Making a Wavefunctional representation of physical states congruent with the false vacuum hypothesis of Sidney Coleman

Andrew . Beckwith

American institute of Beam energy propulsion, Life member,

beckwith@aibep.org, abeckwith@uh.edu

ABSTRACT

We examine quantum decay of the false vacuum in the driven sine-Gordon system and show how both together permit construction of a Gaussian wave functional. This is due to changing a least action integral to be similar with respect to the WKB approximation. In addition we find that the soliton-antisoliton (S-S') separation distance obtained from the Bogomol'nyi inequality permits after rescaling a dominant ϕ^2 contribution to the least action integrand. This is from an initial scalar potential characterized by a tilted double well potential construction.

PACS numbers: 03.75.Lm, 11.27.+d, 71.45.Lr, 75.30.Fv , 85.25.Cp

INTRODUCTION

In this paper, we apply the vanishing contribution to a physical system of a topological charge Q to show how the Bogomol'nyi inequality[6] can be used to simplify a Lagrangian potential energy term. This is so that the potential energy is proportional to a quadratic ϕ^2 scalar field contribution. In doing so, we work with a field theory featuring a Lorenz scalar singlet valued field in D+1 dimensional spacetime. Here, the D is the spatial dimensions of the analyzed system, so if D = 1 we are working with 1 spatial dimension plus a time contribution.

Topological charges and inequalities exist and hold respectively for D+1 dimensional theories featuring scalar and singlet valued fields; only for D = 1. For D > 1, the Lorentz scalar fields must be D-plets! We use the D = 1 dimensional case for describing the dynamics of quasi-one-dimensional metallic materials in our condensed matter example.

We then describe how the quantum decay of a false vacuum[1,2] contributes to our problem. For forming the Gaussian wavefunctionals in our new functional integration presentation of our generalized rate creation problem, we employed a least action principle that Sidney Coleman used for WKB-style modeling of tunneling.[2]

As a sign of its broad scientific interest, for over two decades several quantum tunneling approaches[2] have been proposed to this issue of the quantum decay of the false vacuum. One[2] is to use functional integrals to compute the Euclidean action ("bounce") in imaginary time. This permits inverting the potential and modifying what was previously a potential barrier separating the false and true vacuums into a potential well in Euclidean space and imaginary time. The decay of the false vacuum is a potent paradigm for describing decay of a metastable state to one of lower potential energy. In condensed matter, this decay of the false vacuum method has been used[3] to describe nucleation of cigar-shaped regions of true vacuum with soliton-like domain walls at the boundaries in a charge density wave. Another approach,[4] using the Schwinger proper time method, has been applied by others[5] to calculate the rates of particle-antiparticle pair creation in an electric field for the purpose of simplifying transport problems. Our method fits well with more abstractly presented treatments of transport theory.[6] We also mathematically elaborate upon the S-S' domain wall paradigm[7,8] so that a tunneling Hamiltonian formalism which uses thisGaussian wavefunctional derived here has the kinetic energy information for our rate of transfer problem. But that the wavefunctionals derived here contain the tilted potential contribution the false vacuum hypothesis gives us for physics problems re scaled to a . quadratic ϕ^2 scalar field contribution

BASIC TECHNIQUES USED IN THIS PAPER

In this study, we apply the domain wall physics of S-S' pairs to obtain a quadratic scalar valued potential for transport physics problems involving weakly coupled scalar fields. After this energy/mass representation of the soliton kink is modified by the Bogomol'nyi inequality,[1] we can use the bound on our modified potential to simplify a Euclidian least action integral

If we use Euclidian imaginary time, the least action integral of our wave functional will be changed from Eq. (1a) below to Eq. (1b) by using $time \rightarrow i \cdot (time)$.

$$\int D\phi \cdot \exp\left((i/\hbar) \cdot \int d^d x \cdot \left[\frac{1}{2} \cdot (\partial\phi)^2 - V(\phi) \right] \right) \rightarrow \tag{1a}$$

transforms to

$$\int D\phi \cdot \exp\left((-1/\hbar) \cdot \int d_E^d x \cdot \left[\frac{1}{2} \cdot (\partial\phi)^2 + V(\phi) \right] \right) \tag{1b}$$

We should note that Eq. (1b) has an energy expression of the form

$$\varepsilon(\phi) \equiv \int d_E^d x \cdot \left[\frac{1}{2} \cdot (\partial\phi)^2 + V(\phi) \right] \tag{2a}$$

Eq. (2a) has a potential term that we can write as

$$V(\phi) \equiv C_0 \cdot (\phi - \phi_0)^2 + C_1 \cdot (\phi - \phi_0)^4 + H.O.T. \tag{2b}$$

Furthermore, even after we invert our potentials, we can simplify our expression for the potential by procedures that eliminate the scalar potential terms higher than ϕ^2 by considering the energy per unit length of a soliton kink. This is given by A. Zee,[1] after rescaling to different constants, as

$$\tilde{\varepsilon}(x) = \frac{1}{2} \cdot \left(\frac{d \cdot \phi}{d \cdot x} \right)^2 + \frac{\lambda}{4} \cdot (\phi^2 - \varphi)^2 \tag{3}$$

with a mass of the kink or antikink of this given by

$$M \equiv \int dx \cdot \tilde{\varepsilon}(x) \tag{3a}$$

to be bounded below, namely, by use of the Bogomol'nyi inequality

$$M \geq \int dx \cdot \sqrt{\frac{\lambda}{2}} \cdot \left| \left(\frac{d \cdot \phi}{d \cdot x} \right) \cdot (\phi^2 - \varphi^2) \right| \geq \left| \frac{4}{3 \cdot \sqrt{2}} \cdot \mu \cdot \left(\frac{\mu^2}{\lambda} \right) \cdot Q \right| \tag{4}$$

where Q is a topological charge of the domain wall problem. and $\mu \propto \sqrt{\lambda \cdot \phi_0^2}$ We define conditions for forming a wave functional via the Bogomol'nyi inequality and the vanishing of the topological charge Q, as given by Eq. (5):[1,9]

$$\Psi \equiv c \cdot \exp(-\alpha \cdot \int dx^{(D\equiv 1)} [\phi - \phi_C]^2) \tag{5}$$

We presuppose, when we obtain Eq. (5), a power series expansion of the Euclidian Lagrangian, L_E about $\phi_C \equiv \phi_0$. The first term of this expansion,

$$L_E \,|_{\phi=\phi_O} = \frac{1}{2}\cdot\left(\vec{\nabla}\phi\right)^2 \,|_{\phi=\phi_0} \equiv \varepsilon(\phi)\big|_{\phi\equiv\phi_0} \tag{6}$$

is a comparatively small quantity that we may ignore most of the time. Furthermore, we simplify working with the least action integral by assuming an almost instantaneous nucleation of the S-S' pair. We may then write, starting with a Lagrangian density ζ,

$$\int d\tau \cdot dx \cdot \zeta \rightarrow t_P \cdot \int dx \cdot \zeta \rightarrow \; t_P \cdot \int dx \cdot L \tag{7}$$

Quantity t_P in equation 7 is scaled to unity. Eq. (7) allows us to write our wave functional as a one-dimensional integrand. We called the $t_P \propto 1$ as a unit interval of time in this calculation. Eq. (7) needs considerable explanation. To do this, break up the Lagrangian density as

$$\zeta \equiv (\partial\phi)^2 + V(\phi) \tag{8}$$

with

$$(\partial\phi)^2 = (\partial\phi/\partial\tau_E)^2 + \; \left(\vec{\nabla}\phi\right)^2 \tag{9}$$

where the Euclidian imaginary time is over such a short interval that we, instead, look at the spatial variation according to setting the time varying contribution of the phase as a uniform constant term, so we look at

$$(\partial\phi)^2 \cong + \; \left(\vec{\nabla}\phi\right)^2 \tag{9a}$$

and then look at the integrand as

$$\int d\tau \cdot dx \cdot \zeta = t_P \cdot \breve{\varepsilon}(\phi) \tag{9b}$$

with

$$\breve{\varepsilon}(\phi) \cong \int dx \cdot \left[\frac{1}{2} \cdot (\nabla \phi)^2 + V(\phi) \right] \tag{10}$$

Then we have to look at the behavior of

$$(\nabla \phi) \equiv \delta_n (x - L/2) - \delta_n (x + L/2) \tag{11}$$

which would represent the behavior of test functions converging to Dirac delta functions as $n \to \infty$

Furthermore, we should look at the behavior of, if N is very large

$$\begin{aligned} &\int dx \cdot \left[\frac{1}{2} \cdot (\nabla \phi)^2 \right] \xrightarrow[n \to N]{} \frac{1}{2} \cdot \int dx \cdot \left[\delta_N (x - L/2) - \delta_N (x + L/2) \right]^2 \\ &\equiv \frac{1}{2} \int dx \cdot \left[\delta_N (x - L/2) \right]^2 + \frac{1}{2} \int dx \cdot \left[\delta_N (x + L/2) \right]^2 \\ &\cong \frac{1}{2} \cdot \left[2/\sqrt{2} \right] \equiv \frac{1}{\sqrt{2}} < 1 \end{aligned} \tag{12}$$

where I am, for this example modeling for all N

$$\delta_N (x \pm L/2) \equiv \delta_N (\tilde{x}) \equiv \left(N/2 \cdot \sqrt{\pi} \right) \cdot \exp(-\tilde{x}^2 \cdot N^2/4) \tag{13}$$

where for all N values we have

$$\int_{-\infty}^{+\infty} \delta_N (\tilde{x}) \cdot d\tilde{x} = 1 \tag{14}$$

So, then, we are analyzing this problem according to a finite contribution of $\int dx \cdot \left[\frac{1}{2} \cdot (\nabla \phi)^2 \right] \xrightarrow[n \to N]{} \frac{1}{\sqrt{2}} < 1$ with contributions about the domain walls of $x \cong \pm L/2$ assuming a thin wall approximation, as illustrated by Fig. 1.

[Insert Fig. 1 here]

Introducing domain wall physics via Eq. (6) and Eq. (7) allows us to use a least action integral interpretation of WKB tunneling as the starting point to our analysis. This permits us to write our wave functional as proportional to[1,9]

$$\psi \;\propto c\;\cdot \exp\left(-\tilde{\beta}\cdot\int L\, d\tau\right) \tag{15}$$

with the Lagrangian treated as

$$L_E \approx L_E\,|_{\phi=\phi_O} \;+\; \frac{1}{2}\cdot(\phi-\phi_0)^2\frac{\partial^2\cdot V_E}{\partial\cdot\phi^2}\,|_{\phi=\phi_0} \;+\frac{1}{3!}(\phi-\phi_0)^3\cdot\frac{\partial^3\cdot V_E}{\partial\cdot\phi^3}\,|_{\phi=\phi_0} \;+ \frac{1}{4!}(\phi-\phi_0)^4\cdot\frac{\partial^4\cdot V_E}{\partial\cdot\phi^4}\,|_{\phi=\phi_0} \tag{16}$$

We should be aware that for a wick rotation, when $t=-i\cdot\tau_E$ that for d dimensions $d^d x = -i\cdot d_E^d x$ with $d_E^d x = d\tau_E\cdot d^{d-1}x$, and then we will set $d=2$, effectively leaving us with use of $\tilde{\varepsilon}(x)=\frac{1}{2}\cdot\left(\frac{d\cdot\phi}{d\cdot x}\right)^2+\frac{\lambda}{4}\cdot\left(\phi^2-\varphi\right)^2$ for a soliton kink. We also use a conserved current quantity of[1,10]

$$J^{\mu} = \frac{1}{2\cdot\varphi}\cdot\varepsilon^{\mu\nu}\cdot\partial_{\nu}\cdot\phi \tag{17}$$

with a *topologica*l charge of[1]

$$Q \equiv \int_{-\infty}^{+\infty} dx\cdot J^0(x) = \frac{1}{2\cdot\varphi}\left[\phi\cdot(\infty)-\phi\cdot(-\infty)\right] \tag{18}$$

Note that the denominator φ is not the same as $\phi(x)$! In Zee,[1] the φ term is due to his setting of two minimum positions for ϕ for a double well potential. We find that if we have meson type behavior for the *field* $\phi(x)$, this charge will vanish. It is useful to note that if we look at the mass of a kink via a scaling $\mu \propto \sqrt{\lambda \cdot \phi_0^2}$ with M defined as the same as the energy of a soliton kink given in Eq. (3), with a subsequent mass given in Eq. (3a), that we have, via using $a^2 + b^2 \geq 2 \cdot |a \cdot b|$, an inequality of the form given by Eq. (4). So that[1]

$$M \geq |Q| \tag{19}$$

with *mass* M in terms of units of $\frac{4}{3 \cdot \sqrt{2}} \cdot \mu \cdot \left(\frac{\mu^2}{\lambda} \right)$. If we note that we have $(\phi - \phi_0)^4 = (\phi^2 - \phi_0^2)^2 - 4 \cdot \phi \cdot \phi_0 \cdot (\phi - \phi_0)^2$ in one dimension, we physically use our topological current as a vanishing quantity from the kinetic term and the fourth order term both in a current as a vanishing quantity from the kinetic term and as an expansion of the potential about ϕ_0. Then we can write

$$L_E \geq |Q| \; + \; \frac{1}{2} \cdot (\phi_0 - \phi_C)^2 \cdot \{ \; \} \tag{20}$$

where

$$|Q| \rightarrow 0 \tag{21}$$

Due to a *topological* current argument (S-S' pairs usually being of opposite charge)

and

$$\{ \; \} \equiv \{ \; \}_A - \{ \; \}_B \equiv 2 \cdot \Delta E_{gap} \approx 2 \cdot \alpha^{-1} \tag{22}$$

where if we pick[1] :

$$\frac{(\{ \; \} \equiv \{ \; \}_A - \{ \; \}_B)}{2} \equiv \Delta E_{gap} \equiv V_E(\phi_F) - V_E(\phi_T) \tag{23}$$

This means a wavefunctional with information from a inverted potential as part of a transport problem of weakly coupled systems along the lines suggested by Tekeman.[11] We found our weakly coupled systems eliminated the cross terms in our derivation of a functional integral and for $D = 1$, can write more generally the initial configuration of the form[12]

$$\Psi_i\left[\phi(\mathbf{x})\right]\Big|_{\phi=\phi_{Ci}} = c_i \cdot \exp\left\{-\alpha \int d\mathbf{x}\left[\phi_{c\,i}(\mathbf{x}) - \phi_0\right]^2\right\}, \tag{24a}$$

which is

$$\begin{aligned}\Psi_i\left[\phi(\mathbf{x})\right] &= c \cdot \exp\left\{-\alpha' \int d\ \ \mathbf{x}\ L_E(\mathbf{x})\right\} \\ &= c \cdot \exp\left\{-\alpha' S_E\right\}.\end{aligned} \tag{24b}$$

in addition, we would also have a final state immediately after tunneling,[1,12]

$$\Psi_f\left[\phi(\mathbf{x})\right]\Big|_{\phi=\phi_{Cf}} = c_f \cdot \exp\left\{-\int d\ \ \mathbf{x}\ \beta(\mathbf{x})\left[\phi_{Cf}(\mathbf{x}) - \phi_0(\mathbf{x})\right]^2\right\}, \tag{24c}$$

In the case of a driven sine-Gordon potential system, the initial state is similar to Coleman's false vacuum bounce representation.[1] The final state can be approximated as a modified Gaussian centered about a final field configuration of $\phi_{Cf}(x)$ that includes a bubble in which ϕ_0 has tunneled through the barrier into the true vacuum state, creating one or more soliton domain walls at the boundary between true and false vacuums $\phi_0(x)$ inside the tunnel barrier. Furthermore, we have that[1]

$$\frac{\partial V}{\partial \phi} = 0,$$

$$\Rightarrow \phi_F \ . \approx \left[\frac{\varepsilon^+}{\varepsilon^+ + 1}\right] \approx \varepsilon^+ \tag{25}$$

that is then tied in with the Bogomol'nyi inequality formulation of Eq. (20) where the topological charge $Q \to \varepsilon^+ \approx 0$. We also have in the case of a driven sine-Gordon potential a situation where we can generalize our wave functionals as[1,12]

$$\Psi_i[\phi(\mathbf{x})]\big|_{\phi \equiv \phi_{Ci}} = c_i \cdot \exp\left\{-\alpha \int d\mathbf{x}\left[\phi_{c\,i}(\mathbf{x}) - \phi_0\right]^2\right\} \to$$
$$c_1 \cdot \exp\left(-\alpha_1 \cdot \int d\tilde{x}\left[\phi_F\ \right]^2\right) \equiv \Psi_{initial}\ , \tag{26a}$$

and

$$\Psi_f[\phi(\mathbf{x})]\big|_{\phi \equiv \phi_{Cf}} = c_f \cdot \exp\left\{-\int d\mathbf{x}\ \alpha\left[\phi_{C\,f}(\mathbf{x}) - \phi_0(\mathbf{x})\right]^2\right\} \to$$
$$c_2 \cdot \exp\left(-\alpha_2 \cdot \int d\tilde{x}\left[\phi_T\ \right]^2\right) \cong \Psi_{final}\ , \tag{26b}$$

where a driven sine-Gordon system is of the form [9] (assuming $C_a >> C_b$)

$$V(\phi) \approx C_a \cdot (1 - \cos\phi) + C_b \cdot \left(\phi - \phi_{C\,i,f}\right)^2 \tag{27a}$$

$$\Rightarrow \phi_T \approx 2 \cdot \pi \tag{27b}$$

We also assume that c_i, with the i being either 1 or 2, will take into consideration the contributions denoted from Eq. (12). Furthermore, where $\phi_{ci,cf}(x)$ is the initial and final state equilibrium configuration of phase, the wavefunctionals so obtained permit us to write wavefunctionals that obey the extremal condition of[2,9]

$$\frac{\delta}{\delta\phi(x)}\left(\int L_{i,f}\, d\tau\right)\Big|_{\phi_0 \equiv \phi_{Ci,Cf}} \equiv 0 \qquad (28)$$

which is a further tie in with Sidney Coleman's fate of the false vacuum hypothesis.[2]

CONCLUSION

It is straightforward to construct wavefunctionals that represent creation of a particular event within an embedding space. Diaz and Lemos[13] use this technique as an example of the exponential of a Euclidian action to show how black holes nucleate from nothing. This was done in the context of de Sitter space; Diaz and Lemos[13] used a similar calculation with respect to nucleating a de Sitter space from nothing. The ratio of the modulus of these two wavefunctionals is used to calculate the probability of Black hole nucleation within a de Sitter space, which is the general embedding space of the universe. This trick was also used by Kazumi Maki[14] to observe a field theoretic integration of condensates of S-S' pairs in the context of boundary energy of a two-dimensional bubble of space-time. This two-dimensional bubble action value was minus a contribution to the action due to volume energy of the same two-dimensional bubble of space-time. Maki's[14] probability expression for S-S' pair production is not materially different from what Diaz and Lemos[13] used for black hole nucleation.

What we have done is to generalize this technique to constructing wavefunctional representations of false and true vacuum states in a manner that allows for transport problems to be written in terms of kinetic dynamics as they are given by a functional generalization of a tunneling Hamiltonian. It also allows us to isolate soliton/instanton information in a potential field that overlaps with a Gaussian wavefunctional presentation of soliton/instanton dynamics.[1] We believe that this approach will prove especially fruitful when we analyze nucleation of instanton[15] states that contribute to lower dimensional analysis of the configurations of known physical systems (e.g., $NbSe_3$).[1,9] This approach to wavefunctionals materially contributes to calculations we have performed with respect to I-E curves fitting experimental data quite exactly[1,9] — and in a manner not seen in more traditional renderings of transport problems in condensed matter systems with many weakly coupled fields interacting with each other.[16]

FIGURE CAPTION

FIG. 1: Evolution from an initial state $\Psi_i[\phi]$ to a final state $\Psi_f[\phi]$ for a double-well potential (inset) in a 1-D model, showing a kink-antikink pair bounding the nucleated bubble of true vacuum. The shading illustrates quantum fluctuations about the initial and final optimum configurations of the field, while $\phi_0(x)$ represents an intermediate field configuration inside the tunnel barrier. The upper right hand side of this figure is how the fate of the false vacuum hypothesis gives a difference in energy between false and true potential vacuum values which we tie in with the results of the Bogomol'nyi inequality.

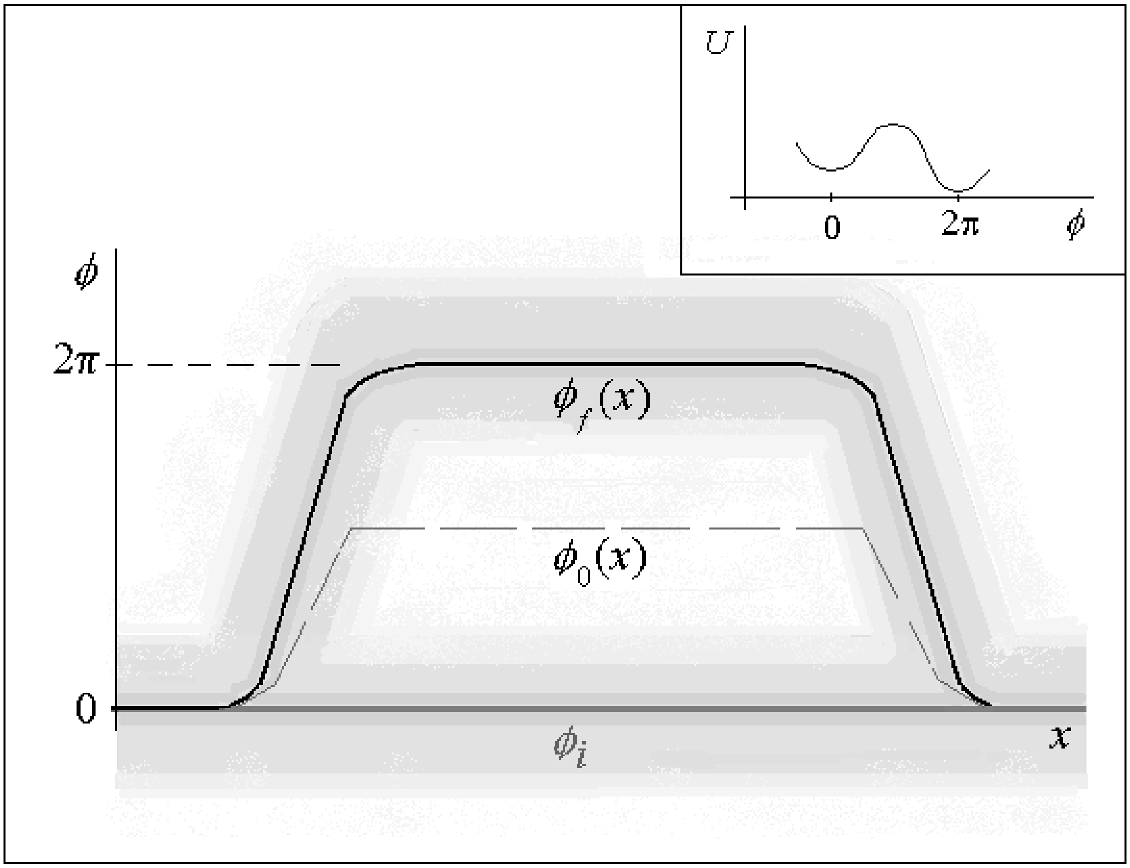

FIGURE 1
BECKWITH

REFERENCES

[6] A. Beckwith; arXIV math-ph/0406053; A. Zee, *Quantum field theory in a nutshell*, Princeton University Press 2003, pp.261-63 and pp.279-281.

[2] S. Coleman; *Phys.Rev*.**D 15**, 2929 (1977).

[3] I.V. Krive and A.S. Rozhavskii; *Soviet Physics* **JETP 69**,552 (1989).

[4] J. Schwinger, *Phys.Rev*.**82**, 664 (1951).

[5] Y. Kluger, J.M. Eisenberg , B. Sventitsky, F. Cooper and E. Mottola; *Phys.Rev.Lett*. **67**,2427 (1991).

[6] For a review, see R. Jackiw's article in *Field Theory and Particle Physics*, O. Eboli, M. Gomes, and A. Samtoro, Eds. (World Scientific, Singapore, 1990); Also see F. Cooper and E. Mottola, *Phys. Rev. D***36**, 3114 (1987); S.-Y. Pi and M. Samiullah, *Phys. Rev. D***36**, 3128 (1987); R. Floreanini and R. Jackiw, *Phys. Rev. D***37**, 2206 (1988); D. Minic and V. P. Nair, *Int. J. Mod. Phys. A* **11**, 2749 (1996).

[7] J. H. Miller,Jr. , C. Ordonez, and E. Prodan, *Phys. Rev. Lett* **84**, 1555(2000).

[8] J. H. Miller, Jr., G. Cardenas, A. Garcia-Perez, W. More, and A. W. Beckwith, *J. Phys. A: Math. Gen*. **36**, 9209 (2003).

[9] A.W. Beckwith, *Classical and quantum models of density wave transport: A comparative study*. PhD Dissertation, 2001.

[10] Davison Soper, 'Classical Field theory' , Wiley, 1976, pp 101-108.,eqn 9.13.

[11] S. Ciraci , E. Tekman ; *Phys.Rev*. **B 40**, 11969 (1989).

[12] Hermann G. Kümmel , *Phys. Rev*. **B 58**, 2620–2625 (1998).

[13] O. Dias, J. Lemos: arxiv :hep-th/0310068 v1 7 Oct 2003.

[14] K. Maki; *Phys.Rev.Lett*. **39**, 46 (1977), K. Maki *Phys.Rev* **B 18**, 1641 (1978).

[15] Javir Casahoran, *Comm. Math. Sci*, Vol 1, No. 2, pp 245-268.

[16] W. Su, J. Schrieffer, and J. Heeger, *Phsy Rev Lett*. **42** , 1698(1979).