High School Publishers' Criteria for the Common Core State Standards for Mathematics

> These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. … It is time to recognize that standards are not just promises to our children, but promises we intend to keep.
>
> –CCSSM, p. 5

The Common Core State Standards were developed through a bipartisan, state-led initiative spearheaded by state superintendents and state governors. The Standards reflect the collective expertise of hundreds of teachers, education researchers, mathematicians, and state content experts from across the country. The Standards build on the best of previous state standards plus a large body of evidence from international comparisons and domestic reports and recommendations to define a sturdy staircase to college and career readiness. Most states have now adopted the Standards to replace previous expectations in English language arts/literacy and mathematics.

Standards by themselves cannot raise achievement. Standards don't stay up late at night working on lesson plans, or stay after school making sure every student learns—it's teachers who do that. And standards don't implement themselves. Education leaders from the state board to the building principal must make the Standards a reality in schools. Publishers too have a crucial role to play in providing the tools that teachers and students need to meet higher standards. This document, developed by the CCSSM writing team with review and collaboration from partner organizations, individual experts, and districts using the K-8 criteria, aims to support faithful CCSSM implementation by providing criteria for materials aligned to the Common Core State Standards for Mathematics. States, districts, and publishers can use these criteria to develop, evaluate, or purchase aligned materials, or to supplement or modify existing materials to remedy weaknesses. Note that an update to this document is planned for Fall 2013.

How should alignment be judged? Traditionally, judging alignment has been approached as a crosswalking exercise. But crosswalking can result in large percentages of "aligned content" while obscuring the fact that the materials in question align not at all to the letter or the spirit of the standards being implemented. These criteria are an attempt to sharpen the alignment question and make alignment and misalignment more clearly visible.

These criteria were developed from the perspective that publishers and purchasers are equally responsible for fixing the materials market. Publishers cannot deliver focus to buyers who only ever complain about what has been left out, yet never complain about what has crept in. More generally, publishers cannot invest in quality if the market doesn't demand it of them nor reward them for producing it.

The High School Publishers' Criteria are structured as follows:

I. Focus, Coherence, and Rigor in the High School Standards
II. Criteria for Materials and Tools Aligned to the High School Standards
III. Appendix: "Lasting Achievements in K–8"

I. Focus, Coherence, and Rigor in the High School Standards

> This finding that postsecondary instructors target fewer skills as being of high importance is consistent with recent policy statements and findings raising concerns that some states require too many standards to be taught and measured, rather than focusing on the most important state standards for students to attain. …
>
> Because the postsecondary survey results indicate that a more rigorous treatment of fundamental content knowledge and skills needed for credit-bearing college courses would better prepare students for postsecondary school and work, states would likely benefit from examining their state standards and, where necessary, reducing them to focus only on the knowledge and skills that research shows are essential to college and career readiness and postsecondary success. …
>
> —*ACT National Curriculum Survey 2009*

> …[B]ecause conventional textbook coverage is so fractured, unfocused, superficial, and unprioritized, there is no guarantee that most students will come out knowing the essential concepts of algebra.
>
> –Wiggins, 2012[1]

For years national reports have called for greater focus in U.S. mathematics education. TIMSS and other international studies have concluded that mathematics education in the United States is a mile wide and an inch deep. A mile-wide inch-deep curriculum translates to less time per topic. Less time means less depth and moving on without many students. In high-performing countries, strong foundations are laid and then further knowledge is built on them; the design principle in those countries is focus with coherent progressions. The U.S. has lacked such discipline and patience.

There is evidence that state standards have become somewhat more focused over the past decade. But in the absence of standards shared across states, instructional materials have not followed suit. Moreover, prior to the Common Core, state standards were making little progress in terms of coherence: states were not fueling achievement by organizing math so that the subject makes sense.

With the advent of the Common Core, a decade's worth of recommendations for greater focus and coherence finally have a chance to bear fruit. Focus and coherence are the two major evidence-based design principles of the Common Core State Standards for Mathematics.[2] These principles are meant to fuel greater achievement in a deep and rigorous curriculum, one in which students acquire conceptual understanding, procedural skill and fluency, and the ability to apply mathematics to solve problems and formulate mathematical models. Thus, the implications of the standards for mathematics education could be summarized briefly as follows:

[1] From http://grantwiggins.wordpress.com/2012/02/01/a-postscript-to-my-comment-about-kids-having-trouble-with-the-distributive-property.

[2] For some of the sources of evidence consulted during the standards development process, see pp. 91–93 of CCSSM.

Focus: focus strongly where the standards focus

Coherence: think across grades/courses, and link to major topics in each course

Rigor: in major topics, pursue with equal intensity

- conceptual understanding,
- procedural skill and fluency, and
- applications

Focus

Focus in high school is important in order to prepare students for college and careers. National surveys have repeatedly concluded that postsecondary instructors value greater mastery of a smaller set of prerequisites over shallow exposure to a wide array of topics, so that students can build on what they know and apply what they know to solve substantial problems. A college-ready curriculum including all of the standards without a (+) symbol in High School should devote the majority of students' time to building the particular knowledge and skills that are most important as prerequisites for a wide range of college majors, postsecondary programs, and careers.

Coherence

Coherence is about making math make sense. Mathematics is not a list of disconnected tricks or mnemonics. It is an elegant subject in which powerful knowledge results from reasoning with a small number of principles.[3] A special character of the mile-wide inch-deep problem in high school is that there are often too many separately memorized techniques, with no overall structure to tie them altogether. Taking advantage of coherence can reduce clutter in the curriculum. For example, if students can see that the distance formula and the trigonometric identity $\sin^2(t) + \cos^2(t) = 1$ are both manifestations of the Pythagorean theorem, they have an understanding that helps them reconstruct these formulas and not just memorize them temporarily. In order to help teachers and curriculum developers see coherence, the High School content standards in the Algebra and Function categories are arranged under headings like "Seeing Structure in Expressions" and Building Functions."

"Fragmenting the Standards into individual standards, or individual bits of standards … produces a sum of parts that is decidedly less than the whole" (Appendix from the K-8 Publishers' Criteria). Breaking down standards poses a threat to the focus and coherence of the Standards. It is sometimes helpful or necessary to isolate a part of a compound standard for instruction or assessment, but not always, and not at the expense of the Standards as a whole. A drive to break the Standards down into 'microstandards' risks making the checklist mentality even worse than it is today. Microstandards would also make it easier for microtasks and microlessons to drive out extended tasks and deep learning. Finally, microstandards could allow for micromanagement: Picture teachers and students

[3] For some remarks by Phil Daro on this theme, see the excerpt at http://vimeo.com/achievethecore/darofocus, and/or the full video available at http://commoncoretools.me/2012/05/21/phil-daro-on-learning-mathematics-through-problem-solving/.

being held accountable for ever more discrete performances. If it is bad today when principals force teachers to write the standard of the day on the board, think of how it would be if every single standard turns into three, six, or a dozen or more microstandards. If the Standards are like a tree, then microstandards are like twigs. You can't build a tree out of twigs, but you can use twigs as kindling to burn down a tree.

Rigor

To help students meet the expectations of the Standards, educators will need to pursue, with equal intensity, three aspects of rigor: (1) conceptual understanding, (2) procedural skill and fluency, and (3) applications. The word "rigor" isn't a code word for just one of these three; rather, it means equal intensity in all three. The word "understand" is used in the Standards to set explicit expectations for conceptual understanding, and the phrase "real-world problems" and the star symbol (★) are used to set expectations and flag opportunities for applications and modeling. (Modeling is a Standard for Mathematical Practice as well as a content category in High School.) The High School content standards do not set explicit expectations for fluency, but fluency is important in high school mathematics.

The Standards for Mathematical Practice set expectations for using mathematical language and representations to reason, solve problems, and model. These expectations are related to fluency: precision in the use of language, seeing structure in expressions, and reasoning from the concrete to the abstract correspond to high orders of fluency in the acquisition of mathematical language, especially in the form of symbolic expressions and graphs. High School mathematics builds new and more sophisticated fluencies on top of the earlier fluencies from K-8 that centered on numerical calculation.

To date, curricula have not always been balanced in their approach to these three aspects of rigor. Some curricula stress fluency in computation without acknowledging the role of conceptual understanding in attaining fluency and making algorithms more learnable. Some stress conceptual understanding without acknowledging that fluency requires separate classroom work of a different nature. Some stress pure mathematics without acknowledging that applications can be highly motivating for students and that a mathematical education should make students fit for more than just their next mathematics course. At another extreme, some curricula focus on applications, without acknowledging that math doesn't teach itself.

The Standards do not take sides in these ways, but rather they set high expectations for all three components of rigor in the major work of each grade. Of course, that makes it necessary that we focus—otherwise we are asking teachers and students to do more with less.

II. Criteria for Materials and Tools Aligned to the High School Standards

> Students deserve pathways to college designed as preparation, not as obstacle courses….
>
> —Daro, in the 2008 IAS-Carnegie Commission Report

Using the criteria

One approach to developing a document such as this one would have been to develop a separate criterion for each mathematical topic approached in deeper ways in the Standards, a separate criterion for each of the Standards for Mathematical Practice, etc. It is indeed necessary for textbooks to align to the Standards in detailed ways. However, enumerating those details here would have led to a very large number of criteria. Instead, the criteria use the Standards' focus, coherence, and rigor as the main themes. In addition, this document includes a section on indicators of quality in materials and tools, as well as a criterion for the mathematics and statistics in instructional resources for science and technical subjects. Note that the criteria apply to materials and tools, not to teachers or teaching.

The criteria can be used in several ways:

- *Informing purchases and adoptions*. Schools or districts evaluating materials and tools for purchase can use the criteria to test claims of alignment. States reviewing materials and tools for adoption can incorporate these criteria into their rubrics.
- *Working with previously purchased materials*. Most existing materials and tools likely fail to meet one or more of these criteria, even in cases where alignment to the Standards is claimed. But the pattern of failure is likely to be informative. States and districts need not wait for "the perfect book" to arrive, but can use the criteria now to carry out a thoughtful plan to modify or combine existing resources in such a way that students' actual learning experiences approach the focus, coherence, and rigor of the Standards. Publishers can develop innovative materials and tools specifically aimed at addressing identified weaknesses of widespread textbooks or programs.
- *Guiding the development of materials*. Publishers currently modifying their programs and designers of new materials and tools can use the criteria to shape these projects.
- *Professional development.* The criteria can be used to support activities that help communicate the shifts in the Standards. For example, teachers can analyze existing materials to reveal how they treat the major work of the grade, or assess how well materials attend to the three aspects of rigor, or determine which problems are key to developing the ideas and skills of the grade.

In all these cases, it is recommended that the criteria for focus be attended to first. By attending first to focus, coherence and rigor may realistically develop.

The Standards do not dictate the acceptable forms of instructional resources—to the contrary, they are a historic opportunity to raise student achievement through innovation. Materials and tools of very different forms can meet the criteria, including workbooks, multi-year programs, and targeted interventions. For example, materials and tools that treat a single important topic or domain might be valuable to consider.

Alignment for digital and online materials and tools. Digital materials offer substantial promise for conveying mathematics in new and vivid ways and customizing learning. In a digital or online format, diving deeper and reaching back and forth across the grades is easy and often useful. That can enhance focus and coherence. But if such capabilities are poorly designed, focus and coherence could also be diminished. In a setting of dynamic content navigation, the navigation experience must preserve the coherence of Standards clusters and progressions while allowing flexibility and user control: Users can readily see where they are with respect to the structure of the curriculum and its basis in the Standards' domains, clusters and standards.

Digital materials that are smaller than a course can be useful. The smallest granularity for which they can be properly evaluated is a cluster of standards. These criteria can be adapted for clusters of standards or progressions within a cluster, but might not make sense for isolated standards.

Special populations. As noted in the Standards (p. 4),

> All students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs.

Thus, **an over-arching criterion** for materials and tools is that they provide supports for special populations such as students with disabilities, English language learners,[4] and gifted students. Designers of materials should consult accepted guidelines for providing these supports.

*

For the sake of brevity, the criteria sometimes refer to parts of the Standards using abbreviations such as A.REI.10 (an individual content standard), MP.8 (a practice standard), F.BF.A (a cluster heading), or N.RN (a domain heading). Readers of the document should have a copy of the Standards available in order to refer to the indicated text in each case.

A note about high school courses: The High School Standards do not mandate the sequence or organization of high school courses. However, curriculum materials and tools based on a course sequence should ensure that the sequence of the courses does not break apart the coherence of the mathematics while meeting focus and rigor as well.

[4] Slides from a brief and informal presentation by Phil Daro about mathematical language and English language learners can be found at http://db.tt/VARV3ebl.

Criteria for Materials and Tools Aligned to the Standards

1. **Focus on Widely Applicable Prerequisites: In any single course, students using the materials as designed spend the majority of their time developing knowledge and skills that are widely applicable as prerequisites for postsecondary education**. Comprehensive materials coherently include all of the standards in High School without a (+) symbol, with a majority of the time devoted to building the particular knowledge and skills that are most *applicable and prerequisite* to a wide range of college majors and postsecondary programs. Materials developed to prepare students for STEM majors ensure that STEM-intending students learn all of the prerequisites in the Standards necessary for calculus and other advanced courses.

   Table 1 lists clusters and standards with relatively wide applicability across a range of postsecondary work. Table 1 is a **subset** of the material students must study to be college and career ready (CCSSM, pp. 57, 84). But to meet this criterion, materials must give especially careful treatment to the domains, clusters, and standards in Table 1, including their interconnections and their applications—amounting to a majority of students' time.

   This criterion also applies to digital or online materials without fixed pacing plans. Such tools are explicitly designed for focus, so that students spend the majority of their time on widely applicable work.

Table 1. Content From CCSSM Widely Applicable as Prerequisites for a Range of College Majors, Postsecondary Programs and Careers*

Number and Quantity	Algebra	Functions	Geometry	Statistics and Probability	Applying Key Takeaways from Grades 6–8**
N-RN, Real Numbers: Both clusters in this domain contain widely applicable prerequisites. **N-Q★, Quantities**: Every standard in this domain is a widely applicable prerequisite. Note, this domain is especially important in the high school content standards overall as a widely applicable prerequisite.	**Every domain in this category** contains widely applicable prerequisites.[o] Note, the **A-SSE** domain is especially important in the high school content standards overall as a widely applicable prerequisite.	**F-IF, Interpreting Functions**: Every cluster in this domain contains widely applicable prerequisites.[o] Additionally, standards **F-BF.1** and **F-LE.1** are relatively important within this category as widely applicable prerequisites.	**The following standards and clusters** are relatively important within this category as widely applicable prerequisites: **G-CO.1** **G-CO.9** **G-CO.10** **G-SRT.B** **G-SRT.C** Note, the above standards in turn have learning prerequisites within the Geometry category, including: G-CO.A G-CO.B G-SRT.A	**The following standards** are relatively important within this category as widely applicable prerequisites: **S-ID.2** **S-ID.7** **S-IC.1** Note, the above standards in turn have learning prerequisites within 6-8.SP.	**Solving problems at a level of sophistication appropriate to high school by**: • Applying ratios and proportional relationships. • Applying percentages and unit conversions, e.g., in the context of complicated measurement problems involving quantities with derived or compound units (such as mg/mL, kg/m^3, acre-feet, etc.). • Applying basic function concepts, e.g., by interpreting the features of a graph in the context of an applied problem. • Applying concepts and skills of geometric measurement e.g., when analyzing a diagram or schematic. • Applying concepts and skills of basic statistics and probability (see 6-8.SP). • Performing rational number arithmetic fluently.

A note about the codes: Letter codes (A, B, C) are used to denote cluster headings. For example, G-SRT.B refers to the *second* cluster heading in the domain G-SRT, "Prove theorems using similarity" (pp. 77 of CCSSM).

* Informed by postsecondary survey data in Conley *et al.* (2011), "Reaching the Goal: The Applicability and Importance of the Common Core State Standards to College and Career Readiness," http://www.epiconline.org/publications/documents/ReachingtheGoal-FullReport.pdf.

** See CCSSM, p. 84: "…some of the highest priority content for college and career readiness comes from Grades 6-8. This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume."

★ Modeling star (present in CCSSM)

[o] Only the standards without a (+) sign are being cited here.

2. **Rigor and Balance: Materials and tools reflect the balances in the Standards and help students meet the Standards' rigorous expectations, by (all of the following, in the case of comprehensive materials; at least one of the following for supplemental or targeted resources):**

    a. **Developing students' <u>conceptual understanding</u> of key mathematical concepts, especially where called for in specific content standards or cluster headings.** Materials amply feature high-quality conceptual problems and questions. This includes brief conceptual problems with low computational difficulty (e.g., 'What is the maximum value of the function $f(t) = 5 - t^2$?'); brief conceptual questions (e.g., 'Is $\sqrt{2}$ a polynomial? How about $\frac{1}{2}\left(x+\sqrt{2}\right)+\frac{1}{2}\left(-x+\sqrt{2}\right)$?'); and problems that involve identifying correspondences across different mathematical representations of quantitative relationships.[5] Classroom discussion about such problems can offer opportunities to engage in mathematical practices such as constructing and critiquing arguments (MP.3). In the materials, conceptual understanding is attended to most thoroughly in those places in the content standards where explicit expectations are set for understanding or interpreting. Such problems and activities center on fine-grained mathematical concepts, such as the correspondence between an equation and its graph, solving equations as a process of answering a question, analyzing a nonlinear equation $f(x) = g(x)$ by graphing f and g on a single set of axes, etc. Conceptual understanding of key mathematical concepts is thus distinct from applications or fluency work, and these three aspects of rigor must be balanced as indicated in the Standards.

    b. **Giving attention throughout the year to <u>procedural skill and fluency</u>**. In higher grades, algebra is the language of much of mathematics. Like learning any language, we learn by using it. Sufficient practice with algebraic operations is provided so as to make realistic the attainment of the Standards as a whole; for example, fluency in algebra can help students get past the need to manage computational details so that they can observe structure (MP.7) and express regularity in repeated reasoning (MP.8).[6] Progress toward procedural skill and fluency is interwoven with students' developing conceptual understanding of the operations in question. Manipulatives and concrete representations are connected to the written and symbolic methods to which they refer. As well, purely procedural problems and exercises are present. These include cases in which opportunistic strategies are valuable, as in solving $(3x - 2)^2 = 6x - 4$, as well as an ample number of generic cases so that students can learn and practice efficient and general methods (e.g., solving $c + 8 - c^2 = 3(c - 1)^2 - 5$). Methods and algorithms are general and based on principles of mathematics, not mnemonics or tricks.

[5] Note that for ELL students, multiple representations also serve as multiple access paths.

[6] See the PARCC Model Content Frameworks for Mathematics for additional examples of specific fluency recommendations: http://www.parcconline.org/mcf/mathematics/parcc-model-content-frameworks-browser.

c. **Allowing teachers and students using the materials as designed to spend sufficient time working with engaging applications/modeling.** Materials include an ample number of contextual problems that develop the mathematics of the course, afford opportunities for practice, and engage students in problem solving. Materials also include problems in which students must make their own assumptions or simplifications in order to model a situation mathematically. Applications take the form of problems to be worked on individually as well as classroom activities centered on application scenarios. Materials attend thoroughly to those places in the content standards where expectations for multi-step and real-world problems are explicit. Students learn to use the content knowledge and skills specified in the content standards in applications, with particular stress on applying widely applicable work. Problems and activities show a sensible tradeoff between the sophistication of the problem and the difficulty or newness of the content knowledge the student is expected to bring to bear.

Note that modeling is a mathematical practice in every grade, but in high school it is also a content category (CCSSM, pp. 72, 73); therefore, modeling is prominent and enhanced in high school materials, with more elements of the modeling cycle present (CCSSM, p. 72). Finally, materials include an ample number of high-school-level problems that involve applying key takeaways from grades K–8; see Table 1.[7] For example, a problem in which students use reference data to determine the energy cost of different fuels might draw on proportional relationships, unit conversion, and other skills that were first introduced in the middle grades, yet still be a high-school level problem because of the strategic competence required.[8]

Additional aspects of the Rigor and Balance Criterion:
(1) *The three aspects of rigor are not always separate in materials.* (Conceptual understanding and fluency go hand in hand; fluency can be practiced in the context of applications; and brief applications can build conceptual understanding.)

(2) *Nor are the three aspects of rigor always together in materials.* (Fluency requires dedicated practice to that end. Rich applications cannot always be shoehorned into the mathematical topic of the day. And conceptual understanding will not always come along for free unless explicitly taught.)

(3) Digital and online materials with no fixed lesson flow or pacing plan are not designed for superficial browsing but rather should be designed to instantiate the Rigor and Balance criterion.

---

[7] From CCSSM, p. 84: "The evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by (+) symbols in these standards. Indeed, some of the highest priority content for college and career readiness comes from Grades 6-8. This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume."

[8] For more on the role that skills first introduced in the middle grades continue to play in high school and beyond, see Appendix, "Lasting Achievements in K–8."

3. **Consistent Content: Materials are consistent with the content in the Standards, by (all of the following):**

    a. **Basing courses on the content specified in the Standards.** Content in materials matches well with the mathematics specified in the Standards for Mathematical Content. (This does not require the table of contents in a book to be a replica of the content standards.) Any discrepancies in high school content enhance the required learning and are clearly aimed at helping students meet the Standards as written, rather than setting up competing requirements or effectively rewriting the standards. Comprehensive materials do not introduce gaps in learning by omitting any content without a (+) symbol that is specified in the Standards.

        Digital and online materials that allow students and/or teachers to navigate content across course levels promote coherence by tracking the structure in the Standards. For example, such materials might link problems and concepts so that teachers and students can browse a cluster.

    b. **Giving all students extensive work with course-level problems.** Previous-grades review and previous-course review is clearly identified as such to the teacher, and teachers and students can see what their specific responsibility is for the current year. The basic model for course-to-course progression involves students making tangible progress during each given course, as opposed to substantially reviewing then marginally extending from previous grades. Differentiation is sometimes necessary, but materials often manage unfinished learning from earlier grades and courses inside course-level work, rather than setting aside course-level work to reteach earlier content. Unfinished learning from earlier grades and courses is normal and prevalent; it should not be ignored nor used as an excuse for cancelling course level work and retreating to below-level work. (For example, the equation of a circle is an occasion to surface and deal with unfinished learning about the correspondence between equations and their graphs.) Likewise, students who are "ready for more" can be provided with problems that take course-level work in deeper directions, not just exposed to later courses' topics.

    c. **Relating course level concepts explicitly to prior knowledge from earlier grades and courses.** The materials are designed so that prior knowledge becomes reorganized and extended to accommodate the new knowledge. Course-level problems in the materials often involve application of knowledge learned in earlier grades and courses. Although students may well have learned this earlier content, they have not learned how it extends to new mathematical situations and applications. They learn basic ideas of functions, for example, and then extend them to deal explicitly with domains. They learn about expressions as recording calculations with numbers, and then extend them to symbolic objects in their own right. The materials make these extensions of prior knowledge explicit. Thus, materials routinely integrate new knowledge with knowledge from earlier grades.

4. **Coherent Connections: Materials foster coherence through connections in a single course, where appropriate and where required by the Standards, by (all of the following):**

   a. **Including learning objectives that are visibly shaped by CCSSM cluster and domain headings.** Cluster headings and domain headings in the High School standards function like topic sentences in a paragraph in that they state the point of, and lend additional meaning to, the individual content standards that follow. Cluster or domain headings in High School also sometimes signal important content-practice connections, e.g., "Seeing Structure in Expressions" connects expressions to MP.7 and "Reasoning with Equations and Inequalities" connects solving to MP.3. Hence an important criterion for coherence is that some or many of the learning objectives in the materials are visibly shaped by CCSSM cluster or domain headings. Materials do not simply treat the Standards as a sum of individual content standards and individual practice standards.

   b. **Including problems and activities that serve to connect two or more clusters in a domain, two or more domains in a category, or two or more categories, in cases where these connections are natural and important**. If instruction only operates at the individual standard level, or even at the individual cluster level, then some important connections will be missed. For example, creating equations (see A-CED) isn't very valuable in itself unless students can also solve them (see A-REI). Materials do not invent connections not explicit in the standards without first attending thoroughly to the connections that are required explicitly in the Standards (e.g., A-REI.11 connects functions to equations in a graphical context.) Not everything in the standards is naturally well connected or needs to be connected (e.g., systems of linear equations aren't well thought of in relation to functions, and connecting these two things is incoherent). Instead, connections in materials are mathematically natural and important (e.g., work with quadratic functions and work with quadratic equations), reflecting plausible direct implications of what is written in the Standards without creating additional requirements.

   c. **Preserving the focus, coherence, and rigor of the Standards even when targeting specific objectives**. Sometimes a content standard is a compound statement, such as 'Do X and do Y.' More intricate compound forms also exist. (For example, see 3.OA.8.) It is sometimes helpful or necessary to isolate a part of a compound standard, but not always, and not at the expense of the Standards as a whole. Digital or print materials or tools are not aligned if they break down the Standards in such a way as to detract from focus, coherence, or rigor. This criterion applies to student-facing and teacher-facing materials, as well as to architectural documents or digital platforms that are meant to guide the development of student-facing or teacher-facing materials.

5. **Practice-Content Connections: Materials meaningfully connect content standards and practice standards.** "Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction." (CCSSM, p. 8.) Over the course of any given year of instruction, each mathematical practice standard is meaningfully present in the form of activities or problems that stimulate students to develop the habits of mind described in the practice standards. These practices are well-grounded in the content standards.

The practice standards are not just processes with ephemeral products (such as conversations). They also specify a set of products students are supposed to learn how to produce. Thus, students are asked to produce answers and solutions but also, in a course-appropriate way, arguments, explanations, diagrams, mathematical models, etc.

Materials are accompanied by an analysis, aimed at evaluators, of how the authors have approached each practice standard in relation to content within each applicable course and provide suggestions for delivering content in ways that help students meet the practice standards in course-appropriate ways. Materials tailor the connections to the content of the grade and to course-level-appropriate student thinking. Materials also include teacher-directed materials that explain the role of the practice standards in the classroom and in students' mathematical development.

6. **Focus and Coherence via Practice Standards: Materials promote focus and coherence by connecting practice standards with content that is emphasized in the Standards**. Content and practice standards are not connected mechanistically or randomly, but instead support focus and coherence. Examples: Materials connect looking for and making use of structure (MP.7) with structural themes emphasized in the standards, such as purposefully transforming expressions, linking the structure of an expression to a feature of the its context, grasping the behavior of a function defined by an expression, etc.; materials use looking for and expressing regularity in repeated reasoning (MP.8) to shed light on algebra and functions, e.g., by summarizing repeated numerical examples in the form of equations or in the form of recursive expressions that define functions. These and other practices can support focus—for example, by moving students from repeated reasoning with the slope formula to writing equations for straight lines in various forms, rather than relying on memorizing all those forms in isolation.

7. **Careful Attention to Each Practice Standard: Materials attend to the full meaning of each practice standard.** For example, MP.1 does not say, "Solve problems." Or "Make sense of problems." Or "Make sense of problems and solve them." It says "Make sense of problems and persevere in solving them." Thus, students using the materials as designed build their perseverance in course-appropriate ways by occasionally solving problems that require them to persevere to a solution beyond the point when they would like to give up.[9] MP.5 does not say, "Use tools." Or "Use appropriate tools." It says "Use appropriate tools strategically." Thus, materials include problems that reward students' strategic decisions about how to use tools, or about whether to use them at all. MP.8 does not say, "Extend patterns." Or "Engage in repetitive reasoning." It says "Look for and express regularity in repeated reasoning." Thus, it is not enough for students to extend patterns or perform repeated calculations. Those repeated calculations must lead to an insight (e.g., "When I substitute $x - k$ for x in a function $f(x)$, where k is any

[9] Curriculum designers might consider how research on motivation and character development has value for designing tools that develop students' perseverance and other mathematical practices. For more information, see, e.g., Dweck (2008), "Mindsets and Math" (http://opportunityequation.org/teaching-and-leadership/mindsets-math-science-achievement); Duckworth et al. (2007), "Grit: Perseverance and Passion for Long-Term Goals" (http://www.sas.upenn.edu/~duckwort/images/publications/DuckworthPetersonMatthewsKelly_2007_PerseveranceandPassion.pdf); and http://www.psychologicalscience.org/index.php/publications/observer/2013/april-13/true-grit.html.

constant, the graph of the function shifts *k* units to the right."). The analysis for evaluators explains how the full meaning of each practice standard has been attended to in the materials.

8. **Emphasis on Mathematical Reasoning: Materials support the Standards' emphasis on mathematical reasoning, by (all of the following):**

   a. **Prompting students to construct viable arguments and critique the arguments of others concerning key course-level mathematics that is detailed in the content standards (cf. MP.3).** Materials provide sufficient opportunities for students to reason mathematically and express reasoning through classroom discussion, written work and independent thinking. Reasoning is not confined to optional or avoidable sections of the materials but is inevitable when using the materials as designed. Materials do not approach reasoning as a generalized imperative, but instead create opportunities for students to reason *about* key mathematics detailed in the content standards. Materials thus attend first and most thoroughly to those places in the content standards setting explicit expectations for explaining, justifying, showing, or proving. Students are asked to critique given arguments, e.g., by explaining under what conditions, if any, a mathematical statement is valid.[10] Teachers and students using the materials as designed spend significant classroom time communicating reasoning (by constructing viable arguments and critiquing the arguments of others concerning key grade-level mathematics)—recognizing that learning mathematics also involves time spent working on applications and practicing procedures. Materials provide examples of student explanations and arguments (e.g., fictitious student characters might be portrayed). Materials follow accepted norms of mathematical reasoning, such as distinguishing between definitions and theorems, not asking students to explain why something is true when it has been defined to be so, etc.

   b. **Engaging students in problem solving as a form of argument**. Materials attend thoroughly to those places in the content standards that explicitly set expectations for multi-step problems; multi-step problems are not scarce in the materials. Some or many of these problems require students to devise a strategy autonomously. Sometimes the goal is the final answer alone (cf. MP.1); sometimes the goal is to lay out the solution as a sequence of well justified steps. In the latter case, the solution to a problem takes the form of a cogent argument that can be verified and critiqued, instead of a jumble of disconnected steps with a scribbled answer indicated by drawing a circle around it (cf. MP.6).

   c. **Explicitly attending to the specialized language of mathematics.** Mathematical reasoning involves specialized language. Therefore, materials and tools address the development of mathematical and academic language associated with the standards. The language of argument, problem solving and mathematical explanations are taught rather than assumed. Correspondences between language and multiple mathematical representations including

[10] As students progress through the grades, their production and comprehension of mathematical arguments evolves from informal and concrete toward more formal and abstract. In early grades students employ imprecise expressions which with practice over time become more precise and viable arguments in later grades. Indeed, the use of imprecise language is part of the process in learning how to make more precise arguments in mathematics. Ultimately, conversation about arguments helps students transform assumptions into explicit and precise claims.

diagrams, tables, graphs, and symbolic expressions are identified in material designed for language development. Note that variety in formats and types of representations—graphs, drawings, images, and tables in addition to text—can relieve some of the language demands that English language learners face when they have to show understanding in math.

The text is considerate of English language learners, helping them to access challenging mathematics and helping them to develop grade level language. For example, materials might include annotations to help with comprehension of words, sentences and paragraphs, and give examples of the use of words in other situations. Modifications to language do not sacrifice the mathematics, nor do they put off necessary language development.

<u>A criterion for the mathematics and statistics in materials for science and technical subjects</u>

Lack of alignment in these subjects could have the effect of compromising the focus and coherence of the mathematics Standards. Instead of reinforcing concepts and skills already carefully introduced in math class, teachers of science and technical subjects would have to teach this material in stopgap fashion.

[S] **Consistency with CCSSM: Materials for science and technical subjects are consistent with CCSSM.** High school materials for these subjects build coherence across the curriculum and support college and career readiness by integrating key mathematics into the disciplines, particularly simple algebra in the physical sciences and technical subjects, and basic statistics in the life sciences and technical subjects (see Table 2 for a possible picture along these lines).

Table 2

Algebraic competencies integrated into materials for high school science and technical subjects	**Statistical competencies integrated into materials for high school science and technical subjects**
• Working with positive and negative numbers (including fractions) to solve problems • Using variables and writing and solving equations to solve problems • Recognizing and using proportional relationships to solve problems • Working with functions and their graphs to solve problems	• Working with distributions and measures of center and variability • Working with simple probability and random sampling • Working with bivariate categorical data (e.g., two-way tables) • Working with bivariate measurement data (e.g., scatter plots) and linear models

Indicators of quality in instructional materials and tools for mathematics

The preceding criteria express important dimensions of alignment to the Standards. The following are some additional dimensions of quality that materials and tools should exhibit in order to give teachers and students the tools they need to meet the Standards:

- Problems in the materials are worth doing:
  - The underlying design of the materials distinguishes between *problems* and *exercises*. Whatever specific terms are used for these two types, in essence the difference is that in solving problems, students learn new mathematics, whereas in working exercises, students apply what they have already learned to build mastery. Problems are problems because students haven't yet learned how to solve them; students are learning from solving them. Materials use problems to teach mathematics. Lessons have a few well designed problems that progressively build and extend understanding. Practice exercises that build fluency are easy to recognize for their purpose. Other exercises require longer chains of reasoning.
  - Each problem or exercise has a purpose—whether to teach new knowledge, bring misconceptions to the surface, build skill or fluency, engage the student in one or several mathematical practices, or simply present the student with a fun puzzle.
  - Assignments aren't haphazardly designed. Exercises are given to students in intentional sequences—for example, a sequence leading from prior knowledge to new knowledge, or a sequence leading from concrete to abstract, or a sequence that leads students through a number of important cases, or a sequence that elicits new understanding by inviting students to see regularity in repeated reasoning. Lessons with too many problems make problems a commodity; they forbid concentration, and they make focus and coherence unlikely.
  - The language in which problems are posed is carefully considered. Note that mathematical problems posed using only ordinary language are a special genre of text that has conventions and structures needing to be learned. The language used to pose mathematical problems should evolve with the grade level and across mathematics content.

- There is variety in the pacing and grain size of content coverage.
  - Materials that devote roughly equal time to each content standard do not allow teachers and students to focus where necessary.
  - The Standards are not written at uniform grain size. Sometimes an individual content standard will require days of work, possibly spread over the entire year, while other standards could be sufficiently addressed when grouped with other standards and treated in a shorter time span.

- There is variety in what students produce: Students are asked to produce answers and solutions, but also, in a course-appropriate way, arguments, explanations, diagrams, mathematical models, etc. In a way appropriate to the grade level, students are asked to answer questions or develop explanations about why a solution makes sense, how quantities are represented in expressions, and how elements of symbolic, diagrammatic, tabular, graphical and/or verbal representations correspond.

- Lessons are thoughtfully structured and support the teacher in leading the class through the learning paths at hand, with active participation by all students in their own learning and in the learning of their classmates. Teachers are supported in extending student explanations and modeling explanations of new methods. Lesson structure frequently calls for students to find solutions, explain their reasoning, and ask and answer questions about their reasoning as it concerns problems, diagrams, mathematical models, etc. Over time there is a rhythm back and forth between making sense of concepts and exercising for proficiency.

- There are separate teacher materials that support and reward teacher study, including:
    - Discussion of the mathematics of the units and the mathematical point of each lesson as it relates to the organizing concepts of the unit.
    - Discussion of student ways of thinking with respect to important mathematical problems and concepts—especially anticipating the variety of student responses.
    - Guidance on interaction with students, mostly questions to prompt ways of thinking.
    - Guidance on lesson flow.
    - Discussion of desired mathematical behaviors being elicited among the students.

- The use of manipulatives follows best practices (see, e.g., *Adding It Up*, 2001):
    - *Manipulatives are faithful representations of the mathematical objects they represent*. For example, algebra tiles can be helpful in representing some features of algebra, but they do not provide particularly direct representations of all of the important mathematics. For example, tiles aren't particularly well suited as models for polynomials having non-integer coefficients and/or high degree.
    - *Manipulatives are connected to written methods*. For example, algebra tiles are a reasonable *model* of certain features of algebra, but not a reasonable *method* for doing algebra. Procedural skill and fluency refers a written or mental method, not a method using manipulatives or concrete representations.

- Materials are carefully reviewed by qualified individuals, whose names are listed, in an effort to ensure:
    - Freedom from mathematical errors[11]

[11] Sometimes errors in materials are simple falsehoods, e.g., printing an incorrect answer to a problem; other errors are more subtle, e.g., asking students to explain why something is so when it has been defined to be so.

    - Age-appropriateness
    - Freedom from bias (for example, problem contexts that use culture-specific background knowledge do not assume readers from all cultures have that knowledge; simple explanations or illustrations or hints scaffold comprehension).
    - Freedom from unnecessary language complexity.

- The visual design isn't distracting or chaotic, or aimed at adult purchasers, but instead serves only to support young students in engaging thoughtfully with the subject.

- Support for English language learners is thoughtful and helps those learners to meet the same standards as all other students. Allowing English language learners to collaborate as they strive to learn and show understanding in an environment where English is used as the medium of instruction will give them the support they need to meet their academic goals. Materials can structure interactions in pairs, in small groups, and in the large group (or in any other group configuration), as some English language learners might be shy to share orally with the large group, but might not have problem sharing orally with a small group or in pairs. (In addition, when working in pairs, if ELLs are paired up with a student who shares the same language, they might choose to think about and discuss the problems in their first language, and then worry about doing it in English.)

Appendix

"Lasting Achievements in K–8"

Essay by Jason Zimba, July 6, 2011[12]

Most of the K–8 content standards trace explicit steps A → B → C in a progression. This can sometimes make it seem as if any given standard only exists for the sake of the next one in the progression. There are, however, culminating or capstone standards (I sometimes call them "pinnacles"), most of them in the middle grades, that remain important far beyond the particular grade level in which they appear. This is signaled in the Standards themselves (p. 84):

> The evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by (+) symbols in these standards. Indeed, some of the highest priority content for college and career readiness comes from Grades 6–8. This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume. Because important standards for college and career readiness are distributed across grades and courses, systems for evaluating college and career readiness should reach as far back in the standards as Grades 6–8. It is important to note as well that cut scores or other information generated by assessment systems for college and career readiness should be developed in collaboration with representatives from higher education and workforce development programs, and should be validated by subsequent performance of students in college and the workforce.

One example of a standard that refers to skills that remain important well beyond middle school is 7.EE.3:

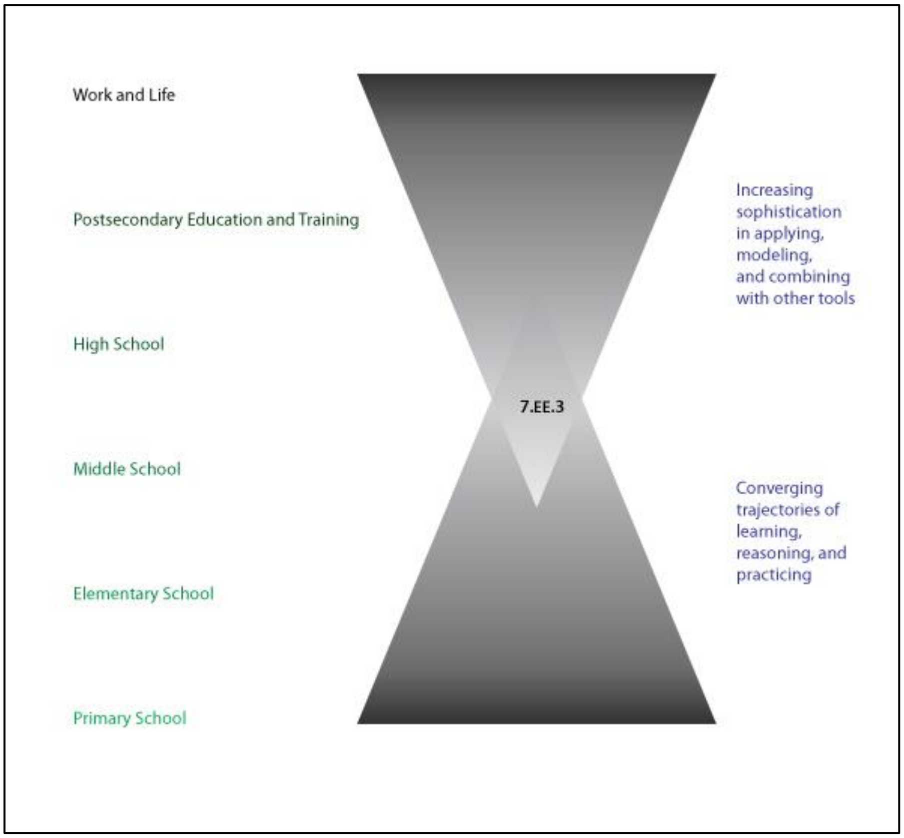
> Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

Other lasting achievements from K–8 would include working with proportional relationships and unit rates (6.RP.3; 7.RP.1,2); working with percentages (6.RP.3e; 7.RP.3); and working with area, surface area, and volume (7.G.4,6).

As indicated in the quotation from the Standards, skills like these are crucial tools for college, work and life. They are not meant to gather dust during high school, but are meant to be applied in increasingly flexible ways, for example to meet the high school standards for Modeling. The illustration below shows how these skills fit in with both the learning progressions in the K–8

[12] http://commoncoretools.me/2011/06/15/essay-by-jason-zimba-on-pinnacle-standards/

standards as well as the demands of the high school standards and readiness for careers and a wide range of college majors.



As shown in the figure, standards like 7.EE.3 are best thought of as descriptions of component skills that will be applied flexibly during high school in tandem with others in the course of modeling tasks and other substantial applications. This aligns with the demands of postsecondary education for careers and for a wide range of college majors. Thus, when high school students work with these skills in high school, they are not working below grade level; nor are they reviewing. Applying securely held mathematics to open-ended problems and applications is a *higher*-order skill valued by colleges and employers alike.

One reason middle school is a complicated phase in the progression of learning is that the pinnacles are piling up even as the progressions A $\rightarrow$ B $\rightarrow$ C continue onward to the college/career readiness line. One reason we draw attention to lasting achievements here is that their importance for college and career readiness might easily be missed in this overall flow.