ERRATA for *An Introduction to Homological Algebra* 2nd Ed.

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Here are all the errata that I know (aside from misspellings). If you have found any errors not listed below, please send them to me at

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Page 3 line 17. change da to dx (two times)

Page 4 line 2. change $\ldots \cup [a, c]$ to $\ldots \cup [a, b]$

Page 26 lines $-4, -3$. should read: For all $C \in \text{obj}(\mathcal{C})$,

$$(\sigma\tau)_C = \sigma_C \tau_C.$$

Page 30 line 3. change t_n to t_{n-1}

Page 38 line 10. ... and $\alpha\beta(y) = \sum_{x \in G} \alpha(x)\beta(x^{-1}y)$.

line 17. Elements of G, when viewed as elements of kG, multiply as

Page 39 Prop 2.4(i) should read:

$$(f + g)_* = f_* + g_* \text{ for all } f, g \in \text{Hom}_R(A, B).$$

Page 40 line 17. Definition of additive functor should be placed on page 39.

Page 47 line 12. should read: $0 \to S/T \to M/T \to M/S \to 0$

Page 48 line -10. interchange "injection" and "projection"

Page 49 line 14. Change φ to ψ.

Page 49 line -10. Should be: (iv) $\Rightarrow$ (v)

Page 51 line 7. should read: $\ker \overline{\rho} = (T + S')/S'$

Page 52 In Prop 2.26, should be $S_i \cap (S_1 + \cdots + \widehat{S}_i + \cdots + S_n) = \{0\}$

Page 53 line -18 should read:

is the map $\lambda_j \colon A_j \to B$, defined by $a_i \mapsto$

Page 65 Exercise 2.8(i) Note that iq is a retraction

Page 78 line 9 should be: $\text{Hom}_S(\square, B)$

Page 83 line 11. change k-trilinear to k-triadditive

Page 85 line -16. should read: $f \colon (a, b'') \mapsto a \otimes b + E$

Page 87 line -17. replace line; should read: $A \otimes_R \left(\bigoplus_i B_i\right)$. There is a ho-

Page 89 Move Prop 2.50 to page 88, just before Prop 2.68.

Page 93 line 8. change $\mathrm{Hom}_S(\Box \otimes_S B, C)$ to $\mathrm{Hom}_S(\Box \otimes_R B, C)$

line 10 $\mathrm{Hom}_S(B \otimes_R \Box, C)$

Page 119 line 7 change E_i to E_k

Page 121 line -10 change $rf(1)$ to $rf(a)/a$

Page 125 line -3 should read: If E is a proper essential

Page 133 line -12 should be: $= \sum \kappa(m_j) \otimes a_j$

Page 142 line 7 change A to B

Page 147 line 3 should read: Let $0 \to A \to$

line -17 change middle row to middle column

Page 148 line -3. change $\{a_i + K : i = 1, \ldots, m\}$ to $\{a_j + K : j = 1, \ldots, n\}$

Page 149 line 5 change a_i to a_1

line 9 Lemma 3.68

change second item in inequalities to: $\sum_j a_j \otimes \left(\sum_i r_{ji} h_i' \right)$

Page 172 line -4 change basis of V to basis of B

Page 173 line -9 change Theorem 4.35 to Theorem 4.34

Page 181 line -1 change g^k to g^{k-1}

Page 185 line -8 change *essential* to *superfluous*

Page 192 line 5 because s'' is not a zero-divisor,

Page 195 line 6 change $\mathfrak{p}R\mathfrak{p}$ to $\mathfrak{p}R_\mathfrak{p}$

Page 196 line -5 change $S^{-1} \otimes M$ to $S^{-1}R \otimes M$

Page 198 line -8. the composite should have another functor on the right, namely, the change of rings functor ${}_{S^{-1}R}\mathbf{Mod} \to {}_R\mathbf{Mod}$ induced by the localization map $h\colon R \to S^{-1}R$

Page 207 line 3 change $f(x)$ to $f(y)$

Page 217 line 1 change C to B (3 times)

Page 218 line 2 change X to C

Page 230 lines 1, 2 should read: **inverse system in $\mathcal{C}$ over I**

Example 5.16(i): interchange A and C in the diagram

line -8 change modules to objects.

Page 233 line -11 change ψ_N^M to ψ_M^N

line -8 change $\psi_M^D f_M$ to $\psi_D^M f_D$

Page 235 line 5 change $b_n + JM$ to $b_n + J^n M$

Page 237 top diagram vertical arrows should point down

line 11 should read: $\varphi^i_j\colon M_i \to M_j$

Page 258 line 11 change $\mathrm{Hom}(A, B \otimes C)$ to $\mathrm{Hom}(A \otimes B, C)$

Page 283 line 23 should read

$$\lim_{\to} {}_{U \ni x} \mathcal{P}_1(U) \to \lim_{\to} {}_{U \ni x} \mathcal{P}_2(U) :$$

Page 286 line -9 should read

because, if x is a closed point, then all the stalks of $x_* A$ are $\{0\}$ except $(x_* A)_x$, which is A.

Page 302 Exercise 5.42 should read

Give an example of a presheaf of abelian groups on a discrete space X which is not a sheaf.

Page 316 line 15 should read $F\colon \mathcal{A} \to {}_R\mathbf{Mod}$ for some ring R.

Page 321 Exercise 5.57 in every abelian category

Page 327 lines 6, 7 change π to p (2 times)

Page 333 line 12, 13 should read $= dic' = id'c$,

Page 335 line 2 change $\mathbf{Comp}(\mathcal{A})$ to $\mathcal{A}$.

Page 336 top diagram remove subscript * from vertical arrows f, g, h

line 3 below top diagram

$$f_* \partial \mathrm{cls}(z'') = \mathrm{cls}(f z').$$

Page 337 Change $1_{\mathbf{C}}$ to $(1_{\mathbf{C}})_*$

Page 343 line 17 change ∂^*_n and ∂^*_{n-1} to ∂^*_{n+1} and ∂^*_n

Page 350 line -3 of first paragraph change K_0 to V (twice)

bottom: Change δ_{n+1} to d_{n+1} (twice)

Page 359 first diagram: interchange ∂ and ∂'

Page 360 line -8. change H_q to H_n

Use Proposition 2.70 to prove the case $n = 1$

Page 361 third diagram change $\tau_{n-1,K}$ on far right to $T_{n-1}g$

Page 365 line -9. $B \in \mathrm{obj}(\mathcal{A})$

Page 366 line 3. change im η to im d_0 and im d^{n-1} to im d^n

Page 373 line 9 change Ext to ext

Prof. A. Azizi found a gap in the proof of Theorem 7.5; on page 408, line 6, why is $H_1(\mathbf{F}_A, B) \cong \operatorname{im}\gamma$? He also sent me a corrected version which I have rewritten.

Page 406 Replace the paragraph beginning "We are going to use" by

The following two results will be used in the next proof. The first is a generalization of the First Isomorphism Theorem.

If $f\colon A \to B$ is a homomorphism and $N \subseteq \ker f$, then $\widetilde{f}\colon A/N \to B$, given by $\widetilde{f}\colon a + N \mapsto fa$, is a well-defined homomorphism with $\ker \widetilde{f} = \ker f/N$ and $\operatorname{im} \widetilde{f} = \operatorname{im} f$.

The second result is a variation of the Snake Lemma, due to Cartier–Weil.

If $f\colon A \to B$ and $g\colon B \to C$ are homomorphisms, then there is an exact sequence

$$0 \to \ker\, f \to \ker(gf) \to \ker\, g \to \operatorname{coker} f \to \operatorname{coker}(gf) \to \operatorname{coker} g \to 0.$$

We sketch a proof. There is a commutative diagram with exact rows

$$\begin{array}{ccccccccc} 0 & \longrightarrow & A & \longrightarrow & A \oplus B & \longrightarrow & B & \longrightarrow & 0 \\ & & \downarrow{\scriptstyle f} & & \downarrow{\scriptstyle h} & & \downarrow{\scriptstyle g} & & \\ 0 & \longrightarrow & B & \longrightarrow & B \oplus C & \longrightarrow & C & \longrightarrow & 0; \end{array}$$

the maps in the rows are the usual injections and projections of direct sums, while $h(a, b) = (fa - b, gb)$. The Snake Lemma gives exactness of

$$0 \to \ker\, f \to \ker\, h \to \ker\, g \to \operatorname{coker} f \to \operatorname{coker} h \to \operatorname{coker} g \to 0,$$

and it is easy to see that $\ker\, h \cong \ker(gf)$ and $\operatorname{coker} h \cong \operatorname{coker}(gf)$.

Page 407, line 11 through Page 408, line 7 Replace with the following.

For $n = 1$, let $h = d_1 \otimes 1\colon F_1 \otimes_R B \to F_0 \otimes_R B$. Since $\operatorname{im}(d_2 \otimes 1) \subseteq \ker(d_1 \otimes 1)$, the generalized First Isomorphism Theorem says that $\widetilde{h}\colon F_1 \otimes_R B/\operatorname{im}(d_2 \otimes 1) \to F_0 \otimes_R B$ has kernel

$$\ker \widetilde{h} = \frac{\ker(d_1 \otimes 1)}{\operatorname{im}(d_2 \otimes 1)} = H_1(\mathbf{F}_A, B). \tag{1}$$

Let $Y = \operatorname{im}\, d_1$, let $i\colon Y \to F_0$ be the inclusion, and let $d_1'\colon F_1 \to Y$ be given by $d_1'\colon x \mapsto d_1(x)$ [so d_1' differs from d_1 only in its target]. Of course, $id_1' = d_1$.

Define $f = d_1' \otimes 1\colon F_1 \otimes_R B \to Y \otimes_R B$. Note that d_1' is surjective, so that right exactness of $\Box \otimes_R B$ gives $f = d_1' \otimes 1$ surjective; that is, $Y \otimes_R B = (\operatorname{im}\, d_1) \otimes_R B = \operatorname{im}(d_1' \otimes 1)$. Since $\operatorname{im}(d_2 \otimes 1) \subseteq \ker(d_1' \otimes 1)$, the generalized First Isomorphism Theorem gives $\widetilde{f}\colon F_1 \otimes B/\operatorname{im}(d_2 \otimes 1) \to Y \otimes_R B$ surjective and

$$\ker \widetilde{f} = \ker(d_1' \otimes 1)/\operatorname{im}(d_2 \otimes 1).$$

Let $g = i \otimes 1 \colon Y \otimes_R B \to F_0 \otimes_R B$. the Cartier–Weil variation gives exactness of

$$\ker \widetilde{f} \to \ker(g\widetilde{f}) \to \ker g \to \operatorname{coker} \widetilde{f}.$$

We have seen that $\widetilde{f}$ is surjective, so that coker $\widetilde{f} = \{0\}$. Moreover, exactness of $F_2 \to F_1 \to Y \to 0$ gives exactness of

$$F_2 \otimes_R B \xrightarrow{d_2 \otimes 1} F_1 \otimes_R B \xrightarrow{d_1' \otimes 1} Y \otimes_R B \to 0,$$

because $\Box \otimes_R B$ is right exact. Hence, $\operatorname{im}(d_2 \otimes 1) = \ker(d_1' \otimes 1)$, and so $\ker \widetilde{f} = \ker(d_1' \otimes 1)/\operatorname{im}(d_2 \otimes 1) = \{0\}$. Thus, $\ker(g\widetilde{f}) \cong \ker g$. But $g\widetilde{f} = \widetilde{h}$, and Eq. (1) gives $\ker \widetilde{h} \cong H_1(\mathbf{F}_A, B)$. Therefore,

$$H_1(\mathbf{F}_A, B) \cong \ker g = \ker(i \otimes 1).$$

Page 411 line -8 change *essential* to *superfluous*

Page 422 line just below diagram. change $GR \cong C$ to $GR \cong A$

Page 427 line -13. change Proposition 6.19 to Exercise 6.19

Page 429, 430 Formula II: change $e(C, A')$ to $\operatorname{Ext}^1(C, A')$

Formula III: change $e(C', A)$ to $\operatorname{Ext}^1(C', A)$

Page 453 line -6. should read

$$\operatorname{Ext}^n_R(A, B) = \frac{\ker d^*_{n+1}}{\operatorname{im} d^*_n},$$

Page 465 line 9. change $\operatorname{fd}(R)$ to $\operatorname{fd}(A)$

Page 466 Exercise 8.5 $0 \to M' \to M \to M'' \to 0$

Page 474 line 1 Change $\operatorname{pd}_R(K^*) \le 2$ to $\operatorname{pd}_R(M^*) \le 2$

Page 475 line 5. Change ${}_R\mathbf{Mod}$ to $\mathbf{Mod}_R$

Page 475 line -1. $a \otimes a' \otimes m \mapsto aa' \otimes m$

Page 477 line -12. change $(d_2, p_2, 0)$ to $(d_2 p_2, 0)$

Page 481 line 12. delete "left" two times

Page 482 lines 16-18 Should read:

$f(x) \in \mathfrak{p} \subseteq R[x]$ of least degree, and consider the exact sequence $0 \to (f) \to \mathfrak{p} \to \mathfrak{p}/(f) \to 0$. Now $(f) \cong R[x]$, since $R[x]$ is a domain, and $\operatorname{ann}(\mathfrak{p}/(f)) \neq \{0\}$: if $Q = \operatorname{Frac}(R)$, then $\mathfrak{p}Q[x]$ is generated by f, so that if $g \in \mathfrak{p}$, there is $c \in R$ with $cg \in Q[x]$.

Page 486 line -5. $\operatorname{pd}(M) \le n$

Page 487 line -2. Change $y_i + \mathfrak{m}$ to $y_i + (x)$.

Page 487 line -1. Change mod $\mathfrak{m}$ to mod $\mathfrak{m}^2$

Page 488 line -6. change $\{f/g \in k[V]$ to $\{f/g \in k(V)$

Page 489 Lemma 8.59 add hypothesis: R is noetherian

Page 490 Proposition 8.61 add hypothesis: R is noetherian

Page 519 line 8. change $\mathrm{Hom}_G(A, B)$ to $\mathrm{Hom}_{\mathbb{Z}}(A, B)$

Page 560 bottom 3 lines should read

$\mathrm{Hom}_S(\mathbb{Z}, A)$ is a (left) G-module as follows: if $y \in G$ and $g\colon \mathbb{Z}G \to A$, define

$$yg\colon x \mapsto g(y^{-1}x).$$

Page 614 line -10. a subscript Q should be q: $\Delta'_P \otimes 1_{B_q}$

Page 629 line -8. change subscript on $(F^{p-1})_n$ to $n-1$

Page 632, 633 interchange the rows in the 2×2 matrices

Page 667 lines $-7, -6$

$$\mathrm{Hom}_S(B \otimes_R A, C) \cong \mathrm{Hom}_R(A, \mathrm{Hom}_S(B, C)).$$

If $G = B \otimes_R \square$ and $F = \mathrm{Hom}_S(\square, C)$,

Page 678 line 10. change "isomorphic" to "chain equivalent"

Page 679 line 3. also assume that the complex $\mathbf{Z}$ of cycles is flat

Page 692 add references

Neukirch, J., Schmidt, A., and Wingberg, K., *Cohomology of Number Fields*, Grundlehren der mathematischen Wissenschaften 323, 2d Ed., Springer, 2008.

Northcott, D. G., *A First Course of Homological Algebra*, Cambridge University Press, 1973.