Lesson 16-3

PLAN

Pacing: 1 class period

Chunking the Lesson

#1 #2 #3–4
#5 #6 #7–8
#9
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

Tell whether each property is a property of every rhombus.

1. opposite sides are congruent [*yes*]
2. all angles are right angles [*no*]
3. diagonals are perpendicular [*yes*]

1 Activating Prior Knowledge Students should recall the definition of a rhombus as they work to prove in Items 1 through 5 that if the diagonals of a quadrilateral are perpendicular, then the quadrilateral is a rhombus.

2 Think-Pair-Share, Group Presentation, Discussion Groups, Debriefing Students make a conjecture to complete the theorem and write an informal proof.

3–4 Think-Pair-Share, Group Presentation, Discussion Groups, Debriefing Students make a conjecture for the theorem and write a paragraph proof. Remind students that although the paragraph form is more informal than a two-column proof, each step must include a reason.

5 Think-Pair-Share, Visualization Students are led through the key steps in a proof of a theorem that they will use to prove that a parallelogram is a rhombus. As students work through this item, have them mark the congruent segments and angles on the diagram. They should note that the two congruent angles are between the two pairs of corresponding sides.

My Notes

Learning Targets:

- Develop criteria for showing that a quadrilateral is a rhombus.
- Prove that a quadrilateral is a rhombus.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Create Representations, Group Presentation, Discussion Groups

1. Complete the following definition.

A rhombus is a parallelogram with **four congruent sides**.

2. a. Complete the theorem.

Theorem If a parallelogram has two consecutive congruent sides, then it has **four** congruent sides, and it is a **rhombus**.

b. Use one or more properties of a parallelogram and the definition of a rhombus to explain why the theorem in Item 2a is true.

Opposite sides of a parallelogram are congruent, so all sides of this parallelogram are congruent (by the transitive property). By definition, this parallelogram is a rhombus.

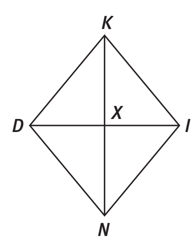
3. Complete the theorem.

Theorem If a quadrilateral is equilateral, then it is a **rhombus**.

4. Write a paragraph proof to explain why the theorem in Item 3 is true.

If all sides of a quadrilateral are congruent, then both pairs of opposite sides are congruent. Hence, the quadrilateral is a parallelogram. By definition, the parallelogram is a rhombus.

5. Given ▱*KIND* with $\overline{KN} \perp \overline{ID}$.



a. List the three triangles that are congruent to $\triangle KXD$, and give the reason for the congruence.

$\triangle KXD \cong \triangle KXI \cong \triangle NXI \cong \triangle NXD$ by SAS

b. List all segments congruent to $\overline{KD}$ and explain why.

$\overline{KD} \cong \overline{KI} \cong \overline{NI} \cong \overline{ND}$ because CPCTC.

c. Complete the theorem.

Theorem If the diagonals of a parallelogram are **perpendicular**, then the parallelogram is a **rhombus**.

Lesson 16-3
Proving a Quadrilateral Is a Rhombus

My Notes

6. Given quadrilateral *BIRD* with coordinates $B(-2, -3)$, $I(1, 1)$, $R(6, 1)$, and $D(3, -3)$.

a. Show that quadrilateral *BIRD* is a parallelogram.

Method 1 Both pairs of opp sides parallel: slope of $\overline{BI}$ = slope of $\overline{RD} = \frac{4}{3}$ and slope of $\overline{IR}$ = slope of $\overline{BD} = 0$

Method 2 Both pairs of opp sides congruent: $BI = RD = 5$ and $IR = BD = 5$

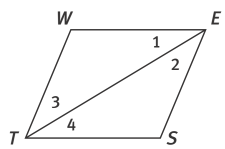
Method 3 1 pair of opp sides ≅ and ||: slope of $\overline{BI}$ = slope of $\overline{RD} = \frac{4}{3}$ and $BI = RD = 5$ or slope of $\overline{IR}$ = slope of $\overline{BD} = 0$ and $IR = BD = 5$

Method 4 Diagonals bisect each other: midpt of $\overline{BR}$ = midpt of $\overline{ID} = (2, -1)$

b. Use the theorem in Item 5 to show ▱*BIRD* is a rhombus.

Diagonals are perpendicular: slope of $\overline{BR} = \frac{1}{2}$ and slope of $\overline{ID} = -2$

7. Given ▱*WEST* with $\overline{TE}$ that bisects ∠*WES* and ∠*WTS*.



a. List all angles congruent to ∠1 and explain why.

∠1 ≅ ∠2 ≅ ∠3 ≅ ∠4; ∠1 ≅ ∠2 and ∠3 ≅ ∠4 (def of angle bisector) and ∠1 ≅ ∠4 and ∠2 ≅ ∠3 (alternate interior angles and def of parallelogram)

b. In △*WET*, $\overline{WT}$ ≅ __**WE**__. In △*SET*, $\overline{ST}$ ≅ __**SE**__. Explain.

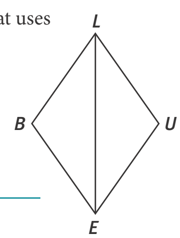
If two angles of a triangle are congruent, the sides opposite those angles are congruent.

c. Complete the theorem.

Theorem If a diagonal bisects ________________ in a parallelogram, then the parallelogram is a ______________.

a pair of opposite angles; rhombus.

8. Construct viable arguments. Write a proof that uses the theorem in Item 7 as the last reason.



Given: ▱*BLUE*

△*BLE* ≅ △*ULE*

Prove: ▱*BLUE* is a rhombus.

Statements	Reasons
1. △*BLE* ≅ △*ULE*	1. Given
2. ∠*BLE* ≅ ∠*ULE* ∠*BEL* ≅ ∠*UEL*	2. CPCTC
3. $\overline{LE}$ bisects ∠*BLU* and ∠*BEU*.	3. Def of bisect
4. ▱*BLUE*	4. Given
5. ▱*BLUE* is a rhombus.	5. If a diagonal bisects a pair of opp ∠s in a parallelogram, the parallelogram is a rhombus.

6 Think-Pair-Share, Group Presentation, Discussion Groups, Debriefing To show that a quadrilateral is a rhombus, students must first use a coordinate argument to show that it is a parallelogram, and then apply the theorem from Item 5 and another coordinate argument to show that a quadrilateral is a parallelogram.

7–8 Think-Pair-Share, Activating Prior Knowledge Students are led through the key steps in the proof that if a diagonal bisects a pair of opposite angles of a quadrilateral, then it is a rhombus. Students apply the theorem for congruent triangles from Item 7 and the definition of an angle bisector to write the proof. Have students mark the congruent parts of the triangles as they develop the proof.

9 Think-Pair-Share, Group Presentation, Discussion Groups, Debriefing This is an opportunity for students to review the third part of this activity. Lead a class discussion on ways of proving that a quadrilateral (or a parallelogram) is a rhombus. Either the teacher or one of the students can continue the "master list" from the previous class discussion at the end of the second part of the activity.

Check Your Understanding

Debrief students' answers to this item to ensure that they understand concepts related to proving that quadrilaterals are rhombuses.

Answers

10. Yes. If a rectangle is a square, it is a rhombus.

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 16-3 PRACTICE

11. $(-4, -2)$
12. $(2, -2)$
13. $(3, -2)$
14. a. $x = \frac{5}{2}$
b. $x = 1$
15. 14.4 cm

ADAPT

Check students' answers to the Lesson Practice to ensure that they understand basic concepts related to proving that quadrilaterals are rhombuses. Students should ask themselves what must be true of the sides and angles of a quadrilateral in order for it to be a rhombus.

My Notes

9. Summarize this part of the activity by making a list of the ways to prove that a quadrilateral is a rhombus.
Show that a quadrilateral (or parallelogram) has four congruent sides. Show that a parallelogram has two consecutive congruent sides. Show that the diagonals of a parallelogram are perpendicular. Show that a diagonal of a parallelogram bisects a pair of opposite angles.

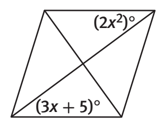
Check Your Understanding

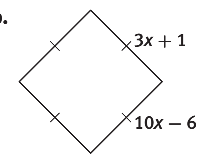
10. Can a rectangle ever be classified as a rhombus as well? Explain.

LESSON 16-3 PRACTICE

Three vertices of a rhombus are given. Find the coordinates of the fourth vertex.

11. $(-2, -8), (3, -3), (-9, -7)$
12. $(-1, 2), (-1, -1), (2, 1)$
13. $(1, 1), (-1, -2), (1, -5)$
14. Find the value of x that makes the parallelogram a rhombus.

a.


b.


15. **Reason quantitatively.** LaToya is using a coordinate plane to design a new pendant for a necklace. She wants the pendant to be a rhombus. Three of the vertices of the rhombus are $(3, 1)$, $(-1, -1)$, and $(1, -2)$. Assuming each unit of the coordinate plane represents one centimeter, what is the perimeter of the pendant? Round your answer to the nearest tenth.