Unit 6, Lesson 1

Simplifying Radicals and Pythagorean Theorem

Choose always the way that seems the best however rough it may be; custom will soon render it easy and agreeable.

-Pythagoras

Radicals should always be in <u>simplest form</u>:

1. No perfect square factor (other than 1) is under the radical. $\sqrt{12} = 2\sqrt{3}$ $\sqrt{4} = 2$
2. No fraction is under the radical. $\sqrt{\frac{7}{4}} = \frac{\sqrt{7}}{\sqrt{4}} = \frac{\sqrt{7}}{2}$
3. No fraction has a radical in its denominator. $\frac{2}{\sqrt{7}}$

Example 1: Simplify

a) $3\sqrt{112}$

112
4 28
4 7

$3(4\sqrt{7})$

$\sqrt{112} = 4\sqrt{7}$

$12\sqrt{7}$

$3\sqrt{112}$
31.74901573
$12\sqrt{7}$
31.74901573

b) $\sqrt{256} = 16$

c) $\sqrt{\frac{7}{4}}$

$\frac{\sqrt{7}}{\sqrt{4}} = \frac{\sqrt{7}}{2}$

It is often helpful to know the perfect squares up to 16.

$1^2 = 1$	$5^2 = 25$	$9^2 = 81$	$13^2 = 169$
$2^2 = 4$	$6^2 = 36$	$10^2 = 100$	$14^2 = 196$
$3^2 = 9$	$7^2 = 49$	$11^2 = 121$	$15^2 = 225$
$4^2 = 16$	$8^2 = 64$	$12^2 = 144$	$16^2 = 256$

How do we simplify $\frac{3}{\sqrt{5}}$?

In example 1, did we change the value of $3\sqrt{112}$ when we simplified? No

What number can we multiply by so the value of an expression does not change? 1

What is the value of $(\sqrt{x})(\sqrt{x})$? x

Using these ideas....

$$\frac{3}{\sqrt{5}} \cdot \frac{1}{1} = \frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

Example 2: Simplify

a) $\sqrt{\frac{3}{6}} = \frac{\sqrt{3}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{18}}{6} = \frac{3\sqrt{2}}{6} = \frac{\sqrt{2}}{2}$

$\sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

18 = 9 · 2

b) $\frac{7}{3\sqrt{14}} \cdot \frac{\sqrt{14}}{\sqrt{14}} = \frac{7\sqrt{14}}{42} = \frac{\sqrt{14}}{6}$

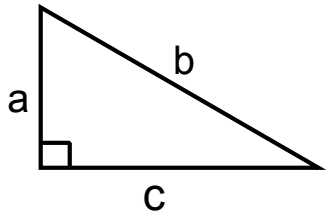
14

c) $\frac{16}{\sqrt{8}} \cdot \frac{\sqrt{8}}{\sqrt{8}} = \frac{16\sqrt{8}}{8} = \frac{2\sqrt{8}}{2} = 2(2\sqrt{2}) = 4\sqrt{2}$

8 = 4 · 2

d) $\frac{3\sqrt{14}}{\sqrt{18}} = \frac{3\sqrt{7}}{\sqrt{9}} = \frac{3\sqrt{7}}{3} = \sqrt{7}$

Write the Pythagorean Theorem using the picture below:

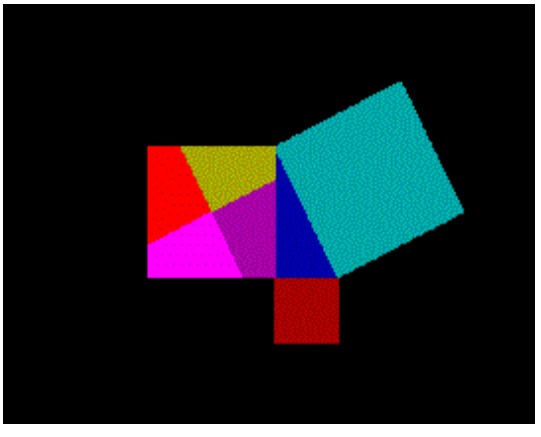


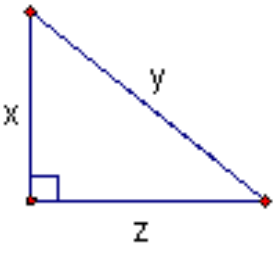
$a^2+b^2=c^2$ $a^2+c^2=b^2$

$leg^2+leg^2=hyp^2$

Pythagorean Theorem (Theorem 8-2)

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.

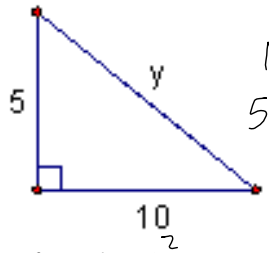




http://upload.wikimedia.org/wikipedia/commons/6/65/Pythag_anim.gif

Example 3: Find the value of the variable.

a)



$5^2+10^2=y^2$

$125=y^2$

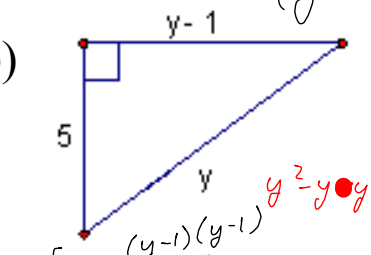
$y=\sqrt{125}$

125

$5 \cdot 25$

$5\sqrt{5}$

b)



$(y-1)^2=y^2+1$

y^2-1

$y^2-y \cdot y+1$

$(y-1)(y-1)$

$5 \cdot 5$

$5^2+(y-1)^2=y^2$

$25+y^2-2y+1=y^2$

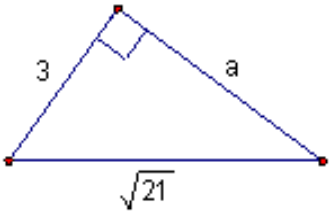
$y^2-2y+26=y^2$

$-2y+26=0$

$26=2y$

$y=13$

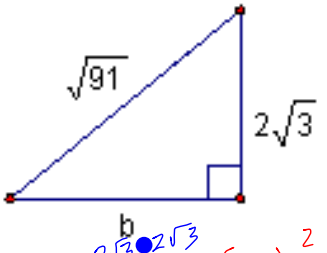
c)



$3^2+a^2=(\sqrt{21})^2$

$9+a^2=21$

d)



$2\sqrt{3} \cdot 2\sqrt{3}$

$b^2+(2\sqrt{3})^2=(\sqrt{91})^2$

$=91$

$b^2+4 \cdot 3$

$b^2+12=91$