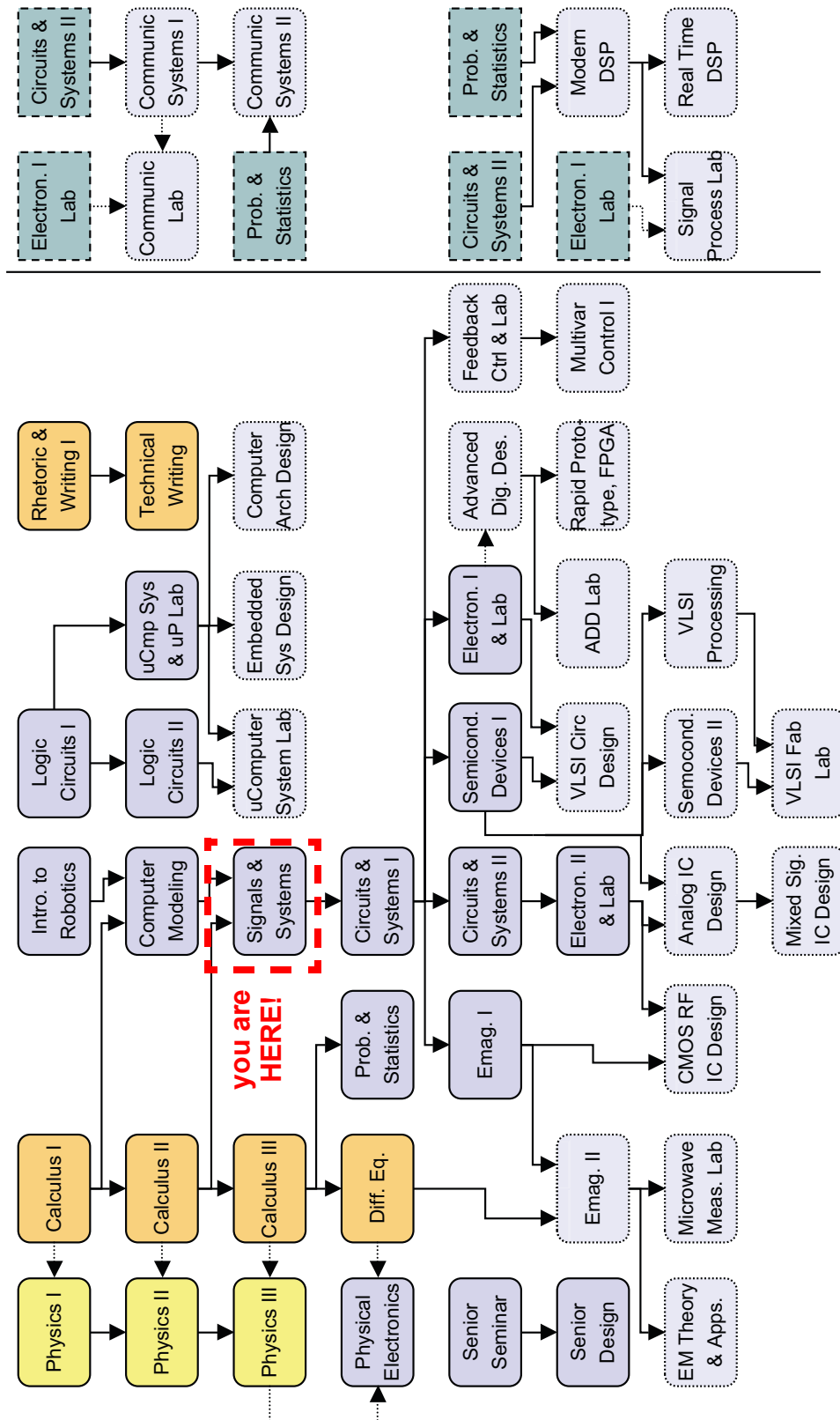
- Signals and systems – what for?
- Course perspective
- Course syllabus
- Instructor policies
- Computer tools
- Introduction to mathematical modeling of signals and systems

Signals and Systems – What for?

- Electronics for audio (iPod) and wireless devices (cell phones, wireless local area networking) are all around us
 - What are some others?
- Signals and systems are an integral part of making these devices perform their intended function
- *Signals* convey information from one point to another
 - They may be generated by electronic means, or by some natural means such as talking, walking, your heart beating, an earthquake, the sun heating the sidewalk

- *Systems* process signals to produce a modified or transformed version of the original signal
 - The transformation may be as simple as a microphone converting a sound pressure wave into an electrical waveform
 - The four campuses of the University of Colorado are often termed the ‘CU System’
- In this class systems are specialized primarily to those that process signals of an electrical nature
 - If we do not have an electrical signal directly we may use a transducer to obtain one, e.g., a *thermistor* to sense the temperature of the heat sink in a computer power supply
- In the traditions of electrical engineering, signals and systems means the mathematical modeling of signals and systems, to assist in the design and development of electronic devices

Course Perspective – From Here to There



Course Syllabus

Spring Semester 2011

Instructor: Dr. Mark Wickert **Office:** EB-292 **Phone:** 255-3500
 wickert@eas.uccs.edu **Fax:** 255-3589
<http://www.eas.uccs.edu/wickert/ece2610/>

Office Hrs: M&W 12:45-1:15am, M&W 3:05pm-4:00pm, others by appointment.

Required Text James McClellan, Ronald Schafer, and Mark Yoder, *Signal Processing First*, Prentice Hall, New Jersey, 2003. ISBN 0-13-090999-8.

Optional Software: The student version of MATLAB 7.x available under general software in the UCCS bookstore. Other specific programming tools will be discussed in class.

Grading:

- 1.) Graded homework worth 20%.
- 2.) Quizzes worth 15% total
- 3.) Laboratory assignments worth 20% total.
- 4.) Mid-term exam worth 15%.
- 5.) Final MATLAB project worth 10%.
- 6.) Final exam worth 20%.

Topics	Text	Weeks
1. Course Overview and Introduction	1.1–1.4	0.5
2. Sinusoids	2.1–2.9	1.0
3. Spectrum Representation	3.1–3.9	1.0
4. Sampling and Aliasing	4.1–4.6	1.0
5. FIR filters	5.1–5.9	1.5
6. Frequency response of FIR filters	6.1–6.9	1.5 (exam)
7. z-Transforms	7.1–7.10	1.0
8. IIR Filters	8.1–8.12	2.0
9. Continuous-Time Signals and Systems	9.1–9.10	1.5?
10. Frequency Response	10.1–10.6	0.5?
11. Continuous-Time Fourier Transform	11.1–11.11	1.5?
12. Filtering, Modulation, and Sampling	12.1–12.4	1.5 (project)

Note: that topics 9–12 will most likely only be overviewed at the end of the semester.

Instructor Policies

- Homework papers are due at the start of class
- If business travel or similar activities prevent you from attending class and turning in your homework, please inform me beforehand
- Grading is done on a straight 90, 80, 70, ... scale with curving below these thresholds if needed
- Homework solutions will be placed on the course Web site in PDF format with security password required; hints pages may also be provided

Computer Tools

- Through-out this semester we will be using MATLAB for modeling and simulation of signals and systems
- MATLAB is a very powerful vector/matrix oriented programming language
- It features an integrated graphics/visualization engine
- MATLAB has an integrated source code editor and debugging environment
- There are specialized toolboxes available for signal processing, communications, image processing, and many other engineering applications
- The text for this course includes a collection of MATLAB functions specialized for the signal processing taught in this course
- The laboratory portion of this course will focus on the use of MATLAB to explore signals and systems
- A very brief introduction to MATLAB follows
 - We will be learning shortly that a signal in mathematical terms can be as simple as a function of time, say a trigonometric function like

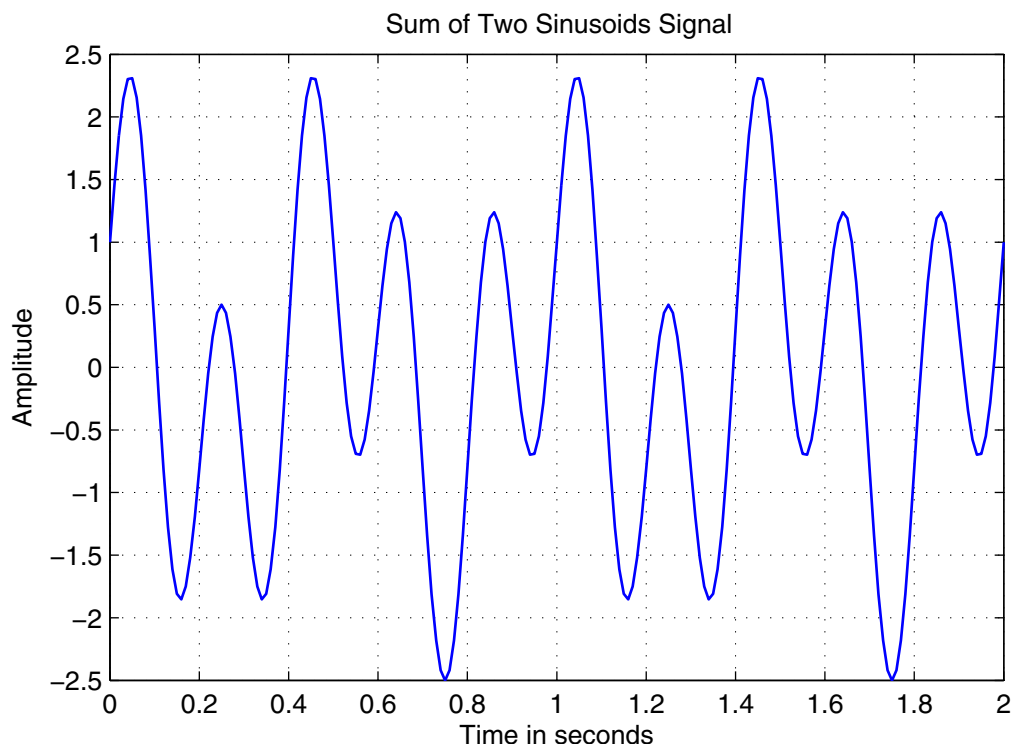
$$x(t) = A \cos(2\pi f_0 t) \quad (1.1)$$

where we call A the amplitude, f_0 the frequency in cycles per second, and t is the independent variable

- MATLAB operates from a *command window* similar to a calculator

```
Command Window
>> t = 0:.01:2;
>> x = cos(2*pi*2*t) + 1.5*sin(2*pi*5*t);
>> plot(t,x)
>> grid
>> xlabel('Time in seconds')
>> ylabel('Amplitude')
>> title('Sum of Two Sinusoids Signal')
```

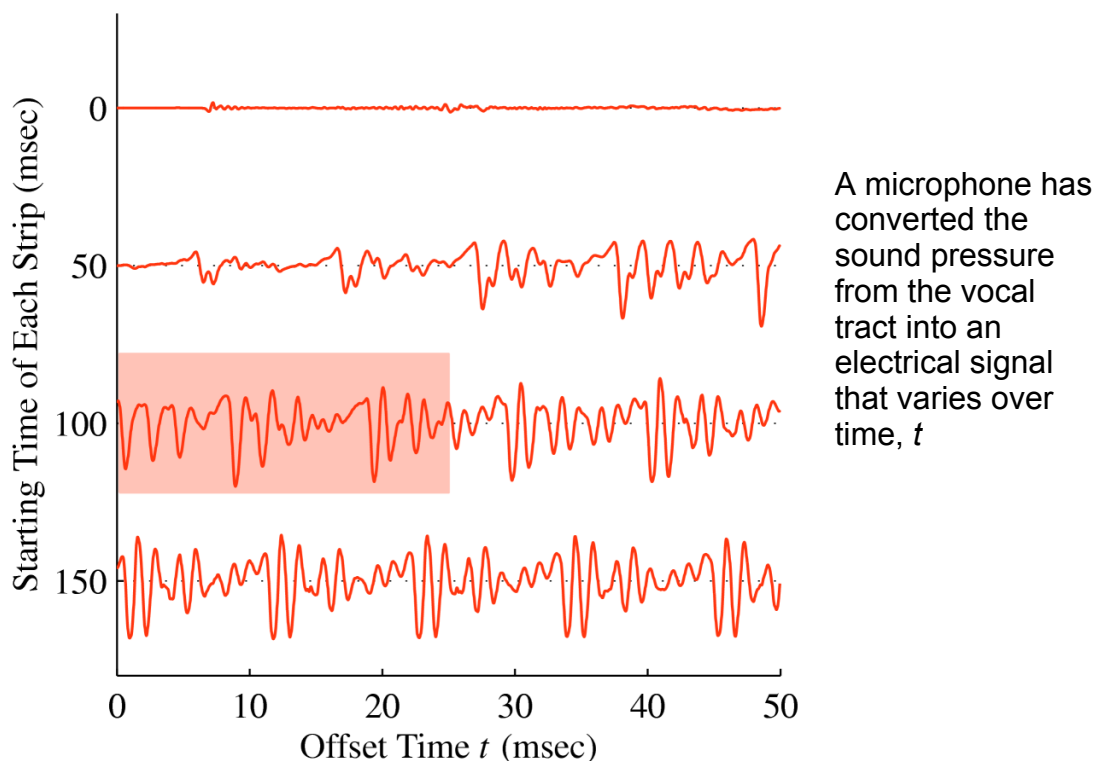
- On the first line we create a time axis vector running from 0 to 2 seconds, with time step 0.01 seconds
- The second line we fill a vector x with functional values that correspond, in this case, to the sum of two sinusoids
- What are the amplitudes and frequencies of these sinusoids?
- Finally we plot the signal using the `plot()` function



Introduction to Mathematical Modeling of Signals and Systems

Mathematical Representation of Signals

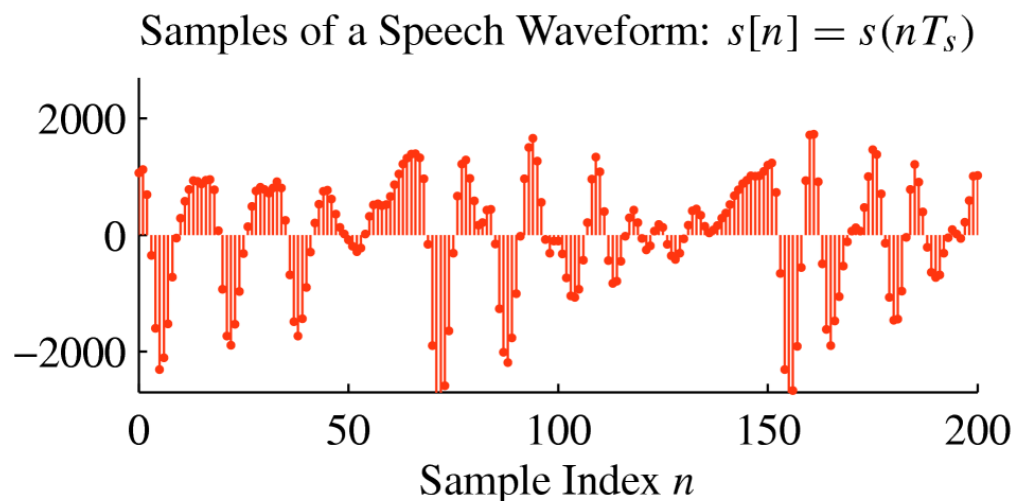
- Signals represent or encode information
 - In communications applications the information is almost always encoded
 - In the probing of medical and other physical systems, where signals occur naturally, the information is not purposefully encoded
 - In human speech we create a waveform as a function of time when we force air across our *vocal cords* and through our *vocal tract*



- Signals, such as the above speech signal, are continuous functions of time, and denoted as a *continuous-time signal*
- The independent variable in this case is time, t , but could be another variable of interest, e.g., position, depth, temperature, pressure
- The mathematical notation for the speech signal recorded by the microphone might be $s(t)$
- In order to process this signal by computer means, we may *sample* this signal at regular interval T_s , resulting in

$$s[n] = s(nT_s) \quad (1.2)$$

- The signal $s[n]$ is known as a *discrete-time signal*, and T_s is the *sampling period*
 - Note that the independent variable of the sampled signal is the integer sequence $n \in \{ \dots, -2, -1, 0, 1, 2, \dots \}$
 - Discrete-time signals can only be evaluated at integer values



- The speech waveform is an example of a one-dimensional signal, but we may have more than one dimension
- An image, say a photograph, is an example of a two-dimensional signal, being a function of two spatial variables, e.g. $p(x, y)$
- If the image is put into motion, as in a movie or video, we now have a three-dimensional image, where the third independent variable is time, (x, y, t)
 - Note: movies and videos are shot in frames, so actually time is discretized, e.g., $t \rightarrow nT_s$ (often $1/T_s = 30$ fps)
- To manipulate an image on a computer we need to sample the image, and create a two-dimensional discrete-time signal

$$p[m, n] = p(m\Delta_x, n\Delta_y) \quad (1.3)$$

where m and n takes on integer values, and Δ_x and Δ_y represent the horizontal and vertical sampling periods respectively

Mathematical Representation of Systems

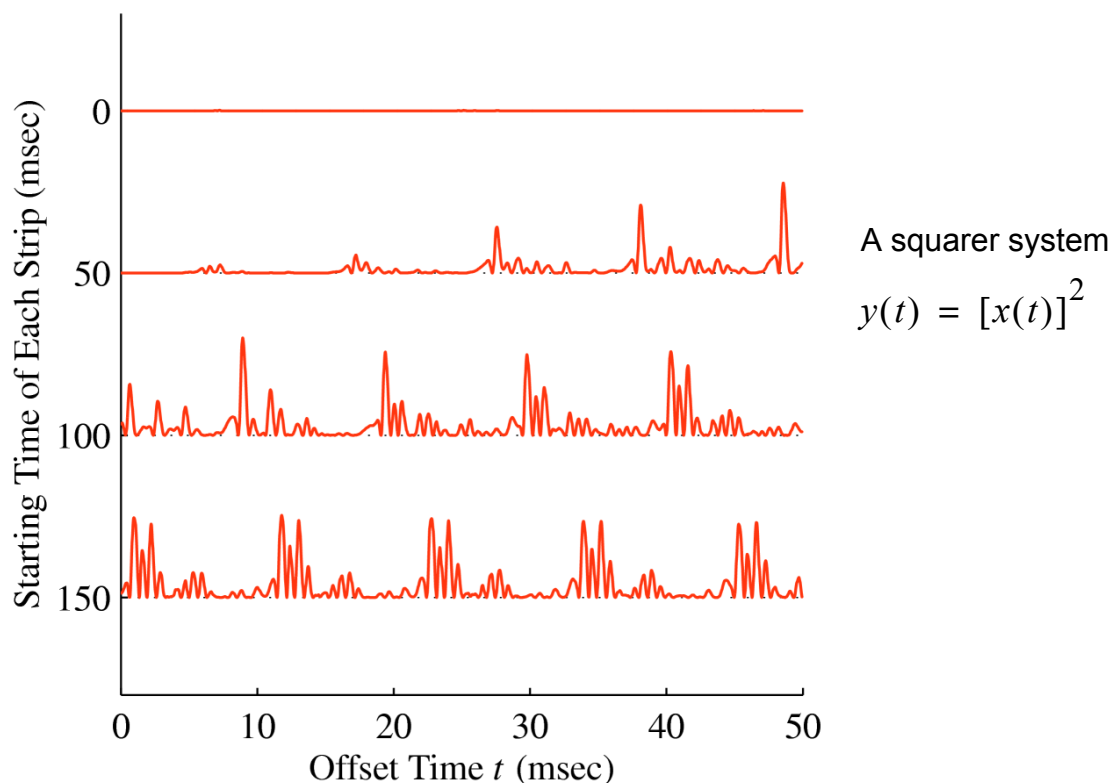
- In mathematical modeling terms a system is a function that transforms or maps the input signal/sequence, to a new output signal/sequence

$$\begin{aligned} y(t) &= T_c\{x(t)\} \\ y[n] &= T_d\{x[n]\} \end{aligned} \quad (1.4)$$

where the subscripts c and d denote continuous and discrete system operators

- Because we are at present viewing the system as a pure mathematical model, the notion of a system seems abstract and distant
- Consider the microphone as a system which converts sound pressure from the vocal tract into an electrical signal
- Once the speech waveform is in an electrical waveform format, we might want to form the square of the signal as a first step in finding the energy of the signal, i.e.,

$$y(t) = [x(t)]^2 \quad (1.5)$$



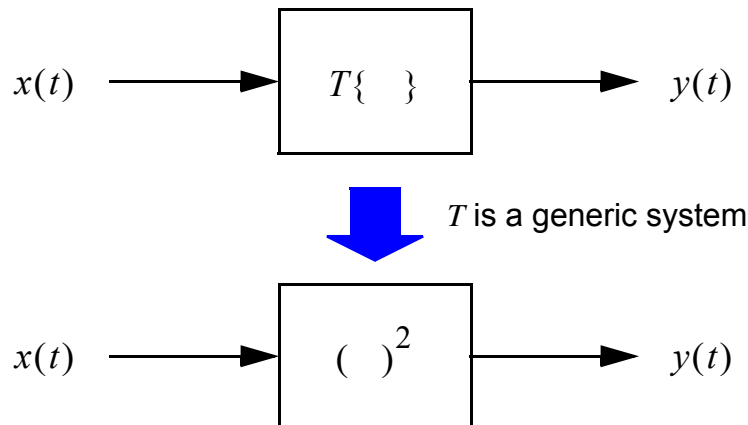
- The squarer system also exists for discrete-time signals, and in fact is easier to implement, since all we need to do is multiply each signal sample by itself

$$y[n] = (x[n])^2 = x[n] \cdot x[n] \quad (1.6)$$

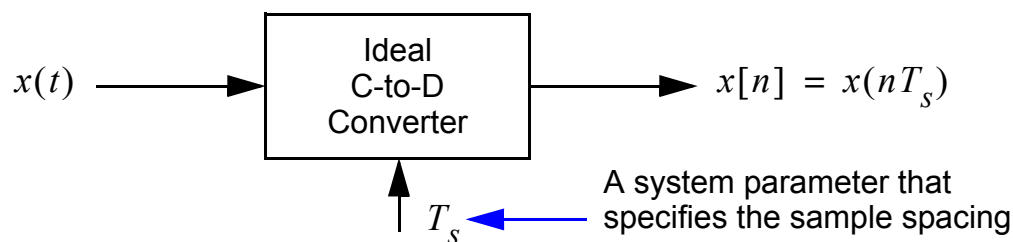
- If we send $y[n]$ through a second system known as a *digital filter*, we can form an estimate of the signal energy
 - This is a future topic for this course

Thinking About Systems

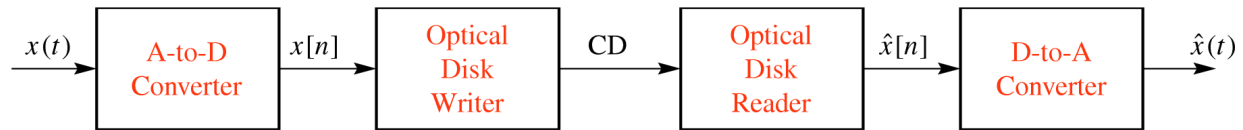
- Engineers like to use block diagrams to visualize systems
- Low level systems are often interconnected to form larger systems or subsystems
- Consider the squaring system



- The ideal sampling operation, described earlier as a means to convert a continuous-time signal to a discrete-times signal is represented in block diagram form as an ideal C-to-D converter



- A more complex system, depicted as a collection of subsystem blocks, is a system that records and then plays back an audio source using a compact disk (CD) storage medium



- The optical disk reader shown above is actually a high-level block, as it is composed of many lower-level subsystems, e.g.,
 - Laser, on a sliding carriage, to illuminate the CD
 - An optical detector on the same sliding carriage
 - A servo control system positions the carriage to follow the track over the disk
 - A servo speed control to maintain a constant linear velocity as 1/0 data is read from different portions of the disk
 - more ...
- If we just considering a CD player, we would only need the last two subsystem blocks (why?)

The Next Step

- Basic signals, composed of linear combinations of trigonometric functions of time will be studied next
- We also consider complex number representations as a means to simplify the combining of more than one sinusoidal signal

