6-6 Trapezoids and Kites

Mathematics Florida Standards

MAFS.912.G-SRT.2.5 Use congruence . . . criteria to solve problems and prove relationships in geometric figures.

MP 1, MP 3, MP 4, MP 6

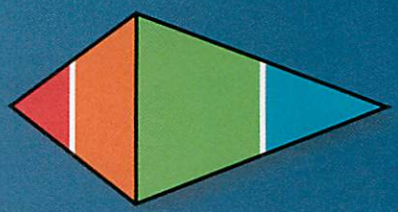
Objective To verify and use properties of trapezoids and kites

Make a sketch and number the angles to help make sense of the problem.

SOLVE IT!

Getting Ready!

Two isosceles triangles form the figure at the right. Each white segment is a midsegment of a triangle. What can you determine about the angles in the orange region? In the green region? Explain.

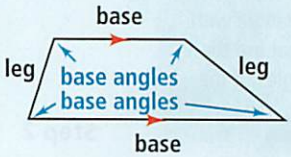


MATHEMATICAL PRACTICES

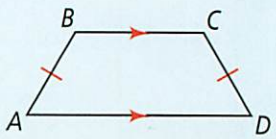
In the Solve It, the orange and green regions are trapezoids. The entire figure is a kite. In this lesson, you will learn about these special quadrilaterals that are not parallelograms.

Essential Understanding The angles, sides, and diagonals of a trapezoid have certain properties.

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides of a trapezoid are called **bases.** The nonparallel sides are called **legs.** The two angles that share a base of a trapezoid are called **base angles.** A trapezoid has two pairs of base angles.



An **isosceles trapezoid** is a trapezoid with legs that are congruent. *ABCD* at the right is an isosceles trapezoid. The angles of an isosceles trapezoid have some unique properties.



Lesson Vocabulary

- trapezoid
- base
- leg
- base angle
- isosceles trapezoid
- midsegment of a trapezoid
- kite

take note

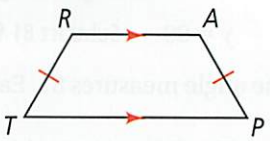
Theorem 6-19

Theorem

If a quadrilateral is an isosceles trapezoid, then each pair of base angles is congruent.

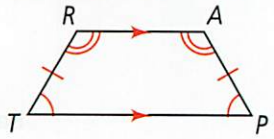
If . . .

TRAP is an isosceles trapezoid with bases $\overline{RA}$ and $\overline{TP}$



Then . . .

$\angle T \cong \angle P$, $\angle R \cong \angle A$



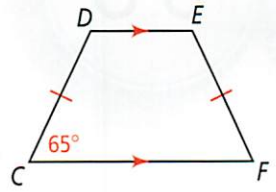
You will prove Theorem 6-19 in Exercise 45.

Problem 1 Finding Angle Measures in Trapezoids

Think

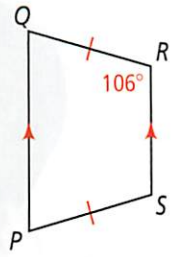
What do you know about the angles of an isosceles trapezoid?
You know that each pair of base angles is congruent. Because the bases of a trapezoid are parallel, you also know that two angles that share a leg are supplementary.

***CDEF* is an isosceles trapezoid and $m\angle C = 65$. What are $m\angle D$, $m\angle E$, and $m\angle F$?**



$m\angle C + m\angle D = 180$	Two angles that form same-side interior angles along one leg are supplementary.
$65 + m\angle D = 180$	Substitute.
$m\angle D = 115$	Subtract 65 from each side.

Since each pair of base angles of an isosceles trapezoid is congruent, $m\angle C = m\angle F = 65$ and $m\angle D = m\angle E = 115$.



Got It? **1. a.** In the diagram, *PQRS* is an isosceles trapezoid and $m\angle R = 106$. What are $m\angle P$, $m\angle Q$, and $m\angle S$?

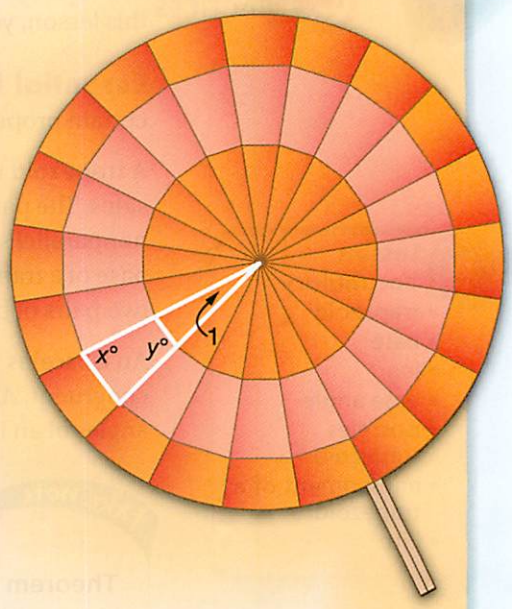
b. Reasoning In Problem 1, if *CDEF* were not an isosceles trapezoid, would $\angle C$ and $\angle D$ still be supplementary? Explain.

Problem 2 Finding Angle Measures in Isosceles Trapezoids

Think

What do you notice about the diagram?
Each trapezoid is part of an isosceles triangle with base angles that are the acute base angles of the trapezoid.

Paper Fans **The second ring of the paper fan shown at the right consists of 20 congruent isosceles trapezoids that appear to form circles. What are the measures of the base angles of these trapezoids?**



Step 1 Find the measure of each angle at the center of the fan. This is the measure of the vertex angle of an isosceles triangle.

$$m\angle 1 = \frac{360}{20} = 18$$

Step 2 Find the measure of each acute base angle of an isosceles triangle.

$18 + x + x = 180$	Triangle Angle-Sum Theorem
$18 + 2x = 180$	Combine like terms.
$2x = 162$	Subtract 18 from each side.
$x = 81$	Divide each side by 2.

Step 3 Find the measure of each obtuse base angle of the isosceles trapezoid.

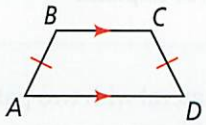
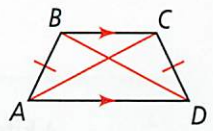
$81 + y = 180$	Two angles that form same-side interior angles along one leg are supplementary.
$y = 99$	Subtract 81 from each side.

Each acute base angle measures 81. Each obtuse base angle measures 99.

Got It? **2.** A fan like the one in Problem 2 has 15 angles meeting at the center. What are the measures of the base angles of the trapezoids in its second ring?

take note

Theorem 6-20

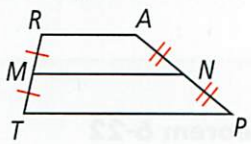
Theorem	If . . .	Then . . .
If a quadrilateral is an isosceles trapezoid, then its diagonals are congruent.	$ABCD$ is an isosceles trapezoid 	$\overline{AC} \cong \overline{BD}$ 

You will prove Theorem 6-20 in Exercise 54.

In Lesson 5-1, you learned about midsegments of triangles. Trapezoids also have midsegments. The **midsegment of a trapezoid** is the segment that joins the midpoints of its legs. The midsegment has two unique properties.

take note

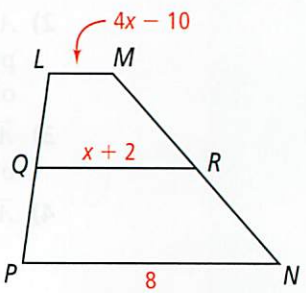
Theorem 6-21 Trapezoid Midsegment Theorem

Theorem	If . . .	Then . . .
If a quadrilateral is a trapezoid, then (1) the midsegment is parallel to the bases, and (2) the length of the midsegment is half the sum of the lengths of the bases.	$TRAP$ is a trapezoid with midsegment $\overline{MN}$ 	(1) $\overline{MN} \parallel \overline{TP}$, $\overline{MN} \parallel \overline{RA}$, and (2) $MN = \frac{1}{2}\left(TP + RA\right)$

You will prove Theorem 6-21 in Lesson 6-9.

Problem 3 Using the Midsegment of a Trapezoid

Algebra $\overline{QR}$ **is the midsegment of trapezoid** $LMNP$. **What is** x**?**



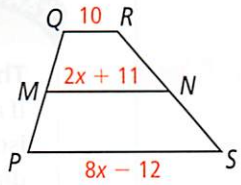
$QR = \frac{1}{2}(LM + PN)$	Trapezoid Midsegment Theorem
$x + 2 = \frac{1}{2}[(4x - 10) + 8]$	Substitute.
$x + 2 = \frac{1}{2}(4x - 2)$	Simplify.
$x + 2 = 2x - 1$	Distributive Property
$3 = x$	Subtract x and add 1 to each side.

Think

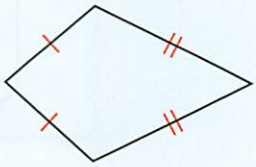
How can you check your answer?
Find LM and QR. Then see if QR equals half of the sum of the base lengths.

Got It? 3. a. **Algebra** $\overline{MN}$ is the midsegment of trapezoid $PQRS$. What is x? What is MN?

b. **Reasoning** How many midsegments can a triangle have? How many midsegments can a trapezoid have? Explain.



A **kite** is a quadrilateral with two pairs of consecutive sides congruent and no opposite sides congruent.

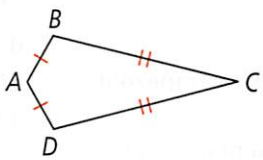


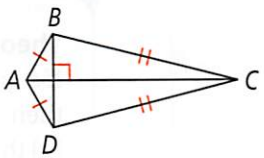
Essential Understanding The angles, sides, and diagonals of a kite have certain properties.

take note

Theorem 6-22

Theorem	If . . .	Then . . .
If a quadrilateral is a kite, then its diagonals are perpendicular.	$ABCD$ is a kite	$\overline{AC} \perp \overline{BD}$

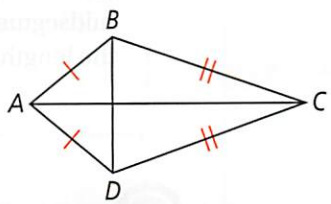




Proof **Proof of Theorem 6-22**

Given: Kite $ABCD$ with $\overline{AB} \cong \overline{AD}$ and $\overline{CB} \cong \overline{CD}$

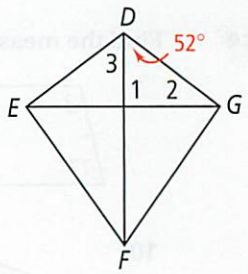
Prove: $\overline{AC} \perp \overline{BD}$



Statements	Reasons
1) Kite $ABCD$ with $\overline{AB} \cong \overline{AD}$ and $\overline{CB} \cong \overline{CD}$	1) Given
2) A and C lie on the perpendicular bisector of $\overline{BD}$.	2) Converse of Perpendicular Bisector Theorem
3) $\overline{AC}$ is the perpendicular bisector of $\overline{BD}$.	3) Two points determine a line.
4) $\overline{AC} \perp \overline{BD}$	4) Definition of perpendicular bisector

Problem 4 Finding Angle Measures in Kites

Quadrilateral *DEFG* is a kite. What are $m\angle 1$, $m\angle 2$, and $m\angle 3$?



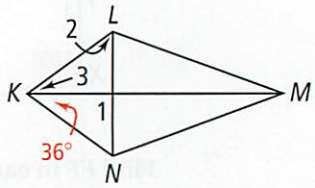
$m\angle 1 = 90$	Diagonals of a kite are $\perp$.
$90 + m\angle 2 + 52 = 180$	Triangle Angle-Sum Theorem
$142 + m\angle 2 = 180$	Simplify.
$m\angle 2 = 38$	Subtract 142 from each side.

Think

How are the triangles congruent by SSS? $\overline{DE} \cong \overline{DG}$ and $\overline{FE} \cong \overline{FG}$ because a kite has congruent consecutive sides. $\overline{DF} \cong \overline{DF}$ by the Reflexive Property of Congruence.

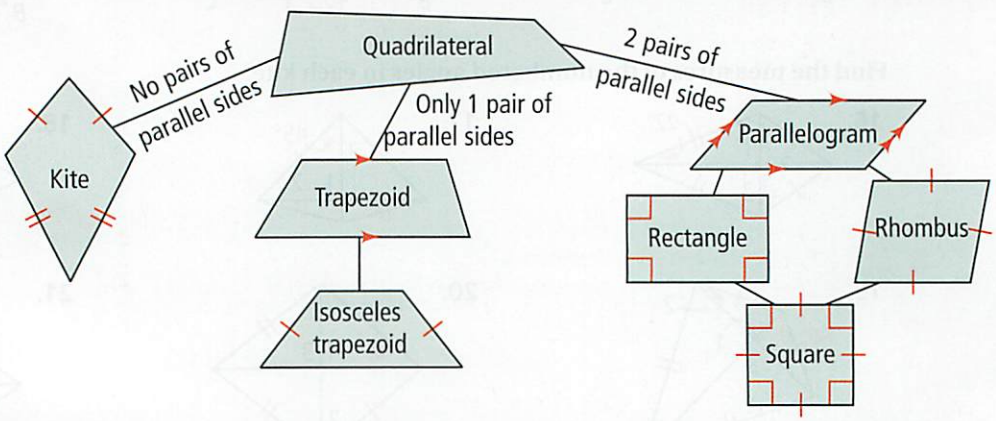
$\triangle DEF \cong \triangle DGF$ by SSS. Since corresponding parts of congruent triangles are congruent, $m\angle 3 = m\angle GDF = 52$.

Got It? **4.** Quadrilateral *KLMN* is a kite. What are $m\angle 1$, $m\angle 2$, and $m\angle 3$?



take note

Concept Summary Relationships Among Quadrilaterals

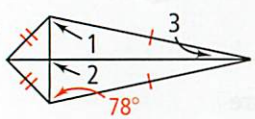


Lesson Check

Do you know HOW?

What are the measures of the numbered angles?

1.



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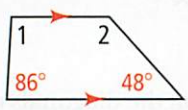


3. What is the length of the midsegment of a trapezoid with bases of length 14 and 26?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. Vocabulary Is a kite a parallelogram? Explain.

5. Compare and Contrast How is a kite similar to a rhombus? How is it different? Explain.

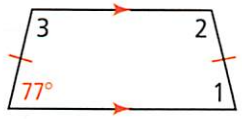
6. Error Analysis Since a parallelogram has two pairs of parallel sides, it certainly has one pair of parallel sides. Therefore, a parallelogram must also be a trapezoid. What is the error in this reasoning? Explain.

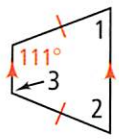
Practice and Problem-Solving Exercises

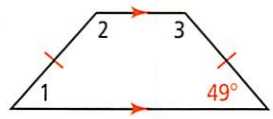
MATHEMATICAL PRACTICES

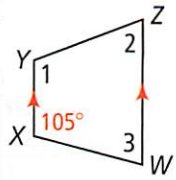
A Practice

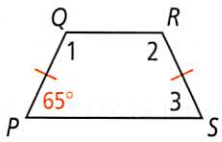
Find the measures of the numbered angles in each isosceles trapezoid. See Problems 1 and 2.

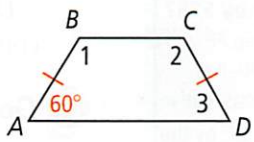
7. 

8. 

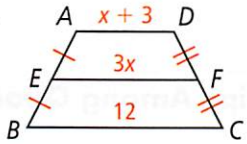
9. 

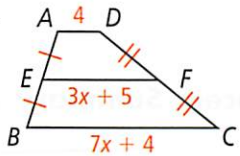
10. 

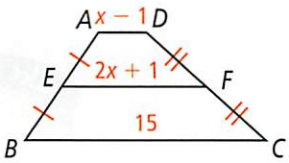
11. 

12. 

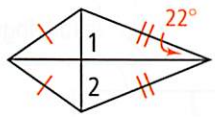
Find *EF* in each trapezoid. See Problem 3.

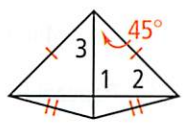
13. 

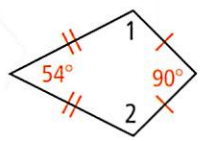
14. 

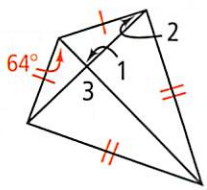
15. 

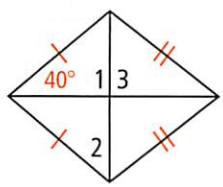
Find the measures of the numbered angles in each kite. See Problem 4.

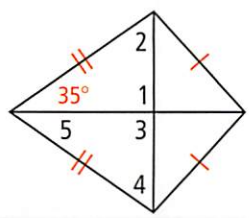
16. 

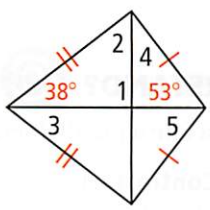
17. 

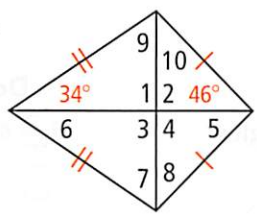
18. 

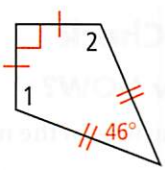
19. 

20. 

21. 

22. 

23. 

24. 

B Apply

25. **Open-Ended** Sketch two noncongruent kites such that the diagonals of one are congruent to the diagonals of the other.

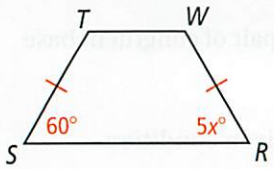
26. Think About a Plan The perimeter of a kite is 66 cm. The length of one of its sides is 3 cm less than twice the length of another. Find the length of each side of the kite.

- Can you draw a diagram?
- How can you write algebraic expressions for the lengths of the sides?

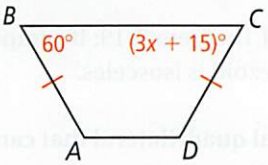
27. Reasoning If $KLMN$ is an isosceles trapezoid, is it possible for $\overline{KM}$ to bisect $\angle LMN$ and $\angle LKN$? Explain.

Algebra **Find the value of the variable in each isosceles trapezoid.**

28.



29.



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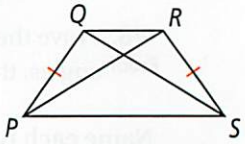
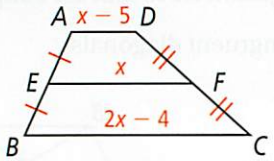


$QS = x + 5$
$RP = 3x + 3$

Algebra **Find the lengths of the segments with variable expressions.**

31.



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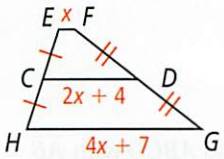

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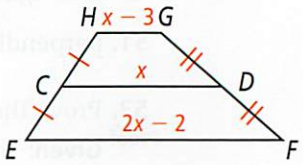
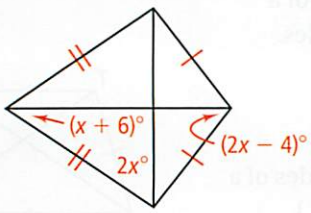


Algebra **Find the value(s) of the variable(s) in each kite.**

34.



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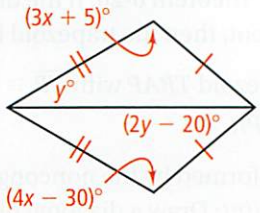
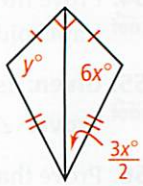
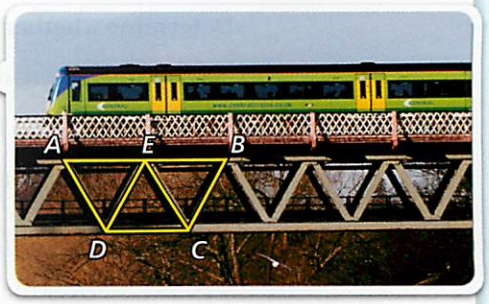


36.



STEM **Bridge Design** **The beams of the bridge at the right form quadrilateral $ABCD$. $\triangle AED \cong \triangle CDE \cong \triangle BEC$ and $m\angle DCB = 120$.**



37. Classify the quadrilateral. Explain your reasoning.

38. Find the measures of the other interior angles of the quadrilateral.

Reasoning **Can two angles of a kite be as follows? Explain.**

39. opposite and acute

40. consecutive and obtuse

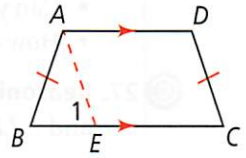
41. opposite and supplementary

42. consecutive and supplementary

43. opposite and complementary

44. consecutive and complementary

45. Developing Proof The plan suggests a proof of Theorem 6-19. Write a proof that follows the plan.



Given: Isosceles trapezoid $ABCD$ with $\overline{AB} \cong \overline{DC}$

Prove: $\angle B \cong \angle C$ and $\angle BAD \cong \angle D$

Plan: Begin by drawing $\overline{AE} \parallel \overline{DC}$ to form parallelogram $AECD$ so that $\overline{AE} \cong \overline{DC} \cong \overline{AB}$. $\angle B \cong \angle C$ because $\angle B \cong \angle 1$ and $\angle 1 \cong \angle C$. Also, $\angle BAD \cong \angle D$ because they are supplements of the congruent angles, $\angle B$ and $\angle C$.

46. Proof Prove the converse of Theorem 6-19: If a trapezoid has a pair of congruent base angles, then the trapezoid is isosceles.

Name each type of special quadrilateral that can meet the given condition. Make sketches to support your answers.

47. exactly one pair of congruent sides

48. two pairs of parallel sides

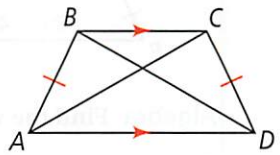
49. four right angles

50. adjacent sides that are congruent

51. perpendicular diagonals

52. congruent diagonals

53. Proof Prove Theorem 6-20.



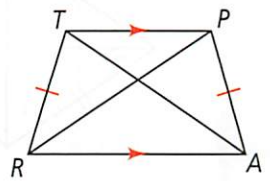
Given: Isosceles trapezoid $ABCD$ with $\overline{AB} \cong \overline{DC}$

Prove: $\overline{AC} \cong \overline{DB}$

54. Proof Prove the converse of Theorem 6-20: If the diagonals of a trapezoid are congruent, then the trapezoid is isosceles.

55. Proof **Given:** Isosceles trapezoid $TRAP$ with $\overline{TR} \cong \overline{PA}$

Prove: $\angle RTA \cong \angle APR$



56. Proof Prove that the angles formed by the noncongruent sides of a kite are congruent. (*Hint:* Draw a diagonal of the kite.)

Determine whether each statement is *true* or *false*. Justify your response.

57. All squares are rectangles.

58. A trapezoid is a parallelogram.

59. A rhombus can be a kite.

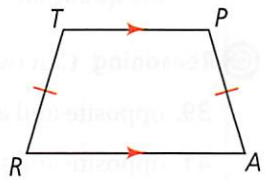
60. Some parallelograms are squares.

61. Every quadrilateral is a parallelogram.

62. All rhombuses are squares.

Challenge

63. Proof **Given:** Isosceles trapezoid $TRAP$ with $\overline{TR} \cong \overline{PA}$; $\overline{BI}$ is the perpendicular bisector of $\overline{RA}$, intersecting $\overline{RA}$ at B and $\overline{TP}$ at I.



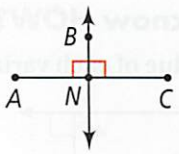
Prove: $\overline{BI}$ is the perpendicular bisector of $\overline{TP}$.

For a trapezoid, consider the segment joining the midpoints of the two given segments. How are its length and the lengths of the two parallel sides of the trapezoid related? Justify your answer.

64. the two nonparallel sides

65. the diagonals

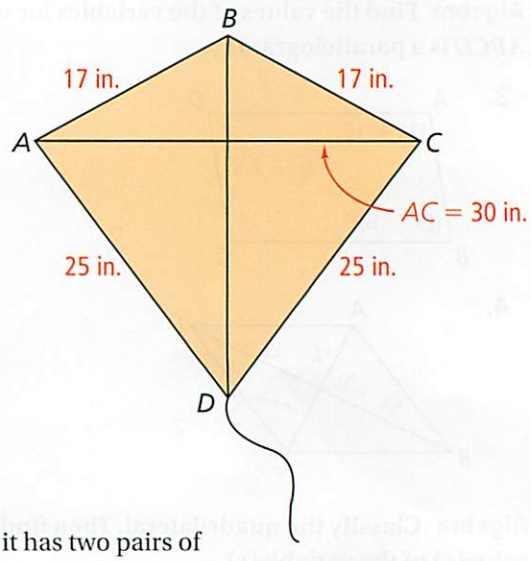
66. $\overleftrightarrow{BN}$ is the perpendicular bisector of $\overline{AC}$ at N. Describe the set of points, D, for which $ABCD$ is a kite.



Apply What You've Learned

MATHEMATICAL PRACTICES
MP 4

Look back at the information about Alejandro's kite on page 351. His sketch is shown again at the right, with the missing vertical support drawn in.



Choose from the following words to complete the sentences below.

diagonals	sides	opposite
consecutive	congruent	perpendicular
parallel	bisects	right

Alejandro's kite fits the geometric definition of a kite because it has two pairs of congruent **a.** ? sides and no pairs of congruent **b.** ? sides.

The vertical and horizontal supports of the kite are its **c.** ? . Vertices B and D are each equidistant from vertices A and C, so the vertical support **d.** ? the horizontal support.

Because the diagonals of a kite are **e.** ? to each other, they divide the kite into four **f.** ? triangles. The kite's vertical support divides it into two **g.** ? triangles.