CLASS II PUC

UNIT I: RELATIONS AND FUNCTIONS

1. Relations and Functions

Types of relations: Reflexive, symmetric, transitive, empty, universal and equivalence relations. Examples and problems.

Types of functions: One to one and onto functions, inverse of a function composite functions, mentioning their properties only , examples and problems.

Binary operations: associative, commutative, identity, inverse with examples

2. Inverse Trigonometric Functions

Definition, range, domain, principal value branches. Mentioning domain and range of trigonometric and inverse trigonometric functions. Graphs of inverse trigonometric functions.

Properties and proofs of inverse trigonometric functions given in NCERT prescribed text book, mentioning formulae for $\sin^{-1}x \pm \sin^{-1} y$, $\cos^{-1}x \pm \cos^{-1} y$,

$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ without proof.

Conversion of one inverse trigonometric function to another w.r.t to right angled triangle. Problems.

UNIT II: ALGEBRA

1. Matrices

Concept, notation, order,

Types of matrices: column matrix, row matrix, rectangular matrix, square matrix, zero matrix, diagonal matrix, scalar matrix and unit matrix.

Algebra of matrices: Equality of matrices, Addition, multiplication, scalar multiplication of matrices, Transpose of a matrix. Mentioning properties with respect to addition, multiplication, scalar multiplication and transpose of matrices.

Symmetric and skew symmetric matrices: Definitions,

properties of symmetric and skew symmetric matrices: proofs of

i) If A is any square matrix A+A′ is symmetric and A-A′ is skew symmetric

ii) Any square matrix can be expressed as the sum of a symmetric and a skew symmetric matrix.

Concept of elementary row and column operations and finding inverse of a matrix restricted to 2x2 matrices only.

Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

2. Determinants

Determinant of a square matrix (up to 3 × 3 matrices): Definition, expansion, properties of determinants, minors , cofactors and problems.
Applications of determinants in finding the area of a triangle.
Adjoint and inverse of a square matrix, definition of singular and non-singular matrices, mentioning their properties:

a)If A and B are nonsingular matrices of same order, then AB and BA are nonsingular matrices of same order

b)A square matrix A is invertible if and only if A is non-singular matrix

Consistency, inconsistency and number of solutions of system of linear equations by examples,
Solving system of linear equations in two and three variables (having unique solution) using inverse of a matrix.

UNIT III: CALCULUS

1. Continuity and Differentiability

Continuity: Definition, continuity of a function at a point and on a domain. Examples and problems,
Algebra of continuous functions, problems ,
continuity of composite function and problems

Differentiability: Definition, Theorem connecting differentiability and continuity with a counter example.
Defining logarithm and mentioning its properties ,
Concepts of exponential, logarithmic functions,
Derivative of e^x , log x from first principles,
Derivative of composite functions using chain rule, problems.
Derivatives of inverse trigonometric functions, problems.
Derivative of implicit function and problems.
Logarithmic differentiation and problems .
Derivative of functions expressed in parametric forms and problems.
Second order derivatives and problems
Rolle's and Lagrange's Mean Value Theorems (without proof) and their geometric Interpretations and problems

2. Applications of Derivatives

Tangents and normal: Equations of tangent and normal to the curves at a point and problems

Derivative as a Rate of change: derivative as a rate measure and problems

Increasing/decreasing functions and problems

Maxima and minima : introduction of extrema and extreme values, maxima and minima in a closed interval, first derivative test, second derivative test.
Simple problems restricted to 2 dimensional figures only

Approximation and problems

3. Integrals

Integration as inverse process of differentiation: List of all the results immediately follows from knowledge of differentiation. Geometrical Interpretation of indefinite integral, mentioning elementary properties and problems.

Methods of Integration: Integration by substitution, examples. Integration using trigonometric identities, examples,

Integration by partial fractions: problems related to reducible factors in denominators only.

Integrals of some particular functions :

Evaluation of integrals of $\int \frac{dx}{a^2 \pm x^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}\ \int \frac{dx}{\sqrt{a^2 - x^2}}$ and problems .

Problems on Integrals of functions like $\int \frac{px+q}{ax^2+bx+c} dx, \int \frac{px+q}{\sqrt{ax^2+bx+c}} dx$

Integration by parts : Problems , Integrals of type $\int e^x [f(x) + f'(x)]dx$ and related simple problems.

Evaluation of Integrals of some more types like $\sqrt{x^2 \pm a^2}, \sqrt{a^2 \pm x^2}$ and problems

Definite integrals: Definition,

Definite Integral as a limit of a sum to evaluate integrals of the form $\int_0^a f(x)dx$ only.

Fundamental Theorem of Calculus (without proof).

Basic properties of definite integrals and evaluation of definite integrals.

4. Applications of the Integrals:

Area under the curve : area under simple curves, especially lines, arcs of circles/parabolas/ellipses (in standard form only),

Area bounded by two above said curves: problems

5. Differential Equations

Definition-differential equation, order and degree, general and particular solutions of a differential equation.

Formation of differential equation whose general solution containing at most two arbitrary constants is given.

Solution of differential equations by method of separation of variables,

Homogeneous differential equations of first order and first degree.

Solutions of linear differential equation of the type

$\frac{dy}{dx}$+ py = q where p and q are functions of x or constant

$\frac{dx}{dy}$+ px = q where p and q are functions of y or constant

(Equation reducible to variable separable , homogeneous and linear differential equation need not be considered)

UNIT IV: VECTORS AND THREE-DIMENSIONAL GEOMETRY

1. Vectors

Definition of Vectors and scalars, magnitude and direction of a vector.
Direction cosines/ratios of vectors: direction angles, direction cosines, direction ratios, relation between direction ratio and direction cosines. Problems.
Types of vectors :Equal, unit, zero, parallel and collinear vectors, coplanar vector position vector of a point, negative of a vector.
Components of a vector,
Algebra of vectors: multiplication of a vector by a scalar
addition of vectors: triangle law, parallelogram law,
properties of addition of vectors,
position vector of a point dividing a line segment in a given ratio(section formula).
Scalar (dot) product of vectors: definition, properties, problems
projection of a vector on a line.
Vector (cross) product of vectors: definition, properties and problems
Scalar triple product: definition, properties and problems.

2. Three-dimensional Geometry:

Direction cosines/ratios of a line joining two points.
Straight lines in space: Cartesian and vector equation of a line passing through given point and parallel to given vector, Cartesian and vector equation of aline passing through two given points, coplanar and skew lines, distance between two skew lines(Cartesian and vector approach), distance between two parallel lines (vector approach). Angle between two lines. Problems related to above concepts.
Plane: Cartesian and vector equation of a plane in normal form, equation of **a** plane passing through the given point and perpendicular to given vector, equation of aplane passing through three non- collinear points, Intercept form of equation of a plane, angle between two planes, equation of plane passing through the intersection of two given planes, angle between line and plane, condition for the coplanarity of two lines, distance of a point from a plane (vector approach) ,Problems related to above concepts.

Unit V: Linear Programming

Introduction of L.P.P. definition of constraints, objective function, optimization, constraint equations, non- negativity restrictions, feasible and infeasible region, feasible solutions, Mathematical formulation-mathematical formulation of L.P.P.
Different types of L.P.P: problems namely manufacturing, diet and allocation problems with bounded feasible regions only, graphical solutions for problem in two variables, optimum feasible solution(up to three non-trivial constraints).

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Unit VI: Probability

Conditional probability – definition, properties, problems.
Multiplication theorem, independent events,
Baye's theorem, theorem of total probability and problems.

Probability distribution of a random variable-definition of a random variable, probability distribution of random variable, Mean , variance of a random variable and problems.

Bernoulli trials and Binomial distribution:
Definition of Bernoulli trial, binomial distribution, conditions for Binomial distribution, and simple problems.

Note: **Unsolved miscellaneous problems given in the prescribed text book need not be considered.**

Design of the Question Paper

MATHEMATICS (35) CLASS : II PUC

Time: 3 hours 15 minute; Max. Mark:100

(*of which 15 minutes for reading the question paper*).

The weightage of the distribution of marks over different dimensions of the question paper shall be as follows:

I. Weightage to Objectives:

Objective	Weightage	Marks
Knowledge	40%	60/150
Understanding	30%	45/150
Application	20%	30/150
Skill	10%	15/150

II. Weightage to level of difficulty:

Level	Weightage	Marks
Easy	35%	53/150
Average	55%	82/150
Difficult	10%	15/150

III. Weightage to content:

Chapter No.	Chapter	No. of teaching Hours	Marks
1	RELATIONS AND FUNCTIONS	11	11
2	INVERSE TRIGONOMETRIC FUNCTIONS	8	8
3	MATRICES	8	9
4	DETERMINANTS	13	12
5	CONTINUITY AND DIFFERENTIABILITY	19	20
6	APPLICATION OF DERIVATIVES	11	10
7	INTEGRALS	21	22
8	APPLICATION OF INTEGRALS	8	8
9	DIFFERENTIAL EQUATIONS	9	10
10	VECTOR ALGEBRA	11	11
11	THREE DIMENSIONAL GEOMETRY	12	11
12	LINEAR PROGRAMMING	7	7
13	PROBABILITY	12	11
	Total	**150**	**150**

IV. Pattern of the question paper:

PART	Type of questions	Number of questions to be set	Number of questions to be answered	Remarks
A	1 mark questions	10	10	Compulsory part
B	2 mark questions	14	10	---
C	3 mark questions	14	10	---
D	5 mark questions	10	6	Questions must be asked from the specific set of topics as mentioned below, under section V
E	10 mark questions (Each question with two subdivisions namely a) 6 mark and b) 4 mark.	2	1	

V. Instructions:

Content areas to select questions for PART – D and PART – E

a) In PART D:

1. Relations and functions: Problems on verification of invertibility of a function and writing its inverse.

For example:

a. Show that the function, $f : R \rightarrow R$ defined by $f(x) = 4x + 3$ is invertible. Hence write the inverse of f.

b. Let R_+ be the set of all non-negative real numbers. Show that the function $f : R_+ \rightarrow [4, \infty)$ defined by $f(x) = x^2 + 4$ is invertible. Also write the inverse of $f(x)$.

c. If R_+ is the set of all non-negative real numbers prove that the function $f : R_+ \rightarrow [-5, \infty)$ defined by $f(x) = 9x^2 + 6x - 5$ is invertible. Write also $f^{-1}(x)$.

2. Matrices: Problems on verifications of basic conclusions on algebra of matrices.

For example:

a. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & -2 \\ 3 & 0 \end{bmatrix}$ verify that $A(BC) = (AB)C$.

b. If $A' = \begin{bmatrix} 1 & 5 \\ 2 & 0 \\ -3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \end{bmatrix}$ find $A + B$ and $B - C$ show that $A + (B - C) = (A + B) - C$.

c. If $A = \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -3 \\ -3 & 4 \end{bmatrix}$ verify that $AB - BA$ is a skew symmetric matrix and $AB + BA$ is a symmetric matrix.

3. Determinants: Problems on finding solution to simultaneous linear equations involving three unknown quantities by matrix method.

For example:

a. Solve the following by using matrix method:
$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$, $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 4$, $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 4$.

b. Solve the following by using matrix method:
$2x + y + z = 1$, $x - 2y - z = \frac{3}{2}$, $3y - 5z = 9$.

c. Use the product $\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix}\begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{pmatrix}$ to solve the system of equations $x - y + 2z = 1$, $2y - 3z = 1$, $3x - 2y + 4z = 9$.

4. Continuity and differentiability: Problems on second derivatives.

For example:

a. If $x = a(\cos t + t.\sin t)$ and $y = a(\sin t - t.\cos t)$ find $\frac{d^2y}{dx^2}$.

b. If $(x-a)^2 + (y-b)^2 = c^2$, $c > 0$, prove that $\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b.

c. If $e^y(x+1) = 1$ prove that $\frac{dy}{dx} = -e^y$. Hence prove that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

5. Application of derivatives: Problems on derivative as a rate measurer.

For example:

a. A ladder $5\,m$ long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall at the rate of $2\,cm/sec$. How fast is its height of the wall decreasing when the foot of the ladder is $4\,m$ away from the wall?

b. Sand is pouring from a pipe at the rate of $12\,cm^3/sec$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is $4\,cm$?

c. A stone is dropped into a quit lake and waves move in circles at a speed of $4\,cm\ per\ second$. At the instant, when the radius of the circular wave is $10\,cm$, how fast is the enclosed area increasing?

6. Integrals: Derivations on indefinite integrals and evaluation of an indefinite integral by using the derived formula.

For example:

a. Find the integral of $\frac{1}{\sqrt{x^2 - a^2}}$ with respect to x and hence evaluate $\int \frac{1}{\sqrt{x^2 - 25}}\, dx$.

b. Find the integral of $\sqrt{a^2 - x^2}$ with respect to x and hence evaluate $\int \sqrt{1 + 4x - x^2}\, dx$.

c. Find the integral of $\frac{1}{x^2 - a^2}$ with respect to x and hence evaluate $\int \frac{x}{x^4 - 16}\, dx$.

7. Application of integrals: Problems on finding the area of the bounded region by the method of integration.

For example:

a. Find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

b. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

c. Find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle, $x^2 + y^2 = 32$.

8. Differential equations: Problems on solving linear differential equations.

For example:

a. Solve the differential equation, $x\frac{dy}{dx} + 2y = x^2 \log x$.

b. Find a particular solution of the differential equation $\frac{dy}{dx} - 3y.\cot x = \sin 2x$, $y = 2$, when $x = \frac{\pi}{2}$.

c. Find the equation a curve passing through the point $(0, 1)$.If the slope of the tangent to the curve at any point (x, y) is equal to the sum of x coordinate and the product of x coordinate and y coordinate of that point.

9. Three dimensional geometry: Derivations on 3 dimensional geometry (both vector and Cartesian form).

For example:

a. Derive a formula to find the shortest distance between the two skew line $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ in the vector form.

b. Derive the equation of a plane passing through three non collinear points both in the vector and Cartesian form.

c. Derive the equation of a line in space passing through two given points both in the vector and Cartesian form.

10. Probability: Problems on Bernoulli trials and binomial distribution.

For example:

- **a.** A die is thrown 6 times. If "getting an odd number" is a success, what is the probability of (i) 5 success? (ii) at least 5 successes?
- **b.** Five cards are drawn successively with replacement from a well shuffled deck of 52 playing cards. What is the probability that (i) all the five cards are spades? (ii) only 3 cards are spades? (iii) none is a spade?
- **c.** The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs (i) none will fuse after 150 days. (ii) at most two will fuse after 150 days.

b) In PART E:

(i) 6 mark questions must be taken from the following content areas only.

1. Integrals: Derivations on definite integrals and evaluation of a definite integral using the derived formula.

For example:

- **a.** Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ and hence evaluate $\int_0^{\pi/2} \log(\sin x)\,dx$.
- **b.** Prove that $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ and hence evaluate $\int_{-1}^2 |x^3 - x|\,dx$.
- **c.** Prove that $\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x) & if\ f(2a-x) = f(x) \\ 0 & if\ f(2a-x) = -f(x) \end{cases}$ and hence evaluate $\int_0^{\pi} \frac{1}{a^2\cos^2 x + b^2 \sin^2 x}\,dx$.

2. Linear programming: Problems on linear programming.

For example:

- **a.** A corporative society of farmers has 50 hectare of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs 10,500 and Rs 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops X and Y at rates of 20 litres and 10 litres per hectare. Further no more than 800 litres of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. How much should be allocated to each crop so as to maximize the total profit of the society?
- **b.** Solve the following linear programming problem graphically:

Minimize and maximize $Z = x + 2y$, subject to constraints
$x + 2y \geq 100,\ 2x - y \leq 0,\ 2x + y \leq 200,\ x,\ y \geq 0$

c. There are two types of fertilizers F_1 and F_2. F_1 consists of 10% nitrogen and 6% phosphoric acid and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that he needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for his crop. If F_1 costs Rs 6/kg and F_2 costs Rs 5/kg, determine how much of each type of fertilizer should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

(ii) 4 mark questions must be taken from the following content areas only.

1. Continuity and differentiability: Problems on continuous functions.

For example:

a. Verify whether the function $f(x) = \begin{cases} x, & \text{if } x \geq 0 \\ x^2, & \text{if } x < 0 \end{cases}$ is continuous function or not.

b. Find the points of discontinuity of the function $f(x) = x - [x]$, where $[x]$ indicates the greatest integer not greater than x. Also write the set of values of x, where the function is continuous.

c. Discuss the continuity of the function $f(x) = \begin{cases} |x| + 3, & \text{if } x \leq -3 \\ -2x, & \text{if} -3 < x < 3 \\ 6x + 2, & \text{if } x \geq 3 \end{cases}$.

2. Determinants: Problems on evaluation of determinants by using properties.

For example:

a. Prove that $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$.

b. Prove that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$

c. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

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SAMPLE BLUE PRINT

II PUC MATHEMATICS (35)

TIME: 3 hours 15 minute **Max. Mark: 100**

Chapter	CONTENT	Number of Teaching hours	PART A	PART B	PART C	PART D	PART E		Total marks
			1 mark	2 mark	3 mark	5 mark	6 mark	4 mark	
1	RELATIONS AND FUNCTIONS	11	1	1	1	1			11
2	INVERSE TRIGONOMETRIC FUNCTIONS	8	1	2	1				8
3	MATRICES	8	1		1	1			9
4	DETERMINANTS	13	1	1		1		1	12
5	CONTINUITY AND DIFFERENTIABILITY	19	1	2	2	1		1	20
6	APPLICATION OF DAERIVATIVES	11		1	1	1			10
7	INTEGRALS	21	1	2	2	1	1		22
8	APPLICATION OF INTEGRALS	8			1	1			8
9	DIFFERENTIAL EQUATIONS	9		1	1	1			10
10	VECTOR ALGEBRA	11	1	2	2				11
11	THREE DIMENSIONAL GEOMETRY	12	1	1	1	1			11
12	LINEAR PROGRAMMING	7	1				1		7
13	PROBABILITY	12	1	1	1	1			11
	TOTAL	**150**	**10**	**14**	**14**	**10**	**2**	**2**	**150**

GUIDELINES TO THE QUESTION PAPER SETTER

1. The question paper must be prepared based on the individual blue print without changing the weightage of marks fixed for each chapter.

2. The question paper pattern provided should be adhered to.

Part A : 10 compulsory questions each carrying 1 mark;

Part B : 10 questions to be answered out of 14 questions each carrying 2 mark ;

Part C : 10 questions to be answered out of 14 questions each carrying 3 mark;

Part D : 6 questions to be answered out of 10 questions each carrying 5 mark

Part E : 1 question to be answered out of 2 questions each carrying 10 mark with subdivisions (a) and (b) of 6 mark and 4 mark respectively.

(The questions for PART D and PART E should be taken from the content areas as explained under section V in the design of the question paper)

3. There is nothing like a single blue print for all the question papers to be set. The paper setter should prepare a blue print of his own and set the paper accordingly without changing the weightage of marks given for each chapter.

4. Position of the questions from a particular topic is immaterial.

5. In case of the problems, only the problems based on the concepts and exercises discussed in the text book (prescribed by the Department of Pre-university education) can be asked. Concepts and exercises different from text book given in Exemplar text book should not be taken. Question paper must be within the frame work of prescribed text book and should be adhered to weightage to different topics and guidelines.

6. No question should be asked from the historical notes and appendices given in the text book.

7. Supplementary material given in the text book is also a part of the syllabus.

8. Questions should not be split into subdivisions. No provision for internal choice question in any part of the question paper.

9. Questions should be clear, unambiguous and free from grammatical errors. All unwanted data in the questions should be avoided.

10. Instruction to use the graph sheet for the question on LINEAR PROGRAMMING in PART E should be given in the question paper.

11. Repetition of the same concept, law, fact etc., which generate the same answer in different parts of the question paper should be avoided.

Model Question Paper – 1

II P.U.C MATHEMATICS (35)

Time : 3 hours 15 minute **Max. Marks : 100**

Instructions :

(i) *The question paper has five parts namely A, B, C, D and E. Answer all the parts.*

(ii) *Use the graph sheet for the question on Linear programming in PART E.*

PART – A

Answer ALL the questions **$10 \times 1 = 10$**

1. Give an example of a relation which is symmetric and transitive but not reflexive.
2. Write the domain of $f(x) = sec^{-1}x$.
3. Define a diagonal matrix.
4. Find the values of x for which, $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$.
5. Find the derivative of $cos(x^2)$ with respect to x.
6. Evaluate: $\int (1-x)\sqrt{x}\, dx$.
7. If the vectors $2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$ and $4\hat{\imath} - m\hat{\jmath} - 12\hat{k}$ are parallel find m.
8. Find the equation of the plane having intercept 3 on the y axis and parallel to ZOX plane.
9. Define optimal solution in linear programming problem.
10. An urn contains 5 red and 2 black balls. Two balls are randomly selected. Let X represents the number of black balls, what are the possible values of X?

PART B

Answer any TEN questions: **$10 \times 2 = 20$**

11. Define binary operation on a set. Verify whether the operation $*$ defined on Z, by $a * b = ab + 1$ is binary or not.
12. Find the simplest form of $tan^{-1}\sqrt{3} - sec^{-1}(-2)$.
13. Evaluate $sin\left\{\frac{\pi}{3} - sin^{-1}\left(-\frac{1}{2}\right)\right\}$.
14. Find the area of the triangle whose vertices are $(3,8)$, $(-4,2)$ and $(5,1)$ using determinants.

15 Check the continuity of the function f given by $f(x) = 2x + 3$ at $x = 1$.

16. Find the derivative of $\left(3x^2 - 7x + 3\right)^{5/2}$ with respect to x.

17. If the radius of a sphere is measured as $9\ cm$ with an error, $0.03\ cm$, then find the approximate error in calculating its volume.

18. Evaluate: $\int \frac{dx}{\sin^2 x \cos^2 x}$.

19. Evaluate: $\int \log x\ dx$

20. Find the order and degree of the differential equation,

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0.$$

21. If the position vectors of the points$A$ and B respectively are $\hat{i} + 2\hat{j} - 3\hat{k}$ and $\hat{j} - \hat{k}$ find the direction cosines of $\overrightarrow{AB}$.

22. Find a vector of magnitude 8 units in the direction of the vector, $\vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}$.

23. Find the distance of the point$(2,\ 3, -5)$ from the plane $\vec{r}\cdot\left(\hat{i} + 2\hat{j} - 2\hat{k}\right) = 9$.

24. A die is thrown. If E is the event 'the number appearing is a multiple of 3' and F is the event ' the number appearing is even', then find whether E and F are independent?

PART C

Answer any TEN questions: **10×3=30**

25. Verify whether the function, $f : A \to B$, where $A = R - \{3\}$ and $B = R - \{1\}$, defined by $f(x) = \frac{x-2}{x-3}$ is one-one and on-to or not. Give reason.

26. Prove that $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ when $xy < 1$.

27. Express $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrices.

28. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2} - 1}{x}\right)$ prove that $\frac{dy}{dx} = \frac{1}{2\left(1+x^2\right)}$.

29. If $x = at^2$ and $y = 2at$ find $\frac{dy}{dx}$.

30. Find the intervals in which the function f given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is

(i) strictly increasing; (ii) strictly decreasing.

31. Find the antiderivative of f(x)given by $f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$.

32. Evaluate: $\int \frac{dx}{x + x\log x}$.

33. Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 4$, $x = 9$ and the x-axis in the first quadrant.

34. Form the differential equation of the family of circles touching the y-axis at origin.

35. If $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath}$ such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ.

36. Find the area of the triangle ABC where position vectors of A, B and C are $\hat{\imath} - \hat{\jmath} + 2\hat{k}$, $2\hat{\jmath} + \hat{k}$ and $\hat{\jmath} + 3\hat{k}$ respectively.

37. Find the Cartesian and vector equation of the line that passes through the points (3, -2, -5) and (3, -2, 6).

38. Consider the experiment of tossing two fair coins simultaneously, find the probability that both are head given that at least one of them is a head.

PART D

Answer any SIX questions: **$6 \times 5 = 30$**

39. Prove that the function, $f : N \to Y$ defined by $f(x) = x^2$, where $Y = \{y : y = x^2, x \in N\}$ is invertible. Also write the inverse of $f(x)$.

40. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$. Calculate AB, AC and A(B+C). Verify that $AB + AC = A(B + C)$.

41. Solve the following system of equations by matrix method:

$x + y + z = 6$; $y + 3z = 11$ and $x - 2y + z = 0$.

42. If $y = 3\cos(\log x) + 4\sin(\log x)$ show that $x^2 y_2 + x y_1 + y = 0$.

43. The length x of a rectangle is decreasing at the rate of 3 cm/minute and the width y is increasing at the rate of 2 cm/minute. When $x = 10\ cm$ and $y = 6\ cm$, find the rates of change of (i) the perimeter and (ii) the area of the rectangle.

44. Find the integral of $\frac{1}{\sqrt{a^2 - x^2}}$ with respect to x and hence evaluate $\int \frac{dx}{\sqrt{7-6x-x^2}}$.

45. Using integration find the area of the region bounded by the triangle whose vertices are $(-1,0)$, $(1,3)$ and $(3,2)$.

46. Solve the differential equation $x\log x \frac{dy}{dx} + y = \frac{2}{x}\log x$.

47. Derive the equation of a plane in normal form(both in the vector and Cartesian form).

48. If a fair coin is tossed 8 times. Find the probability of (i) at least five heads and (ii) at most five heads.

PART E

Answer any ONE question: **$1 \times 10 = 10$**

49. (a) Prove that $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ and evaluate $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

(b) Prove that $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$

50.(a) A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1hour on machine B to produce a package of bolts. He earns a profit of Rs.17.50 per package on nuts and Rs. 7.00 per package on bolts. How many packages of each should be produced each day so as to maximize his profit if he operates his machines for at most 12 hours a day?

(b) Determine the value of k, if $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases}$

is continuous at $x = \frac{\pi}{2}$.

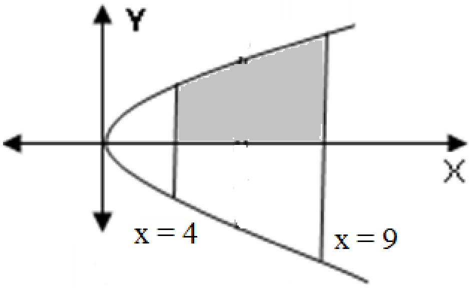
SCHEME OF VALUATION
Model Question Paper – 1

II P.U.C MATHEMATICS (35)

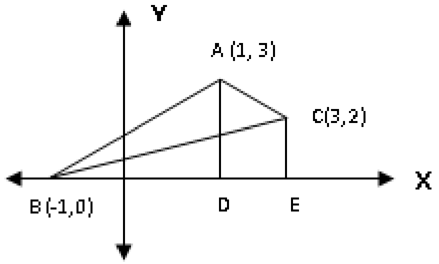
Q.no		Marks
1	Let $A=\{1,\ 2,\ 3\}$. Writing an example of the type $R=\{(1,\ 1),\ (1,\ 2),\ (2,\ 1),\ (2,\ 2)\}$.	1
2	Writing the domain $\|x\|\geq 1$ **OR** $x\geq 1$ *and* $x\leq -1$ **OR** $\{x : x\geq 1\ or\ x\leq -1\}$ **OR** $(-\infty,-1]\cup[1,\ \infty)$.	1
4	Getting: $x=\pm 2\sqrt{2}$	1
5	Getting: $-\sin(x^2).(2x)$ **OR** $-2x\sin\left(x^2\right)$	1
6	Getting: $\frac{2x^{3/2}}{3}-\frac{2x^{5/2}}{5}+c$	1
7	Getting: $m=-6$.	1
8	Getting: Equation of the plane is $y=3$	1
9.	Writing the Definition.	1
10.	Possible values of X are 0, 1, and 2	1
11.	Writing the definition.	1
	Giving the reason, if a and b are any two integers then $ab+1$ is also a unique integer.	1
12.	Writing $\tan^{-1}\sqrt{3}=\frac{\pi}{3}$ **OR** $\sec^{-1}(-2)=\frac{2\pi}{3}$	1
	Getting the answer $-\frac{\pi}{3}$	1
13.	Writing $sin\left\{\frac{\pi}{3}-sin^{-1}\left(-\frac{1}{2}\right)\right\}=sin\left\{\frac{\pi}{3}-\left(-\frac{\pi}{6}\right)\right\}$.	1
	Getting the answer 1.	1
14.	Writing: $\text{Area}=\frac{1}{2}\begin{vmatrix}3&8&1\\-4&2&1\\5&1&1\end{vmatrix}$	1
	Getting: $\text{Area}=\frac{61}{2}$	1
15	Getting: $\lim_{x\to1}f(x)=\lim_{x\to1}(2x+3)=2(1)+3=5$.	1
	Getting : $\lim_{x\to1}f(x)=5=f(1)$	1

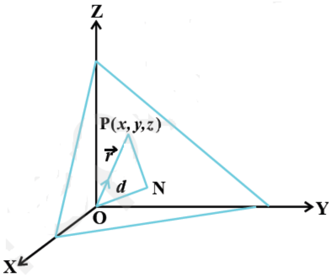
	and concluding f is continuous at $x = 1$.	
16	For writing: $\frac{dy}{dx} = \frac{5}{2}(3x^2 - 7x + 3)^{\frac{5}{2}-1} \times \frac{d}{dx}(3x^2 - 7x + 3)$.	1
	Getting: $\frac{5}{2}(3x^2 - 7x + 3)^{\frac{3}{2}}(6x - 7)$.	1
17.	Writing $dV = \frac{dV}{dr}.\Delta r$ **OR** writing $\frac{dV}{dr} = \frac{4}{3}\pi.3r^2$.	1
	Getting $dV = 4\pi \times 81 \times 0.03 = 9.72\pi\ cm^3$.	1
18.	Writing: $\int(sec^2 x + cosec^2 x)dx$.	1
	Getting: $tan\, x - cot\, x + c$.	1
19	Getting: $\log x \int 1 . dx - \int \frac{d}{dx}.\log x . \int 1\, dx.dx$.	1
	Getting: $x \log x - x + c$.	1
20	Writing: Order = 2.	1
	Writing: Degree = 1.	1
21.	Getting: $\overrightarrow{AB} = -\hat{\imath} - \hat{\jmath} + 2\hat{k}$.	1
	Finding $\lvert\overrightarrow{AB}\rvert = \sqrt{6}$ and writing the direction cosines: $\frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}$.	1
22.	Finding $\lvert\vec{a}\rvert = \sqrt{30}$ **OR** writing $8.\frac{\vec{a}}{\lvert\vec{a}\rvert}$.	1
	Getting: $8\hat{a} = \frac{8(5\vec{\imath} - \vec{\jmath} + 2\vec{k})}{\sqrt{30}}$ or $\frac{40}{\sqrt{30}}\hat{\imath} - \frac{8}{\sqrt{30}}\hat{\jmath} + \frac{16}{\sqrt{30}}\hat{k}$	1
23.	Writing equation of the plane $x + 2y - 2z - 9 = 0$ **OR** writing the formula $\mathrm{d} = \frac{\lvert ax_1 + by_1 + cz_1 + d\rvert}{\sqrt{a^2 + b^2 + c^2}}$ **OR** writing $\left\lvert\frac{1(2) + 2(3) - 2(-5) - 9}{\sqrt{1 + 2^2 + 2^2}}\right\rvert$.	1
	Getting the answer $d = 3$.	1
24.	Writing Sample space $S = \{1,2,3,4,5,6\}$, $E = \{3,6\}$, $F = \{2,4,6\}$ and $E \cap F = \{6\}$ **OR** getting $P(E) = \frac{1}{3}$ **OR** getting $P(F) = \frac{1}{2}$	1
	Getting $P(E \cap F) = \frac{1}{6}$ and $P(E \cap F) = P(E) \cdot P(F)$, and concluding E and F are independent events.	1

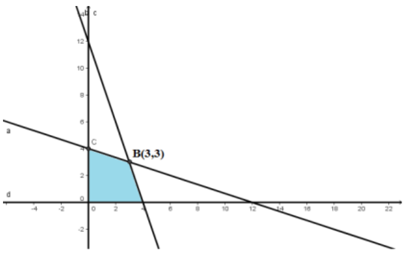
25	$f(x) = \frac{x-2}{x-3}$. Writing $\frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$.	1
	Getting: $x_1 = x_2$.	1
	Proving the function is onto.	1
26	Letting $tan^{-1} x = A,\ tan^{-1} y = B$ and writing $tan\, A = x,\ tan\, B = y$.	1
	Getting $tan\,(A+B) = \frac{x+y}{1-xy}$.	1
	Getting $tan^{-1} x + tan^{-1} y = tan^{-1}\left(\frac{x+y}{1-xy}\right)$ when $xy < 1$	1
27	Writing: $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$	1
	Getting: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 5 \\ 5 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	1+1
28	Taking $x = tan\,\theta$ and getting $y = tan^{-1}\left(\frac{\sqrt{1+tan^2\theta}-1}{tan\,\theta}\right) = tan^{-1}\left(\frac{sec\,\theta - 1}{tan\,\theta}\right)$	1
	Getting $y = tan^{-1}\left(tan\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2}\cdot tan^{-1} x$	1
	Proving $\frac{dy}{dx} = \frac{1}{2}\times\frac{1}{1+x^2} = \frac{1}{2\,(1+x^2)}$.	1
29	Getting $\frac{dx}{dt} = 2a\,t$	1
	Getting $\frac{dy}{dt} = 2a$	1
	Getting $\frac{dy}{dx} = \frac{1}{t}$.	1
30	Getting $f'(x) = 12x^2 - 12x - 72$.	1
	Getting the set of values for strictly increasing, $(-\infty, -2) \cup (3, \infty)$	1
	Getting the set of values for strictly decreasing,$(-2,\ 3)$	1
31	Getting $f(x) = x^4 + \frac{1}{x^3} + c$	1
	Using $f(2) = 0$ and getting $c = -\frac{129}{8}$.	1
	Writing $f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$.	1
32	Writing $I = \int \frac{\frac{1}{x}}{1+\log x} dx$.	1
	Taking $log\, x = t$ and writing $\frac{1}{x}dx = dt$.	1
	$\therefore I = \int \frac{1}{1+t} dt = log(1+t) + c = log(1 + log\, x) + c$	1

33	 Drawing the figure	1
	Writing Area = $\int_4^9 y\,dx$	1
	Getting: Area = $\frac{38}{3}\,sq.units$	1
34	Writing the equation $x^2 + y^2 - 2ax = 0$ **OR** $(x - a)^2 + y^2 = a^2$.	1
	Getting: $2x + 2y\frac{dy}{dx} = 2a$ **OR** $2(x-a) + 2y\frac{dy}{dx} = 0$.	1
	Getting the answer $y^2 - x^2 - 2xy\frac{dy}{dx} = 0$.	1
35	Writing $(\vec{a} + \lambda\vec{b}).\vec{c} = \vec{0}$.	1
	Getting $3(2-\lambda) + 1(2+2\lambda) + 0(3+\lambda) = 0$	1
	Getting $\lambda = 8$.	1
36	Getting $\overrightarrow{AB} = -\hat{\imath} + 3\hat{\jmath} - \hat{k}$ **OR** $\overrightarrow{AC} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$	1
	Getting $\overrightarrow{AB} \times \overrightarrow{AC} = 5\hat{\imath} + 2\hat{\jmath} + \hat{k}$.	1
	Getting $\lvert\overrightarrow{AB} \times \overrightarrow{AC}\rvert = \sqrt{30}$ and area = $\frac{\sqrt{30}}{2}$ sq. units	1
37	Taking $\vec{a}$ and $\vec{b}$ as position vectors of given points and finding $\vec{b} - \vec{a} = 11\hat{k}$. **OR** Writing formula for vector equation of the line $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$.	1
	Getting vector equation of line $\vec{r} = 3\hat{\imath} - 2\hat{\jmath} - 5\hat{k} + \lambda(11\hat{k})$ **OR** $\vec{r} = 3\hat{\imath} - 2\hat{\jmath} - 5\hat{k} + \lambda(-11\hat{k})$ **OR** $\vec{r} = 3\hat{\imath} - 2\hat{\jmath} + 6\hat{k} + m(11\hat{k})$ **OR** $\vec{r} = 3\hat{\imath} - 2\hat{\jmath} + 6\hat{k} + m(-11\hat{k})$.	1
	Writing Cartesian equation of the line $\frac{x-3}{0} = \frac{y+2}{0} = \frac{z+5}{11}$ **OR** $\frac{x-3}{0} = \frac{y+2}{0} = \frac{z+5}{-11}$ **OR** $\frac{x-3}{0} = \frac{y+2}{0} = \frac{z-6}{11}$. **OR** $\frac{x-3}{0} = \frac{y+2}{0} = \frac{z-6}{-11}$.	1
38	Writing , Sample space S = {HH, HT, TH, TT} and events A = { HH}; B = { HH, HT, TH}	1
	Writing $A \cap B = \{HH\}$, $P(A \cap B) = \frac{1}{4}$, $P(B) = \frac{3}{4}$	1

	Getting $P(E)=\frac{1}{3}$, $P(F)=\frac{1}{2}$ and $P(E\cap F)=\frac{1}{6}$ and writing $P(E\cap F)=P(E)\cdot P(F)$ $\therefore$ E and F are independent events.	1
39	Defining $g: Y \to N,\ g(y) = \sqrt{y},\ y \in Y$ **OR** Defining $g: Y \to N,\ g(x) = \sqrt{x},\ x \in Y$ **OR** Writing $x^2 = y \Rightarrow x = \sqrt{y}$.	1
	Getting $gof(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = x$.	1
	Stating $gof = I_N$.	1
	Getting $fog(y) = f(\sqrt{y}) = \sqrt{y}^2 = y,\ y \in Y$ **OR** $fog(x) = f(\sqrt{x}) = \sqrt{x}^2 = x, x \in Y$ and stating $fog = I_Y$.	1
	Writing $f^{-1}(x) = \sqrt{x}$ OR $f^{-1} = \sqrt{x}$.	1
40	Finding: AB	1
	Finding: AC	1
	Finding: B + C	1
	Finding: A(B + C)	1
	Conclusion	1
41	Let $A=\begin{bmatrix}1 & 1 & 1\\ 0 & 1 & 3\\ 1 & -2 & 1\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and $B=\begin{bmatrix}6\\ 11\\ 0\end{bmatrix}$ Getting $\lvert A\rvert=9$.	1
	Getting $\text{adj}A=\begin{bmatrix}7 & -3 & 2\\ 3 & 0 & -3\\ -1 & 3 & 1\end{bmatrix}$ (any 4 cofactors correct award 1 mark)	2
	Writing $X=A^{-1}B=\frac{1}{\lvert A\rvert}(\text{adj}A)B$ **OR** $X = \frac{1}{9}\begin{bmatrix}7 & -3 & 2\\ 3 & 0 & -3\\ -1 & 3 & 1\end{bmatrix}\begin{bmatrix}6\\ 11\\ 0\end{bmatrix}$	1
	Getting $x = 1,\ y = 2,\ z = 3$.	1
42	Getting $y_1 = 3[-\sin(logx)] \times \frac{1}{x} + 4[\cos(\log x)] \times \frac{1}{x}$	1
	Getting $x\,y_1 = -3\ \sin(logx) + 4\cos(\log x)$	1
	Getting $x\,y_2 + y_1.1 = -3[\cos(logx)]\frac{1}{x} + 4[-\sin(\log x)]\frac{1}{x}$	1
	Getting $x^2y_2 + xy_1 = -[3\cos(\log x) + 4\sin(logx)]$	1

	Getting $x^2y_2 + xy_1 + y = 0$	1
43	Writing $\frac{dx}{dt} = -3$ and $\frac{dy}{dt} = 2$.	1
	Writing perimeter $P = 2(x + y)$ and area $a = xy$.	1
	Getting $\frac{dP}{dt} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$ **OR** $\frac{dA}{dt} = x.\frac{dy}{dt} + y.\frac{dx}{dt}$	1
	Getting $\frac{dP}{dt} = -2\ cm/minute$	1
	Getting $\frac{dA}{dt} = 2\ cm^2/minute$	1
44	Taking $x = a\sin\theta$ and writing $dx = a\cos\theta\ d\theta$	1
	Writing $\int \frac{dx}{\sqrt{a^2 - x^2}} = \int \frac{a\cos\theta\ d\theta}{\sqrt{a^2 - a^2\sin^2\theta}}$	1
	Getting the answer $sin^{-1}\left(\frac{x}{a}\right) + c$	1
	Getting $\int \frac{1}{\sqrt{7 - 6x - x^2}}\ dx = \int \frac{1}{\sqrt{16 - (x + 3)^2}}\ dx$	1
	Getting $sin^{-1}\left(\frac{x + 3}{4}\right) + c$	1
45		1
	Getting the equation of the sides AB, AC and BC, y = $\frac{3x+3}{2}$, y = $\frac{-x+7}{2}$, y = $\frac{x+1}{2}$ (any one equation correct award one mark)	2
	Writing area of triangle ABC= $\int_{-1}^{1}\frac{3x + 3}{2}dx + \int_{1}^{3}\frac{-x + 7}{2}dx - \int_{-1}^{3}\frac{x+1}{2}dx$	1
	Area = 4 sq. units.	1
46	Writing: $\frac{dy}{dx} + \frac{y}{x\log x} = \frac{2}{x^2}$	1
	Comparing with standard form and writing P and Q $P = \frac{1}{x\log x}$ and $Q = \frac{2}{x^2}$	1
	Finding I.F : $I.F = e^{\int \frac{1}{x\log x}dx} = e^{\log(\log x)} = \log x.$	1
	Writing solution in the standard form: $y.\log x = \int \frac{2}{x^2}\log x\ dx + C$	1
	Getting: $y.\log x = -\frac{2}{x}(1 + log\lvert x\rvert) + C$	1

47		1
	Getting $\overrightarrow{ON} = d\,\hat{n}$.	1
	Let P(x, y, z) be a point on the plane having p.v. vector. Stating $\overrightarrow{NP} \perp \overrightarrow{ON}$ and getting $(\vec{r} - d\,\hat{n}).\hat{n} = 0$, $\vec{r}.\hat{n} = d$	1
	Let l, m, n be the direction cosines of $\hat{n}$. Writing $\hat{n} = l\hat{i} + m\hat{j} + n\hat{k}$	1
	Getting $lx + my + nz = d$	1
48	Writing $P(X = x) = {}^{n}C_x q^{n-x} p^x$, x = 0, 1, … , n **OR** $n = 8$ $p = 1/2$, $q = 1/2$.	1
	Getting $P(X = x) = {}^{8}C_x \left(\frac{1}{2}\right)^{8-x} . \left(\frac{1}{2}\right)^{x} = {}^{8}C_x \left(\frac{1}{2}\right)^{8}$.	1
	Stating P (at least five heads) $= P(x = 5) + P(x = 6) + P(x = 7) + P(x = 8)$	1
	Getting $= \frac{37}{256}$	1
	Stating $P(\textit{at most five heads}) = P(x \le 5)$ $= P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4) + P(x = 5)$ and getting $\frac{109}{128}$	1
49 (a)	Taking $t = a + b - x$ **OR** $x = a + b - t$ and $dt = -\,dx$;	1
	Proving $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$	1
	Getting $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$	1
	Getting $I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos\left(\frac{\pi}{3} + \frac{\pi}{6} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{3} + \frac{\pi}{6} - x\right)} + \sqrt{\sin\left(\frac{\pi}{3} + \frac{\pi}{6} - x\right)}} dx$	1
	Getting $2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1\, dx = [x]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	1
	Getting $I = \frac{\pi}{12}$	1

49 (b)	Operating $C_1 \to C_1 + C_2 + C_3$ $\text{LHS} = \begin{vmatrix} 2(x+y+z) & x & y \\ 2(x+y+z) & y+z+2x & y \\ 2(x+y+z) & x & z+x+2y \end{vmatrix}$	1
	Taking $2(x+y+z)$ from first column $\text{LHS} = 2(x+y+z)\begin{vmatrix} 1 & x & y \\ 1 & y+z+2x & y \\ 1 & x & z+x+2y \end{vmatrix}$	1
	Operate $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$ $\text{LHS} = 2(x+y+z)\begin{vmatrix} 1 & x & y \\ 0 & x+y+z & 0 \\ 0 & 0 & x+y+z \end{vmatrix}$	1
	Getting: LHS = RHS	1
50 (a)	Formulating and writing the constraints $x + 3y \leq 12$; $3x + y \leq 1$; $x \geq 0, y \geq 0$	1 + 1
		1
	Getting corner points	1
	Writing Maximize $Z = 17.5x + 7y$ and Evaluating objective function Z at each Corner points.	1
	Writing maximum value $Z = 73.5$ at B(3,3)	1
50 (b)	Stating $f\left(\frac{\pi}{2}\right) = \lim_{x \to \frac{\pi}{2}} \left(\frac{k\cos x}{\pi - 2x}\right)$	1
	Taking $x - \frac{\pi}{2} = h$ and stating $h \to 0$	1
	Getting $\lim_{h \to 0} \frac{k(-\sin h)}{-2h}$	1
	Obtaining $k = 6$	1

Model Question Paper – 2

II P.U.C MATHEMATICS (35)

Time : 3 hours 15 minute **Max. Marks : 100**

Instructions :

(i) The question paper has five parts namely A, B, C, D and E. Answer all the parts.

(ii) Use the graph sheet for the question on Linear programming in PART E.

PART – A

Answer ALL the questions **$10 \times 1 = 10$**

1. Define bijective function.
2. Find the principal value of $cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$.
3. Construct a 2×3 matrix whose elements are given by $a_{ij} = |i - j|$.
4. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, find $|2A|$.
5. If $y = tan\,(2x+3)$ find $\frac{dy}{dx}$.
6. Write the integral of $\frac{1}{x\sqrt{x^2-1}}$, $x > 1$ with respect to x.
7. Write the vector joining the points $A(2,\ 3,\ 0)$ and $B(-1, -2, -4)$.
8. Find the equation of the plane which makes intercepts $1,\ -1$ *and* 2 on the x, y and z axes respectively.
9. Define feasible region.
10. If P(B) = 0.5 and P $(A \cap B)$=0.32, find P(A/B).

PART B

Answer any TEN questions **$10 \times 2 = 20$**

11. A relation R is defined on the set $A = \{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$ by $R = \{(x,\ y): y\ \ is\ \ divisible\ \ by\ \ x\}$. Verify whether R is symmetric and reflexive or not. Give reason.
12. Write the simplest form of $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$, $0 < x < \frac{\pi}{2}$.
13. If $\sin\left\{\sin^{-1}\frac{1}{5} + \cos^{-1}x\right\} = 1$, find x.
14. If each element of a row is expressed as sum of two elements then verify for a third order determinant that the determinant can be expressed as sum of two determinants.

15. If $\sqrt{x}+\sqrt{y}=\sqrt{a}$, prove that $\frac{dy}{dx}=-\sqrt{\frac{y}{x}}$.

16. If $y=(\sin^{-1}x)^x$ find $\frac{dy}{dx}$.

17. Find the local maximum value of the function $g(x)=\ x^3-3x$.

18. Evaluate $\int log(\sin x).(\cot x)dx$.

19. Find $\int_0^{\pi/2}\cos 2x\ dx$.

20. Form the differential equation of the family of curves $\frac{x}{a}+\frac{y}{b}=1$ by eliminating the constants a and b.

21. If $\vec{a}$ is a unit vector such that $(\vec{x}-\vec{a}).(\vec{x}+\ \vec{a})=8$ find $|\vec{x}|$.

22. Show that the vector $\hat{\imath}+\ \hat{\jmath}+\ \hat{k}$ is equally inclined to the positive direction of the axes.

23. Find the angle between the pair of lines $\vec{r}=3\hat{i}+5\hat{j}-\hat{k}+\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$ and $\vec{r}=7\hat{i}+4\hat{k}+\mu\left(2\hat{i}+2\hat{j}+2\hat{k}\right)$.

24. Probability distribution of x is

x	0	1	2	3	4
$P(x_1)$	0.1	k	2k	2k	k

Find k.

PART C

Answer any TEN questions **10 $\times$ 3=30**

25. If $*$ is a binary operation defined on $A=N\times N$, by $(a,b)*(c,d)=(a+c,b+d)$, prove that $*$ is both commutative and associative. Find the identity if it exists.

26. Prove that $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$.

27. By using elementary transformations, find the inverse of the matrix $A=\begin{bmatrix}1 & -2\\ 2 & 1\end{bmatrix}$.

28. If $x=a(\theta+\sin\theta)$ and $y=a(1-\cos\theta)$ prove that $\frac{dy}{dx}=\tan\left(\frac{\theta}{2}\right)$.

29. If a function $f(x)$ is differentiable at $x=c$ prove that it is continuous at $x=c$.

30. Prove that the curves $x=y^2$ and $xy=k$ cut at right angles if $8k^2=1$.

31. Evaluate: $\int\sin(ax+b)\cos(ax+b)dx$.

32. Evaluate: $\int\tan^{-1}x\,dx$.

33. Find the area of the region bounded by the curve $y = x^2$ and the line $y = 2$.

34. Find the equation of the curve passing through the point $(-2, 3)$, given that the slope of the tangent to the curve at any point (x, y) is $\frac{2x}{y^2}$.

35. For any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ prove that $\left[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}\right] = 2\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$.

36. Find a unit vector perpendicular to the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ where $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.

37. Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i}+2\hat{j}-2\hat{k}$, both in vector form and Cartesian form.

38. Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed. A reports that a head appears. Find the probability that it is actually head.

PART D

Answer any SIX questions **$6 \times 5 = 30$**

39. Let $f : N \to R$ be defined by $f(x) = 4x^2 + 12x + 15$. Show that $f : N \to S$, where S is the range of the function, is invertible. Also find the inverse of f.

40. If $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$. Calculate AC, BC and $(A+B)C$. Also, verify that $(A+B)C = AC + BC$.

41. Solve the following system of equations by matrix method,

$3x - 2y + 3z = 8$; $2x + y - z = 1$ and $4x - 3y + 2z = 4$.

42. If $y = Ae^{mx} + Be^{nx}$, prove that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$.

43. The volume of a cube is increasing at a rate of 9 cubic centimeters per second. How fast is the surface area increasing when the length of an edge is 10 centimeter?

44. Find the integral of $\sqrt{x^2+a^2}$ with respect to x and evaluate $\int\sqrt{4x^2+9}\,dx$.

45. Solve the differential equation $ydx-\left(x+2y^2\right)dy=0$.

46. Find the area of the circle $4x^2+4y^2=9$ which is interior to the parabola $x^2=4y$.

47. Derive the condition for the coplanarity of two lines in space both in the vector form and Cartesian form.

48. Find the probability of getting at most two sixes in six throws of a single die.

PART E

Answer any ONE question **$1\times$10=10**

49. (a) Minimize and Maximize $z=3x+9y$ subject to the constraints

$x+3y\le 60$
$x+y\ \ge 10$
$x\le y$
$x\ge 0, y\ge 0$, by the graphical method.

(b) Prove that $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix}=(x-y)(y-z)(z-x)(xy+yz+zx)$

50.(a) Prove that $\int_{-a}^{a} f(x)dx=\begin{cases} 2\int_0^a f(x)dx, & \text{if f(x) is even} \\ 0, & \text{if f(x) is odd} \end{cases}$

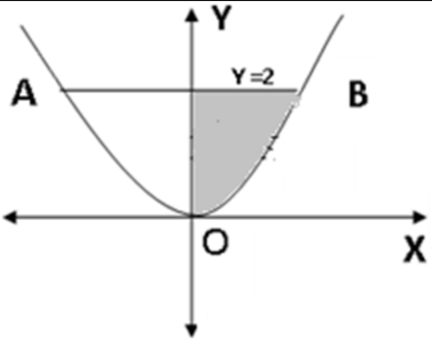
and evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^7 x\,dx$

(b) Define a continuity of a function at a point. Find all the points of discontinuity of f defined by $f(x)=|x|-|x+1|$.

SCHEME OF VALUATION
Model Question Paper – 2
II P.U.C MATHEMATICS (35)

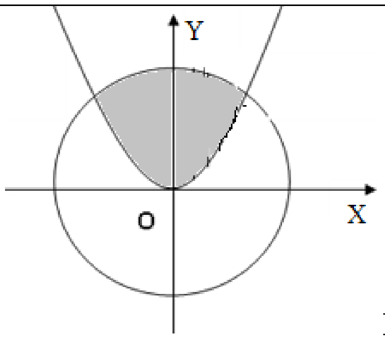
Q.no		Marks
1	Writing the definition.	1
2	Getting $cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$.	1
3	Getting: $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$	1
4	Getting $\|2A\| = -24$	1
5	Getting $sec^2(2x+3).2$ **OR** $2.sec^2(2x+3)$	1
6	Writing $-\text{cosec}^{-1}x + c$ **OR** $\sec^{-1}x + c$	1
7	Getting $\overrightarrow{AB} = -3\hat{i} - 5\hat{j} - 4\hat{k}$	1
8	Writing $\frac{x}{1} + \frac{y}{-1} + \frac{z}{2} = 1$	1
9	Writing the definition	1
10	Getting: $P(A/B) = \frac{P(A \cap B)}{P(B)} = 0.64$	1
11	Stating the reason if y *is divisible by* x then it is not necessary that x *is divisible by* y.	1
	Stating the reason$x$ *is divisible by* x, $\forall\ x \in A$.	1
12	Dividing numerator and denominator by $cosx$ and getting $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right) = \tan^{-1}\left(\frac{1 - \tan x}{1 + \tan x}\right)$.	1
	Getting the answer $\frac{\pi}{4} - x$.	1
13	Writing $\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right\} = \sin^{-1}\left\{\frac{\pi}{3} - \left(-\frac{\pi}{6}\right)\right\}$.	1
	Getting the answer 1.	1
14	Writing $\Delta = \begin{vmatrix} a_1 + x & a_2 + y & a_3 + z \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ and expanding by definition	1
	Getting $\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$	1
15	Getting $\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}.\frac{dy}{dx} = 0$	1

	Getting $\frac{dy}{dx} = -\sqrt{\frac{y}{x}}$.	1
16	Let $v = (sin^{-1}x)^x$. Getting $logv = x.log\,(sin^{-1}x)$	1
	Getting $\frac{dv}{dx} = v\left[x.\frac{1}{sin^{-1}x}.\frac{1}{\sqrt{1-x^2}} + log\,(sin^{-1}x)\right]$	1
17	Getting $g'(x) = 3x^2 - 3$.	1
	Getting the local maximum value $= 2$.	1
18	Substituting $\log\sin x = t$ and writing $\cot x\,.\,dx = dt$	1
	Getting $\frac{(\log\sin x)^2}{2} + c$	1
19	Getting $\left[\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}$	1
	Getting $\frac{1}{2}[\sin\pi - 0] = 0$	1
20	Getting $y = -\frac{b}{a}x + b$ **OR** finding $\frac{dy}{dx}$.	1
	Getting $\frac{d^2y}{dx^2} = 0$	1
21	Writing $\vec{x}.\vec{x} - \vec{a}.\vec{x} + \vec{x}.\vec{a} - \vec{a}.\vec{a} = 8$ **OR** $\vec{x}^2 - \vec{a}^2 = 8$ **OR** $\lvert\vec{x}\rvert^2 - \lvert\vec{a}\rvert^2 = 8$	1
	Getting $\lvert\vec{x}\rvert = 3$	1
22	Getting magnitude $= \sqrt{3}$	1
	Concluding that the direction cosines are equal	1
23	Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + 2\hat{j} + 2\hat{k}$ Getting $\vec{a}\cdot\vec{b} = 6$ **OR** $\lvert\vec{a}\rvert = \sqrt{3}$ **OR** $\lvert\vec{b}\rvert = 2\sqrt{3}$	1
	Getting angle between the vectors $= 0$	1
24	Writing :$\sum P(x_i) = 1$	1
	Getting: $K = 0.15$.	1
25	Proving commutative.	1
	Proving associative.	1
	Proving identity does not exist.	1
26	Writing $2tan^{-1}x = tan^{-1}\frac{2x}{1 - x^2}$ **OR** $2tan^{-1}\frac{1}{2} = tan^{-1}\frac{2\left(\frac{1}{2}\right)}{1 - \left(\frac{1}{2}\right)^2}$.	1
	Getting $2tan^{-1}\frac{1}{2} = tan^{-1}\frac{4}{3}$.	1
	Proving $tan^{-1}\frac{4}{3} + tan^{-1}\frac{1}{7} = tan^{-1}\frac{31}{17}$.	1
27	Writing $A = IA$	1

	Getting any one non diagonal element is zero	1
	Getting the inverse $\frac{1}{5}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$	1
28	Getting $\frac{dx}{d\theta} = a[1+\cos\theta]$ **OR** $\frac{dy}{d\theta} = a\sin\theta$	1
	Getting $\frac{dy}{dx} = \frac{a\sin\theta}{a[1+\cos\theta]}$	1
	Getting $\frac{dy}{dx} = \tan\left(\frac{\theta}{2}\right)$.	1
29	Stating $\lim_{x\to c}\frac{f(x)-f(c)}{x-c} = f'(c)$	1
	Writing $f(x)-f(c) = \frac{f(x)-f(c)}{x-c}.(x-c)$	1
	Getting $\lim_{x\to c} f(x) = f(c)$	1
30	Finding the point of intersection $\left(k^{2/3}, k^{1/3}\right)$	1
	Finding the slope of the tangent to the first curve at point of intersection $m_1 = \frac{1}{2k^{1/3}}$ **OR** similarly to find $m_2 = -\frac{1}{k^{1/3}}$ **OR** writing the orthogonality condition $m_1 m_2 = -1$	1
	To showing the required condition.	1
31	Writing $\frac{1}{2}\int 2.\sin(ax+b).\cos(ax+b)dx$	1
	Writing $\frac{1}{2}\int \sin 2(ax+b)dx$	1
	Getting $\frac{1}{2}\left[-\frac{\cos 2(ax+b)}{2a}\right] + c$	1
32	Writing $\int \tan^{-1} x\, dx$ $= \tan^{-1}x.\int 1\,dx - \int \frac{d}{dx}\tan^{-1}x.\int 1\,dx.dx.$	1
	Getting $x\tan^{-1}x - \int \frac{x}{1+x^2}dx$	1
	Getting $x\tan^{-1}x - \frac{1}{2}\log\lvert 1+x^2\rvert + c$	1
33	 Drawing the figure and explaining it	1
	Stating required area $= 2\int_0^2 \sqrt{y}.dy$	1
	Getting area $= \frac{8\sqrt{2}}{3}$ sq.units	1

34	Writing: $\frac{dy}{dx}=\frac{2x}{y^2}$	1
	Stating $\int y^2 dy = \int 2x\, dx$	1
	Getting $\frac{y^3}{3}=x^2+c$ and getting $c = 5$	1
35	Writing $[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}]$ $=(\vec{a}+\vec{b}).\{(\vec{b}+\vec{c})\times(\vec{c}+\vec{a})\}$	1
	For expanding : $(\vec{a}+\vec{b}).\{\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{c}+\vec{c}\times\vec{a}\}$	1
	Getting $2[\vec{a}\ \ \vec{b}\ \ \vec{c}]$	1
36	Getting $\vec{a}+\vec{b}=2\hat{\imath}+3\hat{\jmath}+4\hat{k}$ **OR** $\vec{a}-\vec{b}=-\hat{\jmath}-2\hat{k}$ **OR** writing the formula $\frac{(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})}{\lvert(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\rvert}$	1
	Getting $(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=-2\hat{\imath}+4\hat{\jmath}-2\hat{k}$ **OR** $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})=2\hat{\imath}-4\hat{\jmath}+2\hat{k}$	1
	Getting the answer $\frac{-2\hat{\imath}+4\hat{\jmath}-2\hat{k}}{\sqrt{24}}$ **OR** $\frac{2\hat{\imath}-4\hat{\jmath}+2\hat{k}}{\sqrt{24}}$	1
37	Writing: $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}+2\hat{j}-2\hat{k}$ **OR** writing the formula $\vec{r}=\vec{a}+\lambda\vec{b}$	1
	Writing: the equation of the line is $=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(3\hat{i}+2\hat{j}-2\hat{k})$	1
	Getting the equation $\frac{x-1}{3}=\frac{y-2}{2}=\frac{z-3}{-2}$	1
38	Writing : $P(E_1)=\frac{1}{2}$ and $P(E_2)=\frac{1}{2}$	1
	Writing: $P(A\mid E_1)=\frac{4}{5}$ and $P(A\mid E_2)=\frac{1}{5}$	1
	Getting: $P(E_1\mid A)=\frac{P(E_1)P(A\mid E_1)}{P(E_1)P(A\mid E_1)+P(E_2)P(A\mid E_2)}=\frac{4}{5}$	1
39	Getting $y=4x^2+12x+15 \Rightarrow x=\frac{\sqrt{y-6}-3}{2}$.	1
	Stating $g(y)=\frac{\sqrt{y-6}-3}{2},\ y\in S$ **OR** $g(x)=\frac{\sqrt{x-6}-3}{2},\ x\in S.$	1
	Proving $gof(x)=g(4x^2+12x+15)=x$ and writing $gof=I_N$.	1
	Proving $fog(y)=f\left(\frac{\sqrt{y-6}-3}{2}\right)=y,\ y\in S$ **OR** $fog(x)=f\left(\frac{\sqrt{x-6}-3}{2}\right)=x,\ x\in S$	1

	and writing $fog = I_S$.	
	Writing $f^{-1}(x) = \frac{\sqrt{x-6}-3}{2}$ **OR** $f^{-1} = \frac{\sqrt{x-6}-3}{2}$.	1
40	Finding : A+B	1
	Finding : (A+B)C	1
	Finding : AC	1
	Finding : BC	1
	Verifying $(A+B)C = AC+BC$	1
41	Let $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$ Getting $\|A\| = -17$.	1
	Getting $\mathrm{adj}A = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$ (any 4 cofactors correct award 1 mark)	2
	Writing $X = A^{-1}B = \frac{1}{\|A\|}(\mathrm{adj}A)B$ **OR** $X = \frac{1}{-17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}\begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$	1
	Getting x =1, y = 2 and z = 3	1
42	Getting $\frac{dy}{dx} = Am.e^{mx} + Bn.e^{nx}$	1
	Getting $\frac{d^2y}{dx^2} = Am.me^{mx} + Bn.ne^{nx}$	1
	Writing LHS $= (Am^2e^{mx} + Bn^2e^{nx})$ $-(m+n)(Am.e^{mx} + Bn.e^{nx}) + mny$	1
	Evaluating the brackets	1
	Getting the answer zero	1
43	Writing $\frac{dV}{dt} = 9$ **OR** $V = x^3$	1
	Getting $\frac{dV}{dt} = 3x^2.\frac{dx}{dt}$	1
	Getting $\frac{dx}{dt} = \frac{1}{300}cm/sec$	1
	Writing $S = 6x^2$ and $\frac{dS}{dt} = 12x.\frac{dx}{dt}$	1
	Getting $\frac{dS}{dt} = \frac{2}{5}cm^2/sec$	1
44	Writing $I = \int\sqrt{x^2+a^2}\,dx$ $= \sqrt{x^2+a^2}\int 1\,dx - \int \frac{d}{dx}\sqrt{x^2+a^2}.\int 1\,dx.dx$	1
	Getting $x\sqrt{x^2+a^2} - \frac{1}{2}\int\frac{2x^2}{\sqrt{x^2+a^2}}dx$	1
	Getting $x\sqrt{x^2+a^2} - \int\frac{x^2+a^2-a^2}{\sqrt{x^2+a^2}}dx$	1

	Getting $I = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left\|x+\sqrt{x^2+a^2}\right\| + c$	1
	Getting $\int \sqrt{4x^2 + 9}dx$ $= \frac{x}{2}\sqrt{4x^2+a^2} + \frac{a^2}{4}\log\left\|2x+\sqrt{4x^2+a^2}\right\| + c$	1
45	Writing $y\frac{dx}{dy} = x + 2y^2$	1
	Getting $\frac{dx}{dy} - \frac{x}{y} = 2y$ and writing $P = -\frac{1}{y},\ Q = 2y$	1
	Getting I.F $= e^{\int -\frac{1}{y}dy} = \frac{1}{y}$	1
	Writing the solution $x\frac{1}{y} = \int (2y)\left(\frac{1}{y}\right)dy + C$	1
	Getting the answer $x = 2y^2 + Cy$	1
46	 Drawing figure	1
	Finding the points of intersection $x = \pm\sqrt{2}$	1
	Writing area of the region $= 2\left\{\int_0^{\sqrt{2}} \frac{1}{2}\sqrt{9-4x^2}\,dx - \int_0^{\sqrt{2}} \frac{x^2}{4}\,dx\right\}$ **OR** area of the region $= \int_{-\sqrt{2}}^{\sqrt{2}} \left(\frac{1}{2}\sqrt{9-4x^2}\right)dx - \int_{-\sqrt{2}}^{\sqrt{2}} \left(\frac{x^2}{4}\right)dx$	1
	Getting the answer $= \left\{\frac{\sqrt{2}}{2} + \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)\right\} - \frac{\sqrt{2}}{3}$	1 + 1
47	Writing the equations $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$	1
	Stating $\overrightarrow{AB} = \vec{a_1} - \vec{a_2}$ is perpendicular to $\vec{b_1} \times \vec{b_2}$	1
	Getting $(\vec{a_1} - \vec{a_2}).(\vec{b_1} \times \vec{b_2}) = 0$	1
	Writing $\overrightarrow{AB} = (x_1 - x_2)\hat{\imath} + (y_1 - y_2)\hat{\jmath} + (z_1 - z_2)\hat{k}$	1
	Getting $\begin{vmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = 0$	1
48	Writing $p = \frac{1}{6},\ q = \frac{5}{6}$ and $n = 6$	1
	Writing P(at most 2 successes) $= P(x=0) + P(x=1) + P(x=2)$	1
	Getting $P(X=0) = \left(\frac{5}{6}\right)^6,\ P(X=1) = \left(\frac{5}{6}\right)^5,$ $P(X=2) = 15\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^4$	2

	Getting the answer $\frac{35}{18}\left(\frac{5}{6}\right)^4$	1
49 (a)	Drawing graph of the system of linear inequalities	2
	Showing feasible region ABCD and getting corner P	1
	Getting corresponding value of Z at each corner point	1
	Obtaining minimum value Z=60 at $x = 5, y = 5$	1
	Obtaining maximum value Z=180, at $x = 15,\ y = 15$ and $x = 0, y = 20$	1
49(b)	Getting $\begin{vmatrix} x & x^2 & yz \\ y-x & y^2-x^2 & zx-yz \\ z-x & z^2-x^2 & xy-yz \end{vmatrix}$ (any one row correct award the mark)	1
	Getting $(y-x)(z-x)\begin{vmatrix} x & x^2 & yz \\ 1 & y+x & -z \\ 1 & z+x & -y \end{vmatrix}$	1
	Getting $(y-x)(z-x)\begin{vmatrix} x & x^2 & yz \\ 1 & y+x & -z \\ 0 & z-y & -y+z \end{vmatrix}$	1
	Getting $(x-y)(y-z)(z-x)(xy+yz+zx)$	1
50 (a)	Writing $\int_{-a}^{a} f(x)dx = \int_{-a}^{0} f(x)dx + \int_{0}^{a} f(x)dx$	1
	Taking $t = -x$ and $dt = -dx$	1
	Getting $\int_{-a}^{a} f(x)dx = \int_{0}^{a} f(-x)dx + \int_{0}^{a} f(x)dx$	1
	Getting $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$ when $f(x)$ is even	1
	Getting $\int_{-a}^{a} f(x)dx = 0$ when $f(x)$ is odd	1
	Writing $\int_{-\pi/2}^{\pi/2} sin^7 x\, dx = 0$ with reason.	1
50 (b)	Definition	1
	Let $g(x) = \lvert x \rvert$ and $h(x) = \lvert x+1 \rvert$. As modulus functions are continuous, therefore g and h are continuous.	1
	As difference of two continuous functions is again continuous function, therefore f is continuous.	1
	There is no point of discontinuity.	1

Model Question Paper – 3

II P.U.C MATHEMATICS (35)

Time : 3 hours 15 minute **Max. Marks : 100**

Instructions :

(i) The question paper has five parts namely A, B, C, D and E. Answer all the parts.

(ii) ***Use the graph sheet for the question on Linear programming in PART E.***

PART – A

Answer ALL the questions **$10 \times 1 = 10$**

1. Let $*$ be a binary operation defined on set of rational numbers, by $a * b = \frac{ab}{4}$. Find the identity element.
2. Write the set of values of x for which $2\tan^{-1} x = \tan^{-1}\frac{2x}{1-x^2}$ holds.
3. What is the number of the possible square matrices of order 3 with each entry 0 or 1?
4. If A is a square matrix with $|A| = 6$, find the values of $|AA'|$.
5. The function $f(x) = \frac{1}{x-5}$ is not continuous at $x = 5$. Justify the statement.
6. Write the antiderivative of e^{2x} with respect to x.
7. Define collinear vectors.
8. Find the distance of the plane $2x - 3y + 4z - 6 = 0$ from the origin.
9. Define Optimal Solution.
10. A fair die is rolled. Consider events $E = \{2,4,6\}$ and $F = \{1,2\}$. Find $P(E|F)$.

PART B

Answer any TEN questions **$10 \times 2 = 20$**

11. Prove that the greatest integer function, $f : R \to R$, defined by $f(x) = [x]$, where $[x]$ indicates the greatest integer not greater than x, is neither one-one nor onto.
12. Prove that $2\sin^{-1} x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$, $-\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$.
13. Find $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$.
14. Find the equation of the line passing through (1, 2) and (3, 6) using the determinants.

15. If $y = \sin(\log_e x)$, prove that $\dfrac{dy}{dx} = \dfrac{\sqrt{1-y^2}}{x}$.

16. Find the derivative of $x^x - 2^{\sin x}$ with respect to x.

17. Find a point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$.

18. Find $\int e^x \sec x(1 + \tan x)dx$.

19. Evaluate $\int \log x \, dx$.

20. Prove that the differential equation $x^2 \dfrac{dy}{dx} = x^2 - 2y^2 + xy$ is a homogeneous differential equation of degree 0.

21. Find k if the vectors $\hat{\imath} + 3\hat{\jmath} + \hat{k}$, $2\hat{\imath} - \hat{\jmath} - \hat{k}$ and $k\hat{\imath} + 7\hat{\jmath} + 3\hat{k}$ are coplanar.

22. Find the area of the parallelogram whose adjacent sides are the vectors $3\hat{\imath} + \hat{\jmath} + 4\hat{k}$ and $\hat{\imath} - \hat{\jmath} + \hat{k}$.

23. Find equation of the plane passing through the line of intersection of the planes $x + y + z = 6$ and $2x + 3y + 4z - 5 = 0$ and the point, $(1, 1, 1)$.

24. Two cards drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

PART C

Answer any TEN questions **$10 \times 3 = 30$**

25. Show that the relation R in the set of all integers, Z defined by $R = \{(a, b) : 2 \ divides \ a - b\}$ is an equivalence relation.

26. If $\tan^{-1}\dfrac{x-1}{x-2} + \tan^{-1}\dfrac{x+1}{x+2} = \dfrac{\pi}{4}$, find x.

27. If A and B are square matrices of the same order, then show that $(AB)^{-1} = B^{-1}A^{-1}$.

28. Verify the mean value theorem for $f(x) = x^2 - 4x - 3$ in the interval $[a, b]$, where $a = 1$ and $b = 4$.

29. If $y = \tan^{-1}\left(\dfrac{3x - x^3}{1-3x^2}\right)$, $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$ find $\dfrac{dy}{dx}$.

30. A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is maximum?

31. Evaluate $\int_0^2 e^x \, dx$ as the limit of the sum.

32. Find $\int \frac{1}{1+\tan x} dx$.

33. Find the area bounded by the parabola $y^2 = 5x$ and the line $y = x$.

34. In a bank, principal p increases continuously at the rate of 5% per year. Find the principal in terms of time t.

35. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

36. Show that the position vector of the point P which divides the line joining the points A and B having position vectors $\vec{a}$ and $\vec{b}$ internally in the ratio $m : n$ is $\frac{m\vec{b} + n\vec{a}}{m+n}$.

37. Find the distance between the parallel lines
$\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + m(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ and
$\vec{r} = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k} + n(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$.

38. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

PART D

Answer any SIX questions **$6 \times 5 = 30$**

39. Verify whether the function, $f : N \to Y$ defined by $f(x) = 4x + 3$, where $Y = \{y : y = 4x + 3, \; x \in N\}$ is invertible or not. Write the inverse of $f(x)$ if exists.

40. If $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & -6 \end{bmatrix}$, verify that $(AB)' = B'A'$.

41. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations $2x - 3y + 5z = 11$; $3x + 2y - 4z = -5$ and $x + y - 2z = -3$.

42. If $y = (\tan^{-1} x)^2$ then show that $(x^2+1)^2 \frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx} = 2$.

43. A particle moves along the curve, $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

44. Find the integral of $\frac{1}{\sqrt{x^2+a^2}}$ with respect to x and hence evaluate $\int \frac{1}{\sqrt{x^2+2x+2}}dx$

45. Find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $(a > b)$ by the method of integration and hence find the area of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$.

46. Find the particular solution of the differential equation $\frac{dy}{dx}+y\cot x=4x\cdot \operatorname{cosec} x$, $x \neq 0$, given that y = 0 when $x = \frac{\pi}{2}$.

47. Derive the equation of the line in space passing through a point and parallel to a vector both in the vector and Cartesian form.

48. A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize at least once and exactly once.

PART E

Answer any ONE question **$1\times$10=10**

49. (a) Prove that $\int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx$ when $f(2a-x) = f(x)$ and hence evaluate $\int_0^{\pi}|\cos x|\,dx$.

(b) Find the values of a and b such that the function defined by

$$f(x)=\begin{cases}5, & \text{if } x\le 2\\ ax+b, & \text{if } 2<x<10\\ 21, & \text{if } x\ge 10\end{cases}$$, is continuous function

50. (a) A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F1 and F2 are available. Food F1 costs Rs 4 per unit food and F2 costs Rs 6 per unit. One unit of food F1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

(b) Prove that $\begin{vmatrix}1 & x & x^2\\ x^2 & 1 & x\\ x & x^2 & 1\end{vmatrix}=(1-x^3)^2$.

SCHEME OF VALUATION

Model Question Paper – 3

II P.U.C MATHEMATICS (35)

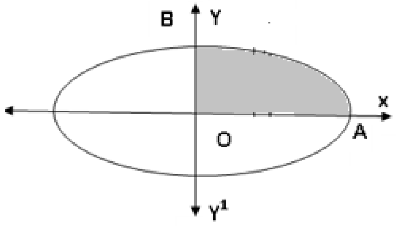
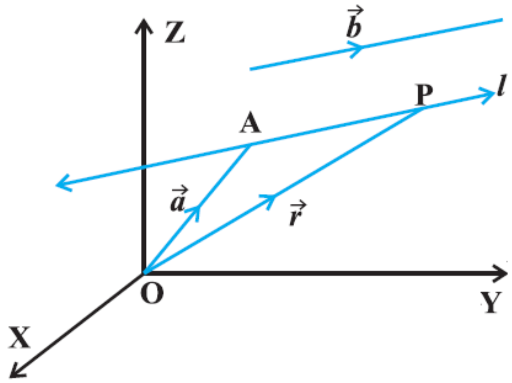
Q.no		Marks
1	Proving identity $e = 4$.	1
2	Writing $-1 < x < 1$ **OR** $\lvert x \rvert < 1$.	1
3	Getting 2^9.	1
4	Getting answer= 36.	1
5	Giving reason: function is not defined at $x = 5$	1
6	Getting $\frac{e^{2x}}{2} + c$.	1
7	Writing the definition.	1
8	Getting: the distance of the plane from the origin = $\frac{6}{\sqrt{29}}$.	1
9	Writing the definition.	1
10	Getting: $P(E\mid F) = \frac{P(E \cap F)}{P(F)} = \frac{1}{2}$	1
11	Giving counter example of the type $f(2.6) = f(2.7) = 2$, but $2.6 \neq 2.7$.	1
	Giving the reason, non integral cannot be an image.	1
12	Letting $x = sin\theta,\ -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$ **OR** using $sin\,2\theta = 2.sin\theta.cos\theta$.	1
	Obtaining LHS = RHS.	1
13	Getting $cos\,\frac{7\pi}{6} = -cos\,\frac{\pi}{6}$ **OR** $cos\,\frac{7\pi}{6} = cos\left(\pi + \frac{\pi}{6}\right) = cos\left(\pi - \frac{\pi}{6}\right)$.	1
	Getting $cos^{-1}\left(cos\frac{7\pi}{6}\right) = \frac{5\pi}{6}$.	1
14	Writing $\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0$.	1
	Getting $4x - 2y = 0$.	1
15	Writing $sin^{-1}y = \log x$.	1
	Getting $\frac{dy}{dx} = \frac{\sqrt{1 - y^2}}{x}$.	1
16	Writing $\frac{d}{dx}x^x = x^x(1 + logx)$ **OR** $\frac{d}{dx}2^{sinx} = 2^{sinx}.(log\,2).cosx$.	1

	Getting the answer $x^x(1+logx) - 2^{sinx}.cosx$.	1
17	Getting $\frac{dy}{dx} = 3x^2 - 11$ **OR** writing slope= 1.	1
	Getting the point $(2, -9)$.	1
18	Writing $\int e^x (\sec x + \sec x \cdot \tan x) dx$ and $\frac{d}{dx}\sec x = \sec x . \tan x$.	1
	Getting $e^x \sec x + c$.	1
19	Writing $\log x \cdot \int 1\, dx - \int 1. \frac{d}{dx} \log x\, dx$.	1
	Getting $= x \log x - x + c$.	1
20	Writing $\frac{dy}{dx} = \frac{x^2 - 2y^2 + xy}{x^2}$ **OR** $F(x, y) = \frac{x^2 - 2y^2 + xy}{x^2}$	1
	Using $F(\lambda x, \lambda y) = \frac{(\lambda x)^2 - 2(\lambda y)^2 + \lambda x \lambda y}{(\lambda x)^2}$ and getting $F(\lambda x, \lambda y) = \lambda^0 F(x, y)$.	1
21	Writing $\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ k & 7 & 3 \end{vmatrix} = 0$ **OR** $\begin{vmatrix} k & 7 & 3 \\ 2 & -1 & -1 \\ 1 & 3 & 1 \end{vmatrix} = 0$.	1
	Getting $k = 0$.	1
22	Taking $\vec{a} = 3\hat{\imath} + \hat{\jmath} + 4\hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$ and writing $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & j & k \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}$ **OR** writing the formula: area of the parallelogram $= \lvert \vec{a} \times \vec{b} \rvert$.	1
	Getting the answer $= \sqrt{42}$ sq. units.	1
23	Writing $(x + y + z - 6) + m(2x + 3y + 4z - 5) = 0$,	1
	Getting $m = 3/4$ and getting the equation of the plane as $10x + 13y + 16z - 39 = 0$.	1
24	Writing: $n(s) = 52$, $n(A) = \frac{26}{52}$ and $n(B) = \frac{25}{51}$	1
	Getting : $P(A \cap B) = P(A) \cdot P(B) = \frac{25}{102}$	1
25	Proving reflexive.	1
	Proving symmetric.	1
	Proving transitive.	1
26	Writing : $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \tan^{-1}\left(\frac{\frac{x-1}{x-2} + \frac{x+1}{x+2}}{1 - \left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right)$ **OR** Writing $tan^{-1}\frac{x-1}{x-2} = tan^{-1} 1 - tan^{-1}\frac{x+1}{x+2}$.	1

	Getting $\tan^{-1}\left(\frac{2x^2-4}{-3}\right)=\frac{\pi}{4}$ **OR** $tan^{-1}\frac{x-1}{x-2} = tan^{-1}\frac{1}{2x+3}$	1
	Getting $x = \pm\frac{1}{\sqrt{2}}$.	1
27	Stating: $(AB)(AB)^{-1} = I$	1
	Pre multiplying by A^{-1} and getting $B(AB)^{-1} = A^{-1}$	1
	Getting $(AB)^{-1} = B^{-1}A^{-1}$.	1
28	Stating $f(x)$ is continuous in [1, 4] **OR** stating differentiable in (1, 4) **OR** Getting $f(1) = -6$ **OR** $f(4) = -3$.	1
	Getting $\frac{f(b)-f(a)}{b-a} = 1$	1
	Getting $c = \frac{5}{2}$	1
29	Taking $x = tant$	1
	Getting $y = tan^{-1}(tan\,3t)$	1
	Getting $\frac{dy}{dx} = \frac{3}{1+x^2}$	1
30	Let x be the height of the box and V the volume of the box. Writing $V = x(18-2x)^2$.	1
	Getting $\frac{dV}{dt}=(18-2x)^2-2x(18-x)$	1
	Getting $x = 4.5$	1
31	Writing $h = \frac{2-0}{n} = \frac{2}{n}$ **OR** writing the formula $\int_a^b f(x)\,dx$ $= (b-a).\lim\limits_{n\to\infty}\frac{1}{n}\{f(a)+ f(a+h)+ \ldots + f(a+(n-1)h\}$ **OR** writing $\int_0^2 e^x\,dx$ $= (2-0).\lim\limits_{n\to\infty}\frac{1}{n}\left\{e^0 + e^{2/n} + e^{4/n} + \cdots + e^{(2n-2)/n}\right\}$	1
	Getting $\int_0^2 e^x\,dx = (2-0).\lim\limits_{n\to\infty}\frac{1}{n}\left\{\frac{e^{2n/n}-1}{e^{2/n}-1}\right\}$	1
	Getting the answer $e^2 - 1$	1
32	Getting $\frac{1}{2}\int\frac{\cos x+\sin x+\cos x-\sin x}{\cos x+\sin x}dx$	1
	Getting $\frac{1}{2}x + \frac{1}{2}\log\lvert\cos x + \sin x\rvert + c$.	1+1
33	Finding points of intersection (0,0) and (5,5)	1
	Writing area= $\int_0^5\sqrt{5x}\,.dx - \int_0^5 x.dx$	1
	Getting the answer $= \frac{50}{3} - \frac{25}{2} = \frac{25}{6}\ sq.units$	1
34	Writing $\frac{dp}{dt}=\frac{5}{100}p$	1

	Writing $\int \frac{1}{p}\, dp = \frac{1}{20} \int dt$	1
	Getting $p = c.e^{t/20}$.	1
35	Knowing $\left\|\vec{a} + \vec{b} + \vec{c}\right\|^2 = \|\vec{a}\|^2 + \left\|\vec{b}\right\|^2 + \|\vec{c}\|^2 + 2\vec{a}.\vec{b} + 2\vec{b}.\vec{c} + 2\vec{c}.\vec{a}$ **OR** writing $\left\|\vec{a} + \vec{b} + \vec{c}\right\| = 0,\ \|\vec{a}\| = 1,\ \left\|\vec{b}\right\| = 1,\ \|\vec{c}\| = 1.$	1
	Writing $0 = 1 + 1 + 1 + 2\left(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}\right)$.	1
	Getting $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -\frac{3}{2}$.	1
36	Writing :Let Pdivide the line joining the points A and B having the position vectors $\vec{a}$ and $\vec{b}$ internally in the ratio $m : n$. (**OR** drawing the figure) Writing $\overrightarrow{AP} = \frac{m}{n}\overrightarrow{PB}$ **OR** $n\overrightarrow{AP} = m\overrightarrow{PB}$.	1
	Getting $n\left(\overrightarrow{OP} - \vec{a}\right) = m\left(\vec{b} - \overrightarrow{OP}\right)$.	1
	Getting $\overrightarrow{OP} = \frac{m\vec{b} + n\vec{a}}{m + n}$.	1
37	Writing $\vec{a}_1 = \hat{\imath} + 2\hat{\jmath} - 4\hat{k},\quad \vec{a}_2 = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k}$ and $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$ **OR** Writing the formula to find the distance $= \left\|\frac{(\vec{a}_2 - \vec{a}_1)\times\vec{b}}{\|\vec{b}\|}\right\|$. **OR** getting $\vec{a}_2 - \vec{a}_1 = 2\hat{\imath} + \hat{\jmath} - \hat{k}$	1
	Finding $(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = 9\hat{\imath} - 14\hat{\jmath} + 4\hat{k}$.	1
	Getting the distance $= \frac{\sqrt{293}}{7}$ units.	1
38	Writing: $\mathrm{P(E_1)} = \frac{1}{2}, \mathrm{P(E_2)} = \frac{1}{2}$	1
	Writing: $\mathrm{P}\left(\mathrm{A} \mid \mathrm{E_1}\right) = \frac{1}{2}$ and $\mathrm{P}\left(\mathrm{A} \mid \mathrm{E_2}\right) = \frac{1}{4}$	1
	Getting: $\mathrm{P}\left(\mathrm{E_1} \mid \mathrm{A}\right) = \frac{\mathrm{P}\left(\mathrm{E_1}\right)\mathrm{P}\left(\mathrm{A} \mid \mathrm{E_1}\right)}{\mathrm{P}\left(\mathrm{E_1}\right)\mathrm{P}\left(\mathrm{A} \mid \mathrm{E_1}\right) + \mathrm{P}\left(\mathrm{E_2}\right)\mathrm{P}\left(\mathrm{A} \mid \mathrm{E_2}\right)} = \frac{2}{3}$	1
39	Defining $g : Y \to N,\ g(y) = \frac{y - 3}{4} y \in Y$ **OR** defining $g : Y \to N,\ g(x) = \frac{x - 3}{4},\ x \in Y$ **OR** writing $y = 4x + 3 \Rightarrow x = \frac{y - 3}{4}$.	1
	Getting $gof(x) = g(4x + 3) = x$.	1
	Stating $gof = I_N$.	1
	Getting $fog(y) = f\left(\frac{y - 3}{4}\right) = y,\ y \in Y$ **OR** $fog(x) = f\left(\frac{x - 3}{4}\right) = x,\ x \in Y$ and stating $fog = I_Y$.	1

	Writing $f^{-1}(x) = \frac{x-3}{4}$ **OR** $f^{-1} = \frac{x-3}{4}$.	1
40	Finding : AB	1
	Finding : $(AB)'$	1
	Finding : A' and B'	1
	Finding : $B'A'$	1
	Conclusion.	1
41	Finding: $\lvert A\rvert = -1$	1
	Finding: $\operatorname{adj}A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$ Any four cofactors are correct award 1 mark.	2
	Getting : $A^{-1} = \frac{\operatorname{adj}A}{\lvert A\rvert} = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$	1
	Finding : $X = A^{-1}B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ Therefore x = 1, y = 2 and z =3.	1
42	Getting $\frac{dy}{dx} = 2\tan^{-1}x \frac{d}{dx}\tan^{-1}x$.	1
	Getting $\frac{dy}{dx} = 2\tan^{-1}x \frac{1}{1+x^2}$.	1
	Writing $(1+x^2)\frac{dy}{dx} = 2\tan^{-1}x$	1
	Differentiating $(1+x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}2x = 2\frac{1}{1+x^2}$	1
	Getting $(x^2+1)^2\frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx} = 2$.	1
43	Writing $\frac{dy}{dt} = 8.\frac{dx}{dt}$	1
	Getting $6.\frac{dy}{dt} = 3x^2.\frac{dx}{dt}$	1
	Getting $x = \pm 4$.	1
	Finding $y = 11$ and $y = \frac{-31}{3}$	1
	Writing the points $(4, 11)$, $\left(-4, \frac{-31}{3}\right)$	1
44	Substituting $x = a\tan\theta$ and writing $dx = a\sec^2\theta\, d\theta$	1
	Getting $\int \sec\theta\, d\theta$	1
	Writing integral $= \log\lvert\sec\theta + \tan\theta\rvert + c$	1
	Getting $\log\left\lvert x + \sqrt{x^2 + a^2}\right\rvert + c$	1
	Getting $\int \frac{1}{\sqrt{x^2+2x+2}}dx = \log\left\lvert x + 1 + \sqrt{x^2+2x+2}\right\rvert + c$	1

45	 Drawing the figure **OR** stating: Ellipse is a symmetrical closed curve centered at the origin. Hence area of the ellipse is 4 times the area of the region in the first quadrant.	1
	Writing area of the ellipse $= 4\int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\ dx$	1
	Knowing $\int \sqrt{a^2 - x^2}\,.dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$ **OR** putting $x = a.\sin t$ and $dx = a.\cos t.dt$	1
	Getting area $= \pi ab$ sq. units	1
	Getting area of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1 = 12\pi$ sq. units.	1
46	Stating: The given differential equation is a linear differential equation **OR** $P = \cot x$ and $Q = 4x.cosecx$	1
	Getting I. F. $= e^{\int \cot x dx} = sinx$	1
	Getting $y.sinx = \int 4x \operatorname{cosec} x.sinx.dx + C$	1
	Getting $y.sinx = 2x^2 + C$	1
	Taking $x = \frac{\pi}{2}$, $y = 0$ and getting $C = -\frac{\pi^2}{2}$	1
47	Drawing figure with explanation 	1
	Concluding $\overrightarrow{AP} = \vec{r} - \vec{a}$.	1
	Getting $\vec{r} = \vec{a} + \lambda\vec{b}$.	1
	Writing $P \equiv (x, y, z)$, $A \equiv (x_1, y_1, z_1)$ and $\vec{b} = (a, b, c)$, $\vec{r} - \vec{a} = \lambda\vec{b} \Rightarrow (x - x_1, y - y_1, z - z_1) = \lambda(a, b, c)$.	1
	Getting $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = \lambda$.	1
48	Writing: $n = 50$, $p = \frac{1}{100}$, $q = \frac{99}{100}$	1
	Writing: $P(x = x) = {}^{n}C_x q^{n-x} p^x$, $n = 0, 1, 2, \cdots, 50$.	1

	Writing: $P(x \geq 1) = 1 - P(x = 0)$	1
	Getting: $P(x \geq 1) = 1 - \left(\frac{99}{100}\right)^{50}$	1
	Getting: $P(x = 1) = \frac{1}{2}\left(\frac{99}{100}\right)^{49}$	1
49(a)	Writing $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_a^{2a} f(x)dx.$	1
	Substituting $x = 2a - t,\ dx = -dt.$	1
	Getting $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a - t)dt.$	1
	Getting $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx.$	1
	Proving $\int_0^{\pi} \lvert\cos x\rvert\, dx = 2\int_0^{\pi/2} \lvert\cos x\rvert\, dx.$	1
	Getting the answer = 2 .	1
49(b)	Stating LHL = RHL at x = 2 and x = 10.	1
	Getting: $5 = 2a + b$	1
	Getting $21 = 10a + b$	1
	Solving to get a = 2 and b = 1.	1
50(a)	Writing: To minimize $z = 4x + 6y$	1
	Writing: constraints $3x + 6y \geq 80$ $4x + 3y \geq 100$ $x \geq 0, y \geq 0$	1
	Drawing graph and identifying the feasible region	2
	Writing: corner point Getting corresponding value of z at each corner point	1
	Getting minimum value of $Z = 104$ at $x = 24,\ y = \frac{4}{3}$	1
50(b)	Getting $LHS = \begin{vmatrix} 1 + x + x^2 & x & x^2 \\ 1 + x + x^2 & 1 & x \\ 1 + x + x^2 & x^2 & 1 \end{vmatrix}$	1
	Getting $= (1 + x + x^2)\begin{vmatrix} 1 & x & x^2 \\ 1 & 1 & x \\ 1 & x^2 & 1 \end{vmatrix}$	1
	Getting $= (1 + x + x^2)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 - x & x - x^2 \\ 0 & x^2 - 1 & 1 - x \end{vmatrix}$	1
	Getting $(1 - x^3)^2$	1